STANDARD 9 — MEASUREMENT

K-12 Overview

All students will develop an understanding of and will use measurement to describe and analyze phenomena.

Descriptive Statement

Measurement helps describe our world using numbers. We use numbers to describe simple things like length, weight, and temperature, but also complex things such as pressure, speed, and brightness. An understanding of how we attach numbers to those phenomena, familiarity with common measurement units like inches, liters, and miles per hour, and a practical knowledge of measurement tools and techniques are critical for students' understanding of the world around them.

Meaning and Importance

Measurement is important because it helps us to quantify the world around us. Although it is perfectly natural to think about length, area, volume, temperature, and weight as attributes of objects that we measure, a little reflection will produce many other measurable quantities: speed, loudness, pressure, and brightness, to name just a few. An understanding of the processes of measurement, the concept of a unit, and a familiarity with the tools and common units of measurement, are all critical for students to develop an understanding of the world around them.

This standard is also, in many ways, the prototypical "integrated" standard because of its strong and essential ties to almost every one of the other content standards. Measurement is an ideal context for dealing with numbers and with numerical operations of all sorts and at all levels. Fractions and decimals appear very naturally in real-world measurement settings. Metric measures provide perhaps the most useful real-world model of a base-ten numeration system we can offer to children. Similarly geometry and measurement are almost impossible to consider separately. For instance, treatments of area and perimeter are called "measurement" topics in some curricula and "geometry" topics in others because they are, quite simply, measurements of geometric figures. Another of the content standards which is inextricably linked to measurement is estimation. Estimation of measures should be a focus of any work that students do with measurement. Indeed, the very concept that any continuous measurement is inexact — that it is at best an "estimate" — is a concept that must be developed throughout the grades.

Think about how many different content standards are incorporated into one simple measurement experience for middle school students: the measurement of a variety of circular objects in an attempt to explore the relationship between the diameter and circumference of a circle. Clearly involved are the *measurement* and *geometry* of the situation itself, but also evident are opportunities to deal with *patterns* in

the search for regularity of the relationship, *estimation* in the context of error in the measurements, and *number sense* and *numerical operations* in the meaning of the ratio that ultimately emerges.

K-12 Development and Emphases

Throughout their study and use of measurement, students should be confronted explicitly with the important **concept of a measurement unit.** Its understanding demands the active involvement of the learner; it is simply not possible to learn about measurement units without measuring things. The process of measurement can be thought of as *matching* or *lining up* a given unit, as many times as possible, with the object being measured. For instance, in its easiest form, think about lining up a series of popsicle sticks, end to end, to see how many it takes to cover the width of the teacher's desk; or consider how many pennies it takes to balance the weight of a small box of crayons on a pan balance. At a slightly more sophisticated level, multiple units and more standard units might be used to add precision to the answers. The desk might be measured with as many orange Cuisenaire Rods as will fit completely and then with as many centimeter cubes as will fit in the space remaining; the crayons, with as many ten-gram weights as can be used and then one-gram weights to get a better estimate of its weight. These types of activities — this active iteration of units — make the act of measurement and the relative sizes of units significantly more meaningful to children than simply reading a number from a measurement instrument like a meter stick or a postal scale. Of course, as the measures themselves become the focus of study, rather than the act of measurement or the use of actual physical units, students should become knowledgeable in the use of a variety of instruments and processes to quickly and accurately determine measurements.

Much research has dealt with the development of children's understanding of measurement concepts, and the general agreement in the findings points to a need for coherent sequencing of curriculum. Young children start by learning to identify the attributes of objects that are measurable and then progress to direct comparisons of those attributes among a collection of objects. They would suggest, for instance, that this stick is *longer than* that one or that the apple is *heavier than* the orange. Once direct comparisons can consistently be made, informal, non-standard units like pennies or "my foot" can be used to quantify how heavy or how long an object is. Following some experiences which illustrate the necessity of being able to replicate the measurements regardless of the measurer or the size of the measurer's foot, these non-standard units quickly give way to **standard, well-defined units** like inches and grams.

Older students should continue to develop their notions of measurement by delving more deeply into the process itself and by measuring more complex things. Dealing with various measurement instruments, they should consider questions concerning the inexact nature of their measures, and to adjust for, or account for, the inherent **measurement error** in their answers. Issues of the **degree of precision** should become more important in their activities and discussion. They need to appreciate that no matter how accurately they measure, more precision is always possible with smaller units and better instrumentation. Decisions about what level of precision is necessary for a given task should be discussed and made before the task is begun, and revised as the task unfolds.

Older students should also begin to develop procedures and formulas for determining the measures of attributes like area and volume that are not easily measured directly, and to develop **indirect measurement techniques** such as the use of similar triangles to determine the height of a flagpole. Their universe of measurable attributes should expand to include measures of a whole variety of physical phenomena (sound, light, pressure) and a consideration of rates as measures (pulse, speed, radioactivity).

Connections are another strong focus of students' work with measurement. The growth of technology in schools opens up a wide range of new possibilities for students of all ages. Inexpensive instruments attached to appropriately programmed graphing calculators and computers are capable of making and recording measurements of temperature, distance, sound and light intensity, and many other physical phenomena. The calculators and computers may then be used to graph those measurements with respect to time or any other measure, to present them in tabular form, or to manipulate them in other ways. These opportunities for scientific data collection and analysis using this technology are unlike any that have been available to mathematics and science teachers in the past and hold great promise for real-life investigations and for the integration of these two disciplines.

IN SUMMARY, measurement offers us the challenge to actively and physically involve students in their learning as well as the opportunity to tie together seemingly diverse components of their mathematics curriculum like fractions and geometry. It is also one of the major vehicles by which we can bring the real worlds of other disciplines such as the natural and social sciences, health, and physical education into the mathematics classroom.

NOTE: *Although each content standard is discussed in a separate chapter, it is not the intention that each be treated separately in the classroom. Indeed, as noted in the Introduction to this Framework, an effective curriculum is one that successfully integrates these areas to present students with rich and meaningful cross-strand experiences.*

Standard 9 — Measurement — Grades K-2

Overview

Students can develop a strong understanding of measurement and measurement systems from consistent experiences in classroom activities where a variety of manipulatives and technology are used. The key components of this understanding, as identified in the K-12 Overview, are: **the concept of a measurement unit; standard measurement units; connections to other mathematical areas and to other disciplines; indirect measurement**; and, for older students, **measurement error** and **degree of precision**.

Students in the early grades encounter measurement in many situations, from their daily work with the calendar, to situations in stories that they are reading, to describing how quickly they are growing. Hands-on science activities often require students to measure objects or compare them directly. Daily calendar activities frequently offer work with temperature, time, and money, in addition to number. Thus, many opportunities for **connections** present themselves in a natural way.

The study of measurement also encourages students to develop their number sense and to practice their counting skills. By using measures, students can recognize that numbers are often used to describe and compare properties of physical objects. Students in the early grades should make estimates not only of discrete objects like marbles or seeds but also of continuous properties like the length of a jumprope or the number of children's feet which might fit in a dinosaur's footprint.

Students need to focus on identifying the property that they wish to measure. They should understand what is meant by the length of an object or its weight or its capacity. Concrete experiences in describing the properties of objects, in sorting objects, and in comparing and contrasting objects provide them with opportunities to develop these concepts.

Students begin by making direct comparisons. *Which string is longer? Which child is taller? Which rock is heavier? Which glass holds more?* Making comparisons will help children better to understand the properties which they are discussing. They also begin to make some **indirect measurements.** For example, in order to compare the height of the blackboard with the height of a window, they might measure both objects using links and then compare the number of links used for each. In this way, they start to see a need for a **measurement unit,** a unit that they can use over and over to compare to a variety of objects.

In grades K-2, students should use a variety of non-standard units to measure objects. *How many links long is a desk? How many erasers high are you? How many pennies balance a Unifix cube?* In each case, students should first be asked to make an estimate and then proceed to actually measure the object. Students should also use different units to measure the same object. They should begin to understand that when the size of a measuring unit increases, the number of units needed to measure the object decreases.

In these grades, students also begin to use **standard measurement units** and standard measurement devices such as rulers and scales. It is important that the students see the use of the standard devices as simply an extension of their earlier activities. For example, the use of an inch ruler is just a more efficient procedure than lining up a series of cubic inch blocks. Students should explore length using inches, feet, centimeters and meters; liquid capacity using quarts, pints, cups, and liters; mass/weight using pounds, ounces, grams,

and kilograms; time using days, weeks, months, years, seconds, minutes, and hours; and temperature using degrees Fahrenheit and Celsius.

Whether making direct comparisons, using non-standard units, or using standard measurement units, students in the early grades should always estimate a measure first and then perform the measurement. In this way, their estimation and number sense skills will be reinforced.

Standard 9 — Measurement — Grades K-2

Indicators and Activities

The cumulative progress indicators for grade 4 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in kindergarten and grades 1 and 2.

Experiences will be such that all students in grades K-2:

1. **Use and describe measures of length, distance, capacity, weight, area, volume, time, and temperature.**
    - Students find out how many cubes long their hand is. The class can then generate a graph showing the results.
    - Using a large map of the school community, students estimate and then use paper clips or links to measure who lives farthest from school. This type of activity might be related to a specific story that was used in the Social Studies unit on community.
    - Students name objects big enough to hold a football or too small to hold a soccer ball.
    - Students lay out a model zoo with several toy animals, using boxes of different sizes for their cages or yards. They also cut doors of appropriate sizes in the boxes for the animals.
    - Students listen to and look at the book *Let's Find Out about What's Light and What's Heavy* by Martha and Charles Shapp. The simple text and humorous illustrations lead to the conclusion that weighing things using a standard unit of weight helps answer the question in the title.
    - Students name objects they can lift and ones that they cannot lift.
    - Students estimate and then use balances to find out how many pennies balance a small familiar object.
    - Students cut strips of paper to fit around a pumpkin or to make Santa's belt.
    - Students fill a large bottle with water using first a 4 ounce cup and then an 8 ounce cup. They then compare the results.
    - Students make their own measuring jug using a large plastic jar. They pour in one cupful of water and mark the water level on the jar with a marker; they repeat this procedure with one cupful after another until no further cupfuls will fit inside the jar.
    - Students read *The Little Gingerbread Man* and make a gingerbread village. In doing so, they measure lengths and capacities.
    - Students make their own paper clip ruler. First they make a paper clip chain and then paste it down on a long cardboard strip. They draw a small vertical line where each paper clip ends.
    - Students estimate and measure the distance around an object using Unifix cubes or a paper clip chain.

- Students conduct experiments using timers: *How many times can you bounce a ball before all the water runs out of the can? How many times can you clap your hands before the sand runs out of the timer? How many times can you blink your eyes before the second hand goes all the way around the clock?*
- Students read or listen to *The Very Hungry Caterpillar* by Eric Carle which shows the time of day at which various activities occur.
- Students make a book describing their day at school. On each page, they stamp a clock face and write underneath a time that the teacher has written on the board. They then draw the hands on the face to show the time. When the actual time of day on the classroom clock matches a time in their book, students draw a picture of what they are doing next to the correct clock face.
- Students line up Cuisenaire Rods in different combinations to measure the width of a sheet of paper.

2. Compare and order objects according to some measurable attribute.

- Kindergarten students listen to and look at the book *Big Friend, Little Friend* by Eloise Greenfield. In it a young boy and his two friends explore situations that clearly demonstrate what it means to be big and what it means to be little. As a nice follow-up assessment activity to this and other manipulative activities, the teacher has the students draw pictures which illustrate "big and little."
- Students compare the lengths of pencils to find out which is longest. They arrange a set of pencils in order from longest to shortest.
- Students use water, rice, or sand to fill different objects, pouring from one object into another to find out which object holds more. They explain how the shape of each object plays a role in the amount it holds.
- Students line up in order, from tallest to shortest.
- Children make stick drawings of a family: father, mother, school-aged child, and baby. They discuss which stick drawings are taller or shorter than others, and relate these to the relative size of the individuals in the family.
- Students collect a large variety of cardboard boxes and arrange them in order from smallest to largest.
- Each group of students is given a cup and several containers of different sizes, plain white paper, some uncooked rice, and 1 inch graph paper. They find out how many cups of rice will fit in each container and show the number of cups as a bar on the graph paper under a picture of the container. After completing the graph, they arrange the containers in order from largest to smallest.
- Students work through the *Will a Dinosaur Fit?* lesson that is described in the First Four Standards of this *Framework.* They arrange the dinosaurs in order from tallest to smallest according to their height, and from longest to shortest according to their length.

3. Recognize the need for a uniform unit of measure.

- Students measure the width of their desks by counting how many widths of their hands it

would take to go from one end of the desk to the other. They compare their results and discuss what would happen to the number of hands if the teacher's hand were used instead.

- Students read and discuss *How Big Is a Foot?* by Rolf Myllar. The king wishes to give the queen a special bed for her birthday and measures the size using his own foot. He gives the measurements to the carpenter, who gives them to the little apprentice. The bed that he makes is too small, but the apprentice solves the problem and everyone lives happily ever after. The students use their own feet to measure the width or length of the hallway and compare their results. Finally, they measure the hallway using meter sticks.
- As an assessment of the students' understanding of units, the teacher has the students measure the length of their math book using paper clips, unifix cubes, and yellow Cuisenaire Rods. They write about their results and explain why they are different.

4. **Develop and use personal referents for standard units of measure (such as the width of a finger to approximate a centimeter).**
    - Students identify parts of their body that are the same length as the unit cube from a base tens block set (1 centimeter).
    - Students make a list of foods or drinks that come in quarts and others that come in liters.
    - Students find out that ten pennies weigh about an ounce.
    - Students find that first-graders are a little taller or a little shorter than a meter.

5. **Select and use appropriate standard and non-standard units of measurement to solve real-life problems.**
    - Students decide whether they should use paper clips or pennies to measure the weight of a pencil.
    - Students discuss whether they should use links or meter sticks to measure the length of the gym.
    - Students write about how they might measure the distance from the cafeteria to their classroom.

6. **Understand and incorporate estimation and repeated measures in measurement activities.**
    - Students estimate how many of their shoes will fit in a giant's footprint (left conveniently on the classroom blackboard!) and write their estimates. They trace around their shoes and cut out the tracings. After the teacher has pasted a few shoes onto the giant's footprint, the students revise their estimate. They then check the accuracy of their estimates by pasting as many shoes as will fit into the footprint.
    - Students estimate the weight of various objects in beans and then use a balance scale to check the accuracy of their measurements.

References

Carle, Eric. *The Very Hungry Caterpillar*. New York: Philomel Books, 1987.

Greenfield, Eloise. *Big Friend, Little Friend.* New York: Black Butterfly Children's Books,

1991.

Myllar, Rolf. *How Big is a Foot?* New York: Dell Publishing, 1962.

Shapp, Martha, and Charles Shapp. *Let's Find Out about What's Light and What's Heavy*. New York: Franklin Watts, 1975.

General reference

Burton, G., and T. Coburn. *Curriculum and Evaluation Standards for School Mathematics: Addenda Series: Kindergarten Book*. Reston, VA: National Council of Teachers of Mathematics, 1991.

One-Line Resources

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Standard 9 — Measurement — Grades 3-4

Overview

Students can develop a strong understanding of measurement and measurement systems from consistent experiences in classroom activities where a variety of manipulatives and technology are used. The key components of this understanding, as identified in the K-12 Overview, are: **the concept of a measurement unit; standard measurement units; connections to other mathematical areas and to other disciplines; indirect measurement;** and, for older students, **measurement error** and **degree of precision**.

Students in grades 3 and 4 continue to encounter measurement situations in their daily lives and in their schoolwork. They investigate how much weight different structures will support or make a model of the solar system in science class, they make maps in social studies, and they read and discuss stories in which people measure objects. Measurement continues to provide opportunities for making mathematical **connections** among subject areas.

Measurement also helps students make **connections** within mathematics. For example, as students begin to develop their understanding of fraction concepts, they extend their understanding of measurement to include fractions of units as well. Measurement is interwoven with developing understanding of the geometric concepts of perimeter, area, and volume. Furthermore, students develop their estimation skills as they develop their understanding of measurement.

Students also continue to learn about more attributes of objects that can be measured. In addition to length, distance, capacity, weight, area, volume, time, and temperature, they now are able to discuss the size of angles and the speed of a car or a bike. Students begin to make more **indirect measurements** as well. For example, they will measure a desk to find out whether it will fit through a door, or measure how far a toy car goes in a minute and divide to find its speed in feet per second.

The emphasis in these grades is on building on the students' earlier experience with non-standard units and their developing **concept of measurement unit** to the use of more sophisticated **standard units of measurement.** They solidify their understanding of the basic units introduced in the earlier grades and begin to use fractional units. Students use half-inches, quarter-inches, and eighths of an inch, for example, in measuring the lengths of objects. Students also begin to use some of the larger units: miles, kilometers, and tons.

Some students may begin to discover formulas to help count units. For example, students may use shortcuts to find out how many squares cover a rectangle, multiplying the number of rows times the number of squares in each row. Or they may find the distance around an object by measuring each side and then adding the measures.

In summary, in grades 3 and 4, it is important that all students get extensive hands-on experience with measuring the properties of a variety of physical objects. They will learn to measure by actually doing so with an appropriate measuring instrument.

Standard 9 — Measurement — Grades 3-4

Indicators and Activities

The cumulative progress indicators for grade 4 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in grades 3 and 4.

Building upon knowledge and skills gained in the preceding grades, experiences in grades 3-4 will be such that all students:

1. **Use and describe measures of length, distance, capacity, weight, area, volume, time, and temperature.**
    - Students find out how many inches long their hand is. The class then generates a graph showing the results.
    - Students use rulers to measure the length of the room in feet and inches and then in metric units.
    - Students move thermometers to different parts of the school, recording the temperature at each location. For example, it may be hot in the cafeteria and cold in the gym. They learn to identify appropriate reference points on both Celsius and Fahrenheit scales (e.g., 30° C is a hot day).
    - Students investigate truth-in-packaging by reading labels, estimating weights, and then using balances to weigh foods.
    - Students investigate how many cups in a pint, how many pints in a quart, and how many quarts in a gallon by making lemonade and filling various sizes of containers.
    - Students make their own rulers, marking off intervals equal in length to one centimeter.
    - Students estimate and measure the distance around an object using a length of string which they then measure with centimeter cubes.
    - Students conduct experiments using timers: *how many times can you bounce a ball, clap your hands, or blink your eyes in one minute?* They discuss how many times each would occur in 10 minutes, in an hour, or in a day, if they continued at the same rate, and why their answers might be different.
    - Students measure all sorts of performances in their physical education class: the time it takes to run 100 meters, the length of a long jump in inches, and the length of a softball throw in meters.
    - Students read *Time to . . .* by Bruce McMillan. In this book about a farm boy and his daily activities, clock faces are always there to remind the reader what time it is.
    - Students use calculators to help them find out how many days old they are.
    - When going on a field trip, students determine how much time they will have available at a museum by considering when they will arrive and when they must leave.

- Students use cubes to fill rectangular boxes of various sizes as they explore the concept of volume.

2. Compare and order objects according to some measurable attribute.

- Students compare the areas of different leaves and order them from smallest to largest. They use a variety of strategies; some students cover the leaves with centimeter cubes, others make a copy of the leaf on grid paper, and still others just "eyeball" it. They discuss the different strategies used, comparing their advantages and disadvantages.
- Students bring in a variety of cereal boxes from home and estimate their order from smallest volume to largest volume. They then check their accuracy by filling the boxes with cubic inch blocks, with cubic centimeter blocks, and with sand, and discuss the reasons for the differences in their results.
- Students build bridges using straws and pipe cleaners, estimate how many round metal washers their bridge will hold, and then place the washers on their bridge until it buckles or breaks. They compare different types of bridges to determine what type is strongest.
- Students estimate and then weigh objects, putting them in order from heaviest to lightest.

3. Recognize the need for a uniform unit of measure.

- Students measure the length of their classroom using their paces and compare their results. They discuss what would happen if the teacher measured the room with his or her pace.
- Students read and study the illustrations in the book *Long, Short, High, Low, Thin, Wide* by James Fey. There are many activities in this attractive book that take the students through an historical account of the development of standard units.

4. Develop and use personal referents for standard units of measure (such as the width of a finger to approximate a centimeter).

- Students identify parts of their body that are the same length as ten centimeters and use them to measure the length of their pencil.
- Students find things in their environment that weigh about an ounce.
- Students use a meter stick to identify a personal referent which is approximately a meter. For example, for one child it may be an armspan, for another it might be the distance between a kneecap and the top of the head.
- Students measure the length of their pace in inches and use that information, along with a measurement of the length of the room in paces, to find the length of the room in inches.

5. Select and use appropriate standard and non-standard units of measurement to solve real-life problems.

- Students decide what units they should use to measure the weight of a textbook.
- Students discuss what units they should use to measure the length of the hallway outside their classroom.

- Students write about how they might measure the distance from the cafeteria to their classroom or the area of the gym.
- A nice approach to assessment of students' skills with this topic is to make a list of items that they can measure, such as the length of a piece of notebook paper, the weight of a teacher, the amount of water a bucket can hold, and the distance between Trenton and Newark, and ask them to name a measurement unit with which it would be appropriate to measure the given item. They discuss their choice of unit, estimate the measure of each item, and then actually measure it and compare it to their estimate and to the results of other students.

6. Understand and incorporate estimation and repeated measures in measurement activities.

- Students read and laugh about the pictures in *Counting on Frank* by Rod Clement. Frank is a dog whose young owner, challenged by his father to use his brain, estimates and imagines all sorts of measurements leading to some pretty silly situations. Possible extensions are plentiful and easy to devise. As an open-ended assessment follow-up to the story, the teacher asks each of the students to make a "Counting-on-Frank-like" estimate of how many of something (television, little sister, car, dog) would fit into their bedroom and to draw a picture showing all of them there.
- Students estimate the weight of various objects in grams and then use a balance scale to check the accuracy of their measurements.
- Students estimate the weight of, and then weigh the contents of a box of animal crackers, graphing their results and comparing them to the weights indicated on the packages.

References

Clement, Rod. *Counting on Frank*. Milwaukee, WI: Gareth Stevens Publishing, 1991.

Fey, James. *Long, Short, High, Low, Thin, Wide*. New York, NY: Thomas Y. Crowell Publishers, 1971.

McMillan, Bruce. *Time to . . .* Lothrop, Lee and Shepard, 1986.

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One-Line Resources

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Standard 9 — Measurement — Grades 5-6

Overview

Students can develop a strong understanding of measurement and measurement systems from consistent experiences in classroom activities where a variety of manipulatives and technology are used. The key components of this understanding, as identified in the K-12 Overview, are: **the concept of a measurement unit; standard measurement units; connections to other mathematical areas and to other disciplines; indirect measurement;** and, for older students, **measurement error** and **degree of precision**.

Why teach measurement? The ability to measure enables students to **connect** mathematics to the environment and offers opportunities for interdisciplinary learning in social studies, geography, science, music, art, and other disciplines. In addition, measurement tools and skills have a variety of uses in everyday adult life.

However, in the most recent international assessment of mathematics achievement, 13-year-olds in the United States performed very poorly in comparison to other nations. The results of this study indicated that, while students are given instruction on measurement, they do not learn the concepts well. For example, some students have difficulty recognizing two fundamental ideas of measurement, the concept of a unit and the iteration of units. A common error is counting number marks on a ruler rather than counting the intervals between the marks. Another difficult concept is that the size of the unit and the number of units needed to measure an object are inversely related; as one increases the other decreases. In the fifth and sixth grades, students begin to encounter both very small and very large **standard measurement units** (such as milligrams or tons), and these ideas become increasingly critical to understanding measurement.

Students must be involved in the act of measurement; they must have opportunities to use measurement skills to solve real problems if they are to develop understanding. Textbooks by themselves can only provide symbolic activities. Middle grade teachers must take responsibility for furnishing hands-on opportunities that reinforce measurement concepts with all common measures.

Using measurement formulas as a more efficient approach to some types of direct measurement is an important part of fifth and sixth grade mathematics. It represents the first formal introduction to **indirect measurement**. Multiplying the length by the width of a rectangle is certainly an easier way to find its area than laying out square units to cover its surface. The formulas should develop, however, as a result of the students' exploration and discovery; they should be seen as efficient ways to count iterated units. Having students memorize formulas that, for them, have no relation to reality or past direct measurement experiences will be unsuccessful. Fewer than half of the U.S. seventh grade students tested in the international competition could figure out the area of a rectangle drawn on a sheet of graph paper, and only slightly more than half could compute the area given the dimensions of length and height. Often length and width are taught separately and how the two measurements combine to form the square units of area is not emphasized in instruction. In addition, area and perimeter are often confused with each other by middle grade students. Limiting students' experiences with measurement to the printed pages of textbooks restricts flexibility so that their understanding cannot be developed or generalized.

In order to further strengthen students' understanding of measurement concepts, it is important to provide

connections of measurement to other ideas in mathematics and to other areas of learning. Students should measure objects, represent the information gathered visually (e.g., in a graph), model the situation with symbols (e.g., with formulas), and apply what they have learned to real-world events. For example, they might collect information about waste in the school lunchroom and present their results to the principal with suggestions for reducing waste. Integrating across mathematical topics helps to organize instruction and generates useful ideas for teaching the important content of measurement.

In summary, measurement activities should require a dynamic interaction between students and their environment, as students encounter measurement outside of school as well as inside school. Students should use each measuring instrument until its use becomes second nature. The curriculum should focus on the development of understanding of measurement rather than on the rote memorization of formulas. This approach can be reinforced by teaching students to estimate and to be aware of the context whenever they make an estimate. For example, when buying carpeting it is advisable that the estimate be too high rather than too low. Students must be given the opportunity to extend their learning to new situations and new applications.

Standard 9 — Measurement — Grades 5-6

Indicators and Activities

The cumulative progress indicators for grade 8 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in grades 5 and 6.

Building upon knowledge and skills gained in the preceding grades, experiences in grades 5-6 will be such that all students:

7. Use estimated and actual measurements to describe and compare phenomena.

- Students estimate the number of square centimeters in a triangle. Then they enclose the triangle in a rectangle and use centimeter cubes or a transparent square centimeter grid to find the area of the rectangle. They also count squares to find the area of the triangle and that of any other triangles formed by the rectangle. They look for a pattern in their results and compare their results to their estimates.
- Students explain why the following is or is not reasonable: An average person can run one kilometer in one minute.
- Students measure how long it takes to go 10 meters, first using "baby steps," then using normal steps, and finally using "giant steps." They then compare their rates.
- Students measure the area of their foot by tracing around it on centimeter graph paper and counting the number of squares covered. To ease the counting task, students can color the squares completely inside the outline blue, those that are one-half inside green, those that are one-third inside yellow, and those that are one-fourth inside orange. Then all of the like-colored squares can be counted more easily and the various totals added to each other.
- An interesting open-ended group assessment project to use after the previous foot-tracing activity has been completed is to tell the students that Will Perdue (of the Chicago Bulls) wears a size 18 ½ shoe which measures 21 ½ inches long. Students are asked to use what they know about the areas of their own feet to estimate the area of his foot. Students who make a good estimate will deal with several issues: the fact that they have information about the length of their own feet, but only about the length of Will's shoe; the fact that as the foot gets longer, it also gets wider; and the issue of how to set up a proportion between appropriate quantities.

8. Read and interpret various scales, including those based on number lines and maps.

- Students use a given scale to compute the actual length of a variety of illustrated dinosaurs.
- Students make a scale drawing of their classroom and use two-dimensional scale models of its furniture in order to propose new ways of arranging the classroom so that they can work more efficiently in cooperative groups.
- Students use a map to find the distance between two cities.
- Students work through the *Short-Circuiting Trenton* lesson that is described in the

Introduction to this *Framework*. Using a map of Trenton and a ruler, students determine the distances between various sites, and then find the most efficient walking tour for their class trip.

9. Determine the degree of accuracy needed in a given situation and choose units accordingly.

- Students plan a vegetable garden, determining the unit of measure appropriate for the garden, estimating its size, and then computing the perimeter (for fencing) and area (for fertilizer).
- Groups of students use a scale drawing of an apartment (1 cm = 1 foot) to find out how many square yards of carpeting are needed for the rectangular (9' x 12') living room and other rooms.
- Statements work through the *Mathematics at Work* lesson that is described in the Introduction to this *Framework*. A parent discusses a problem which her company faces regularly: to determine how large an air conditioner will be needed for a particular room. To solve this problem, the company has to estimate the size of the room, in terms of its volume and the areas of any windows, after determining the appropriate units.

10. Understand that all measurements of continuous quantities are approximate.

- Students measure a specific distance in the room and compare their results, focusing on the idea that any measurement is approximate.
- Before a cotton ball toss competition, students discuss what units should be used to measure the tosses. They decide that measuring to the nearest centimeter should be close enough, even though the actual tosses will probably be slightly more or slightly less.

11. Develop formulas and procedures for solving problems related to measurement.

- Students complete a worksheet showing several rectangles on grids that are partially obscured by inkblots. In order to find the area of each rectangle, they must use a systematic procedure involving multiplying the length of the rectangle by its width.
- Students develop the formula for finding the volume of a rectangular prism by constructing and filling boxes of various sizes with centimeter or inch cubes and looking for patterns in their results. As a journal entry, they describe the "shortcut" way to find the volume.

12. Explore situations involving quantities which cannot be measured directly or conveniently.

- Students work in groups to estimate the number of bricks needed to build the school building. They explain their results in a class presentation, describing the strategies they used.
- Students are asked to estimate how many heads tall they are. Then they work in groups to develop a procedure for finding out how many heads tall each student is.
- Students construct a measuring tool that they can use to find the height of trees, flagpoles, and buildings when they are standing a fixed distance from the object to be measured.

13. Convert measurement units from one form to another, and carry out calculations that involve various units of measurement.

- Students are asked to find how many pumpkin seeds there are in a kilogram. They decide to measure how much 50 seeds weigh and use this result to help them find the answer.
- Students use approximate "rules of thumb" to help them convert units. For example:

  1 km is about 6/10 of a mile
  1 liter is a little bigger than a quart
  1 meter is a little bigger than a yard
  1 kg is about 2 pounds
  20° C is about 70° F (room temperature)
  1000 ml of water normally weighs about 1 kg

14. Understand and apply measurement in their own lives and in other subject areas.

- Students measure the heights of bean plants at regular intervals under different conditions. Some are in sunlight and some are not. The students discuss their results and make a graph of their findings.
- Students estimate and weigh cups filled with jellybeans, raisins, dried beans, peanuts, and sand to find out that equal volumes of different objects do not always weigh the same.
- Students learn how much water is in different foods by first trimming pieces of 5 different foods to a standard 15 grams, then measuring their weights again the next day. *Where did the water go?*
- Students estimate what fraction of an orange is edible, then weigh oranges, peel them and separate the edible parts. They weigh the edible part and then compute what fraction is actually edible and compare that fraction to their estimate.
- Students create their own food recipes.
- An ice cube is placed on a plastic tray in five different parts of the classroom. One group of students is assigned to each ice cube and tray. The students are asked to estimate how long it will take each ice cube to melt. They then observe the ice cube at five-minute intervals, recording their observations. After the ice cubes have melted, the groups share their observations and compare the length of time it took for the ice cubes to melt. They make a conjecture about the warmest spots in the classroom and then measure the temperature in each location to confirm their conjecture.
- Students read *Anno's Sundial* by Mitsumasa Anno. This sophisticated three-dimensional pop-up book presents an extraordinary amount of information about the movement of the earth and the sun, the relationship between those movements, and how people began to tell time. It is an ideal kick-off for an integrated, multi-disciplinary unit with upper-elementary students, incorporating reading, social studies, science, and mathematics.
- Students estimate and then develop a plan to find out how many pieces of popped popcorn will fit in their locker.
- Students work in pairs to design a birdhouse that can be made from a single sheet of wood (posterboard) that is 22" x 28". The students use butcher paper to lay out their plans so that the birdhouse is as large as possible. Each pair of students must show how the pieces can be laid out on the posterboard before cutting.

- Students compare the measurements of an object to those of its shadow on a wall as the distance between the object and the wall increases.

15. Understand and explain the impact of the change of an object's linear dimensions on its perimeter, area, or volume.

- Students use pattern blocks to see how the area of a square changes when the length of its side is doubled. They repeat the experiment using equilateral triangles.
- Students use cubes to explore how the volume of a cube changes when the length of one side is doubled, then when the lengths of two sides are doubled, and, finally, when the lengths of all three sides are doubled.
- Students use graph paper to draw as many rectangles as they can that have a perimeter of 16 units. They find the area of each rectangle, look for patterns, and summarize their results.

16. Apply their knowledge of measurement to the construction of a variety of two- and three-dimensional figures.

- Students use paper fasteners and tagboard strips with a hole punched in each to investigate the rigidity of various polygon shapes. For shapes that are not rigid, they determine how they can be made so.
- Students design and carry out an experiment to see how much water is wasted by a leaky faucet in an hour, a day, a week, a month, a year.
- Students use straw and string to construct models of the two simplest regular polyhedra, the cube and the tetrahedron.

References

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Bright, G. W., and K. Hoeffner. "Measurement, Probability, Statistics, and Graphing," in D. T. Owens, Ed. *Research Ideas for the Classroom: Middle Grades Mathematics.* New York: Macmillan Publishing Company, 1993.

LaPoint, A. E., N. A. Mead, and G. W. Phillips. *A World of Differences: An International Assessment of Mathematics and Science.* Princeton, NJ: Educational Testing Service, 1989.

One-Line Resources

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Standard 9 — Measurement — Grades 7-8

Overview

Students can develop a strong understanding of measurement and measurement systems from consistent experiences in classroom activities where a variety of manipulatives and technology are used. The key components of this understanding, as identified in the K-12 Overview, are: **the concept of a measurement unit; standard measurement units; connections to other mathematical areas and to other disciplines; indirect measurement;** and, for older students, **measurement error** and **degree of precision**.

In grades seven and eight, students begin to look at the measurement process more abstractly while continuing to develop their actual measurement skills and using measurement in connection with other subjects and other topics in mathematics.

All measurement activities should involve both estimation and actual measurement at these grade levels. Estimation strategies should include (1) having a model or referent (e.g., a doorknob is about one meter from the floor), (2) breaking an object to be estimated into parts that are easier to measure (*chunking*), and (3) dividing the object up into a number of equal parts (*unitizing*). Students should also discuss when an estimate is appropriate and when an actual measurement is needed and they should have opportunities to select appropriate measuring tools and units.

Especially in the context of making measurements in **connection with other disciplines**, the approximate nature of measure is an aspect of number that needs particular attention. Because of students' prior experience with counting and with using numerical operations to obtain exact answers, it is often difficult for them to develop the concept of the approximate nature of measuring. Only after considerable experience do they recognize that when they correctly measure to the nearest "unit," the maximum possible error would be one-half of that unit. Teachers must help students to understand that the **error of a measurement** is not a mistake but rather a result of the limitations of the measuring device being used. Only through measurement activities can students discover and discuss how certain acts, such as the selection and use of measuring tools, can affect the **degree of precision** and accuracy of their measurements.

Students in grades seven and eight should expand their understanding of measurement to include new types of measures, especially those involving **indirect measurement**. For example, they learn about density and force and how these characteristics are measured in science class. Middle school students also should develop a deeper understanding of the concept of rate, by experiencing and discussing different rates. Constructing scale drawings and scale models or relating biological growth and form provide excellent opportunities for students to use proportions to solve problems, as does using a variety of measuring tools to find the measures of inaccessible objects. Such personal experiences help students recognize and appreciate the use of measurement concepts in other real-world settings.

Standard 9 — Measurement — Grades 7-8

Indicators and Activities

The cumulative progress indicators for grade 8 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in grades 7 and 8.

Building upon knowledge and skills gained in the preceding grades, experiences in grades 7-8 will be such that all students:

7. Use estimated and actual measurements to describe and compare phenomena.

- Students estimate the number of square centimeters in a trapezoid. Then they use a transparent grid and count squares to find the area. They compare that result to the area of a rectangle whose base is the average of the two bases of the trapezoid and whose height is the same as that of the trapezoid. They look for a pattern in their results and compare their results to their estimates.
- Students build a bridge out of paper to connect two bricks and place weights on the bridge until it breaks, noting how much weight it held.
- Students read and discuss sections of *This Book Is About Time* by Marilyn Burns. Its many engaging activities and experiments are interspersed with an historical treatment of time and the instruments designed to measure it.

8. Read and interpret various scales, including those based on number lines and maps.

- Students use objects shown in a movie poster for *King Kong* to determine how tall the ape is.
- As a long-term assessment project, students make a three-dimensional scale model of their classroom.
- Students use a map to plan an auto trip across the United States, finding the distance traveled each day and the amount of time required to drive each day's route.

9. Determine the degree of accuracy needed in a given situation and choose units accordingly.

- Students plan a school garden, determining the unit of measure appropriate for the garden, estimating its size, and then computing the perimeter (for fencing) and area (for fertilizer).
- Groups of students make and use a scale drawing (1/4 inch = 1 foot) of an apartment and use scale models of the furniture to furnish the living room and dining room.
- Students make a floor plan for a small restaurant furnished with round tables.

10. Understand that all measurements of continuous quantities are approximate.

- Students measure a given hallway in school and compare their results, noting that their results are different because any measurement is approximate. They discuss how accurate

their individual measurements are (degree of precision) and, after reviewing all of their measurements, determine the likely errors in their individual measurements. They also discuss how more precise measures may be obtained and what degree of precision is needed in this situation.

- Each student in a group measures the circumferences and diameters of several round objects using a tape measure or ruler and string. They compare their measurements and decide what the most accurate set of measurements is for each of the objects. They use a calculator to find the ratio of the circumference to the diameter for each object.

11. Develop formulas and procedures for solving problems related to measurement.

- Students develop a formula for finding the surface area of a rectangular prism by constructing boxes of various sizes using graph paper, finding the area of each side and adding them, and looking for patterns in their results. They describe their findings in their journals.
- Students construct different parallelograms whose base and height have the same length on their geoboards. They sketch each parallelogram and record its area (found by counting squares). They discuss their results.
- Students work through the *Sketching Similarities* lesson that is described in the First Four Standards of this *Framework.* They use a computer program and measure the sizes of the corresponding sides and corresponding angles of similar figures. They conclude that similar figures have equal corresponding angles and their corresponding sides have the same ratio.
- Students use plastic models of a pyramid and a prism, each having the same height and polygonal base, to investigate the relationship between their volumes.

12. Explore situations involving quantities which cannot be measured directly or conveniently.

- Students and parents in the McKinley School have been dropping pennies into the very large plastic cylinder in the school lobby in an effort to raise money for new playground equipment. The students are challenged to devise a method to estimate how much money is in the cylinder as a function of the height of the pennies at any given time.
- Students construct a measuring tool that they can use to find the height of trees, flagpoles, and buildings, using cardboard, graph paper, straws, string, and washers.
- Students use proportions to find the height of the flagpole in front of the school.

13. Convert measurement units from one form to another, and carry out calculations that involve various units of measurement.

- Students are given a ring and asked to find the height of the person who lost the ring. They measure their own fingers and their heights, plotting the data on a coordinate graph. They use a piece of spaghetti to fit a straight line to the plotted points and make a prediction about the height of the person who owns the ring, based on the data they have collected.

- Students plan the *weird name Olympics* by renaming standard events in different measurement units. For example, “the hundred meter dash” becomes the “100,000

millimeter marathon" and the "ten meter dive" could become the "one-hundredth of a kilometer splash."

- Students continue to use approximate "rules of thumb" learned in earlier grades to help them convert units. For example:

  1 km is about 6/10 of a mile
  1 liter is a little bigger than a quart
  1 meter is a little bigger than a yard
  1 kg is about 2 pounds
  1 inch is about 2.5 centimeters
  20° C is about 70° F (room temperature)
  1000 ml of water normally weighs about 1 kg

14. Understand and apply measurement in their own lives and in other subject areas.

- Students design and carry out an experiment to see how much water is wasted by a leaky faucet in an hour, a day, a week, a month, and a year.
- Students are told that when people get out of a bath, a film of water about 0.05 cm thick clings to their skin. They are then challenged to find what volume of water clings to the skin of an average eighth grader. In order to make the estimate, of course, they need to estimate the surface area of the body. They can do so by considering a collection of cylinders and spheres that approximates a human.
- Students investigate the concept of density by finding objects for which they can find both the volume and the weight, measuring both, and dividing the latter by the former. Interesting objects to use include an orange, a block of wood, a textbook, a rubber sponge ball, and an air-filled rubber ball. Discussions of the results should lead to interesting conjectures about density which can then be confirmed with additional experimentation.

15. Understand and explain the impact of the change of an object's linear dimensions on its perimeter, area, or volume.

- Students use the computer program *The Geometric SuperSupposer* to explore the relationship in similar triangles between corresponding sides and the perimeters of the triangles. They also analyze the relationship between corresponding sides and the areas of the triangles.
- Students build a "staircase" using wooden cubes. Then they double all of the dimensions and compare the number of cubes used in the second staircase to the number used in the original staircase.
- Students work through the *Rod Dogs* lesson that is described in the First Four Standards of this *Framework*. They discover that if an object is enlarged by a scale factor, its new surface area is the scale factor squared times the original area, and its new volume is the scale factor cubed times the original volume.

16. Apply their knowledge of measurement to the construction of a variety of two- and three-dimensional figures.

- Students use straw and string to construct models of the five regular polyhedra: the cube, the tetrahedron, the octahedron, the icosahedron, and the dodecahedron.
- Students use cardboard and tape to construct a model that demonstrates that the volume of a pyramid is one-third that of a prism with the same base and height.
- Students build scale models of the classroom, the school, or a monument.

References

Burns, Marilyn. *This Book is About Time*. Boston, MA: Little, Brown & Co., 1987.

Software

The Geometric SuperSupposer. Sunburst Communications.

General references

Geddes, D. *Curriculum and Evaluation Standards for School Mathematics: Addenda Series: Measurement in the Middle Grades*. Reston, VA: National Council of Teachers of Mathematics, 1991.

Owens, D. T., Ed. *Research Ideas for the Classroom: Middle Grades Mathematics*. New York, NY: MacMillan, 1993.

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Standard 9 — Measurement — Grades 9-12

Overview

Students can develop a strong understanding of measurement and measurement systems from consistent experiences in classroom activities where a variety of manipulatives and technology are used. The key components of this understanding, as identified in the K-12 Overview, are: **the concept of a measurement unit; standard measurement units; connections to other mathematical areas and to other disciplines; indirect measurement;** and, for older students, **measurement error** and **degree of precision**.

Building upon the measurement skills and understandings developed in grades K-8, high school students move to a more routine use of measurement. They examine measurement as a more abstract process, focusing on **measurement error** and **degree of precision.** They spend much more time on **indirect measurement techniques** than they did in earlier grades, expanding their repertoire to include not only the use of proportions and similarity but also the use of the Pythagorean Theorem and basic right triangle trigonometric relationships.

Students at the high school level will frequently use measurement to help develop **connections** to other mathematical concepts. For example, students may use a computer program that measures angles to help them discover the relationship between the measures of two vertical angles formed by intersecting lines or the measures of inscribed angles intercepting the same arc of a circle. They may also develop algebraic techniques to help them find measures, as, for example, when they develop a formula for finding the distance between two points in the coordinate plane.

High school students also use measurement frequently in **connection** with other subject areas. Science experiments require a precise use of measurement. Social studies activities often require students to read and interpret maps and/or scale drawings. In technology classes, woodshop, drafting, sewing, and cooking, students must also use a variety of measuring tools and techniques; and in physical education, students frequently will need to measure distances and rates.

Standard 9 — Measurement — Grades 9-12

Indicators and Activities

The cumulative progress indicators for grade 12 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in grades 9, 10, 11, and 12.

Building upon knowledge and skills gained in the preceding grades, experiences in grades 9-12 will be such that all students:

17. Use techniques of algebra, geometry, and trigonometry to measure quantities indirectly.

- Students use coordinate geometry techniques to determine the distance between two points.
- Students use similar figures and proportions to measure the height of a tree or a flagpole.
- Students use the Pythagorean Theorem to determine how long a ladder is needed to climb a wall, including a determination of a safe angle at which to place the ladder.
- Students use right-triangle trigonometry to measure the width of a canyon or the height of a waterfall.
- Students work through the *Ice Cones* lesson that is described in the First Four Standards of this *Framework*. They use the formula for the volume of a cone and a graphing calculator to determine the maximum volume of a cone made from a paper circle of radius 10 cm which is cut along a radius.

18. Use measurement appropriately in other subject areas and career-based contexts.

- Students investigate how the volume of a cereal box changes with its area by finding the volume and surface area of a box of their favorite kind of cereal. They also discuss how the shape of the box affects its volume and surface area and why the volume of the box is so large for the amount of cereal it contains.
- As a question on their take-home final exam in Algebra, students are asked to measure their own and gather data about the length of other people's femurs and overall height in an effort to determine whether there is a relationship between the two lengths. They plot the resulting ordered pairs on a coordinate plane and find a line of best fit for their data. For extra credit, they make a prediction of the height of a male with a 22 inch femur.
- Students discuss the possible meaning of "light year" as a unit of measure and find units equivalent to it.

19. Choose appropriate techniques and tools to measure quantities in order to achieve specified degrees of precision, accuracy, and error (or tolerance) of measurements.

- Students use significant digits appropriately in measuring large distances, such as the distance from one school to another, from one city to another, and from one planet to another.

- Students find the distance between two cities by adding the numbers given on a road map for the segments of the trip, by measuring the segments and using the mileage scale, and by referring to a published mileage table. They explain the different results by referring to the degrees of prevision of the different measurements.
- In making a scale drawing of a house, students discuss the degree of accuracy of their measurements.
- Students read and discuss the photographs in *Powers of Ten* by Phillip and Phylis Morrison and the office of Charles and Ray Eames, and view the associated videotape. This well-known book takes the reader on a trip through perspectives representing forty-two powers of ten, from the broadest view of the universe to the closest view of the nucleus of an atom. The measurement units used and the progression from one to another highlight the range and power of our system of measurement.
- Students use computer drawing and measuring utilities to discover geometric concepts. They also discuss the limitations of such a program. For example, a program may give 14.7 for the length of the base of a triangle and 7.3 for its midline (the segment joining the midpoints of the other two sides); however, because of the program's measurement limitations, its answer for the length of the midline may not be exactly half the length of the base, as is the case in reality.
- Students determine what kind of measuring instrument needs to be used to measure ingredients for pain-relievers, for cough syrup, for a cake, and for a stew. They bring to class a variety of empty bottles and packages and note how the ingredients are measured: What does 325 mg (of acetaminophen) mean, or one fluid ounce (of cough syrup) as opposed to 1/4 cup of oil (for a cake). They discuss the accuracy, error, and tolerance of each measurement.

References

Morrison, Philip, Phylis Morrison, and the office of Charles and Ray Eames. *Powers of Ten*. Scientific American, 1988. (Revised, 1994. See also reference under Video.)

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Video

Powers of Ten. Philip Morrison, Phylis Morrison, and the office of Charles and Roy Eames. New York: Scientific American Library, 1991.

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