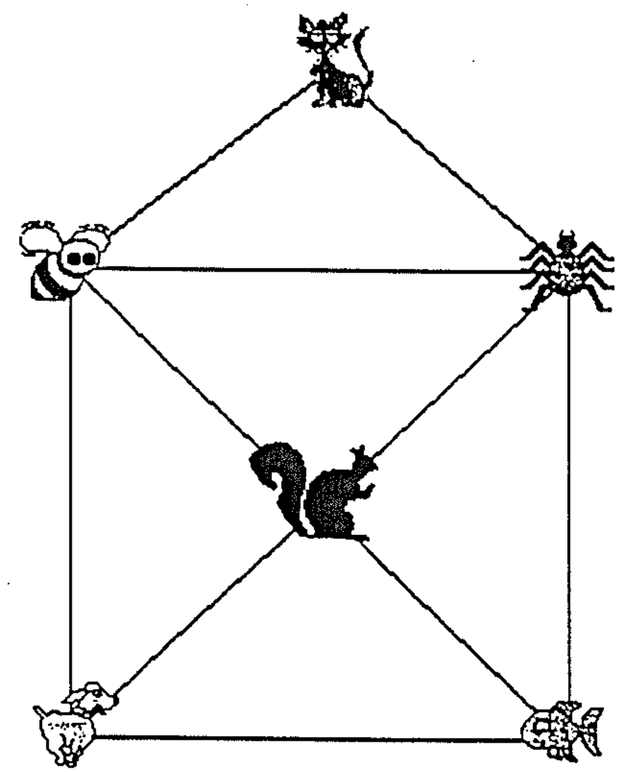
Exploring Discrete Mathematics in the Classroom

Getting It Together: Paths & Graphs K-4

A Workshop for Teachers About Paths & Graphs



Developed by Joseph G. Rosenstein and Valerie A. Debellis

in collaboration with the following participants in the Rutgers Leadership Program in Discrete Mathematics: Lynda Chatzel, Ann Lawrence, Darlene Mixer, and Susan Weiss

Getting It Together: Paths and Graphs K-4

Workshop Overview

Summary

Participants will engage in several motivating activities involving paths and graphs. The focus of these activities is on developing understanding rather than using algorithms to solve problems.

Workshop Outline

I. Objectives

Participants will:

*work together to solve several problems that focus on paths and graphs to solve problems

* be exposed to several methods for teaching these concepts, and

* be provided with materials to implement the activities in their classrooms.

II.	Activity #1 — Getting Tied Up		15 Minutes
III.	Activity #2 — The Robber's on the Loose		20 Minutes
IV.	Activity #3 — Tracing Pictures		20 Minutes
V.	Activity #4 — Seven Bridges		10 Minutes
VI.	Activity #5 — Euler Family Zoo		15 Minutes
VII.	Extension (Optional) Exploring Probability Through Children's Literature		(25 Minutes)
VII.	Workshop Summary		05 Minutes
IX.	Workshop Evaluation		05 Minutes
		Total Workshop Time:	90 Minutes

Workshop Environment

I. Room Arrangement

Ideally, participants should be seated at round tables in groups of four to six persons; round tables help facilitate collaboration and discussion among group members. Each participant should have a reasonable view of the presenter and the screen.

II. Equipment

Overhead projector and screen

III. Workshop Materials List

A. Activity #1 — Getting Tied Up
   - Yarn for every three participants
   - Blank transparencies and transparency markers

B. Activity #2 — The Robber's on the Loose
   - Shower liner and/or masking tape (optional)
   - Blank transparencies and transparency markers

C. Activity #3 — Tracing Pictures
   - Shower liner and/or masking tape (optional)
   - Blank transparencies and transparency markers

D. Activity #4 — Seven Bridges
   - Shower liner and/or masking tape (optional)
   - Blank transparencies and transparency markers

E. Activity #5 — Euler Family Zoo
   - Shower liner and/or masking tape (optional)
   - Blank transparencies and transparency markers

F. Extension Activity: Two for the Road
   - Shower liner and/or masking tape (optional)
   - Blank transparencies and transparency markers

Instructor's Notes

Introduction

Show the "Title Transparency" for this workshop and indicate that this workshop was developed by K-8 teachers who participated in the Rutgers University Leadership Program in Discrete Mathematics. Introduce yourself and mention that you spent three weeks over two summers learning discrete mathematics at the Leadership Program, and have been using materials from the program in your classroom.

Show the "Discrete Mathematics Transparency" and review the information there — discussing five major themes of discrete mathematics. Indicate that the workshops offered in the "Workshops in Your District" program reflect four of these major themes. The theme that we will be focusing on in this workshop is that of "Mathematical Modeling"; the title of this workshop, as noted on the "Title Transparency" is "Getting It Together: Paths and Graphs K-4".

Although you may have been given this information beforehand, find out the grade levels of the participants in your workshop (say, for K, 1-2, 3-4, 5-6, and 7-8) and, if the distribution is not what you expected, be sure to modify your workshop appropriately. Also, please tell the participants that all the information and all the activities in this workshop will be included in a packet of materials that they will receive after the program.

Activity #1 — Getting Tied Up
(Allocated time = 15 minutes)

Start by saying: "We will use the first activity to become acquainted with some important vocabulary related to graphs." Ask participants to form groups of three. Use TSP #1 to review the rules for the activity with the participants: Each group of three will receive a bag of yarn containing either 5 or 6 strands of yarn each about one foot long and all of the same color. (About half of the groups should receive 5 strands and the other half 6 strands.) Ask the shortest person in each group to stand in the middle and grasp the center of the bundle of yarn, leaving both ends of each piece of yarn exposed. The person on the right should take any two ends on his/her side of the middle person's hand and tie these ends together. He/she should continue to choose two ends and tie them together until all possible pairs have been tied. The person on the left end should follow the same procedure. The person in the middle continues to hold the yarn in the middle until the others are finished, and until instructed to let go. Once everyone seems to be finished, ask each group with 6 strands whether all ends are tied — they should be, and ask each group with 5 strands whether there is one loose end on each side — there should be. Then ask: "What do you think the yarn will look like when the person in the middle finally lets go?" Write the predictions on the chalkboard or a blank transparency for future reference. Instruct the middle people to let go of the yarn and see what has happened, then ask each group to share their results. Ask whether participants are surprised by the results. Then compare the results to the earlier predictions. Finally, ask whether any other possible results could have happened. Then show TSP #2 which has on it all the possible outcomes — did each of the outcomes actually happen? Each of the figures in TSP #2 is an example of a "graph" — where the knots are vertices and the pieces of strings are "edges"; review these terms using TSP #3.

For some groups, you may need to demonstrate the procedure participants should use for the activity using three people and five or six pieces of yarn. Also, when participants are making predictions, they will probably use descriptors like "a single strand of yarn", "a single loop of yarn", or "several circles of yarn". Be sure to emphasize that they do not need to use technical vocabulary in their descriptions.

Option: *At the close of an activity (either here or later in the workshop), you may want to emphasize the importance of communication, especially writing, in the mathematics curriculum. Ask the participants to write what they think students might learn from the activity, and ask a few participants to share what they wrote.*

Activity #2 — The Robber's on the Loose
(Allocated time = 15 minutes)

Read the following directions as you show TSP #4: "It's a snowy evening in a small town. The bank has just been robbed by an "America's Most Wanted" who left a distinctive trail of footprints in the snow through various homes and businesses. (All of the footprints were made after the robbery and there's only one set of footprints on each road.) There's a huge reward for her capture. From this map can you provide the tip that will lead to locating and arresting the criminal?" Distribute HO #1 and ask participants to work alone for a couple of minutes to try to solve the problem. Then let them work in pairs to share any ideas they may have. Finally, ask for a volunteer to mark on a blank transparency laid over TSP #4 where they think the robber is and how they think he got there. Several participants should be asked to provide their solutions. During the discussion of the problem, reinforce the terms vertex and edge as they apply to this activity: each home or business is a vertex and each road is an edge. Elicit from the participants that, although a number of different routes are possible, all of them end up with the robber at the gas station. Ask: "Why can't the robber be at any other location?" Elicit the answer that at each other location (except the bank itself) there are two or four edges that emerge from that location, which means that each time the robber arrived there, she also left — so that she can't be there now. On the other hand, at the gas station, there are three edges, so the second time the robber arrived at the gas station, she stayed there. Use TSP #5 to introduce the ideas of a "path" in a graph, the concept of the "degree" of a vertex, and "odd" and "even" degrees. Review these ideas using TSP #2. Ask participants if they can find another vertex of odd degree in TSP #4.

If desired, you can draw the graph of the town on a shower curtain liner or tarp before the workshop. Alternatively, you can use masking tape for roads with small paper plates as vertices on the floor of the room in which the workshop is being held. With any of the large renditions you can have a participant "walk the robber's path" to his hiding place. If you do this, you should use markers of some kind (strips of construction paper work nicely) laid on each road as the volunteer traverses it to designate that it has been traveled. If time or space does not permit you to do the activity in this way, it is helpful to suggest this alternative to participants for their use back in the classroom, especially with younger children.

Activity #3 — Tracing Pictures
(Allocated time = 20 minutes)

Say: "In the previous activity, you found a path for the robber which covered each edge in that graph exactly once. Such a path is called an "Euler path" in honor of the mathematician Leonhard Euler. Now we are going to do an activity which builds on what we did there. Let us first look at the following example. Place TSP #6 on the overhead and ask a volunteer to read the instructions. Elicit from them that this is a graph where the vertices are "critters" and the edges are the lines joining them. Point out that drawing the diagram without repeating edges and without removing pencil from paper is exactly the same as finding a path for the robber which accounted for all of her movements. Pass out HO #2 and a sheet of tracing paper — tell them that the purpose of the tracing paper is so that they can draw paths without messing up the original sheet. (Mention that they might want to provide their students with shoe-boxes containing sand or sugar on which they can trace their paths.) After participants have had a few minutes to work, let a volunteer trace the path on a blank transparency laid over TSP #6-9. Point out that some of the paths end where they begin — such a path is called a "circuit" and an Euler path which is also a circuit is called ... an "Euler circuit".

Circulate around the room as the participants work. Remind them to be looking for any patterns that might be related to when an Euler path or an Euler circuit can be found in a given graph. With some groups, you may want to review vertex of odd and even degree and or ask such question as, "Is there anything special about the vertex where your Euler path begins? 'Where it ends?

Option: If time allows, you can use HO #3 with TSP #10 to provide more practice with these concepts before the participants complete the table below. Alternatively, you may want to use this hand-out after the table is completed and discussed in order to reinforce the conclusions from the table. This will require less time. For example, you can ask questions like: "Without tracing, can someone tell whether the first figure can be drawn as a circuit?" followed by, "How do you know?" In this case, *participants still need to trace figures b and d to verify that a circuit can be found in those figures. Again, let participants trace each circuit on a blank transparency laid over TSP #10.*

Now pass out HO #4 as you place TSP #11 on the overhead. Say, "Let's try to organize our findings in a table to find a pattern. If you have already discovered the pattern, please don't take away someone else's 'aha'. We will share observations as soon as we complete the table." Let volunteer participants provide the information to complete the table, as shown below. Then ask for any patterns they observe.

Look for a Pattern

Sketch of Graph	Number of Vertices	Number of Even Vertices	Number of Odd Vertices	Circuit or Path or Impossible

What patterns did you find?

____

____

____

When it appears that the participants have arrived at the correct conclusion about when there is an Euler circuit and when there is an Euler path, draw a graph which consists of two disconnected triangles. In this example, every vertex has even degree, so that there should be

an Euler circuit — but there isn't! Elicit that this happens because the graph is not connected, in which case it cannot be traced without removing pencil from paper; if, however, the graph is connected, then the conclusions they obtained are correct. Summarize all the terminology and conclusions on TSP #12-13.

Activity #4 — Seven Bridges of Konigsberg
(Allocated time = 10 Minutes)

The city of Konigsberg (formerly Kaliningrad) spans the Pregel River and includes two islands. Ask participants to see if they can find a tour of the city of Konigsberg which crosses each of its seven bridges exactly once (see TSP #14 = HO #5). After they work on this for a while, elicit that this is the same as the previous activity. Use a blank transparency superimposed on TSP #14 to underscore this point by creating a graph which includes a vertex on each of the four land masses and an edge across each bridge. Note that a tour of the city would involve an Euler circuit on this graph. Elicit that this graph has lots of vertices of odd degree — so an Euler circuit (and even an Euler path) is impossible. Note that Euler circuits are called that because Euler was the mathematician in the 17th century who explained exactly why it was impossible to solve the problem of "The Seven Bridges of Konigsberg".

Activity #5 — Euler Family Zoo
(Allocated time = 15 minutes)

For the last activity, pose the question, "What are some real world problems that might be solved using what you have learned today?" Read the problem on HO #6 as you display TSP #15: "By the time the Euler Family Zoo closes each day, there is a great deal of litter that needs to be cleaned from every trail. As the person in charge of this clean-up, you want to know whether it is possible for you to start at the Entrance, clean each trail without retracing any, and finish at the Entrance." Have the participants work in pairs to try to find an Euler circuit.

Based on earlier activities participants should be able to conclude that an Euler circuit is not possible since there are two odd vertices.

Now ask, "Since we can't complete the task without retracing any edges (or trails), what is the smallest number of trails we need to retrace in order to do the clean-up job?" If needed, add, "Can we create a circuit with just one repeated edge? If so, where would it be?" Participants should realize that a circuit could be created by simply repeating the trail between the Entrance and the Petting Zoo. Have a participant trace the circuit on a blank transparency laid over TSP #15.

The two odd vertices at the Entrance and the Petting Zoo determine where the new edge *should be added. This means the trail between those two locations would need to be traveled twice. This is the only way of completing the clean-up job with only one repeated trail.*

As with other examples, you may want to draw the map or graph on a shower curtain liner or tarp. Certainly suggest to participants that they might like to do this with their students.

Extension Activity — Exploring Paths and Graphs through Children's Literature: Two for the Road (This activity has not yet been reviewed!)
(Allotted time = 30 minutes)

You may use the activity "Two for the Road" in connection with the book Frog and Toad are Friend by Arnold Lobel from Workshop Workshop's *Connecting Children's Literature With Discrete Mathematics Topics* K-4. For details of this activity, see Lit-IN #4-5, Lit-TSP #2-3, and Lit-HO #2-4. This activity provides experience with paths and circuits. They work with a map of Toad's neighborhood to find the best route to use in searching for a lost button.

Teachers in grades K-4 often can use children's literature as a vehicle to motivate their students and enrich the curriculum. Such cross-curriculum connections can be very effective. Other books appropriate for the topics in this workshop and these grade levels include the following:

Ahlberg, Alan and Janet. *The Jolly Postman.* New York: Wilteinemann Little, 1987.
Burton, Virginia Lee. *Katy and the Big Snow.* Boston: Houghton Mifflin, 1943.
Celsi, Teresa. *The Fourth Little Pig.* New York: Steck Vaughn, 1992.
Dubois, William. *The Twenty-One Balloons.* New York: Puffin Books, 1989.
Gardner, John R. *Stone Fox.* New York: Harper Collins, 1980.
Irons, Cal. *Sherlock Bones.* San Francisco: Mimosa, 1996.
Lowell, S. *The Three Little Javelinas.* Sunbelt Press, 1992.
Milton, Nancy. *The Giraffe That Walked to Paris.* New York: Crown, 1992.
Shur, Maxine R. *The Marvelous Maze.* New York: Stemmer House, 1995.
Williams, Vera. *Three Days on a River in a Red Canoe.* New York: Greenwillow Books, 1981.

Workshop Summary

Allow some time for participant questions, then explain that this workshop has only "touched the tip of the iceberg"! Including problems that focus on paths and graphs in the mathematics curriculum in the early grades helps students expand their problem solving skills and therefore fosters mathematical power. This workshop provides a sampling of meaningful activities that can be used in K-4 classrooms.

Remind them that they will soon receive a packet of "Take-home Materials" (TKHM) which contains the materials used in the workshop and (in some cases) additional materials. Encourage participants to review these materials and use the activities in their classrooms. In addition, they should be encouraged to develop other activities through which they can weave paths and graphs throughout their curriculum.

Workshop Evaluation

Pass out the "Participant Evaluation Forms". Encourage participants to add their constructive suggestions for future presentations of this workshop. Be sure to provide them with five minutes to complete the form.

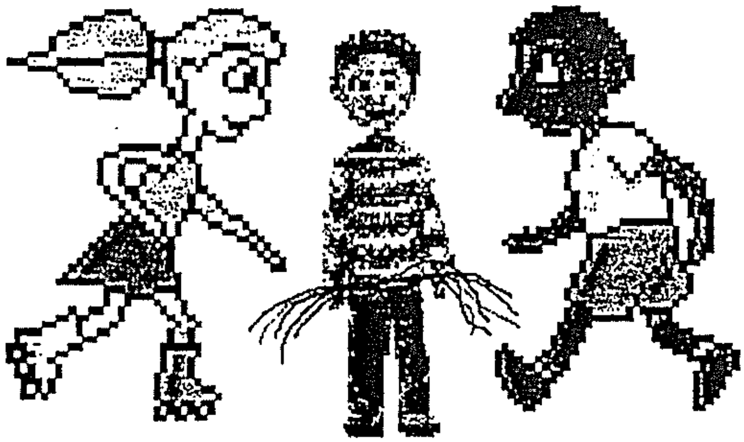
Collect forms from participants and give them the "Take-home Materials". Draw their attention to the last page of the "Take-Home Materials", and encourage them to record on that page their impressions of the workshop and their thoughts about how they might use the workshop materials in their classrooms.

Immediately following the workshop (or at the latest that evening when you get home) please complete the "Presenter Report Form" and then the "Participant Summary Form".

Within three days of your workshop, please mail all "Participant Evaluation Forms", the Presenter Report Form", and the "Participant Summary Form" — together with an annotated copy of these Instructor's Notes and copies of any supplementary materials that you used — to the following address:

K-8 Workshop-Workshop Evaluation Information
P.O. Box 10867
New Brunswick, NJ 08906

GETTING TIED UP



Materials Needed for Each Group:

Five or six pieces of yarn, each one foot long

For this activity, participants work in groups of three.

(1) The shortest person will be in the middle; that person should grasp the bundle of yarn in the middle, leaving both ends of each piece of yarn exposed.

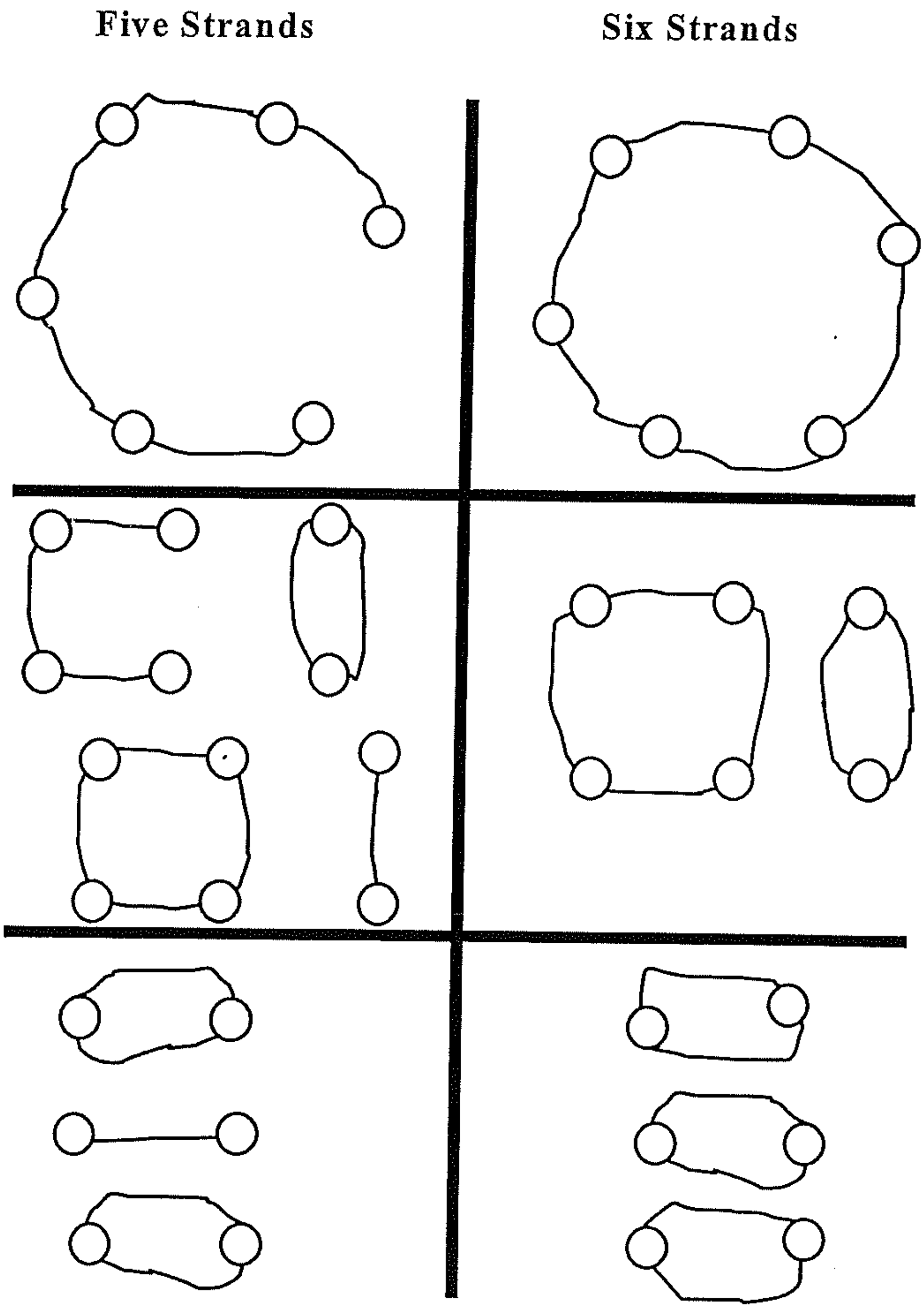
(2) The person on the right end should take any two ends on his/her side of the middle person's hand and tie these ends together. He/she should continue to choose two ends and tie them together until all possible pairs have been tied.

(3) The person on the left end should follow the same procedure.

(4) The person in the middle continues to hold the yarn in the middle until the others are finished.

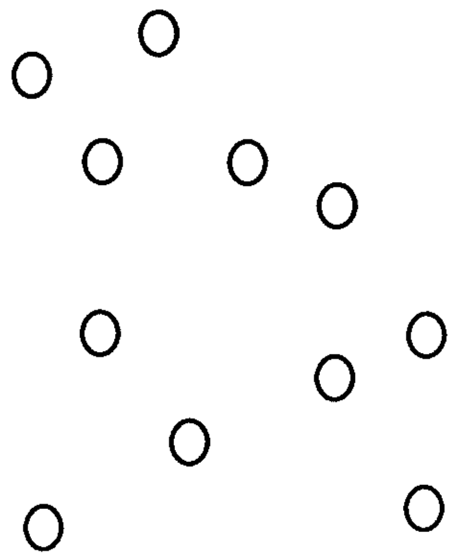
GETTING TIED UP — WHAT CAN HAPPEN?

Here are all the possible configurations that can result.



- A graph or network is a collection of points some of which are joined by lines.

- The points are called "vertices" (singular is "vertex") and are shown as small circles.

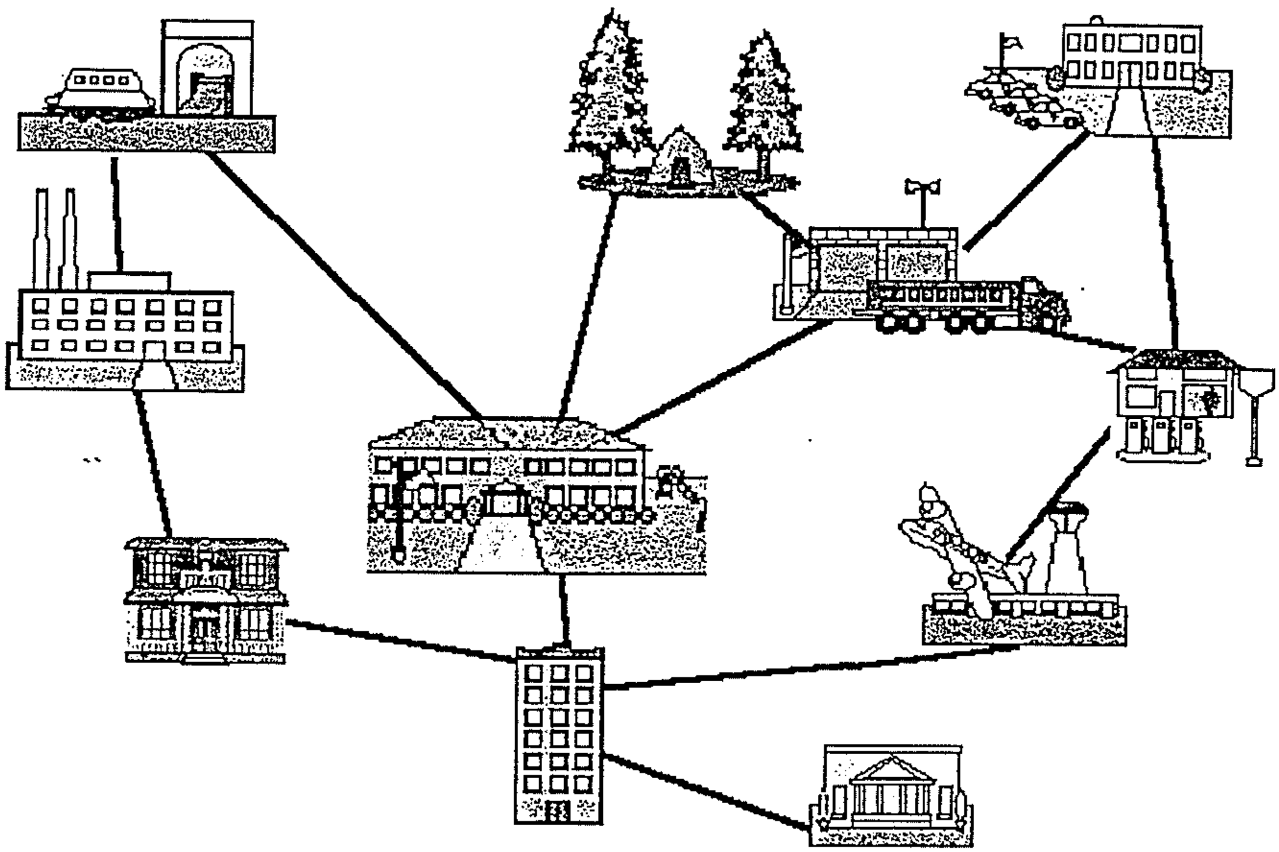


- The lines are called "edges" and are shown as lines which can be straight, curved, or wiggly.

- Each edge joins two different vertices. Two vertices may or may not be joined by an edge.

- Vertices joined by an edge are called "adjacent" or "neighbors".

THE ROBBER'S ON THE LOOSE



It's a snowy evening in a small town. The bank has just been robbed by an "America's Most Wanted" who left a trail of footprints in the snow through various homes and businesses. (All of the footprints were made after the robbery, and there's only one set of footprints on each road.) There's a huge reward for his capture. From this map can you provide the tip that will lead to locating and arresting the criminal?

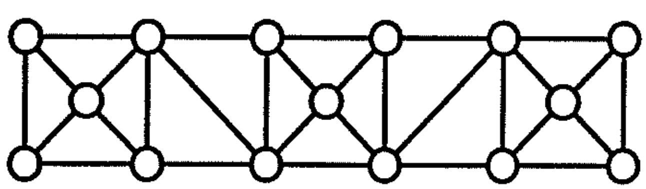
- A "path" is a sequence of edges in a graph, each of which begins where the previous one ends.

- A path which covers each edge exactly once is called an "Euler path". This is named in honor of a mathematician whose name was Leonhard Euler (pronounced "Oiler").

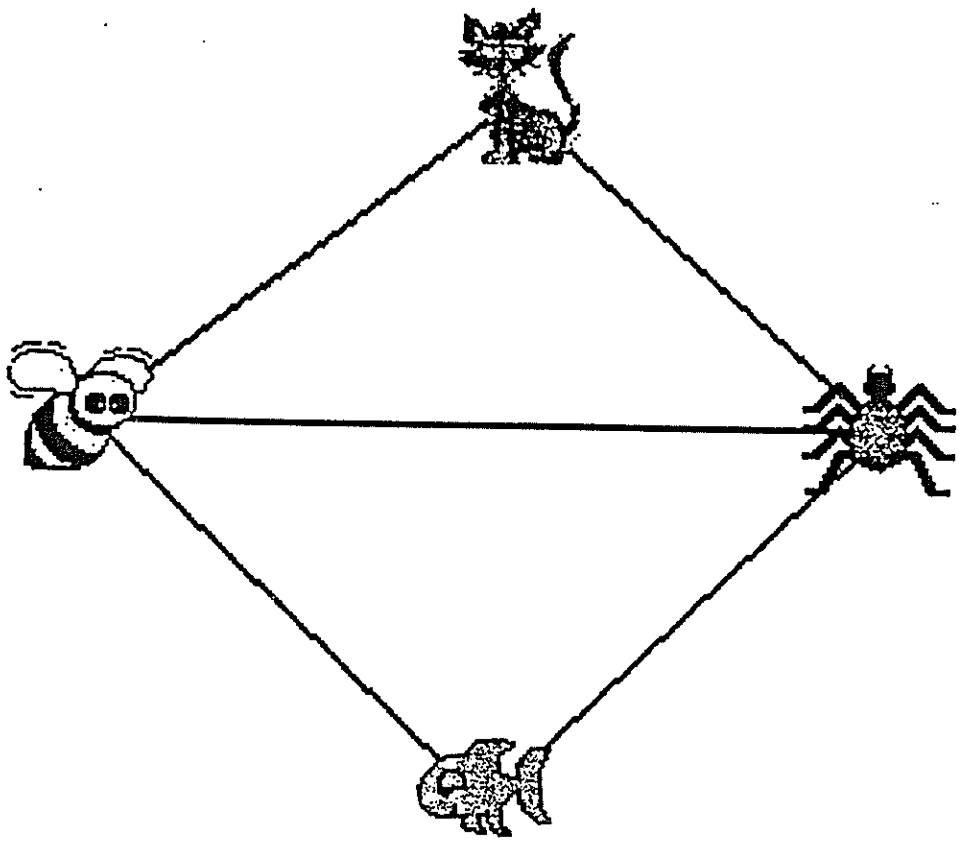
- The "degree" of a vertex is the number of edges which have an end at that vertex.

- A vertex has "odd degree" if an odd number of edges have an end at the vertex. A vertex has "even degree" if an even number of edges have an end at the vertex.

- Which vertices in the graph below have odd degree, and which have even degree?

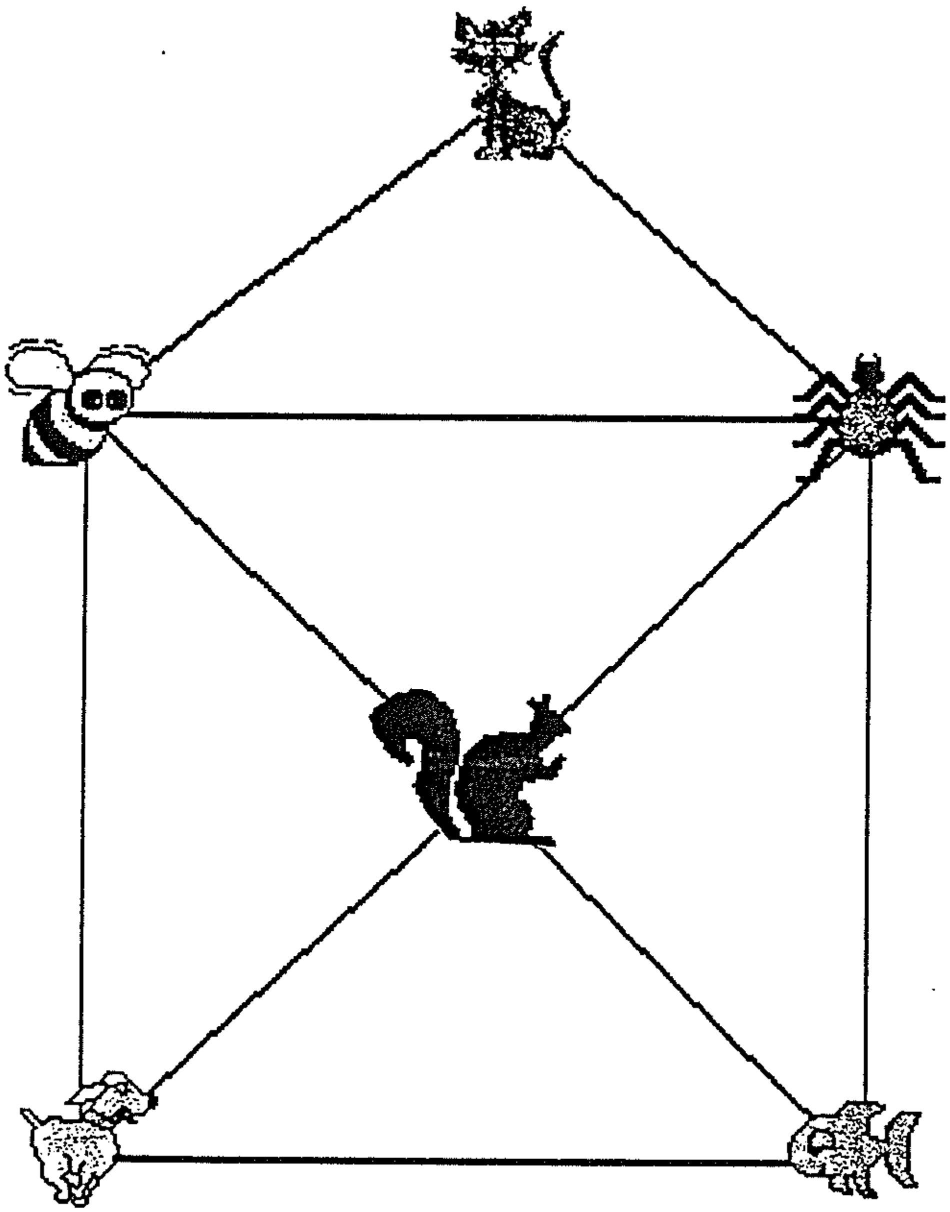


TRACING PICTURES



Without lifting your pencil or tracing any line twice, can you draw each picture? For each picture, circle the animal where you choose to start.

TRACING PICTURES



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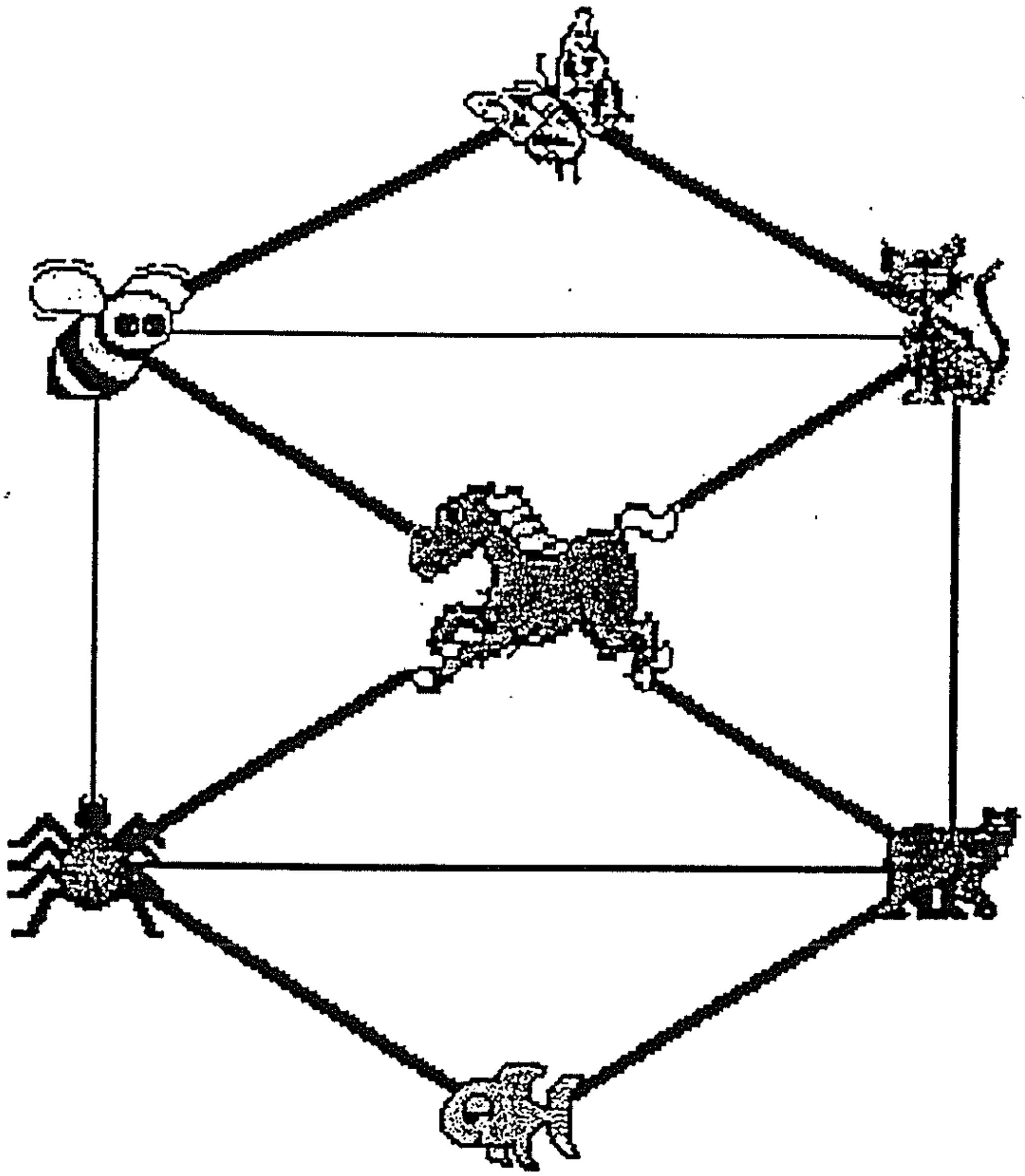

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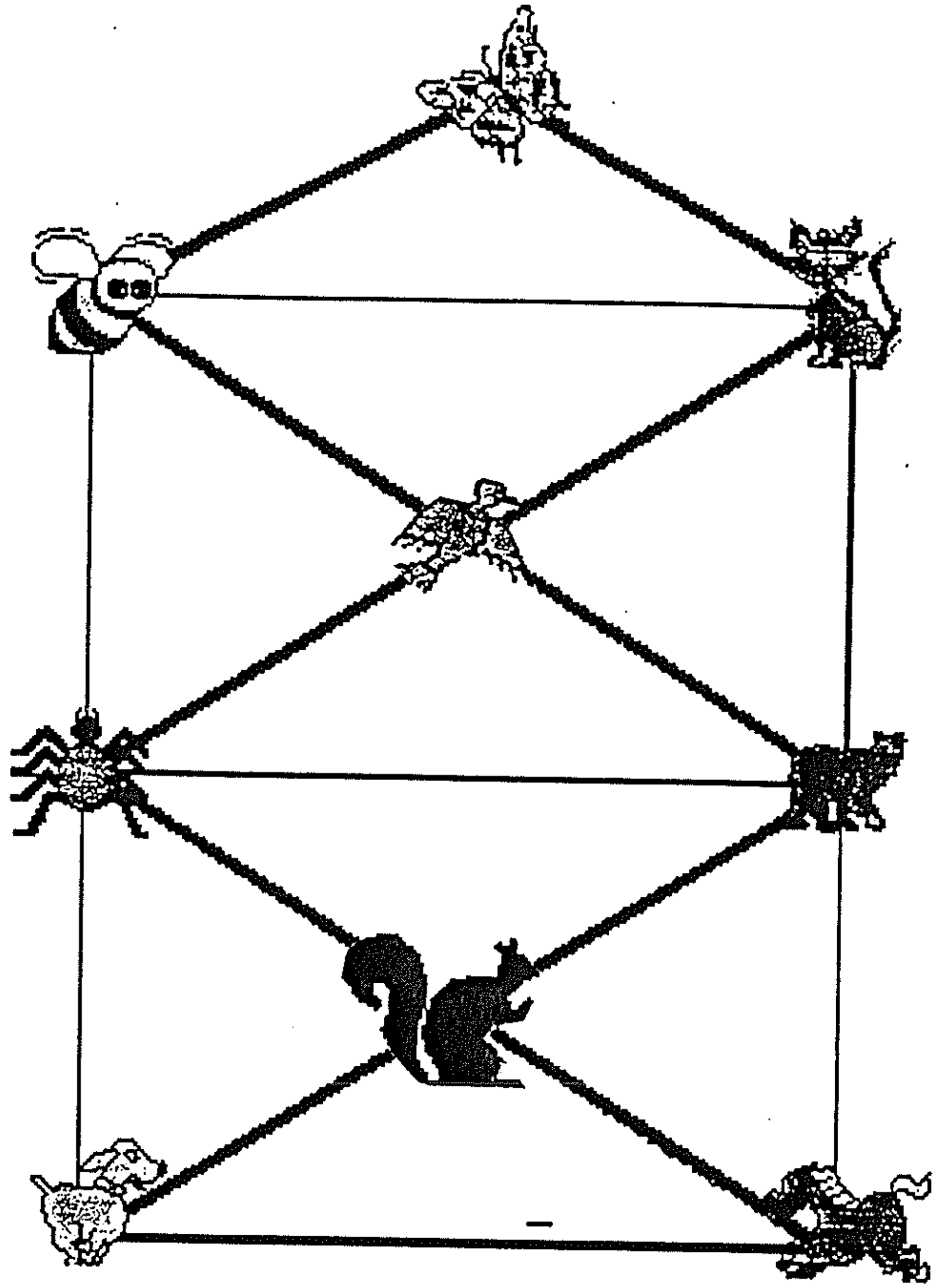

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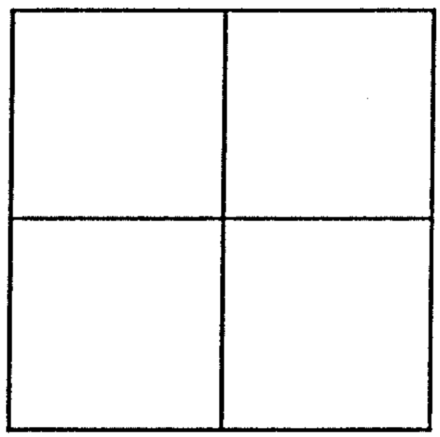
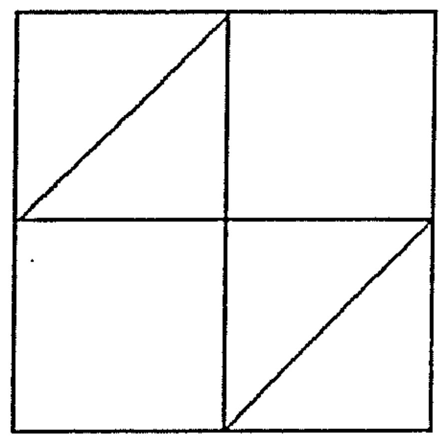
TRACING PICTURES

Which of the following pictures can you trace without lifting your pencil from the paper and without retracing any part of the picture?

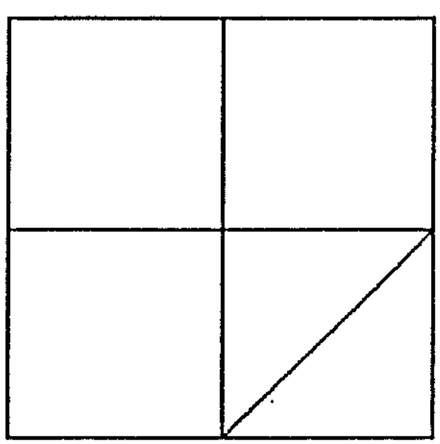
a.



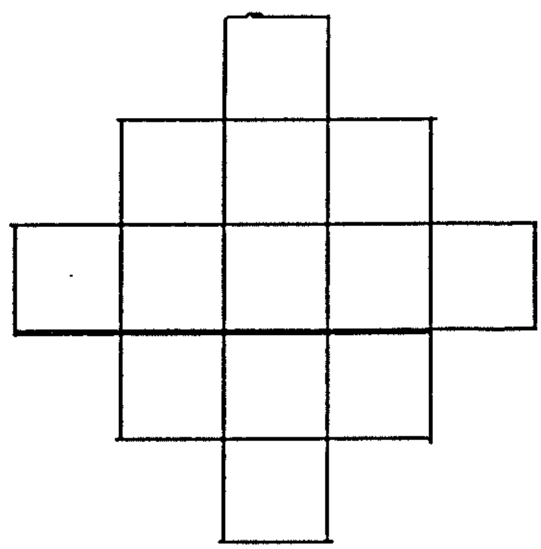
b.



c.



d.

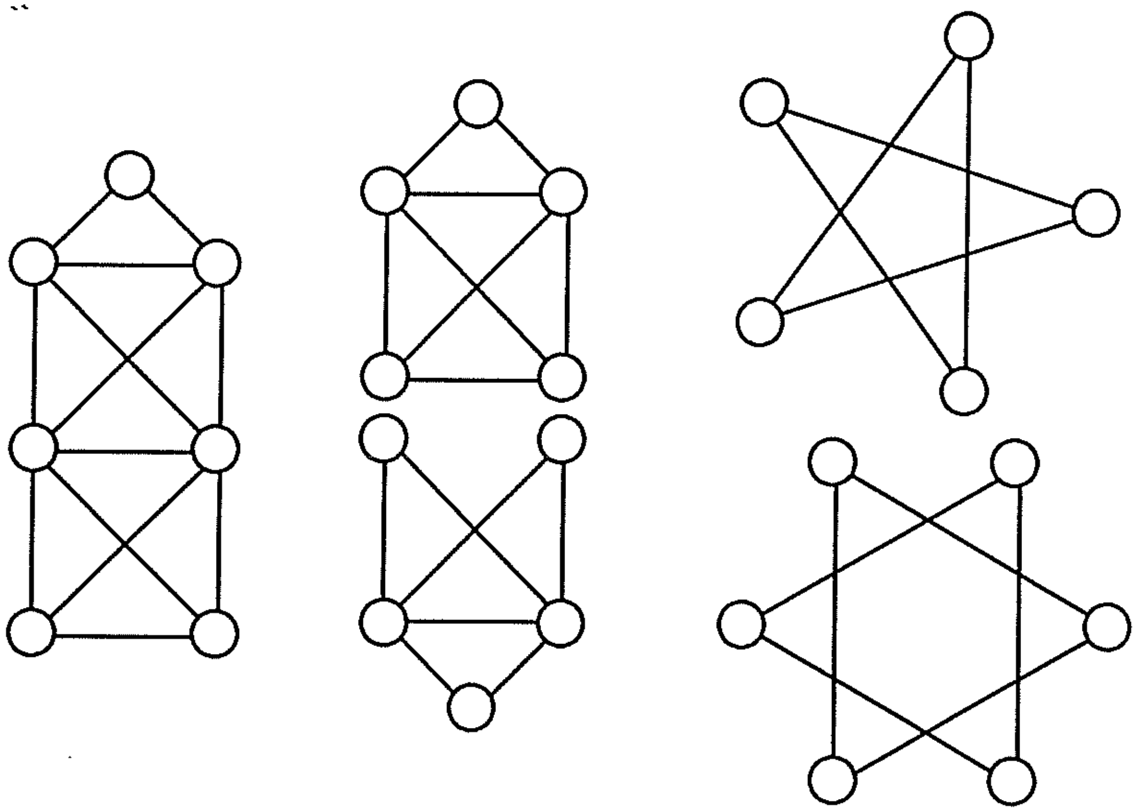


TRACING PICTURES

Look for a Pattern				
Sketch of Graph	Number of Vertices	Number of Even Vertices	Number of Odd Vertices	Circuit or Path or Impossible

Connected A graph is connected if it is possible to get from any vertex to any other vertex along a path.

Disconnected A graph is disconnected if you can find two vertices with no path joining them.



Requirements for an Euler path or circuit:

In order for a graph to have an Euler path or circuit, it must be connected and it must have :

✓ either two vertices of odd degree

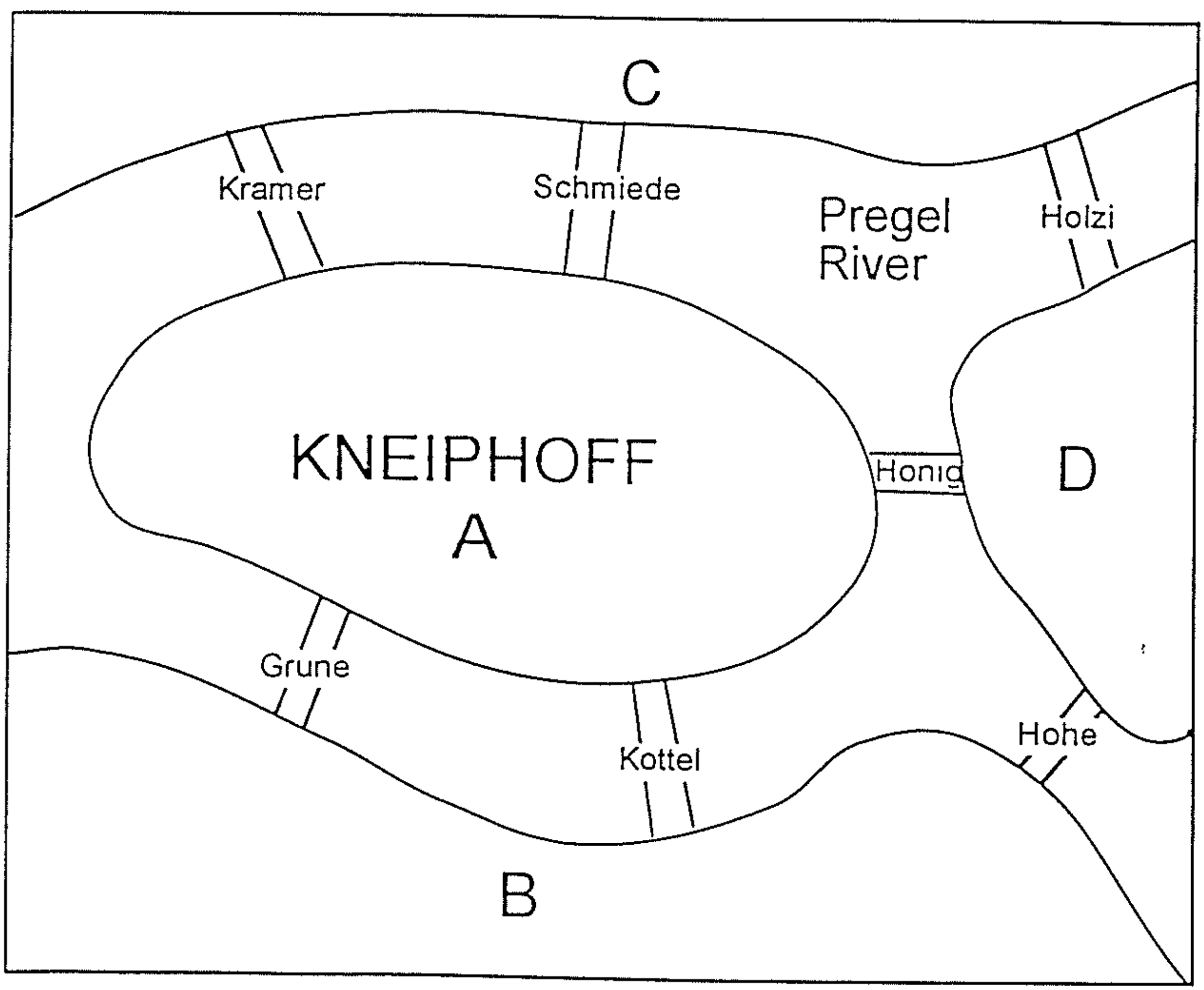
in which case there is an Euler path which starts at one vertex of odd degree and ends at the other

✓ or no vertices of odd degree

in which case there is an Euler circuit which starts at any vertex, and ends at the same vertex

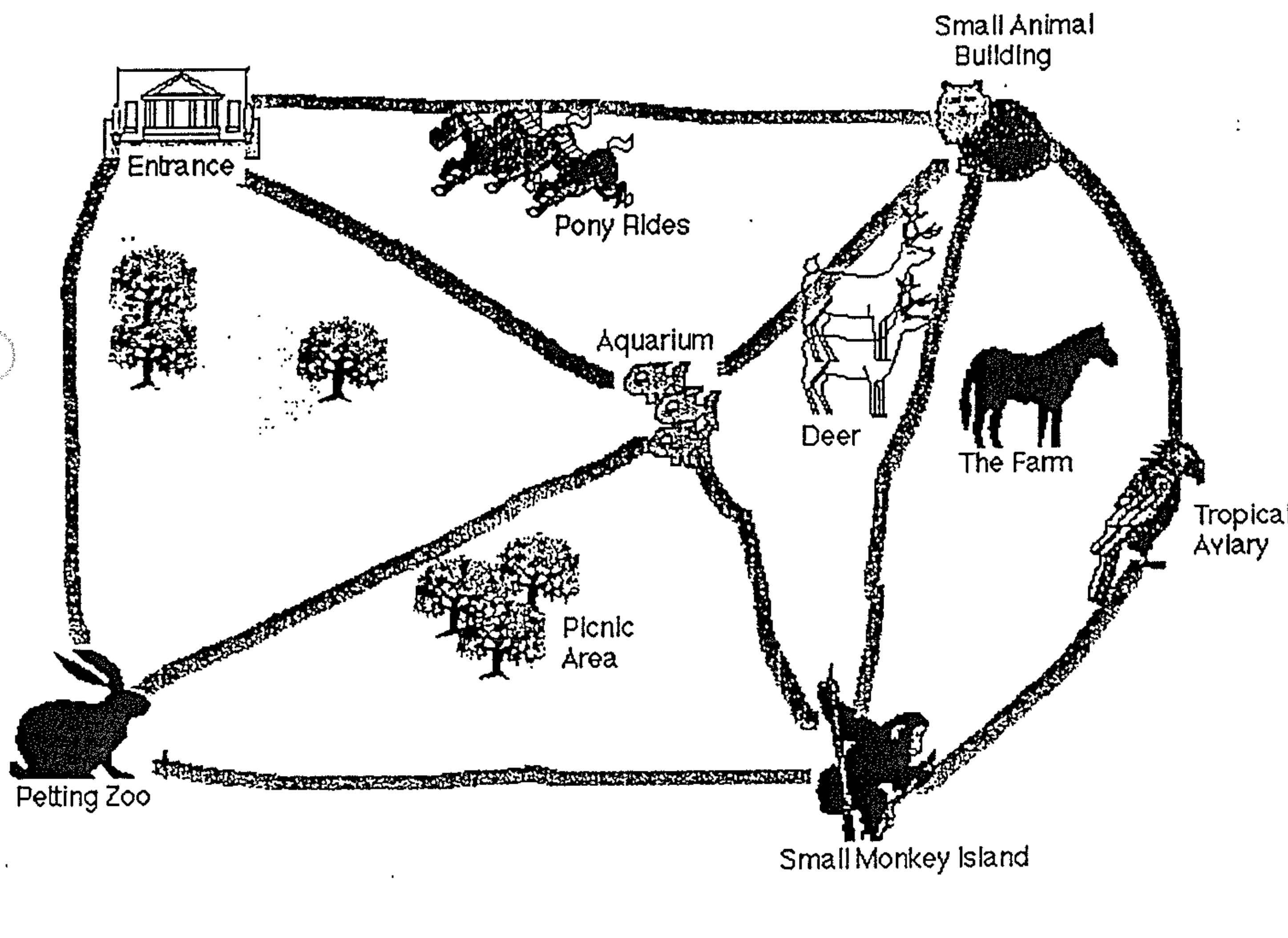
The Seven Bridges of Konigsberg
... or ... Why is Euler anyway?

The city of Konigsberg in East Prussia (Kaliningrad on USSR maps) is located on the banks of the Pregel River, and includes two islands. The four parts of the city were connected in the eighteenth century by seven bridges. On Sundays, the burghers would take a promenade around the town. A question often discussed was whether it was possible to plan the promenade so that one could walk across each bridge exactly once. Euler showed that it was impossible. Can you? (He also explained the circumstances when it would be possible, so this theorem is named in his honor.)



THE EULER FAMILY ZOO

By the time the Euler Family Zoo closes each day, there is a great deal of litter that needs to be cleaned from every trail. As the person in charge of this clean-up, you want to know whether it is possible for you to start at the entrance, clean each trail without retracing any, and finish at the entrance.



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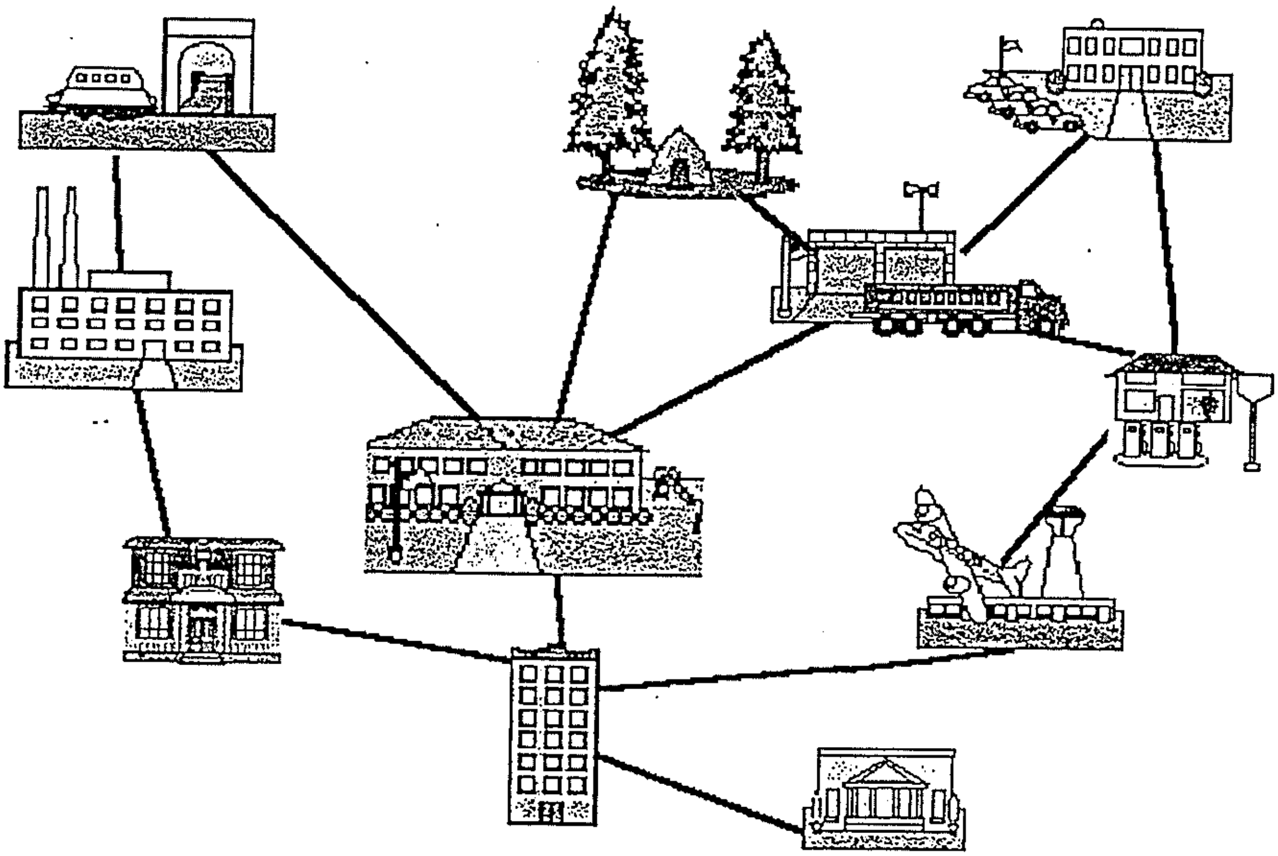

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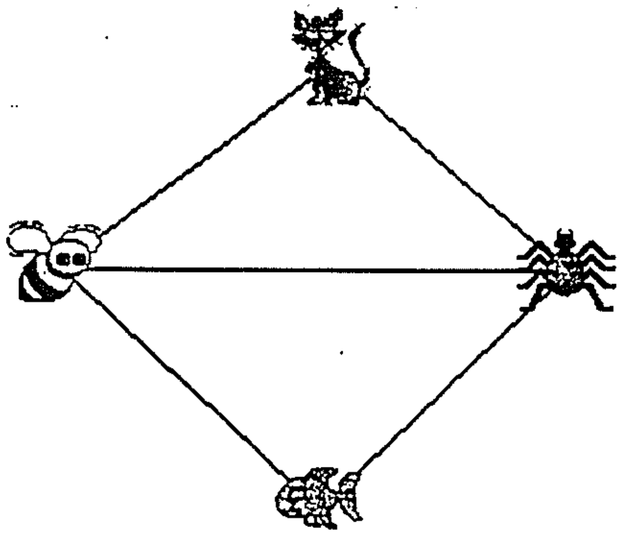

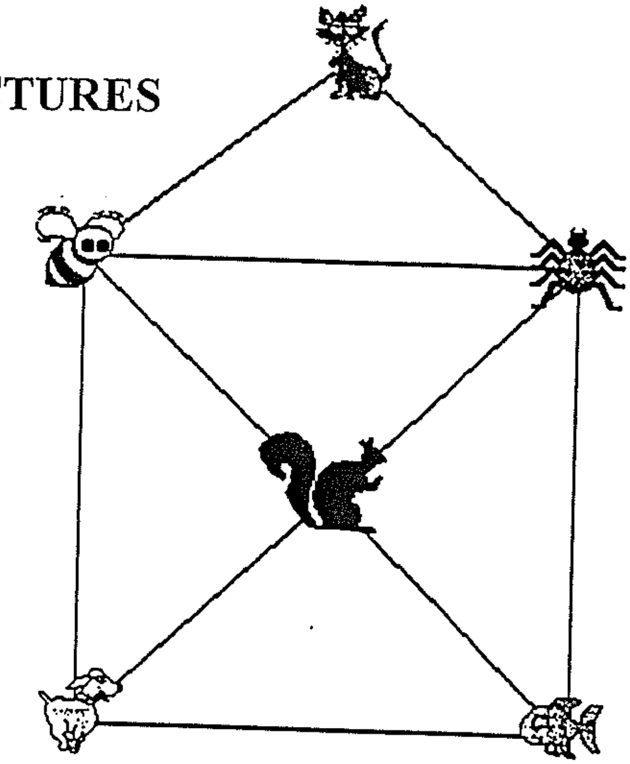

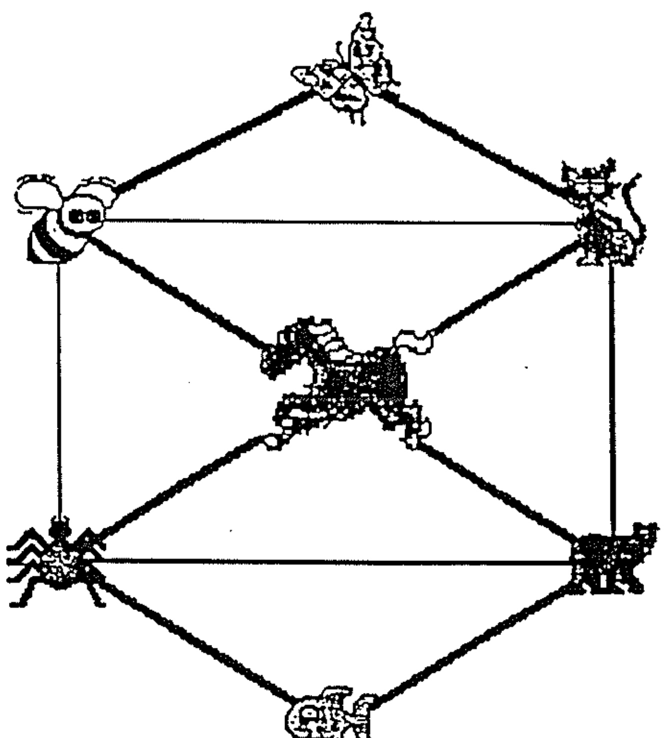

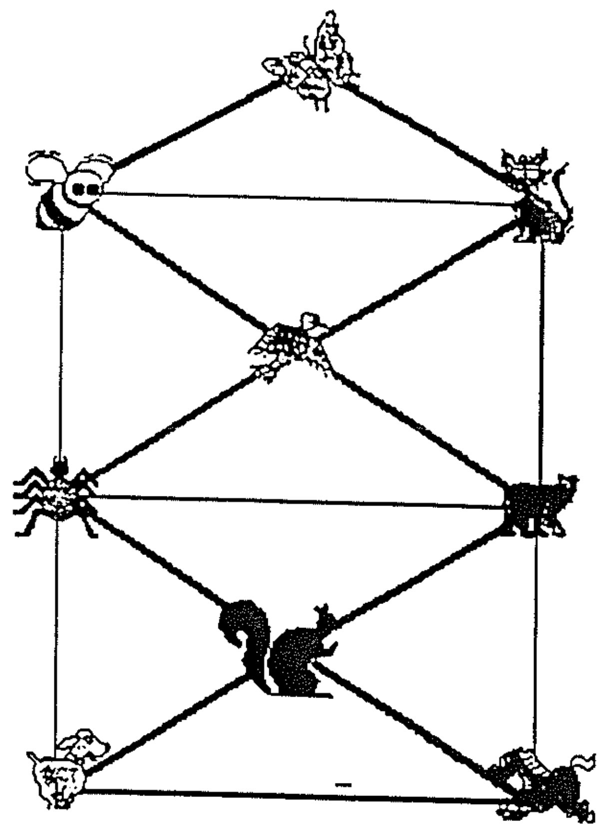

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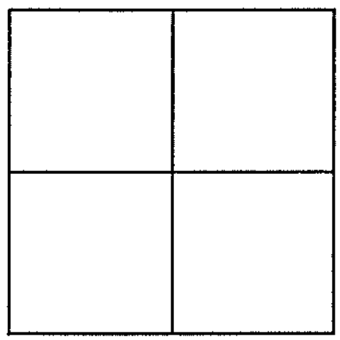
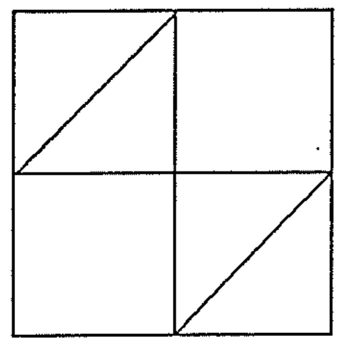


b.



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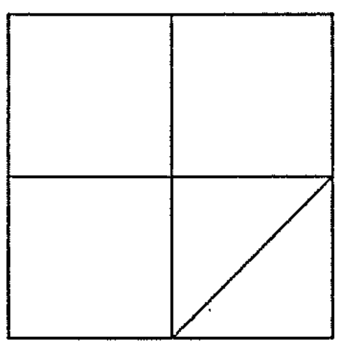

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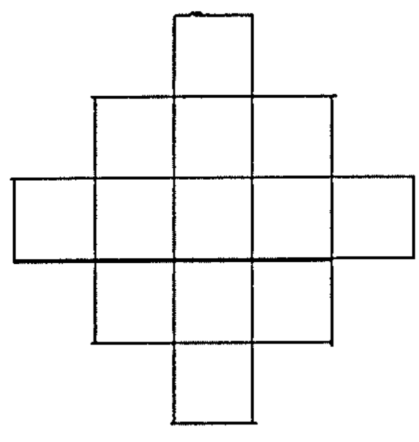

TRACING PICTURES

Look for a Pattern				
Sketch of Graph	Number of Vertices	Number of Even Vertices	Number of Odd Vertices	Circuit or Path or Impossible

What patterns did you find?

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The Seven Bridges of Konigsberg
... or ... Why is Euler anyway?

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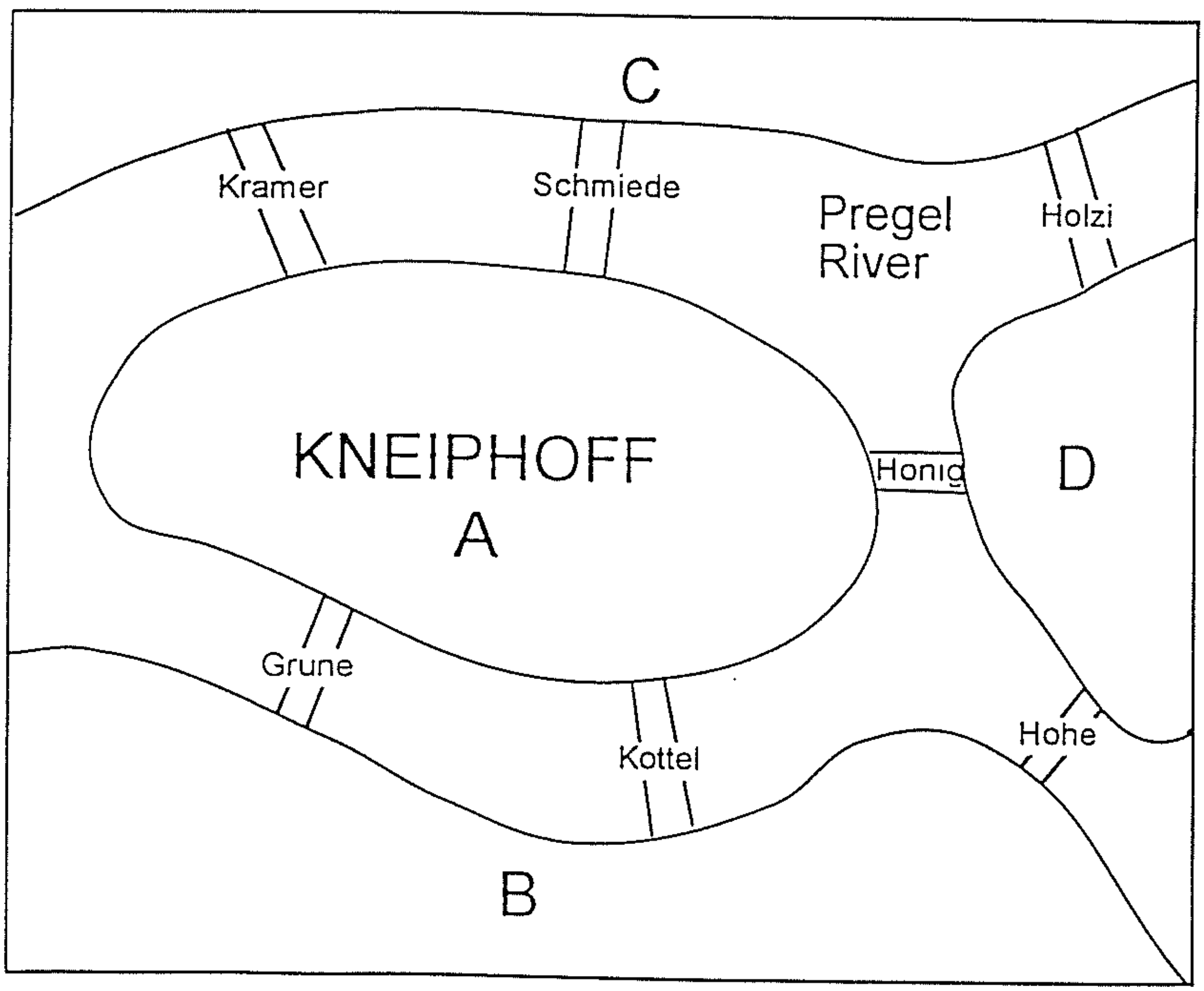

THE EULER FAMILY ZOO

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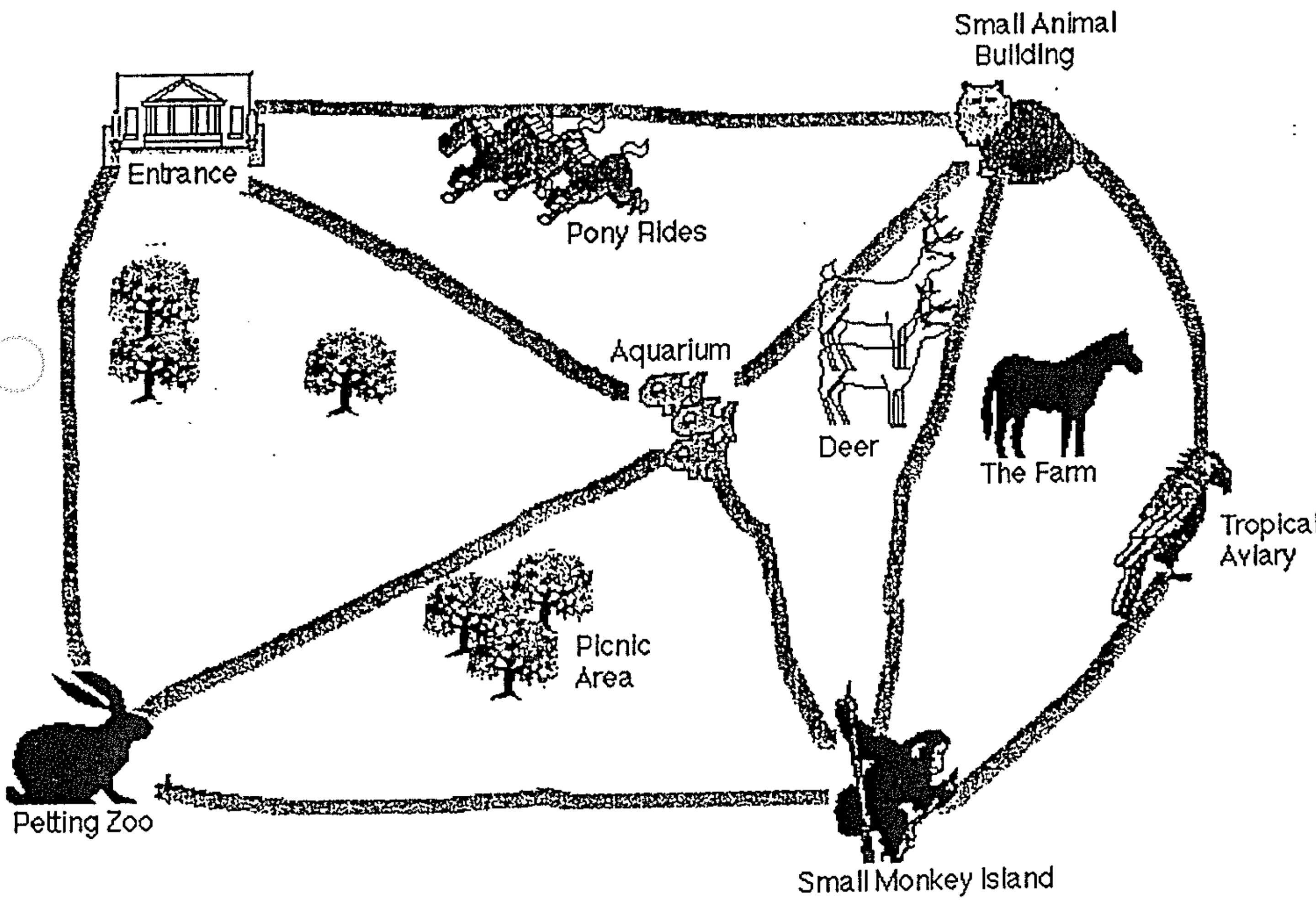
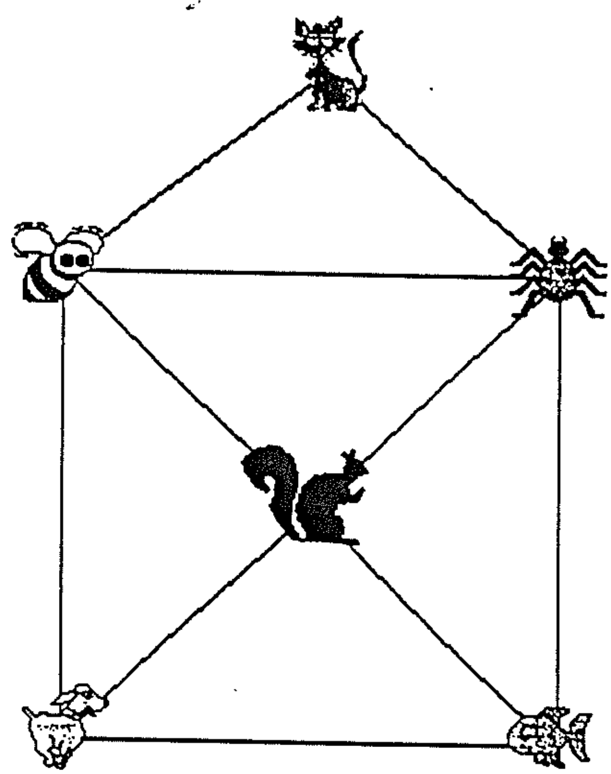


Exploring Discrete Mathematics in the Classroom

Getting It Together: Paths & Graphs K-4

A Workshop for Teachers About Paths & Graphs

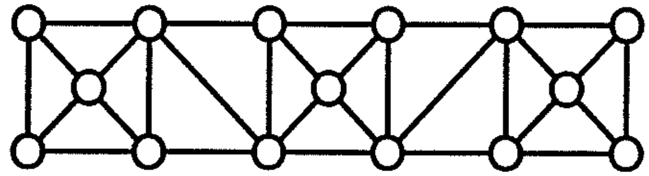


Developed by Joseph G. Rosenstein and Valerie A. Debellis

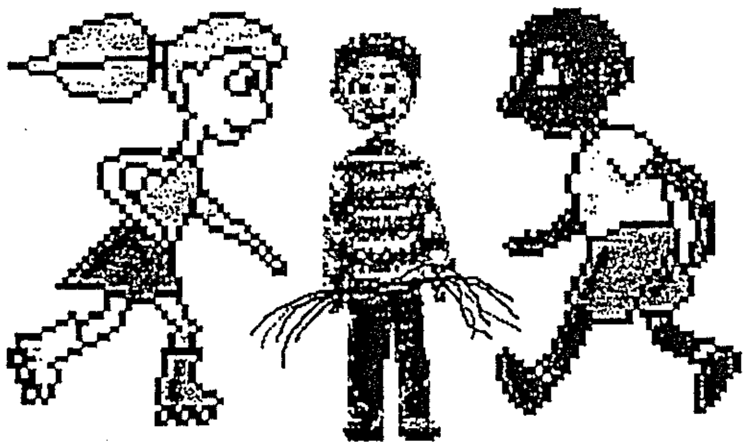
in collaboration with the following participants in the Rutgers Leadership Program in Discrete Mathematics: Lynda Chatzel, Ann Lawrence, Darlene Mixer, and Susan Weiss

Terminology

- A "graph" or "network" is a collection of points some of which are joined by lines.
- The points are called "vertices" (singular is "vertex").
- The lines are called "edges" and are shown as lines which can be straight or curved.
- Each edge joins two different vertices. Two vertices may or may not be joined by an edge.
- Vertices joined by an edge are called "adjacent" or "neighbors".
- A "path" is a sequence of edges in a graph, each of which begins where the previous one ends.
- A path which covers each edge exactly once is called an "Euler path" — named in honor of a mathematician whose name was Leonhard Euler (pronounced "Oiler").
- The "degree" of a vertex is the number of edges which have an end at that vertex.
- A vertex has "odd degree" if an odd number of edges have an end at the vertex. A vertex has "even degree" if an even number of edges have an end at the vertex. (Which vertices in the graph below have odd degree, and which have even degree?)
- A graph is "connected" if it is possible to get from any vertex to any other vertex along a path. A graph is "disconnected" if you can find two vertices with no path joining them.
- Requirements for an Euler path or circuit: In order for a graph to have an Euler path or circuit, it must be connected and it must have
    - either two vertices of odd degree, in which case there is an Euler path which starts at one vertex of odd degree and ends at the other
    - or no vertices of odd degree, in which case there is an Euler circuit which starts at any vertex, and ends at the same vertex



GETTING TIED UP



Materials Needed for Each Group:

Five or six pieces of yarn, each one foot long

For this activity, participants work in groups of three.

(1) The shortest person will be in the middle; that person should grasp the bundle of yarn in the middle, leaving both ends of each piece of yarn exposed.

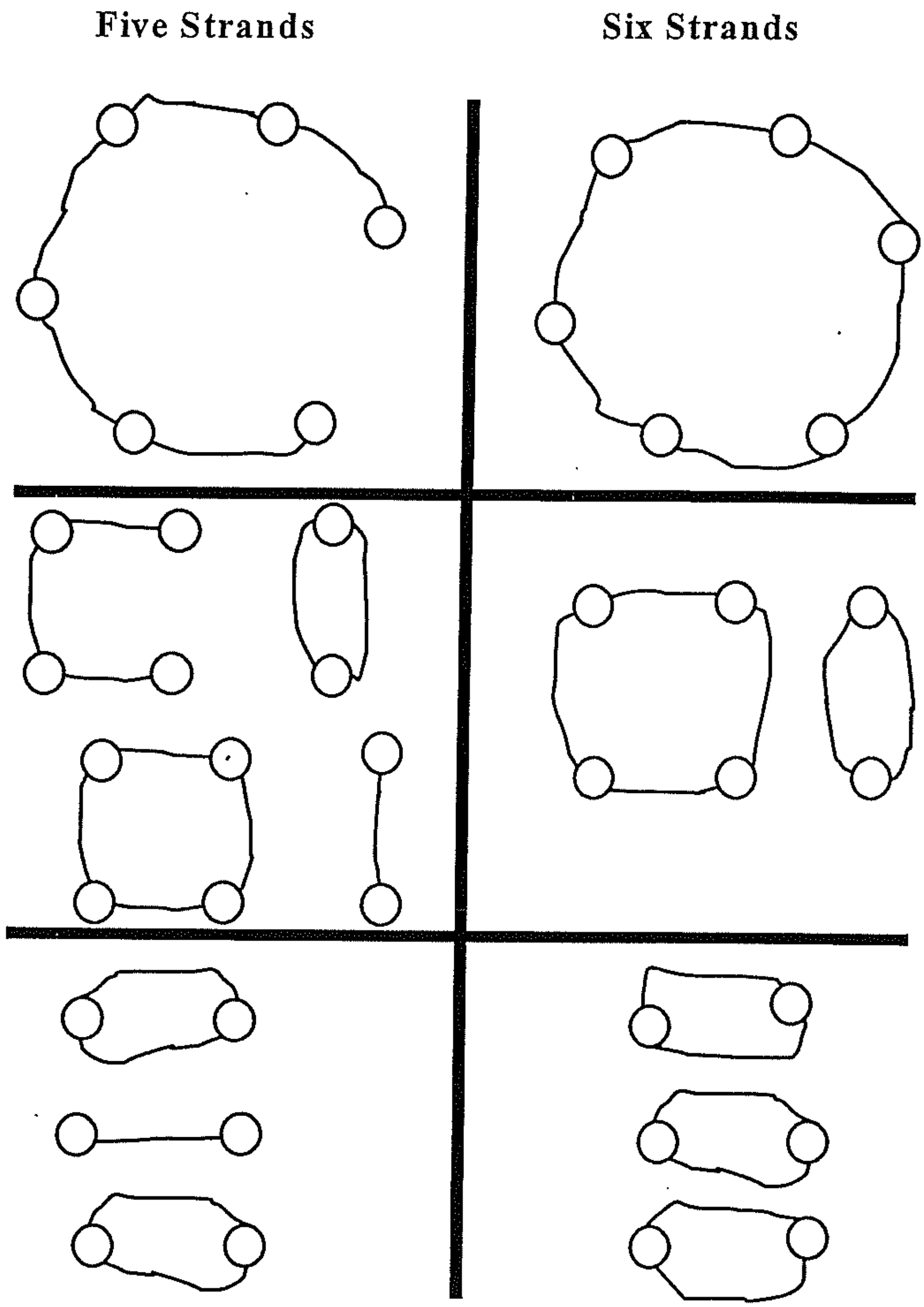
(2) The person on the right end should take any two ends on his/her side of the middle person's hand and tie these ends together. He/she should continue to choose two ends and tie them together until all possible pairs have been tied.

(3) The person on the left end should follow the same procedure.

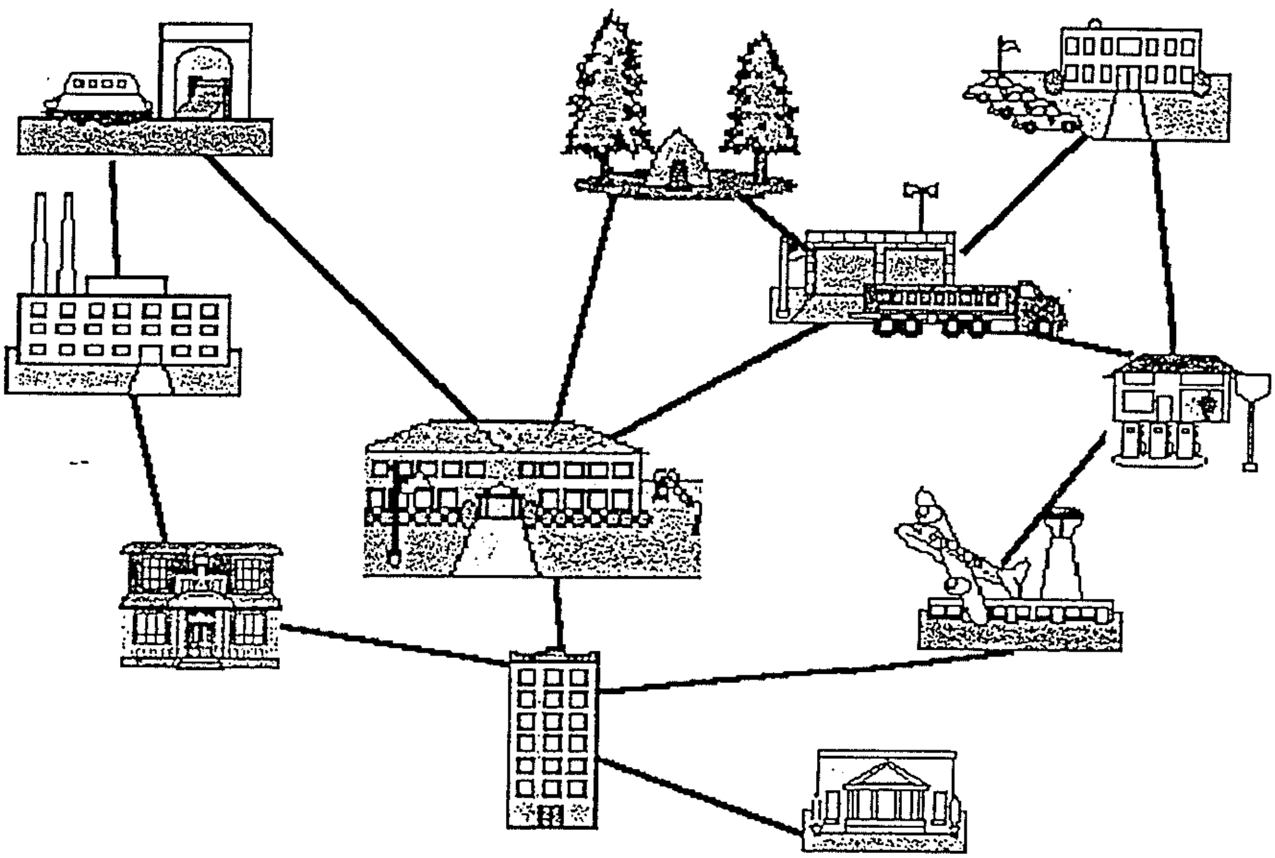
(4) The person in the middle continues to hold the yarn in the middle until the others are finished.

GETTING TIED UP — WHAT CAN HAPPEN?

Here are all the possible configurations that can result.

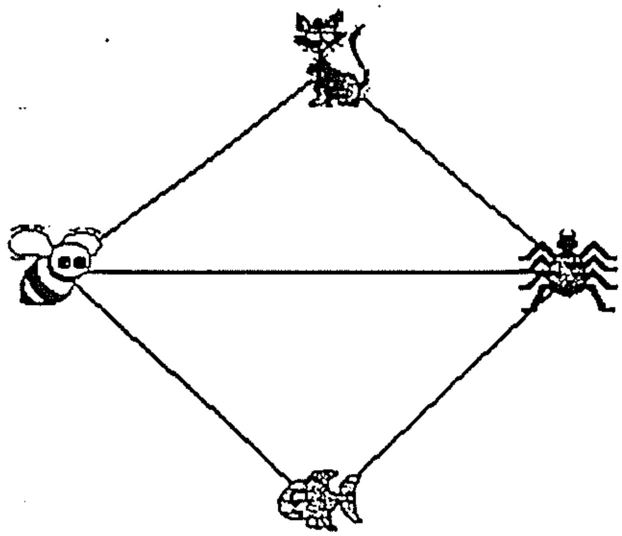


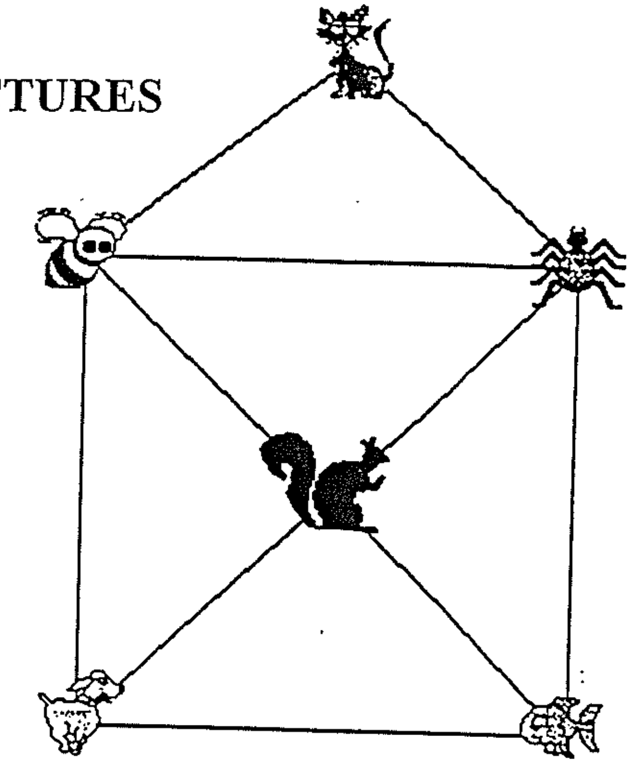
THE ROBBER'S ON THE LOOSE

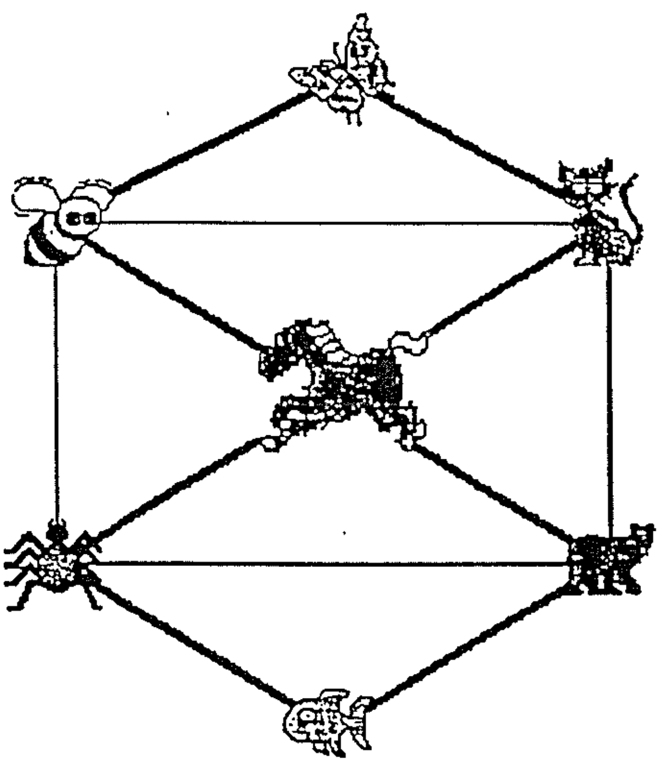


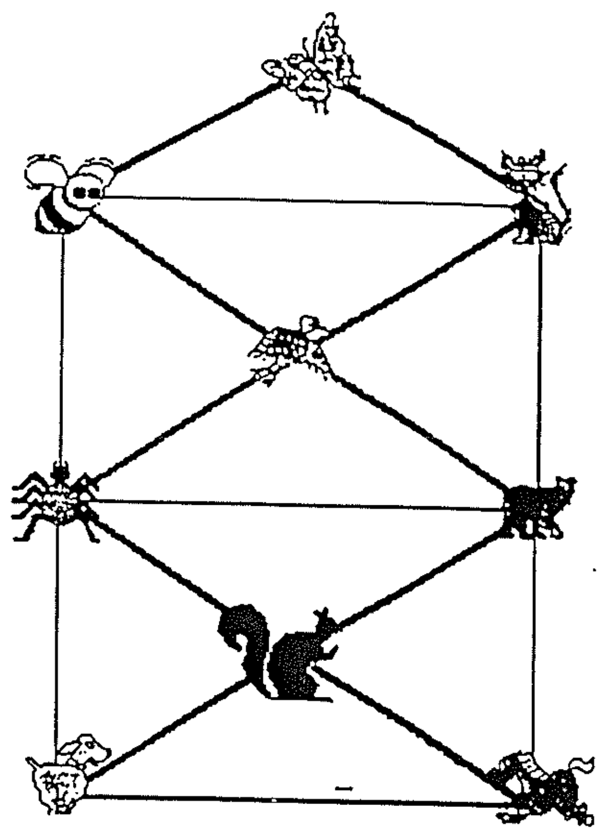
It's a snowy evening in a small town. The bank has just been robbed by an "America's Most Wanted" who left a trail of footprints in the snow through various homes and businesses. (All of the footprints were made after the robbery, and there's only one set of footprints on each road.) There's a huge reward for his capture. From this map can you provide the tip that will lead to locating and arresting the criminal?

TRACING PICTURES





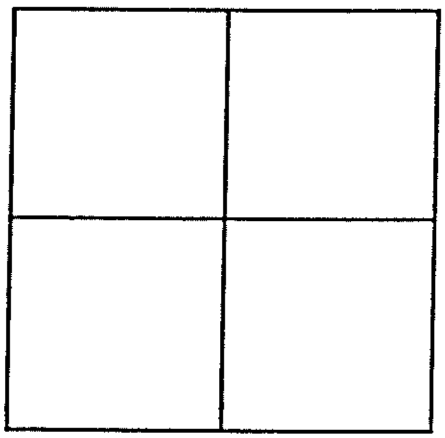




Without lifting your pencil or tracing any line twice, can you draw each picture? For each picture, circle the animal where you choose to start.

TRACING PICTURES

Which of the following pictures can you trace without lifting your pencil from the paper and without retracing any part of the picture?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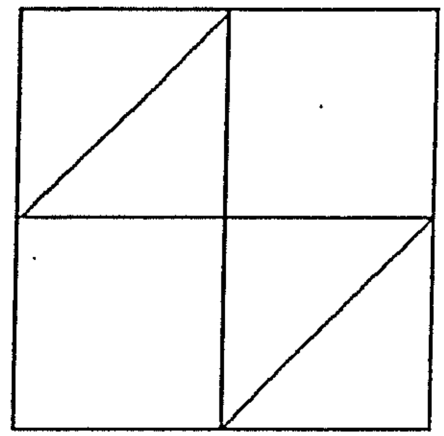
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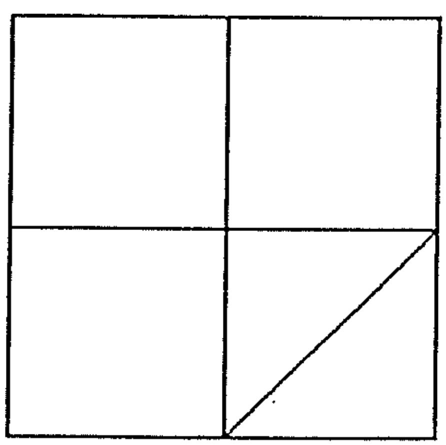
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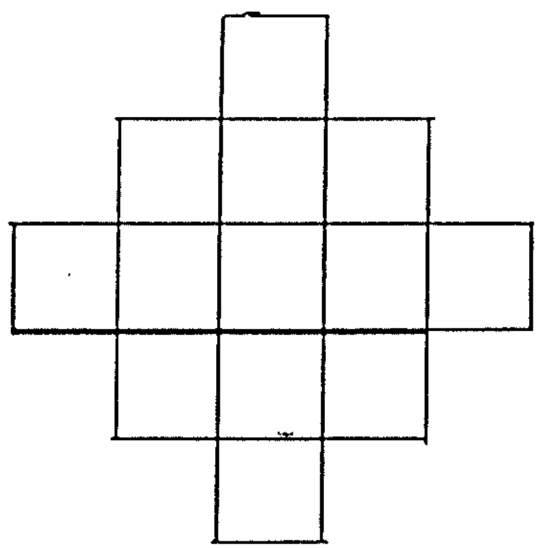
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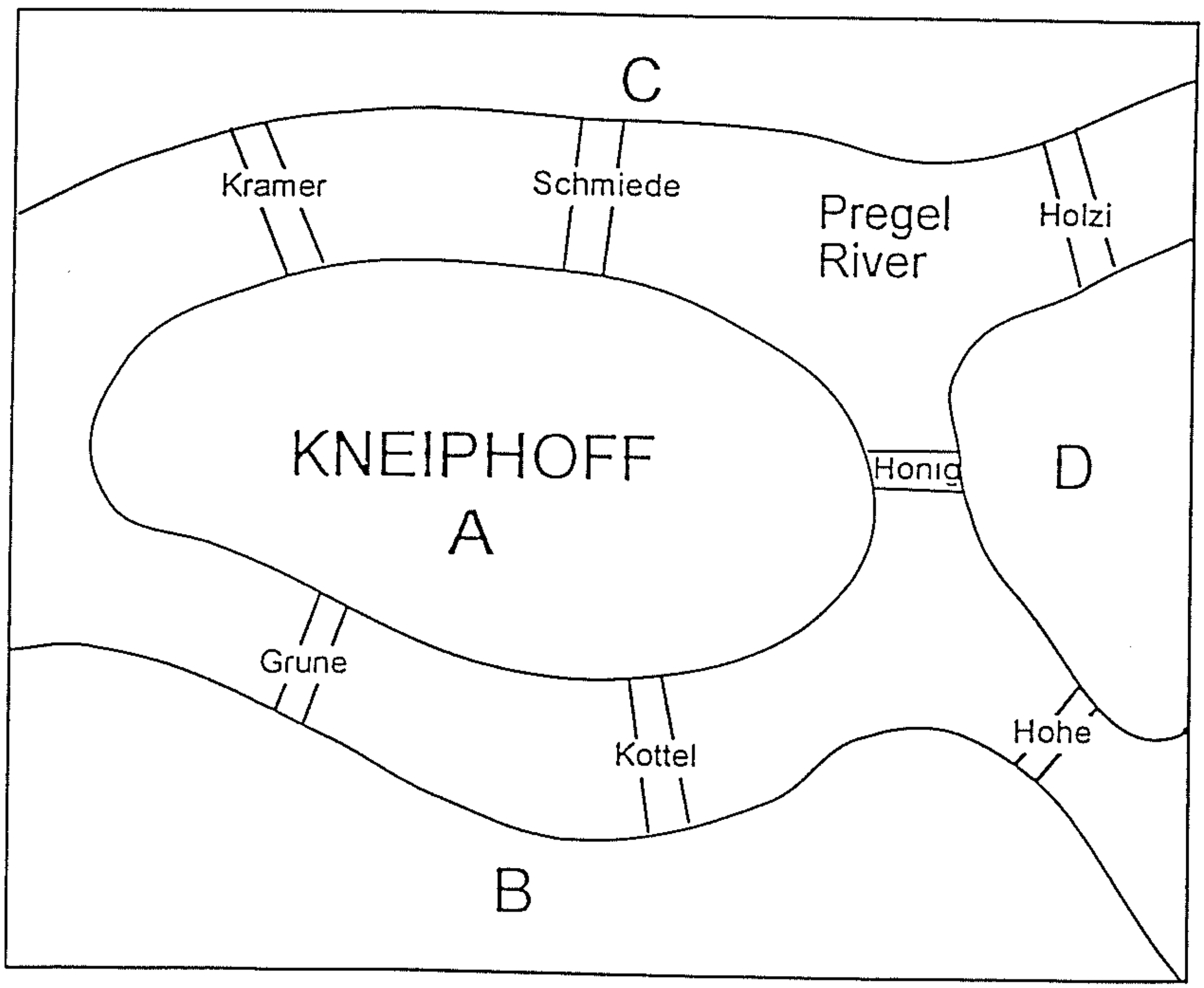

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