Prisoner's dilemma on scale-free networks

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Abstract. In this work, we study via computer simulations the spatial prisoner's dilemma (PD) game for the general case where the distribution of the connections between the individuals playing the game obeys a power law. This distribution has been shown to describe many aspects of social acquaintances, while the PD game is a powerful tool for studying mutual trust and cooperation among individuals. We study this model under different conditions, such as varying degree of connectivity and payoff value. Depending on the exact conditions of the game, we observe a plethora of behaviors for the percentage of cooperating agents. For example, the same network may settle in an equilibrium configuration of either low or high percentage of cooperators, or induce a transition between these two regimes.

The Prisoner's Dilemma (PD) game [1] has attracted a lot of interest during the recent years. It is a very simple, yet powerful, game used mainly for social studies. It can describe the conflict in the behavior of a person between cooperative and selfish attitudes. The selfish behavior is manifested by a defecting strategy, where the individual tries to obtain the greatest possible benefit from interactions with other individuals. Cooperation, on the other side, leads to a smaller personal benefit but to a greater average benefit for all the community.

In its simplest form, PD is a zero-dimensional game where two players can choose between two strategies, i.e. to either cooperate (C) or defect (D), without knowing the strategy chosen by the other player. The payoff for each player depends on the joined responses. If both players cooperate, they each gain a reward of 1. If they both defect, they are both punished, and earn 0. If one chooses to defect while the other cooperates, the defector receives the temptation gain b (where $b > 1$), while the cooperator is left with the sucker payoff (i.e. 0). It is important to notice that an individual cares about maximizing his own payoff, rather than gaining more than his opponent, in which case he would always defect.

An important extension of the game is the spatial prisoner's dilemma [2], where a large number of players is placed on a lattice structure. During a game round each individual plays the game with his neighbors. The total payoff from all these interactions is summed up and computed during each turn for all players. All individuals exhibit an imitation behavior of following the most successful strategy accessible to them. For the next round, thus, each player adopts the strategy of his neighbor that received the largest total payoff (including himself).

The spatial version of the game answers questions related to the conditions under which cooperation can be stable. Since there is no memory for the encounters between players, the cooperation is due to their spatial organization. A number of papers addresses the influence of the underlying topology. Thus, Kim et al [3] studied the spatial

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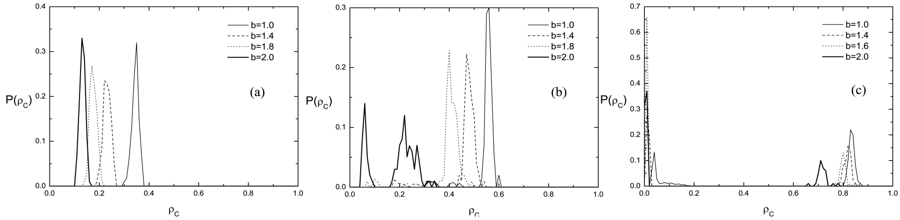

FIGURE 1. Distribution of the percentage ρ_C of cooperators for varying temptation values b, in networks with (a) $\gamma = 3.5$, (b) $\gamma = 3.0$, and (c) $\gamma = 2.5$

PD game on small-world networks, while Holme et al [4] played the same game on a number of empirical scale-free networks. In Ref. [4] the authors used a probability for irrational behavior in order to drive the system, where an individual choses not to follow the winning strategy with a probability p_m. In this work we show that, in general, the probability p_m is not necessary for observing interesting behavior in scale-free networks.

A scale-free network consists of N nodes and L links between them. A node i has k_i links, where the distribution of the connectivity for all nodes follows a power law: $P(k) \sim k^{-\gamma}$. Thus, players are placed on the network nodes and play the PD game with their neighbors (players located on nodes that are connected with a direct link). In this way, a small number of players are placed on hubs (i.e. the most connected nodes) and interact with a large number of other agents, while most agents in the system are located on low-connectivity nodes. We monitor the fraction ρ_C of players that choose to cooperate during one round. This value asymptotically stabilizes to a constant value for a given realization.

It is known [2] that when starting with a random initial distribution of cooperators and defectors the spatial PD game on lattices always leads to a constant asymptotic percentage of cooperators for a fixed b value. For example, considering 8 neighbors interaction yields $\rho_C = 1$ for $1 < b < 1.8$ and $\rho_C = 0.318$ for $1.8 < b < 2$. On scale-free networks we have observed in our simulations a very different behavior. The percentage ρ_C may vary significantly for networks of the same γ exponent and temptation value b. Moreover, even playing the game on the exact same network with different initial random distribution of C and D can lead to different ρ_C values. It is natural, thus, to study the distributions of the asymptotic ρ_C values, in addition to the average $\langle \rho_C \rangle$ value.

In Fig. 1a we can see that the distributions of ρ_C for $\gamma = 3.5$ have a roughly gaussian shape. Their width is roughly constant as we vary b with a range of the values of around 10%. As we increase the temptation value b, the distributions move towards smaller cooperator densities. For $\gamma = 3.0$ (Fig. 1b) there are again some predominant peaks for low b values, but now there is a number of peaks with much lower height, denoting the presence of some low-cooperation configurations. When $b = 2.0$ the distribution becomes bi-modal with two main peaks. For the ‘denser’ networks of $\gamma = 2.5$ the picture is quite different. For all b values the distributions are bi-modal, with a peak at large ρ_C values and a peak at almost zero cooperator density. Variation of b is not inducing important changes, except for $b = 2$, when the right wing of the distribution moves to lower values.

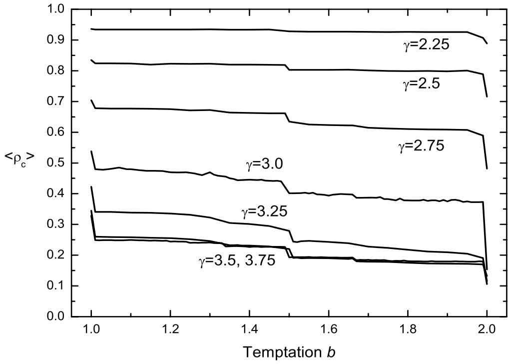

FIGURE 2. Average percentage of cooperators $\langle \rho_C \rangle$ as a function of b, for networks of varying connectivity (the exponent γ is shown on the plot).

In Fig. 2 we plot $\langle \rho_C \rangle$ as a function of b for networks with different exponents γ. At $\gamma = 2.25$ more than 90% of the population opts to cooperate. This emergent cooperation implies a 'globalization' of the prevailing attitude, where spreading over the entire network is very efficient. As we increase γ, the cooperation density decreases, with an average percentage of $\langle \rho_C \rangle = 0.2$ at $\gamma = 3.5$. This shows that the game outcome has become more 'local'. At large γ values there are not nodes with a large number of connections, and a defector cannot influence many neighbors, or be influenced by them. This leads to the survival of many clusters with defecting attitude.

In short, we have seen that the density of cooperators on a scale-free network cannot be characterized by a single value. We have studied the distributions of ρ_C and we found that, depending on the γ and b values, it is possible to have uni-modal or bi-modal distributions. The average $\langle \rho_C \rangle$ density does not greatly vary with b, but increasing the exponent γ causes $\langle \rho_C \rangle$ to decrease significantly.

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