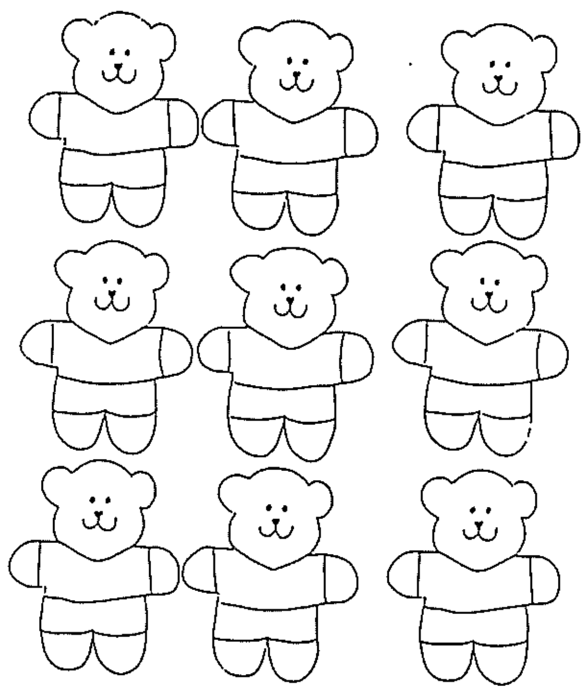
Exploring Discrete Mathematics in the Classroom

Counting and Probability K-4

A Workshop for Teachers About Systematic Listing, Counting, and Probability



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Counting and Probability K-4

Workshop Overview

Summary

Participants will engage in several motivating activities involving systematic counting and listing as well as topics in probability. Most of the activities explore these mathematical concepts through situations in children's literature which give rise to the problems posed.

Outline

I. Workshop Objectives

Participants will:

* solve problems that focus on systematic counting and listing
* be introduced to simple probability problems
* be exposed to several methods for teaching probability to young children, and
* be provided with materials to implement the activities in their classrooms.

II.	Activity #1 — Bobbie Bear I	20 Minutes
III.	Activity #2 — Bobbie Bear II	20 Minutes
IV.	Activity #3 — Rub-a-dub-dub I	20 Minutes
V.	Activity #4 — Rub-a-dub-dub II	20 Minutes
VI.	Extension Activities (Optional)	
	A. Buddy, Can You Spare a Quarter	(15 minutes)
	B. A Three Hat Day	(15 Minutes)
	C. Caps For Sale	(15 Minutes)
VII.	Workshop Summary	05 Minutes
VIII.	Workshop Evaluation	<u>05 Minutes</u>
	Total Workshop Time:	90 Minutes

Workshop Environment

I. Room Arrangement

- Ideally, participants should be seated at round tables in groups of four to six persons; round tables help facilitate collaboration and discussion smong group members. Each participant should have a reasonable view of the presenter and the screen.

II. Equipment

Overhead projector and screen
Flannel board or magnetic board

III. Workshop Materials List

A. Activity #1 — Bobbie Bear I
Magnetic or flannel board
1 large Bobbie Bear (for use on magnetic or flannel board)
4 shadow images of Bobbie Bear (for use on magnetic or flannel board)
Bobbie Bear suitcase (cardboard storage for Bobbie Bear materials)
2 red, 2 blue, and 2 yellow Bear shirts and 2 red, 2 blue, and 2 yellow Bear shorts
Blue, red, and yellow crayons/markers for each participant

B. Activity #2 — Bobbie Bear II
Same as above
Spinner with three colored sections (red, blue, yellow)

C. Activity #3 — Rub-a-dub-dub I
Six cut-out copies of each of three tub characters for each group
Three tub characters cut out from transparencies
Marking Pen and blank transparencies

D. Activity #4 — Rub-a-dub-dub II
Same as above
Extra tub character cut out from transparency

E. Extension Activity I — Buddy, Can You Spare a Quarter
Plastic coins or counters of four different colors
Blank transparencies and transparency markers

F. Extension Activity II — A Three Hat Day
Scissors, blank transparencies, and transparency markers

G. Extension Activity III — Caps for Sale
Crayons, blank transparencies, and transparency markers

Instructor's Notes

Introduction

Show the "Title Transparency" for this workshop and indicate that this workshop was developed by K-8 teachers who participated in the Rutgers University Leadership Program in Discrete Mathematics. Introduce yourself and mention that you spent three weeks over two summers learning discrete mathematics at the Leadership Program, and have been using materials from the program in your classroom.

Show the "Discrete Mathematics Transparency" and review the information there — discussing five major themes of discrete mathematics. Indicate that the workshops offered in the "Workshops in Your District" program reflect four of these major themes. The theme that we will be focusing on in this workshop is that of "Systematic Counting and Listing"; the title of this workshop, as noted on the "Title Transparency" is "Counting and Probability".

Although you may have been given this information beforehand, find out the grade levels of the participants in your workshop (say, for K, 1-2, 3-4, 5-6, and 7-8) and, if the distribution is not what you expected, be sure to modify your workshop appropriately. Also, please tell the participants that all the information and all the activities in this workshop will be included in a packet of materials that they will receive after the program.

Activity #1 — Bobbie Bear I: Counting Bobbie Bear's Outfits
(Allocated time = 20 minutes)

Introduce Bobbie Bear to the participants by placing a large cardboard bear (the pattern for this bear can be found in the TKHM materials) on a flannel or magnetic board and reading the following problem as you show TSP #1 on the overhead projector. "Bobbie Bear is going on vacation. She must pack her own clothes. Bobbie Bear has three shirts — which are red, blue, and yellow — and two pairs of shorts — which are red and blue. She wants to wear a different outfit each day. For how many days can Bobbie Bear wear a different outfit each day?" Ask participants to work together to color the bears on HO #1 (= TSP #1) in as many different ways as possible. Allow groups to color and discuss for a few minutes, then place the shadow images of Bobbie Bear (the pattern for clothes for these shadow images can also be found in the TKHM materials) on the flannel or magnetic board and have volunteer participants come forward to show the different outfits until all six (red shirt with red shorts, red shirt with blue shorts, blue shirt with red shorts, blue shirt with blue shorts, yellow shirt with red shorts, and yellow shirt with blue shorts) are displayed.

Be sure that there are enough red, blue, and yellow crayons or markers at each table before the workshop starts so that each participant can color his/her outfits on HO #1. (Remind *participants that if they use cut-out clothes with their students, then swapping one item for a similar item of the same color — e.g., a red shirt for a red shirt — does not count as a new outfit.)*

Ask participants how they know that they haven't left out any of the possibilities. You want to elicit from them that by listing the possibilities systematically, they have guaranteed that all have been included. One possible way of listing systematically is — start with red shirt, which can be accompanied by red pants or blue pants, then go to blue shirt, with which you can have red pants or blue pants, then go to yellow shirt, which can be accompanied by red pants or blue pants. (Emphasize that there are usually many ways of generating a systematic list, and encourage participants to share other methods.)

One way of showing this systematic listing is by organizing the six possibilities in a rectangular format, as on TSP #2, with three rows and two columns (or the reverse), with each row corresponding to a shirt color and each column corresponding to a shorts color. Be sure that participants understand that the total number of possibilities is 3x2 because each of the three shirt colors can be accompanied by shorts of two different colors. Participants should now be able to explain how many outfits there would be altogether if Bobbie Bear had 7 shirts and 5 shorts, or if these were any other numbers. Point out that this is called the multiplication law of counting — if you can do one thing in 7 ways and a second thing in 5 ways, then there are a total of 7x5 or 35 ways altogether of doing both things. The seven shirts can form the seven rows of a rectangle, and the five shirts can form its five columns, just as in TSP #2. Review all of this using TSP #3. Another way of demonstrating that there are six possibilities by systematic listing involves using a "tree diagram"; this is illustrated on TSP #4.

Option: At the close of an activity (either here or later in the workshop) you may want to emphasize the importance of communication, especially writing, in the mathematics curriculum. Ask the participants to write what they think students might learn from the activity, and ask a few participants to share what they wrote.

Activity #2 — Bobbie Bear II: Bobbie Bear's Odds
(Allocated time = 20 minutes)

Now we look at some questions in probability. Bobbie Bear has two pairs pf shorts in the drawer — one red and one blue. Simulate the following probability questions by pulling shirts and/or shorts out of Bobbie Bear's suitcase (a shoe box designed to look like a dresser drawer). Before you begin, show everyone that inside the "suitcase" are three shirts (one red, one blue, and one yellow) and two shorts (one red, one blue).

(1) If Bobbie takes out a pair of shorts, what is the probability that they will be red? The answer is "one out of two" — which, with children who know about fractions, can be translated into "the probability is one-half". Ask: In what other situation do we have a

probability of one-half (or one out of two). The answer should be "if we toss a coin" — and you should note that instead of picking a pair of shorts out of the drawer without looking, Bobbie could have tossed a coin, and wore red shorts if the coin landed "heads" and blue shorts if the coin landed "tails".

(2) If Bobbie takes out a shirt, what is the probability that it will be red? The answer is "one out of three" — which, with children who know about fractions, can be translated into "the probability is one-third". Ask: In what other situation do we have a probability of one-third (or one out of three). One possible answer is "if we use a spinner" — and you should then show them a spinner which has three regions — marked blue shirt, red shirt, and yellow shirt. You should note that instead of picking a shirt out of the drawer without looking, Bobbie could have spun the spinner, and worn whatever color shirt it indicated. These conclusions are summarized on TSP #5.

Show the top of TSP #6 (keep the bottom covered for now) and ask: What is the probability that if Bobbie Bear picks a shirt and a pair of shorts that one of them will be red? Let them work in groups of 2-3 to discuss this question. Ask the groups what answers they obtained. You should expect to get two answers — 2/3 and 5/6. Ask one of the groups who got 5/6 to explain their answer. The response should be something like the following: the probability of getting a red shirt is 1/3 and the probability of getting a red pair of shorts is ½, so the total is the sum of those two numbers, 2/6 + 3/6, which is 5/6. This reasoning sounds very good but is incorrect. But don't tell them that yet!

Instead ask them to work on a different problem , the one that is on the top of TSP #7 (keep the bottom covered for now). This is the same as the previous problem, except now there are only two shirts — a red shirt and a blue shirt. Those groups who before got 5/6 will now get a probability of ½ + ½ = 1. Ask them what a probability of 1 means, and elicit that if an event has probability 1, it always happens. Ask: Does it always happen that, in this situation, Bobbie Bear ends up with red shirt or red shorts? Of course not, if in this situation Bobbie Bear happens to pick blue shirt and blue shorts, then neither would be red. So simple addition of ½ and ½ does not give the right answer. How do we get the right answer? We have to look at the four possible outfits. (This is on the bottom of TSP #7.) Of the four possible outfits, three have red shirt or red shorts, so the correct answer is "three out of four" or 3/4. (You might point out that in when you take ½ + ½ you are counting "red shorts and red shirt" twice.) One important assumption that we are making here is that the four possible outfits are all "equally likely" — that is, any one is as likely to be picked as any other. Go back to the problem on TSP #6 and check that the correct answer is "4 out of 6" or 2/3. Give them HO #2 (= TSP #8), and review it after they have a chance to work on these problems.

Activity #3 — Rub-a-dub-dub I: How many possible ways can they sit in that tub? (Allocated time = 20 minutes)

Point out that this activity can be done in the classroom using any story which has three or more characters, and can be repeated any time another story is read. For younger children, you would use stories with three characters, and for older children, stories with more characters. The story that we are going to use (and that gives this activity its name) has altogether seven characters. Introduce the activity by placing TSP #9 on the overhead projector and reading the following summary: ""The Tub People" by Pam Conrad is a story about six toy people and a dog in a tub. The father, the mother, the grandfather, the doctor, the policeman, the child, and the dog are neatly lined up on the edge of the bathtub each day. One evening the unspeakable happens, and the tub people must unite to save the day." Point out that teachers would, of course, read the entire story to the class. What we would like to know is "How many ways are there for the seven characters to be lined up all in a row?"

Pass out HO #3 and explain to participants that one way of solving difficult problems is to first try a simpler problem and see if that helps you solve the more difficult one. Let's start by trying a simpler problem — "How many ways are there for three of the characters ("rub-a-dub-dub, three men in a tub") to be lined up in a row?" Ask them to work together to find all the ways to arrange three tub characters ("dog", "mom", and "police") in a row. The answer should involve a systematic list, so that each person can explain why there are no other possible orders of the three characters. Those who finish early should try to find all the ways to arrange four of the tub characters in a row. Those who see a pattern should try to explain how many ways there are to arrange the seven tub characters in a row.

Provide each group of participants with six cut-out copies of each of three of the tub characters so that they use them as manipulatives in working together on the first of these problems — that is, they should be encouraged to use these figures to develop the six possible arrangements of the three tub characters. You may also want to cut the tub characters from a transparency (before the workshop) so that you can demonstrate the different arrangements on the overhead projector.

Circulate among the groups while they work, particularly noting the different problem-solving strategies that are being used (i.e., using the cut-outs, making an organized list, or making a tree diagram).

Be aware that some groups may have difficulty and need help in getting started. Using the cut-out tub characters to physically create each possible arrangement and record as they go will nearly always produce accurate (if somewhat slow) results.

Ask participants to describe their solutions, and use TSP #10-11 to reinforce their solutions. Discuss any patterns participants discover and a rule for generating a systematic list. Be sure to note that there are many "systematic" ways to list. Note also how knowing the number of ways of arranging three characters helps you find the number of ways of arranging four characters; this is described on TSP #12. Then ask the participants to predict the number of possible arrangements for 5, 6, and 7 tub characters, if appropriate, in which case you may also review TSP #13 which provides the answer for 5 characters and introduces factorial notation.

Activity #4 — Rub-a-dub-dub II: Who is where how often?
(Allocated time = 20 minutes)

Show the top of TSP #14 (keep the rest covered for now) and ask: What is the probability that "dog" will be to the left of "mom" in an arrangement of "dog", "mom", and "police"? Elicit from the participants that we have to look at all the possible arrangements and count the ones in which "dog" is to the left of "mom". Show TSP #10 and ask them to do that — the answer is of course "3 out of 6" or one-half. Return now to TSP #14 and reveal the second question: What is the probability that "mom" will be to the left of "dog". After they review the arrangements on TSP #10, they will respond "one-half". Ask them if they're surprised by the answers to these two questions, and why they think this happened. Someone should note that the situation is symmetrical — "dog" should be to the right of "mom" exactly as often as "mom" is to the right of "dog" — for each arrangement where one is to the right of the other, if you turn the arrangement backwards, you have an arrangement where they are in the opposite order. Now reveal the third question on TSP #14: What is the probability that in an arrangement of all seven people, "mom" will be to the right of "dog"? Again the answer is one-half; counting isn't necessary, since one can again appeal to symmetry. Now reveal the final question on TSP #14: "In an arrangement of "child", "dog", "mom", and "police", what is the probability that "child" will be next to "mom"? Distribute HO #4, which includes the list of all possible arrangements of "child", "dog", "mom", and "police". Ask participants to determine this probability individually, and then discuss their solutions with others at their table. Participants will find that "child" is next to "mom" in 12 out of the 24 arrangements; the answer is therefore 1/2. Ask participants whether they think this will always be true — after all, it turned out that the probability that "child" is to the left of "mom" is 1/2 no matter how many characters are lined up. Perhaps it's also true that the probability that "child" is next to "mom" is 1/2 no matter how many characters are lined up? Elicit the response that the more characters there are, the less likely it is that "child" is next to "mom" — and note that with three characters (see TSP #10) the probability is actually 2/3. (There is actually an interesting pattern here which you may want to mention to the participants — if there are n characters lined up, then the probability that two specific ones are next to one another is exactly 2/n. For n=3 this gives 2/3 and for n=4, this gives 1/2, as we learned above. For n=6, the probability is 1/3, and for n=10, for example, the probability is 2/10.)

Extension Activity #1. Buddy, Can You Spare a Quarter?
(Allocated time = 15 minutes)

"A friend wants to borrow 25 cents. You have plenty of pennies, nickels, dimes, and quarters. How many different ways are there to lend your friend the money he/she wants?" Ask a participant to volunteer one way to accomplish this task and write the response on a blank transparency which overlays TSP #15. Hand out HO #5 as you instruct participants to work with a partner to generate all possible ways to total 25 cents. After some time, each pair should share their results with another pair at their table to verify that all possible ways are listed. Each group of four should discuss a method which when employed would systematically generate the possibilities. Ask each group how many possibilities they found and how they knew that they had found them all. One systematic method of counting the different ways of providing 25 cents is given on TSP #16 — in this method, we start with those ways which use quarters, then those ways that use two dimes, then those that use one dime, then those that use five nickels, then those that use four nickels, etc.

You may want to have plastic coins or counters of several different colors on each table before the workshop begins. If you circulate among groups and find participants having difficulty generating all possibilities, encourage them to use an organized list, a model, a picture, or a table to help them consider all cases. It is important that you don't initially provide them with the answer of a systematic method because part of the thinking process involved in systematic listing and counting is to determine a way to "know" that you have all included all cases. Each solution should be equivalent to the solution on TSP #16.

Now ask, "How could you make this problem easier for your students?" (e.g., ask the number of ways to make 20 cents) and "How could you make it harder? (e.g. ask the number of ways to make 50 cents).

Extension Activity #2 — A Three Hat Day
(Allotted time = 15 minutes)

This activity can be done in conjunction with the book "A Three Hat Day", but can also be adapted for use with other books which have similar situations. Pass out HO #6 as you place TSP #17 on the overhead. Say, "Your task is to find every possible combination of three hats from the four on your sheet. Draw, as best you can, each different combination you find and list your total in the blank at the bottom of your sheet."

Point out that participants may want to pass out several copies of TKHM #17 to each student and a pair of scissors so that they can create manipulatives to facilitate the task. (You may even want to do this with the participants.) You will supply each participant with more materials than needed but you will also make it harder for participants to use the amount of materials to help them guess what the solution is.

Tell participants that: "A combination does not before a different combination just because you change the order of the hats; for example, the fireman's hat, the bonnet, and the top hat is the same combination as the top hat, the fireman's hat, and the bonnet." Allow time for participants to work in groups as you circulate among them. Volunteers can share their solutions using blank transparencies as overlays on TSP #17.

As you circulate, be sure participants do not designate a new group by changing the order of one they already have listed. After participants have shared their solutions (four total combinations: fire hat, bonnet, plume hat; fire hat, bonnet, top hat; fire hat, plume hat, top hat; and bonnet, plume hat, top hat), emphasize that these groups are the possible combinations, in which order is not important. Mention that older students might work with finding the total number of combinations(10) of three hats from a group of five. Also, solicit appropriate follow-up questions that can help clarify and assess student understanding, e.g., "How did you know that you found all the possible combinations of three hats?" and "So, what is a combination?"

Extension Activity #3 — Caps For Sale.
(Allocated time = 15 minutes)

This activity can be done in conjunction with the book "Caps For Sale". but can also be adapted for use with other books which have similar situations. (This activity is of course identical to that of Activity #3 above.) Pass out HO #7 as you place TSP #18 on the overhead. Say, "Your task is to find every possible order for four hats, each of a different color. Use crayons to draw, as best you can, each different order you find and list your total in the blank at the bottom of your sheet. Allow time for participants to work in groups as you circulate among them. Volunteers can share their solutions using blank transparencies as overlays on TSP #18.

As you circulate, be sure participants designate a new arrangement by changing only the order of one they already have. Be sure they consider only one hat of each color. After participants have shared their solutions (twenty-four total orders), emphasize that these orders are the possible permutations, in which order is important. Discuss making this problem easier or harder to fit the skill levels of students. Also, solicit appropriate follow-up questions that can help clarify and assess student understanding, e.g., "How did you know that you found all the possible orders for four colors?" and "So, what is a permutation?"

K-4 classroom teachers often use literature as the vehicle to instruct other curriculum areas, including mathematics. Books appropriate for this topic, in addition to those mentioned in this workshop, include the following:

Anderson, Wayne. *The Perfect Match.* NY: Simon and Schuster Books for Young Readers, 1990.

Anno, Mitsumasa. *Anno's Mysterious Multiplying Jar.* NY: Philomel Books, 1983.
Axelrod, Amy. *Pigs Will Be Pigs.* New Jersey: Four Winds Press, 1994.
Barett, Judith. *Cloudy with a Chance of Meatballs.* NY: Boston: Anthenum Press, 1978.
Benjamin, Alan. *1000 Silly Sandwiches.* NY: Simon and Schuster, 1995.
Gardner, John R. *Stone Fox.* NY: Harper Collins, 1980.
Juster, Norman. *The Phantom Tollbooth.* NY: Knopf, 1961.
Lord, John. *Giant Jam Sandwich.* NY: Houghton Mifflin, 1987.
Reid, Margaret S. *The Button Box.* NY: Penguin Books, 1990.
Scleszka, Jon and Smith, Lane. *Math Curse.* NY: Viking, 1995.
Seuss, Dr. *The 500 Hats of Bartholomew Cubbins.* NY: Vanguard, 1938.

Workshop Summary

Allow some time for participant questions, then explain that this workshop has only "touched the tip of the iceberg"! Solving problems that focus on systematic counting and listing helps students to develop organizational and problem-solving skills and therefore fosters mathematical power. This workshop provides a sampling of meaningful mathematical activities that can be used in K-4 classrooms.

Remind them that they will soon receive a packet of "Take-home Materials" (TKHM) which contains the materials used in the workshop and (in some cases) additional materials. Encourage participants to review these materials and use the activities in their classrooms. In addition, they should be encouraged to develop other activities through which they can weave counting, listing, and probability throughout their curriculum.

Workshop Evaluation

Pass out the "Participant Evaluation Forms". Encourage participants to add their constructive suggestions for future presentations of this workshop. Be sure to provide them with five minutes to complete the form.

Collect forms from participants and give them the "Take-home Materials". Draw their attention to the last page of the "Take-Home Materials", and encourage them to record on that page their impressions of the workshop and their thoughts about how they might use the workshop materials in their classrooms.

Immediately following the workshop (or at the latest that evening when you get home) please complete the "Presenter Report Form" and then the "Participant Summary Form".

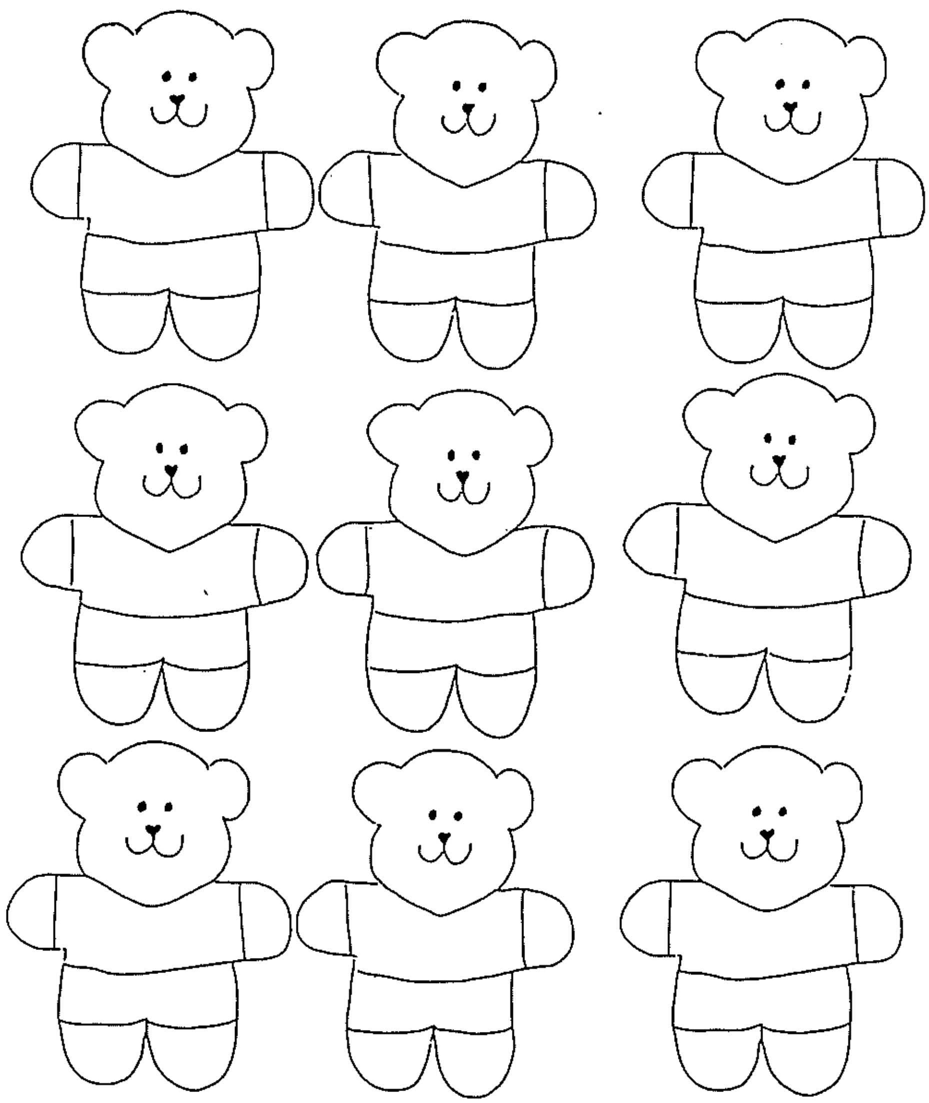
Within three days of your workshop, please mail all "Participant Evaluation Forms", the

Presenter Report Form", and the "Participant Summary Form" — together with an annotated copy of these Instructor's Notes and copies of any supplementary materials that you used — to the following address:

K-8 Workshop-Workshop Evaluation Information
P.O. Box 10867
New Brunswick, NJ 08906

Bobbie Bear's Outfits

Bobbie Bear is going on vacation. She must pack her own clothes. Bobbie Bear has three shirts — which are red, blue, and yellow — and two pairs of shorts — which are red and blue. She wants to wear a different outfit each day. For how many days can Bobbie Bear wear a different outfit each day?



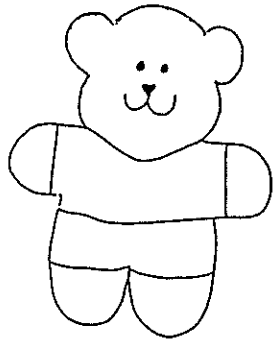
Six Outfits

Shorts

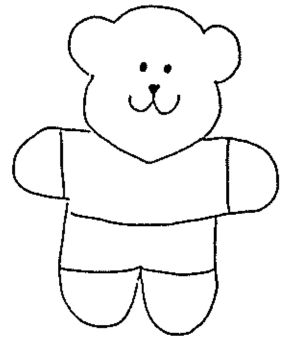
Red

Blue

Red



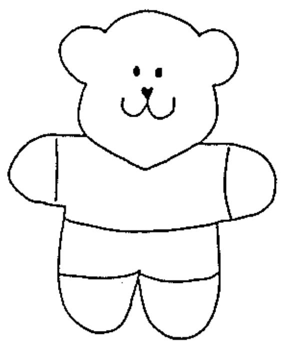
Red shirt
Red shorts



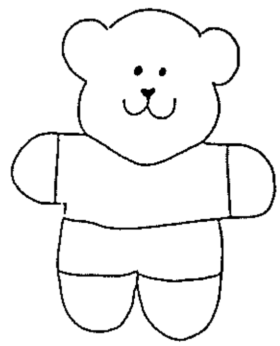
Red shirt
Blue shorts

Blue

Shirts

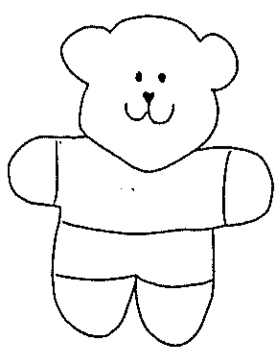


Blue shirt
Red shorts

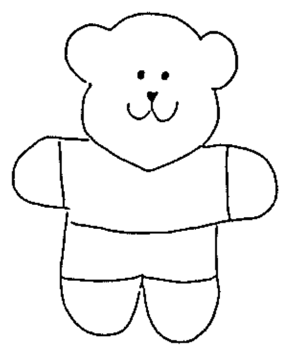


Blue shirt
Blue shorts

Yellow



Yellow shirt
Red shorts



Yellow shirt
Blue shorts

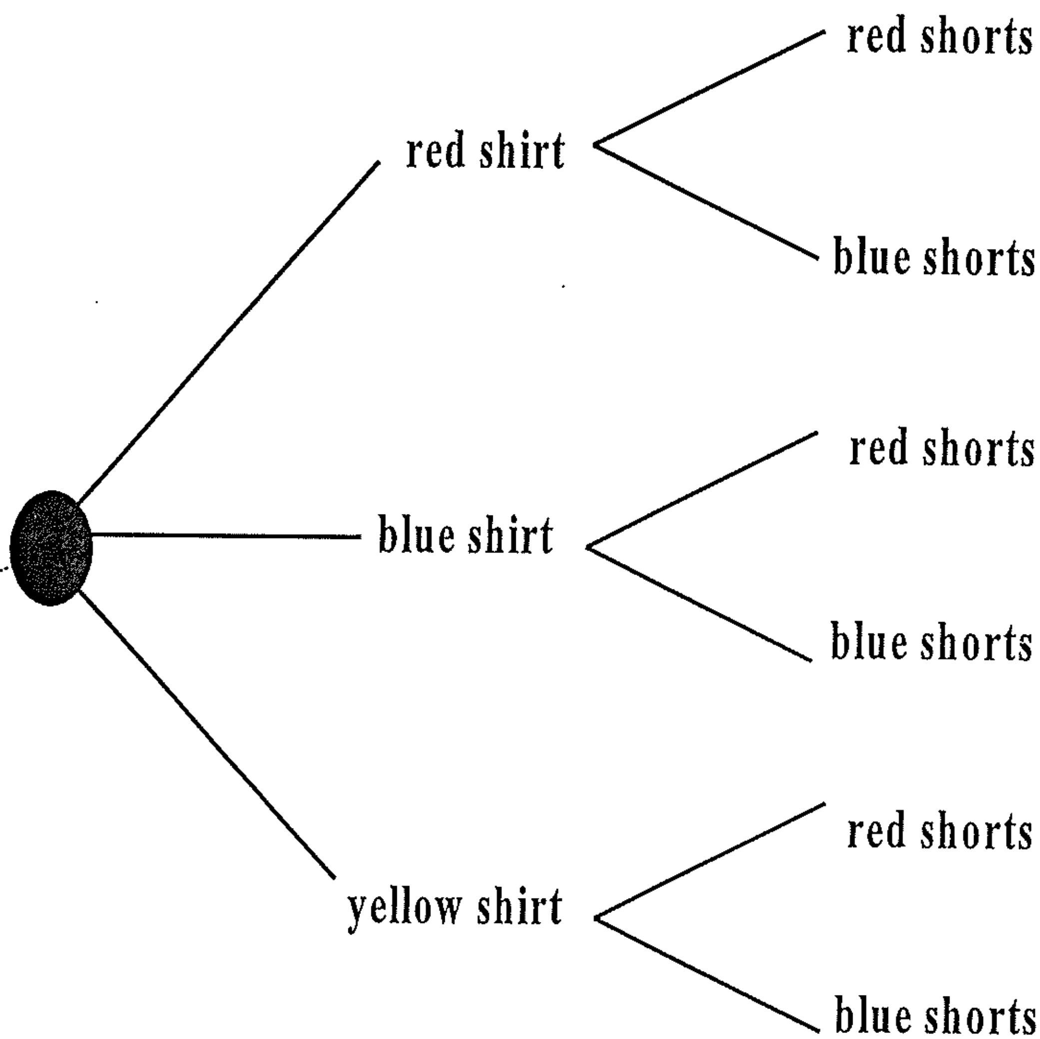
Question: Bobbie Bear wants to pick an outfit consisting of a shirt and a pair of shorts. She has seven different shirts and five different pairs of shorts. How many different outfits can she wear?

The answer is 35.

This is an example of "the multiplication law of counting":

If you can do one thing in 7 different ways and a second thing in 5 different ways, then there is a total of 7x5 ways of doing both things.

More generally, if you can do one thing in K different ways, a second thing in L different ways, a third thing in M different ways, etc., then the total number of possibilities altogether is K x L x M x ...



A Tree Diagram of Bobbie Bear's Outfits

Example #1. Bobbie Bear has two pairs of shorts — one red and one blue. What is the probability that she will pick the red shorts?

Answer: 1 out of 2

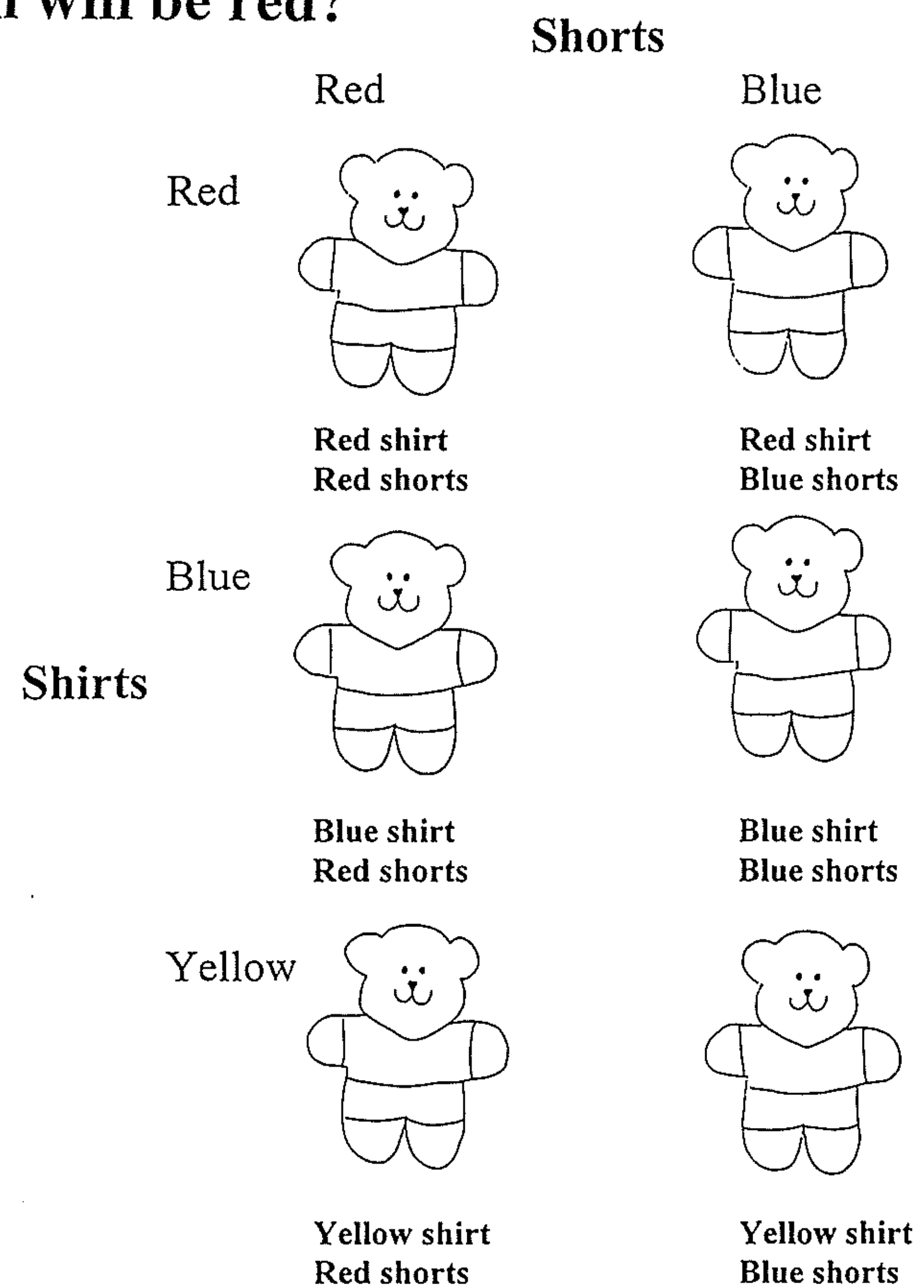
This is just the same as tossing a coin, and choosing blue shorts if heads comes up and red shorts if it's tails.

Example #2. Bobbie Bear has three shirts — one red, one blue, one yellow. What is the probability that she will pick the red shirt?

Answer: 1 out of 3

This is just the same as spinning a spinner with three equal regions (red, blue, yellow), and picking the shirt with the same color as the region indicated by the spinner.

Example #3. Bobbie Bear has three shirts (red, blue, yellow) and two pairs of shorts (red, blue). What is the probability that if she picks a shirt and a pair of shorts that at least one of them will be red?



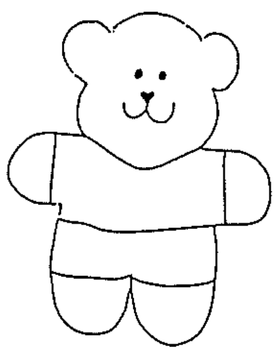
Example #4. Bobbie Bear has two shirts (red, blue) and two pairs of shorts (red, blue). What is the probability that if she picks a shirt and a pair of shorts that at least one of them will be red?

Shorts

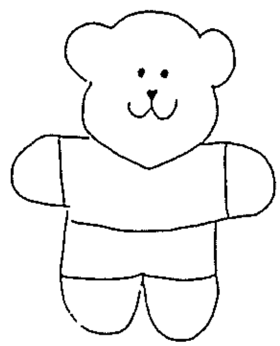
Red | Blue

Shirts

Red

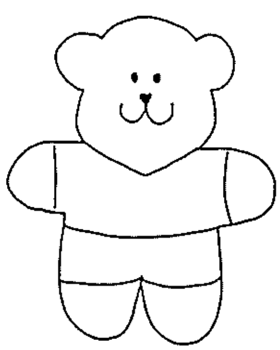


Red shirt
Red shorts

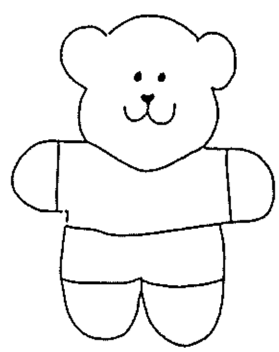


Red shirt
Blue shorts

Blue



Blue shirt
Red shorts



Blue shirt
Blue shorts

Example #5:

Bobbie Bear has three shirts (red, blue, yellow) and three pairs of shorts (red, blue, yellow).

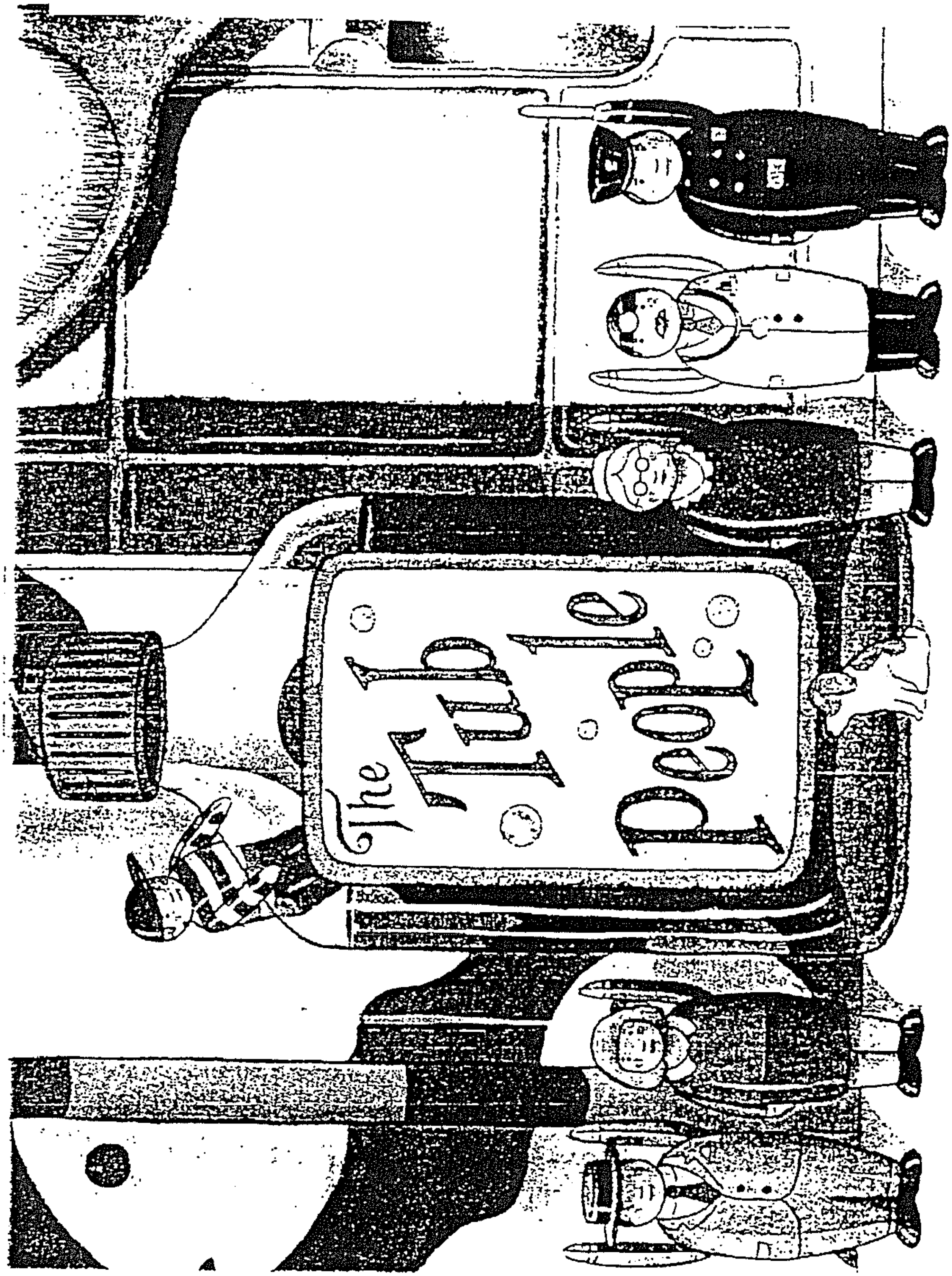
a. What is the probability that an outfit will have something that is blue?

b. What is the probability that an outfit will have nothing that is blue?

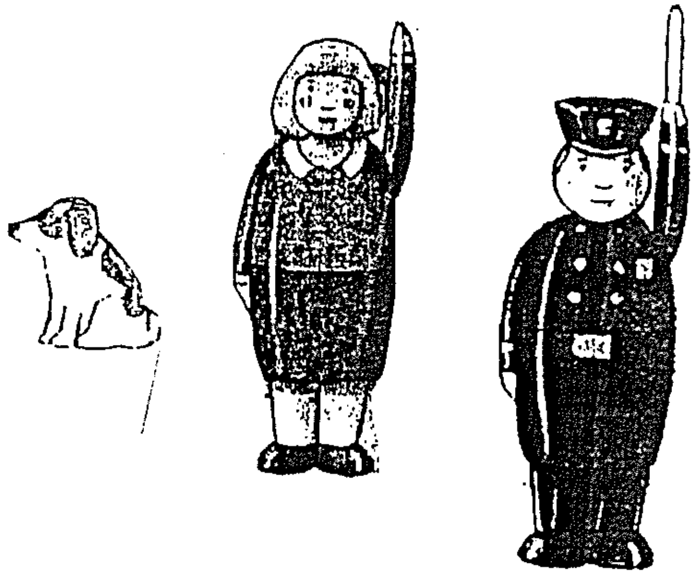
c. What is the probability that an outfit will have something that is either blue or yellow?

d. What is the probability that an outfit will have nothing that is either blue or yellow?

e. Verify your answers in parts a-d above by generating all the possible outfits using both a rectangular array and a tree diagram.



BY PAM CONRAD ILLUSTRATIONS BY RICHARD EGIELSKI



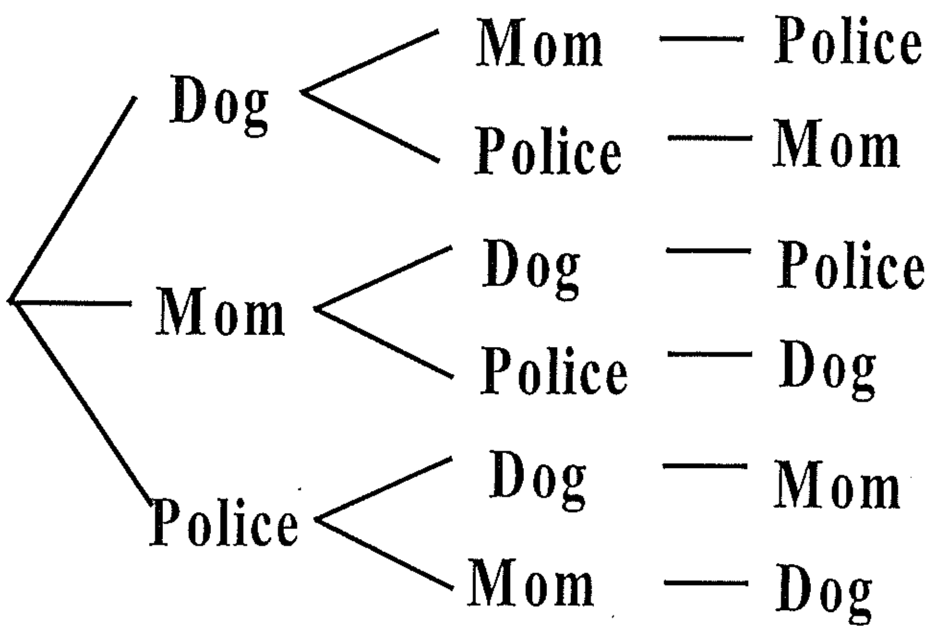
How many ways are there for the three characters to be lined up in a row?

One method of solving this is to construct a systematic list — in which "d" is "dog", "m" is "mother", and "p" is "police":

d-m-p
d-p-m
m-d-p
m-p-d
p-d-m
p-m-d

This is an "alphabetical list" — the entries are in alphabetical order.

Another way is to construct a tree diagram:



You can see that these two methods are very similar, although different in appearance — if you read the branches in the tree consecutively, you get the entries on the list.

There are altogether six ways of arranging three people.

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How many ways are there to arrange 4 people?

If you know there are six ways of arranging three people, you can find the number of ways of arranging four people very easily.

Each of the four people could be first. Then afterwards there are six ways of arranging the remaining three people.

So the total number of ways of arranging four people is 4x6 or 24. These are shown in the following alphabetical arrangement, with "child" added to "dog", "mom", and "police".

c-d-m-p	d-c-m-p	m-c-d-p	p-c-d-m
c-d-p-m	d-c-p-m	m-c-p-d	p-c-m-d
c-m-d-p	d-m-c-p	m-d-c-p	p-d-c-m
c-m-p-d	d-m-p-c	m-d-p-c	p-d-m-c
c-p-d-m	d-p-c-m	m-p-c-d	p-m-c-d
c-p-m-d	d-p-m-c	m-p-d-c	p-m-d-c

The "child" column has the six arrangements of "dog", "mom", and "police" on the previous transparency.

How many ways are there of arranging five tub characters?

Each of the five characters could appear first. Then afterwards there are 24 ways of arranging the remaining four characters. So the total number of ways of arranging five characters is 5x24 or 120.

Summary:

For three characters, there are 3x2 arrangements.

For four characters, there are 4x3x2 arrangements

For five characters, there are 5x4x3x2 arrangements.

How many arrangements are there for six characters, for seven characters?

Factorial notation: 5x4x3x2x1 is denoted 5! and read "five factorial".

What is the total number of ways of arranging all seven characters of "The Tub People"?

What is the probability that "dog" will be located somewhere to the left of "mom" in an arrangement of "dog", "mom", and "police"?

What is the probability that "mom" will be located somewhere to the left of "dog" in an arrangement of "dog", "mom", and "police"?

What is the probability that "mom" will be somewhere to the left of "dog" in an arrangement of all seven characters?

What is the probability that "child" will be next to "mom" in an arrangement of "child", "dog", "mom", and "police"?

BUDDY, CAN YOU SPARE A QUARTER?

A friend wants to borrow 25 cents. You have lots of pennies, nickels, dimes, and quarters. How many different ways are there to lend your friend 25 cents?

Example: One way to get 25 cents is to use five nickels:

Show all the different ways in the space below.

I found ________ different ways to make 25 cents.

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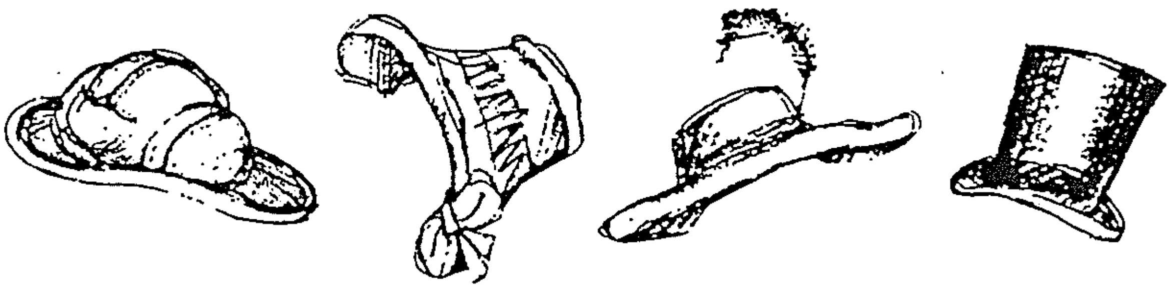
One systematic list

quarters	dimes	nickels	pennies
1	0	0	0
0	2	1	0
0	2	0	5
0	1	3	0
0	1	2	5
0	1	1	10
0	1	0	15
0	0	5	0
0	0	4	5
0	0	3	10
0	0	2	15
0	0	1	20
0	0	0	25

A THREE HAT DAY

R.R. Pottle had many hats. One morning there were four hats on the hat rack and Mr. Potter wanted to choose a combination of three of them to wear — all at the same time!



How many different combinations of three hats could he choose? Draw all the different combinations of three hats below.

I found ________ ways of choosing a combination of three hats.

For use with Laura Geringer's *A Three Hat Day*

CAPS FOR SALE

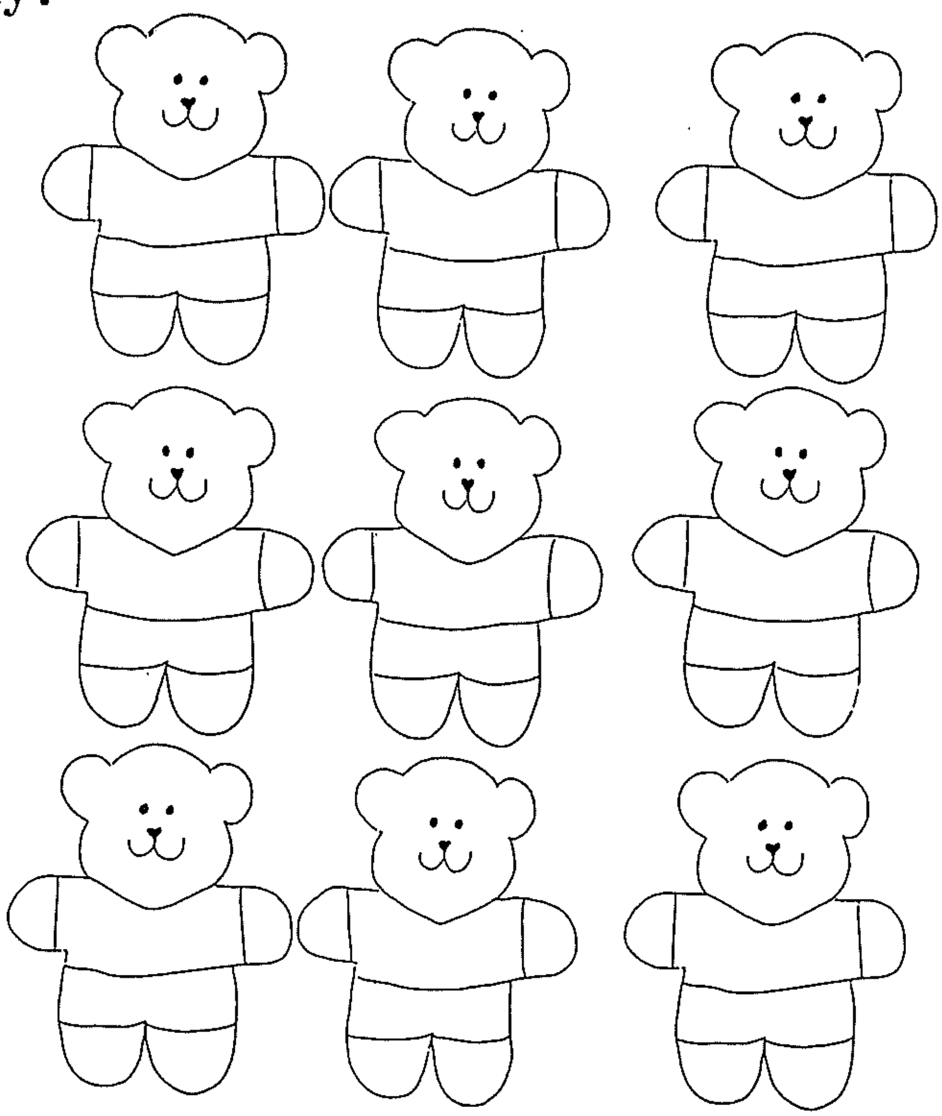
There are caps of four colors — red, blue, brown, and gray. You want to stack one hat of each color on your head. How may ways are there of doing this?

Show your answers in the space below. (Use colors!)

I found __________ different ways of stacking four hats.

Bobbie Bear's Outfits

Bobbie Bear is going on vacation. She must pack her own clothes. Bobbie Bear has three shirts — which are red, blue, and yellow — and two pairs of shorts — which are red and blue. She wants to wear a different outfit each day. For how many days can Bobbie Bear wear a different outfit each day?



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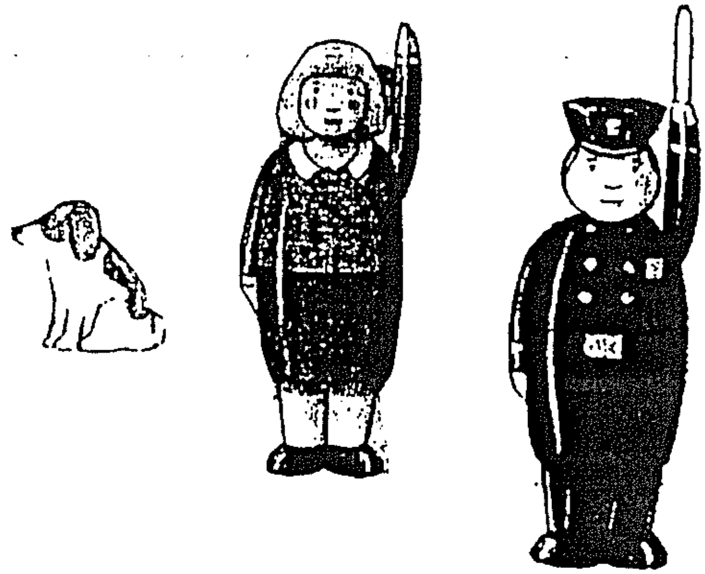

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c-d-m-p	d-c-m-p	m-c-d-p	p-c-d-m
c-d-p-m	d-c-p-m	m-c-p-d	p-c-m-d
c-m-d-p	d-m-c-p	m-d-c-p	p-d-c-m
c-m-p-d	d-m-p-c	m-d-p-c	p-d-m-c
c-p-d-m	d-p-c-m	m-p-c-d	p-m-c-d
c-p-m-d	d-p-m-c	m-p-d-c	p-m-d-c

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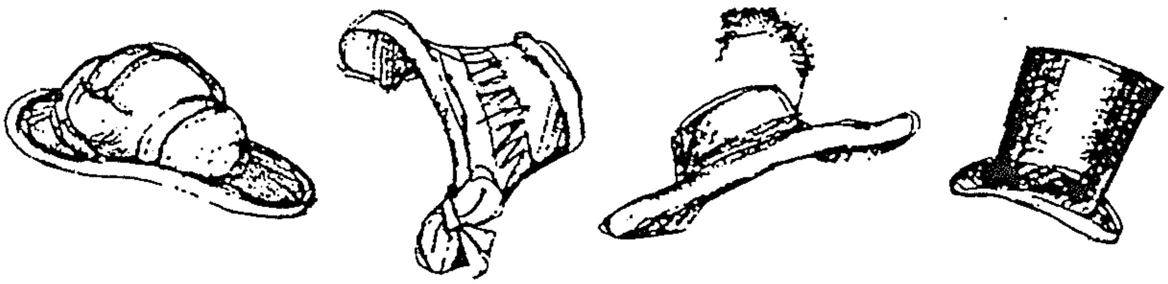

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For use with Laura Geringer's *A Three Hat Day*

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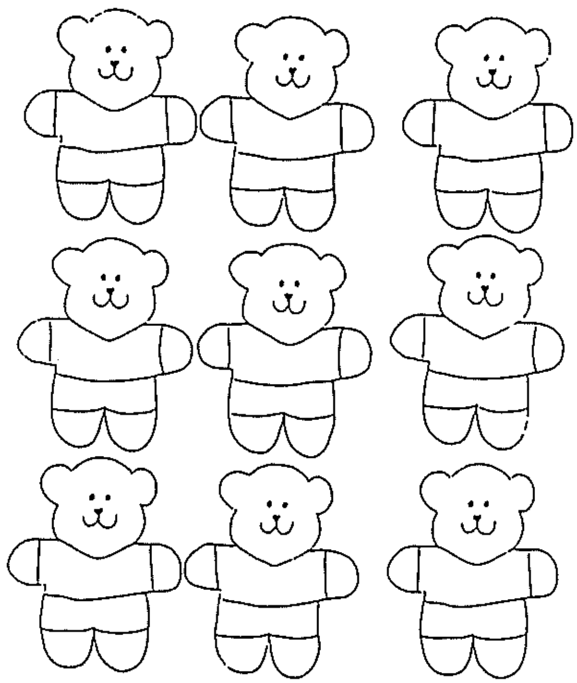
Exploring Discrete Mathematics in the Classroom

Counting and Probability K-4

A Workshop for Teachers About Systematic Listing, Counting, and Probability



Developed by Joseph G. Rosenstein and Valerie A. DeBellis

in collaboration with the following participants in the Rutgers Leadership Program in Discrete Mathematics: Doris Abraskin, Angela Deeney, Judith Gugel, and Ann Lawrence

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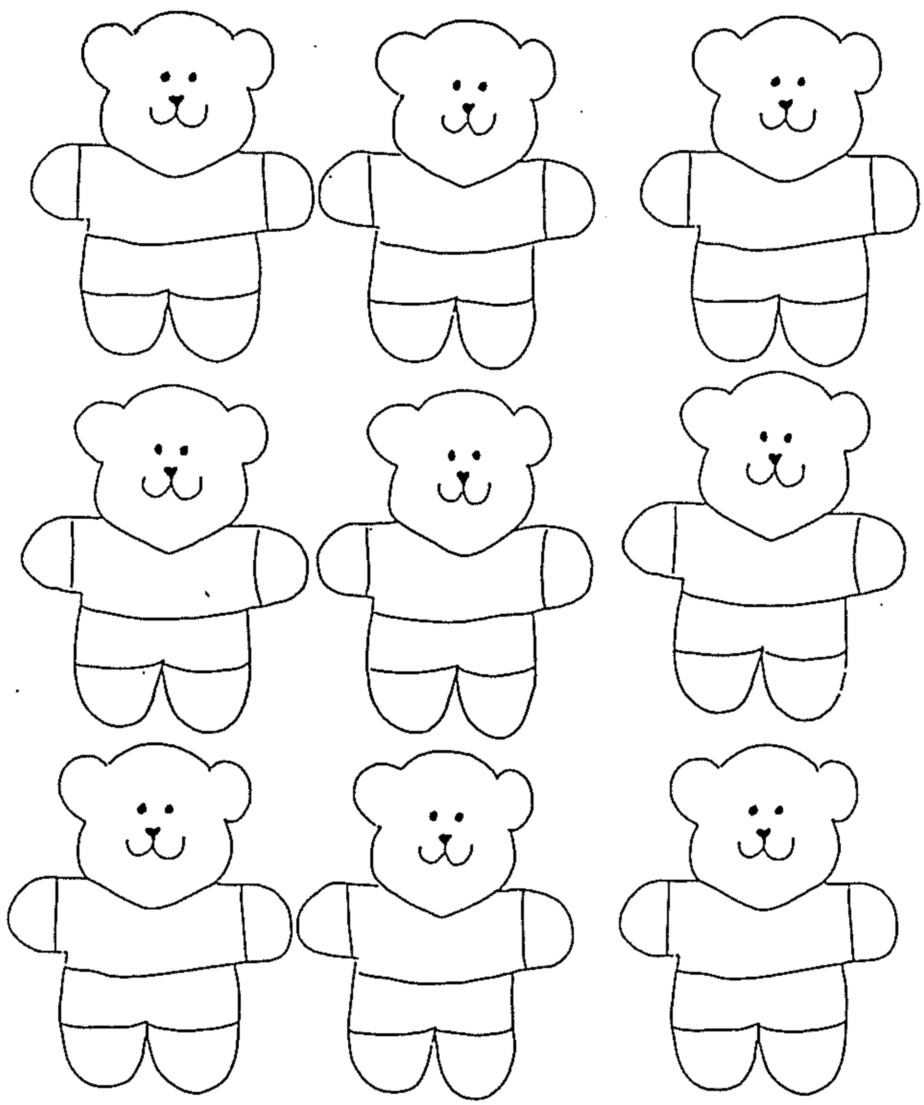

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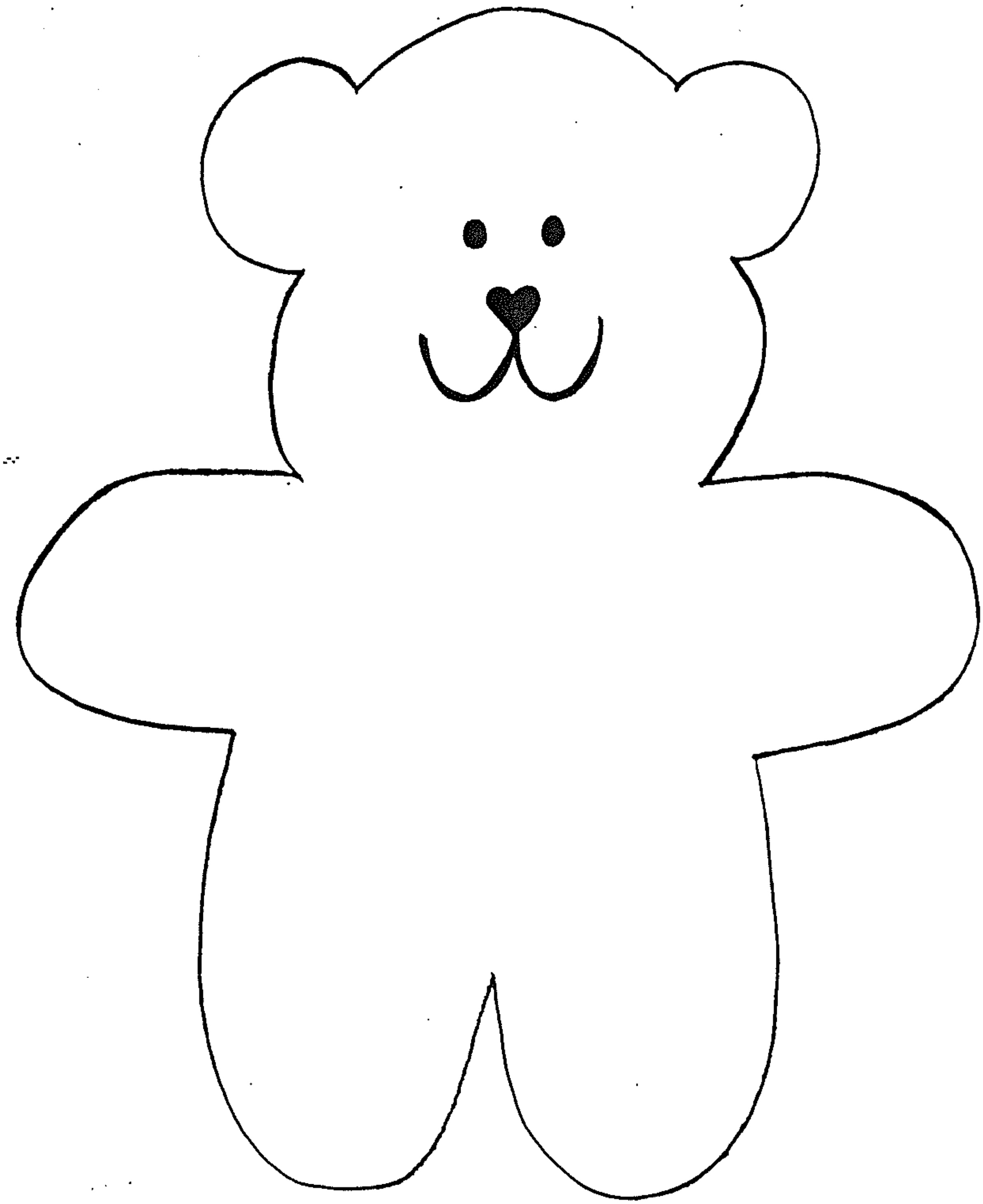
BOBBIE BEAR



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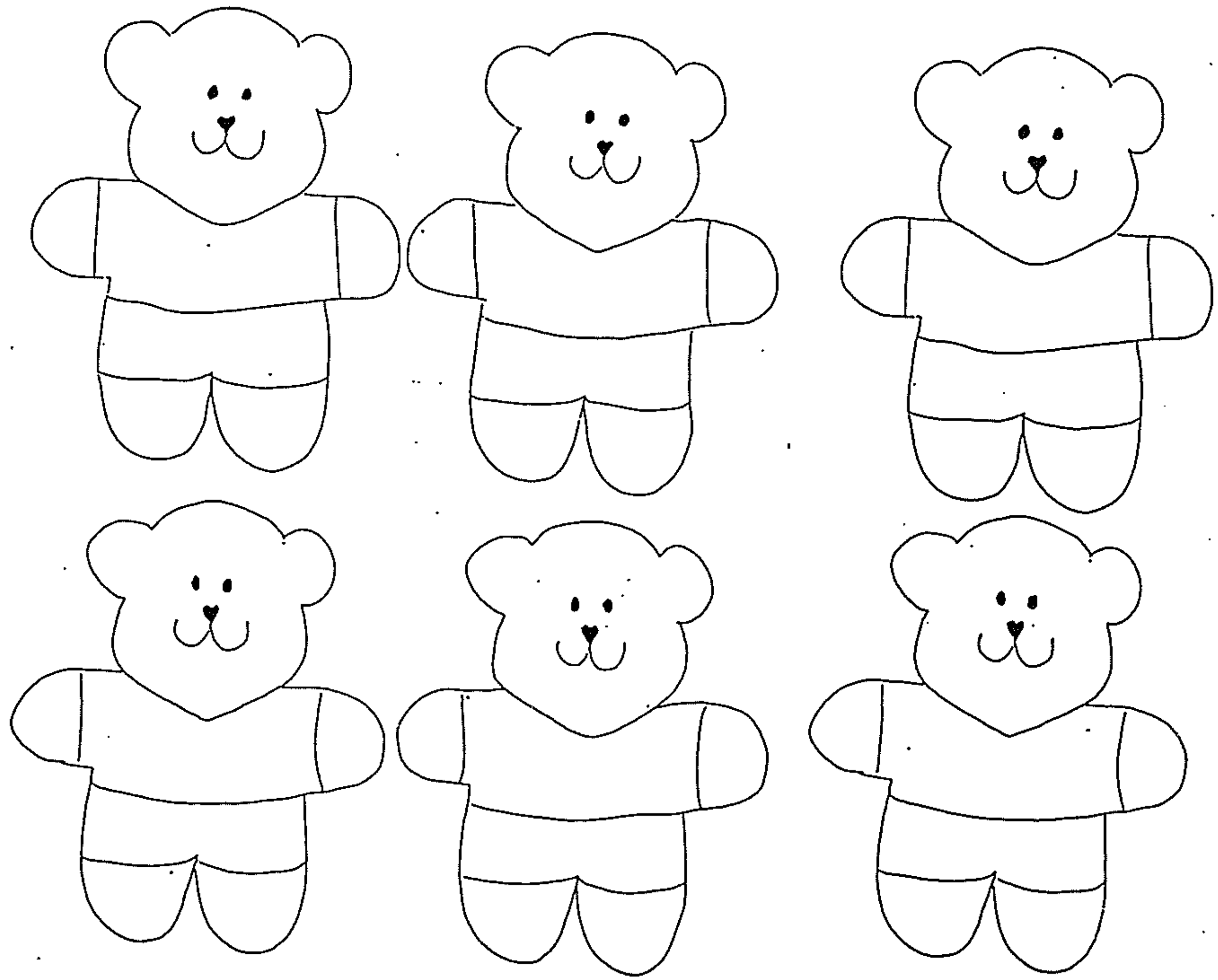
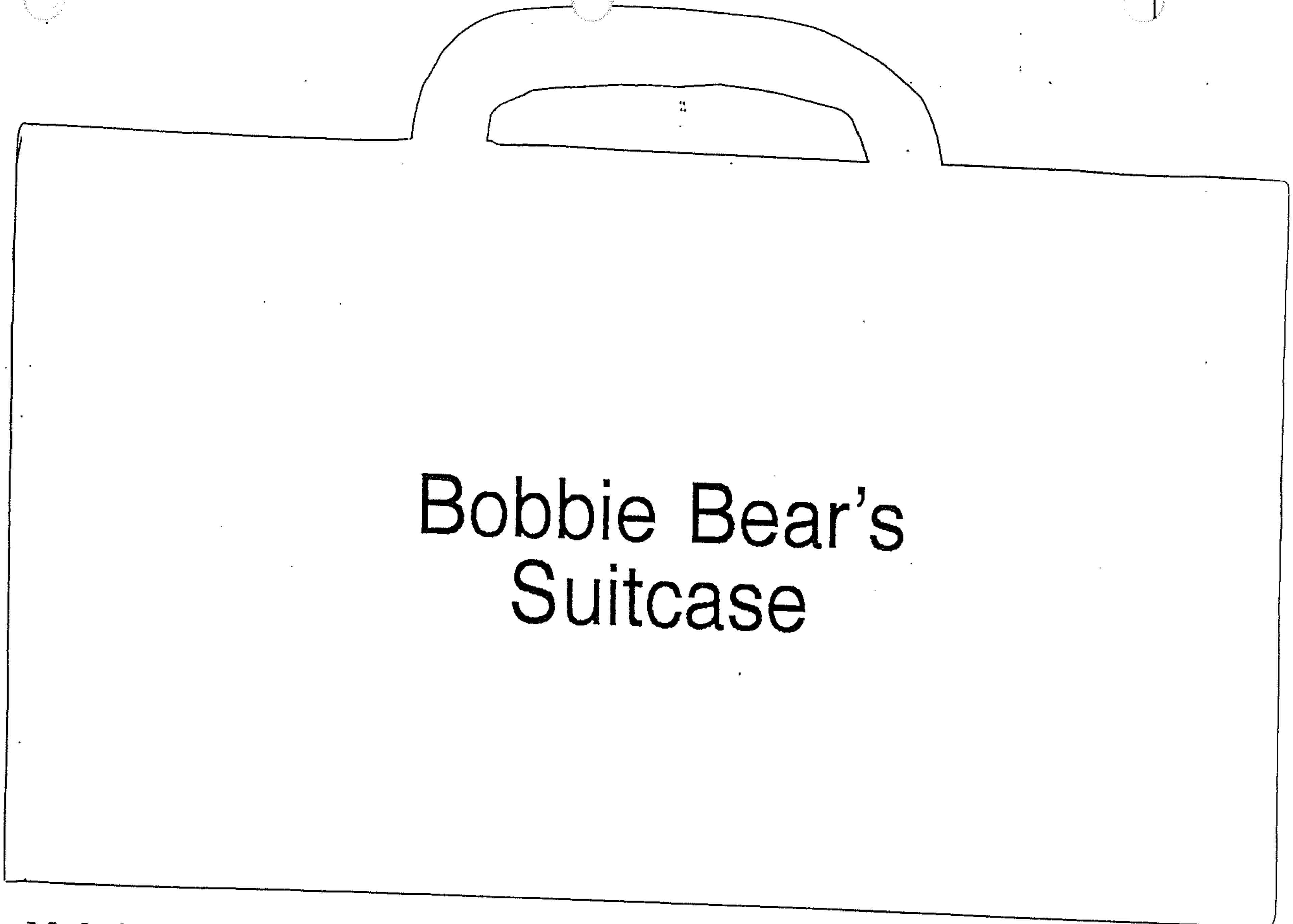
DRESS BOBBIE BEAR

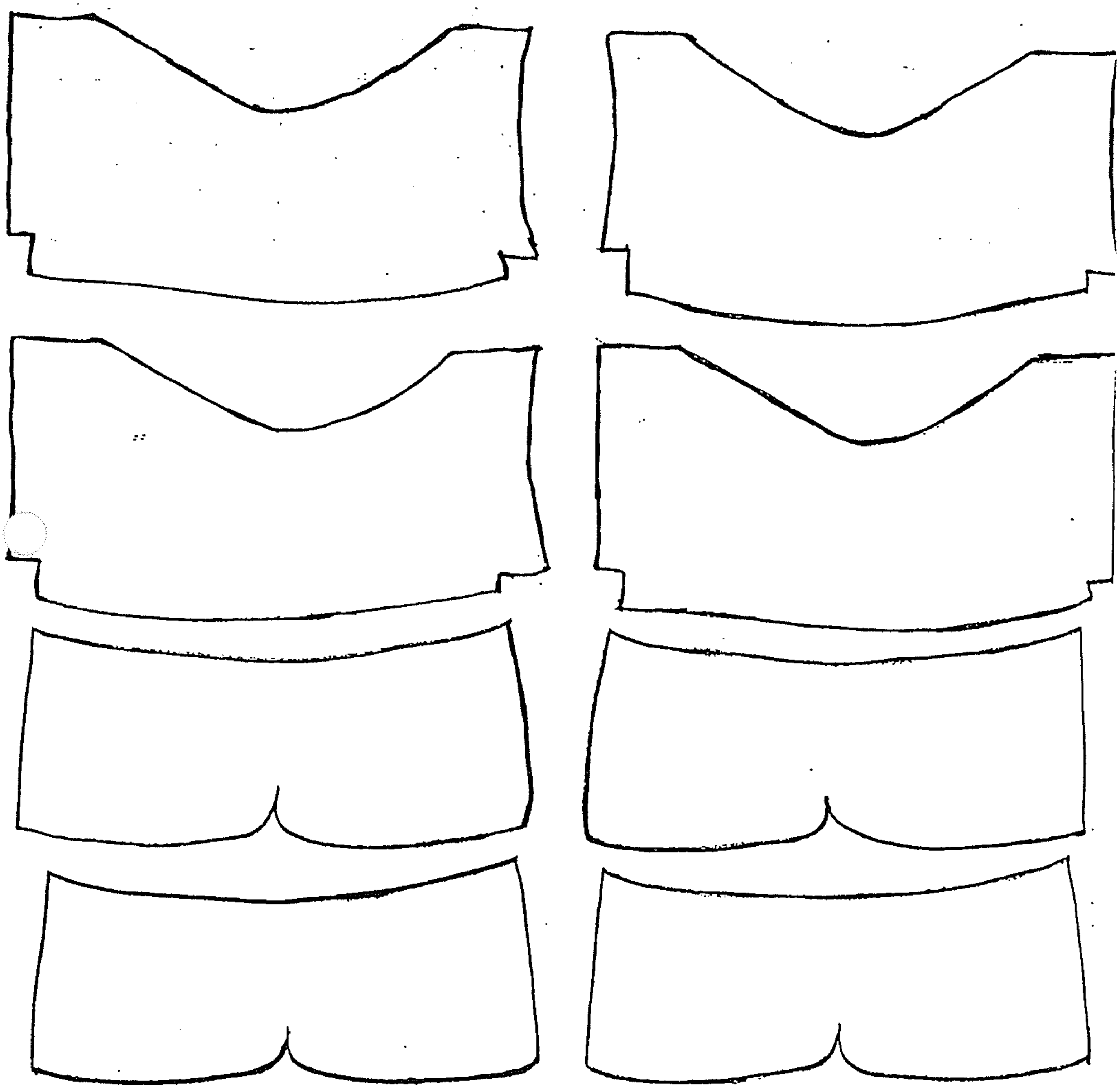


How many different ways can you dress Bobbie?



Made from a pocket folder.

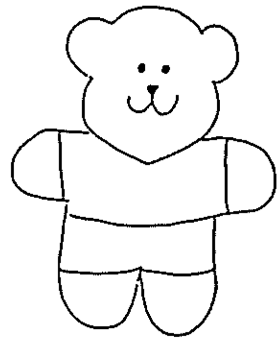
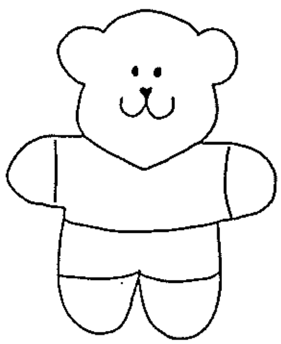
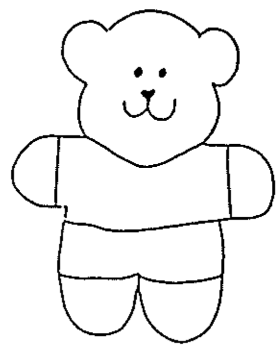
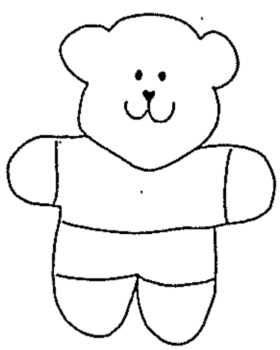
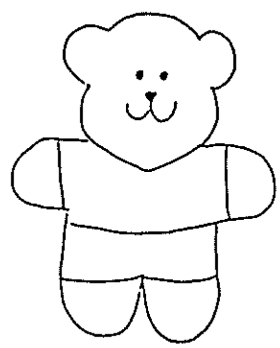
BOBBIE BEARS'S CLOTHES

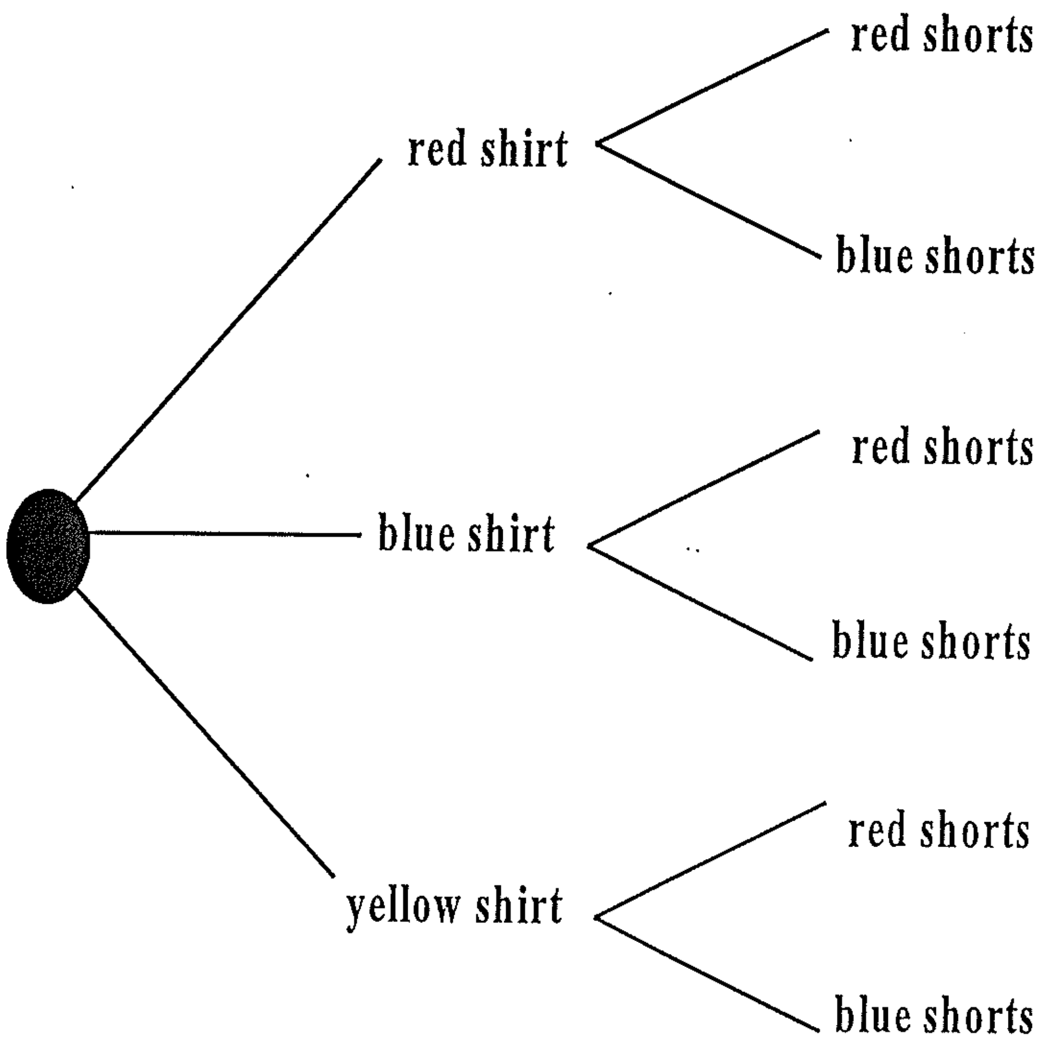


Six Outfits

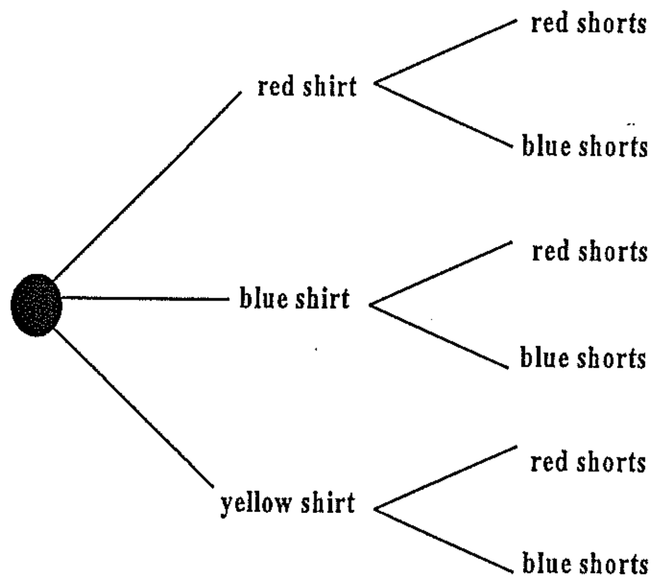
Shorts

Shirts

	Red	Blue
Red	 **Red shirt** **Red shorts**	 **Red shirt** **Blue shorts**
Blue	 **Blue shirt** **Red shorts**	 **Blue shirt** **Blue shorts**
Yellow	 **Yellow shirt** **Red shorts**	 **Yellow shirt** **Blue shorts**



A Tree Diagram of Bobbie Bear's Outfits



The above tree diagram shows the six outfits available to Bobbie Bear.

Question: Bobbie Bear wants to pick an outfit consisting of a shirt and a pair of shorts. She has seven different shirts and five different pairs of shorts. How many different outfits can she wear?

The answer is 35.

This is an example of "the multiplication law of counting":

If you can do one thing in 7 different ways and a second thing in 5 different ways, then there is a total of 7x5 ways of doing both things. More generally, if you can do one thing in K different ways, a second thing in L different ways, a third thing in M different ways, etc., then the total number of possibilities altogether is K x L x M x ...

Example #1. Bobbie Bear has two pairs of shorts — one red and one blue. What is the probability that she will pick the red shorts?

Answer: 1 out of 2. This is just the same as tossing a coin, and choosing blue shorts if heads comes up and red shorts if it's tails.

Example #2. Bobbie Bear has three shirts — one red, one blue, one yellow. What is the probability that she will pick the red shirt?

Answer: 1 out of 3. This is just the same as spinning a spinner with three equal regions (red, blue, yellow), and picking the shirt with the same color as the region indicated by the spinner.

Example #3. Bobbie Bear has three shirts (red, blue, yellow) and two pairs of shorts (red, blue). What is the probability that if she picks a shirt and shorts that at least one of them will be red?

Example #4. Bobbie Bear has two shirts (red, blue) and two pairs of shorts (red, blue). What is the probability that if she picks a shirt and a pair of shorts that at least one of them will be red?

Example #5:

Bobbie Bear has three shirts (red, blue, yellow) and three pairs of shorts (red, blue, yellow).

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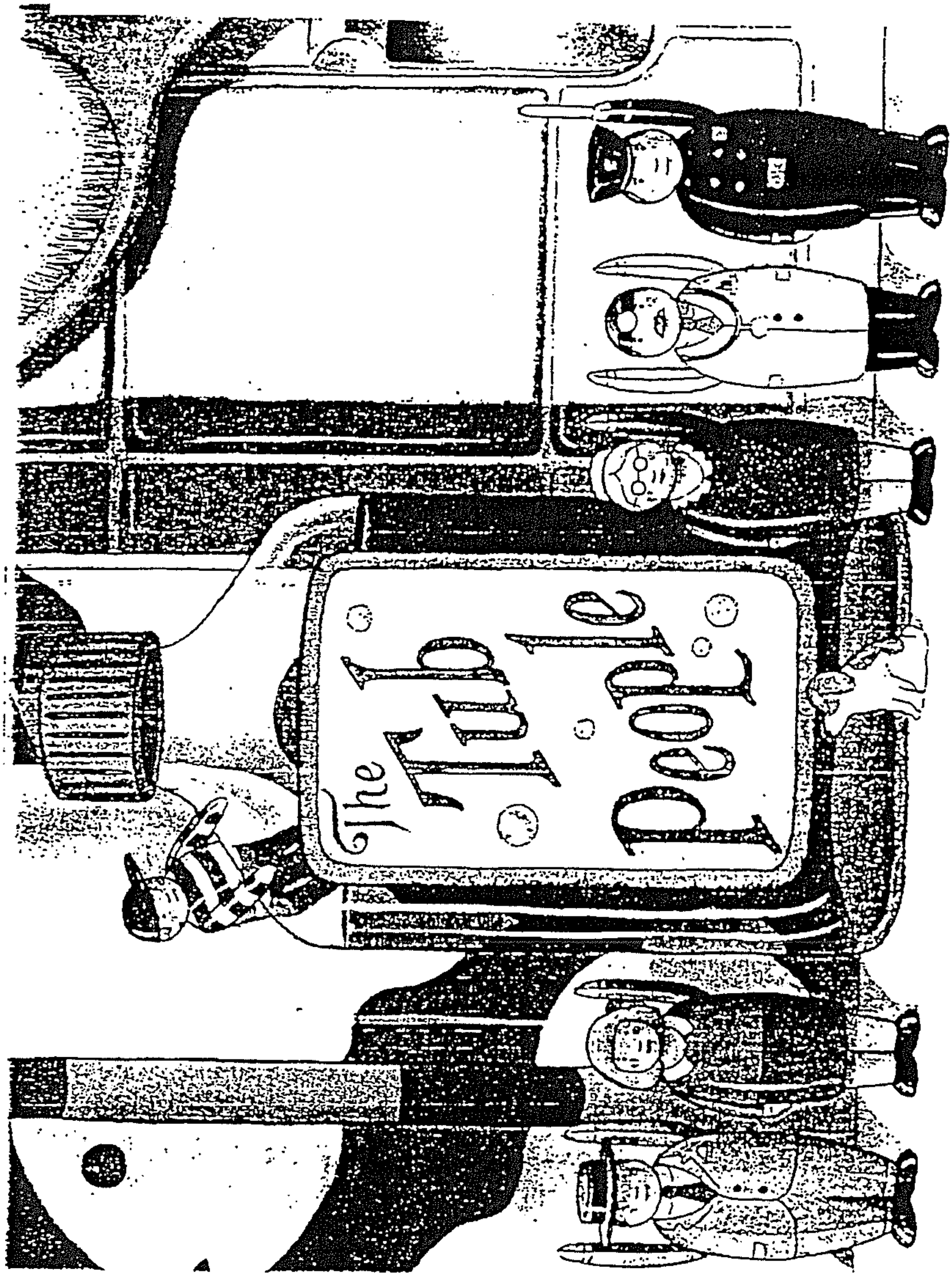

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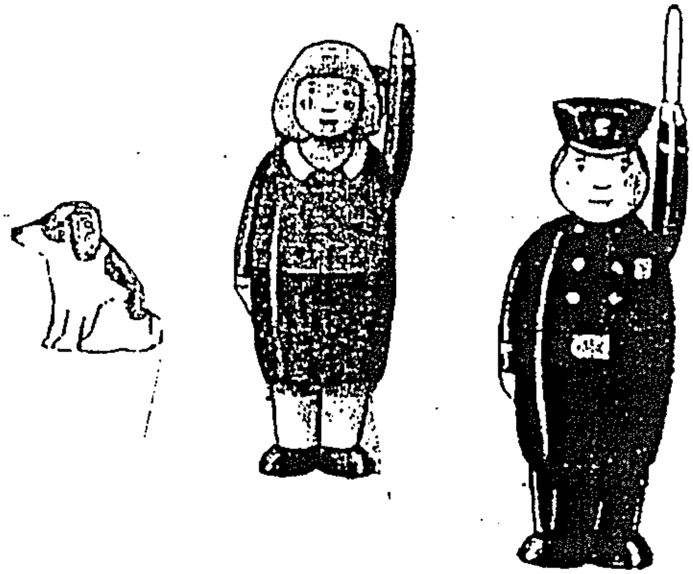

How many ways are there for the three characters to be lined up in a row?

One method of solving this is to construct a systematic list — in which "d" is "dog", "m" is "mother", and "p" is "police":

d-m-p
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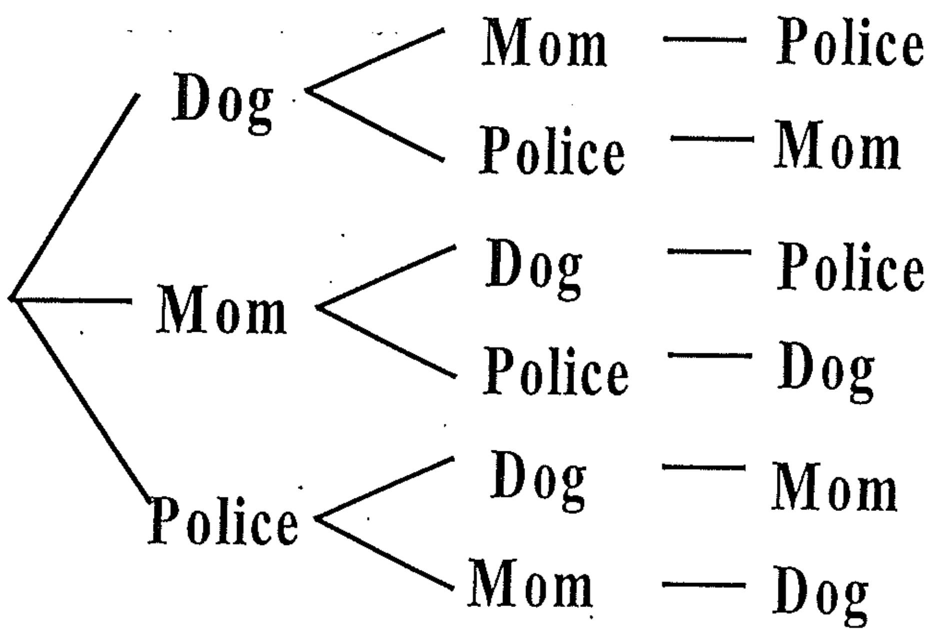
This is an "alphabetical list" — the entries are in alphabetical order.

Another way is to construct a tree diagram:



You can see that these two methods are very similar, although different in appearance — if you read the branches in the tree consecutively, you get the entries on the list.

There are altogether six ways of arranging three people.

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Each of the four people could be first. Then afterwards there are six ways of arranging the remaining three people. So the total number of ways of arranging four people is 4x6 or 24. These are shown in the following alphabetical arrangement, with "child" added to "dog", "mom", and "police".

c-d-m-p	d-c-m-p	m-c-d-p	p-c-d-m
c-d-p-m	d-c-p-m	m-c-p-d	p-c-m-d
c-m-d-p	d-m-c-p	m-d-c-p	p-d-c-m
c-m-p-d	d-m-p-c	m-d-p-c	p-d-m-c
c-p-d-m	d-p-c-m	m-p-c-d	p-m-c-d
c-p-m-d	d-p-m-c	m-p-d-c	p-m-d-c

The "child" column has the six arrangements of "dog", "mom", and "police" on the previous transparency.

How many ways are there of arranging five tub characters?

Each of the five characters could appear first. Then afterwards there are 24 ways of arranging the remaining four characters. So the total number of ways of arranging five characters is 5x24 or 120.

Summary: For three characters, there are 3x2 arrangements.
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For five characters, there are 5x4x3x2 arrangements.

How many arrangements are there for six characters, for seven characters?

Factorial notation: 5x4x3x2x1 is denoted 5! and read "five factorial".

What is the total number of ways of arranging all seven characters of "The Tub People"?

What is the probability that "dog" will be located somewhere to the left of "mom" in an arrangement of "dog", "mom", and "police"?

What is the probability that "mom" will be located somewhere to the left of "dog" in an arrangement of "dog", "mom", and "police"?

What is the probability that "mom" will be somewhere to the left of "dog" in an arrangement of all seven characters?

What is the probability that "child" will be next to "mom" in an arrangement of "child", "dog", "mom", and "police"?

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0	2	0	5
0	1	3	0
0	1	2	5
0	1	1	10
0	1	0	15
0	0	5	0
0	0	4	5
0	0	3	10
0	0	2	15
0	0	1	20
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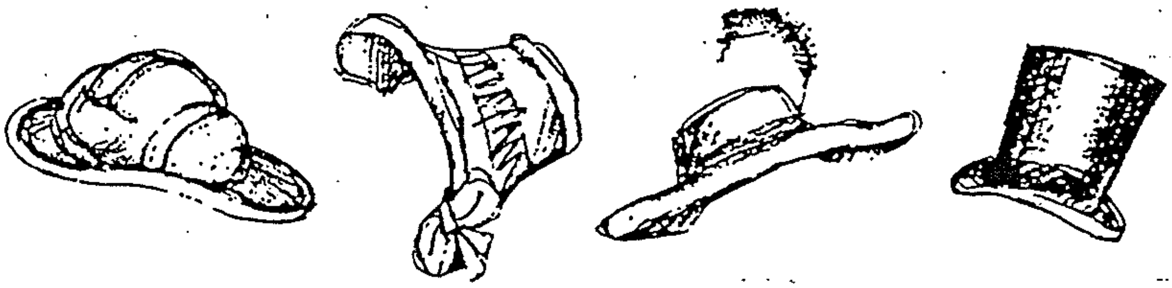

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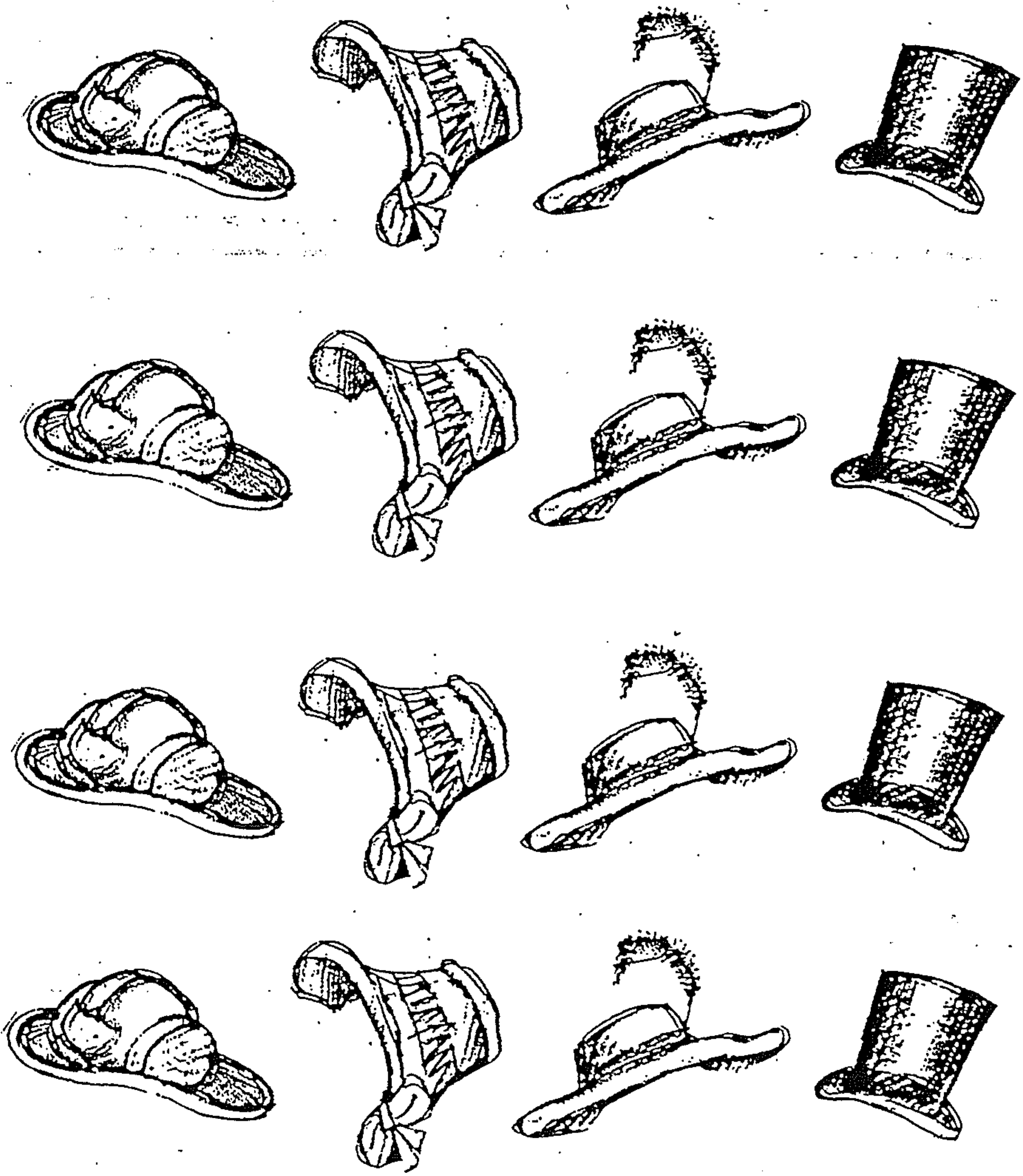
For use with Laura Geringer's *A Three Hat Day*

HATS



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