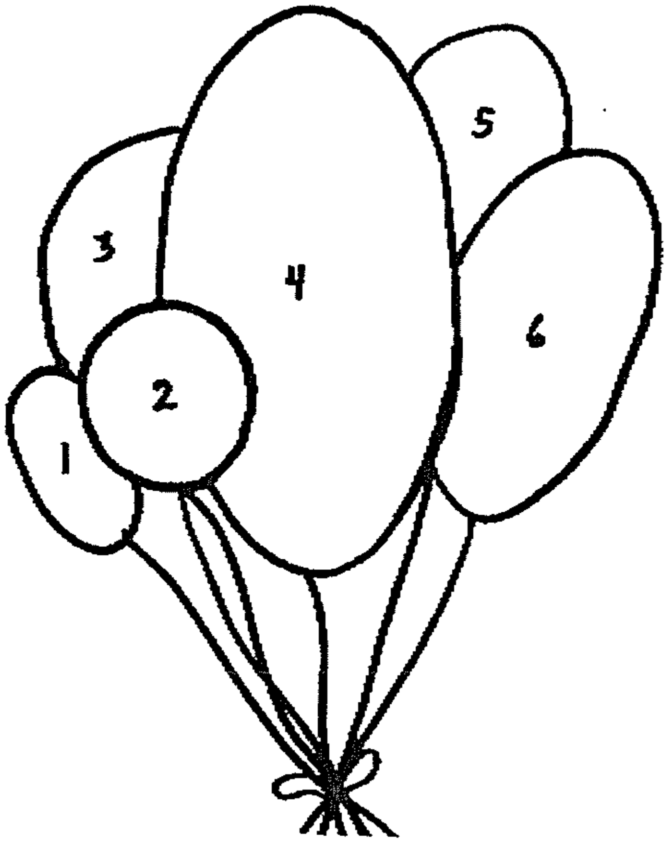
Exploring Discrete Mathematics in the Classroom

Colorful Solutions: Graph Coloring K-4

A Workshop for Teachers About Graph Coloring



Developed by Joseph G. Rosenstein and Valerie A. DeBellis

in collaboration with the following participants in the Rutgers Leadership Program in Discrete Mathematics: Pat Eisemann, Razia Hassan, and Ann Lawrence

Colorful Solutions: Graph Coloring K-4

Workshop Overview

Summary

Participants will engage in several motivating activities involving map coloring, vertex coloring, and resolving conflicts using graphs. The focus of these activities is on developing a mathematical understanding of graph coloring rather than using algorithms to solve problems.

Workshop Outline

I. Objectives

Participants will:

*work together to solve map coloring problems, vertex coloring problems, and graphs to resolve conflicts,

* experience discovery methods for learning new mathematical content, and

* be provided with materials to implement the activities in their classrooms.

I.	Activity #1 — Balloons	20 Minutes
II.	Activity #2 — Stars	20 Minutes
III.	Activity #3 — USA Map Sections	20 Minutes
IV.	Activity #4 — Using Graph Coloring to Resolve Conflicts: An Introduction	20 Minutes
V.	Extension Activity #1 — Using Graph Coloring to Resolve Conflicts	(20 Minutes)
VI.	Extension Activity #2 — Exploring Graph Coloring Through Children's Literature	(20 Minutes)
VII.	Workshop Summary	05 Minutes
VIII.	Workshop Evaluation	05 Minutes
	Total Workshop Time:	90 Minutes

Workshop Environment

I. Room Arrangement

Ideally, participants should be seated at round tables in groups of four to six persons; round tables help facilitate collaboration and discussion among group members. Each participant should have a reasonable view of the presenter and the screen.

II. Equipment

Overhead projector and screen
Monitor and VCR

III. Workshop Materials List

A. Activity #1 — Balloons
   Balloon Display
   Six balloon shapes in each of six colors
   Colored transparency markers
   Eight pieces of thick yarn (or something to represent edges on the balloon display)
   Six large black dots, about two inches in diameter (to represent vertices on the balloon display)

B. Activity #2 — Stars
   M&Ms or colored disks, etc.
   Colored pencils or crayons, etc.
   Blank transparencies and transparency markers

C. Activity #3 — USA Map Sections
   M&Ms or colored disks, etc.
   Colored pencils or crayons, etc.
   Blank transparencies and transparency markers

D. Activity #4 — Using Graph Coloring to Resolve Conflicts: An Introduction
   COMAP Video: Connecting the Dots — Vertex Coloring
   Colored pencils or crayons, etc.
   Blank transparencies and transparency markers

E. Extension Activity #1 — Using Graph Coloring to Resolve Conflicts

F. Extension Activity #2 — Exploring Graph Coloring Through Children's Literature
   Blank transparencies and transparency markers

Instructor's Notes

Introduction

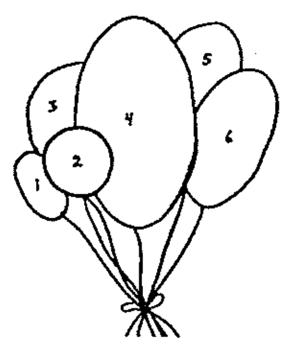
Show the "Title Transparency" for this workshop and indicate that this workshop was developed by K-8 teachers who participated in the Rutgers University Leadership Program in Discrete Mathematics. Introduce yourself and mention that you spent three weeks over two summers learning discrete mathematics at the Leadership Program, and have been using materials from the program in your classroom.

Show the "Discrete Mathematics Transparency" and review the information there — discussing five major themes of discrete mathematics. Indicate that the workshops offered in the "Workshops in Your District" program reflect four of these major themes. The theme that we will be focusing on in this workshop is that of "Modeling using Graphs"; the title of this workshop, as noted on the "Title Transparency" is "Colorful Solutions — Graph Coloring K-4".

Although you may have been given this information beforehand, find out the grade levels of the participants in your workshop (say, for K, 1-2, 3-4, 5-6, and 7-8) and, if the distribution is not what you expected, be sure to modify your workshop appropriately. Also, please tell the participants that all the information and all the activities in this workshop will be included in a packet of materials that they will receive after the program.

Activity #1 — Balloons
(Allocated time = 20 minutes)

Call attention to the display of balloons you have previously hung on the wall. Point out that each of the six balloons is a different color. Say, "We are planning a party and have hired Dottie-the-Decorator to create just the right touch. She has created this balloon bouquet and asks us what we think. Unfortunately, every time she includes a different color balloon in the bouquet, it costs us more money! We don't want two of the same colored balloons touching each other, but we also don't want to spend a fortune on balloons! How can we change the colors in Dottie's arrangement so as to use the fewest possible number of colors?" Ask a volunteer participant to change any balloon to help us minimize the number of colors we must buy. Allow participants to come forward until the display uses only three colors.



In advance you will need to prepare pieces of balloon shapes cut from six different colors of construction paper. For each color, cut out the six balloon shapes. Repeat this for all six colors for a total of 36 balloon shapes. The initial balloon bouquet should be *arranged as pictured above using six different colors. You will have many more balloon pieces than you need but it will not suggest any particular solution to participants. The patterns for the balloon shapes can be found in your Take-home materials.*

Explain that mathematics, and in particular graph theory, can help us determine the minimum number of colors to use in such situations. Take a dot (cut from black construction paper, roughly 2 inches in diameter) and place it over each number on your balloon display explaining that in graph theory this dot is called a "vertex." Now take a piece of thick yarn or string and connect two vertices which share a border.

When arranging these edges on your display, be sure to attach them under each vertex, not over them, so participants don't get confused (see Figures 1 and 2 below). Be sure to cut each edge length in advance, so that it fits nicely between two vertices.

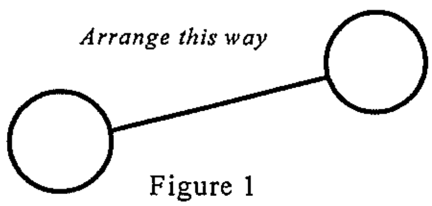


Figure 1

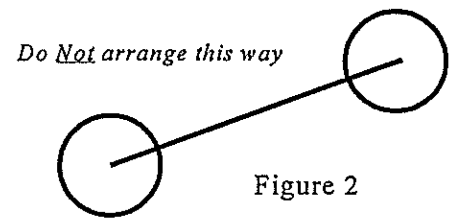


Figure 2

Explain to participants that two vertices are connected by an edge if their balloons share a common border. Place a second edge between two such vertices to show another example. Elicit other examples from participants until you have completed the graph (you should have used a total of eight edges to connect the six vertices). Explain that this collection of vertices and edges is called "a graph" and graphs are used to model many real life situations.

In our case, each vertex represents a balloon and each edge connects two vertices (or balloons) if they share a common border. Because we want two balloons which share a common border to be of different colors, all we have to do is color two vertices with different colors if they are connected by an edge. Show TSP #1 and repeat the dot/string construction on a blank overlay on TSP #1 — using empty circles in the balloons for the vertices. (If your graph comes out looking too messy, you can use TSP #2 for the next step.) When completed, remove TSP #1, leaving just the graph on the overhead projector. The next step is to color the vertices; say, "Since we have to start somewhere, suppose I color this vertex blue (choose a random vertex and color it using a color transparency marker) and ask, "What do we know about any other vertices?" Elicit from participants that the neighbors of this vertex cannot be blue. Participants will probably point to adjacent vertices because they do not know the proper terminology. Use this opportunity to introduce the word "neighbor". So we will have to use a second color to color this neighbor (choose one of its adjacent vertices) and ask, "Do we know anything about any other vertices?" Continue coloring the graph in this way until you are finished. Ask them "What does this imply for the balloon display?" Ask participants if they can see a reason why this graph must use *at least* three colors. Elicit that when three vertices each share an

edge between them (i.e., the three vertices are connected into the shape of a triangle) we need at least three colors to color the graph.

> *Option: At the close of an activity (either here or later in the workshop) you may want to emphasize the importance of communication, especially writing, in the mathematics curriculum. Ask the participants to write what they think students might learn from the activity, and ask a few participants to share what they wrote.*

Activity #2 — Stars
(Allocated time = 20 Minutes)

Place TSP #3 on the overhead as you pass out HO #1. Say, "For this activity, you are to color the stars using the fewest number of colors possible. Remember that any two stars that share a common border must have different colors." Participants should work in groups, first concluding that they can't color this picture with three colors and then, after drawing the graph, explaining why four colors are indeed necessary.

> *Participants may want to use M&Ms or colored disks to manipulate on their graphs prior to coloring them. Mention that such "homemade" manipulatives can be helpful to young children as they try to determine how to best color a simple object.*

Circulate as the participants complete the coloring. Mention that the process of coloring vertices in a graph so that two vertices connected by an edge have different colors is called, "vertex coloring." Have a volunteer use color transparency markers (or transparent colored chips designed for use with an overhead projector) to display his/her solution. Ask whether anyone has a solution requiring fewer colors and, if so, ask that person to display that solution. Ask, "Why did this graph require four colors?" Elicit from participants that every vertex is connected to every other vertex so four colors are needed. Show TSP #4 and distribute HO #2 (= TSP #4). Ask participants to work in their groups to answer the following question: "How many colors are needed for a vertex coloring of each of these graphs?" When they complete that question, ask them: "Do you see any patterns in your solutions?" At an appropriate time when reviewing HO #2, introduce and define the words, "cycle", "complete graph" and "chromatic number". Tell participants that they will have a vocabulary sheet in their take-home materials to use as a resource.

Activity #3 — Sections of the USA Map
(Allocated time = 20 Minutes)

Place TSP #5 on the overhead as you pass out HO #3. Say, "Imagine you are a cartographer (that is, a map-maker) and your boss has asked you to color the states in each regional map with the fewest number of colors possible. It costs more money every time a new color is used to make a map and 'the boss' wants to keep this expense to a minimum.

Since it is difficult to see the border of neighboring states that are colored the same color, two states which share a border must be colored different colors. What is the least number of colors needed for such a coloring?" Allow participants to work in groups to complete this task.

Have available M&Ms (or colored disks) for participants to manipulate while they work out their coloring plan and before they actually color. Ask participants to predict the fewest number of colors they think are needed for each regional map. Circulate as the participants complete the coloring. For the first regional map, when you see groups who have used four colors to color "the Four Corners" (where AZ, NM, UT, and CO meet) do not tell them that two colors will suffice; rather, ask them, "Can you use fewer colors and still see the different states? Why? or Why not?" Most groups will decide for themselves that you can color this region using two colors.

After the groups are done, ask volunteers to use color transparency markers (or transparent colored chips designed for use with an overhead projector) to display each solution. Inquire if anyone has a solution requiring fewer colors and, if so, ask the person to display their solution. Compare the final solution with earlier predictions. By the end of your discussion, be sure to point out explicitly that in this context, a point does not constitute a border. Now ask, "How can you be sure that you have used the fewest number of colors to color a map?"

At this point in the workshop, participants will not have enough experience to discuss odd and even wheels, or even know what they are. For the first section, they will probably explain, "that there are triangles so we need three colors." Reinforce that this is not a true statement. We need AT LEAST three colors, but we don't know that it is exactly three colors just because "a triangle" exists somewhere in our graph; sometimes it may take three colors, but sometimes it may take four.

Point out the following about the map section examples: one map required two colors (Four Corners), one map required three colors (states surrounding Tennessee), and one map required four colors (states surrounding Kentucky). Ask participants, "What do the graphs look like for each of these maps?" Elicit that for Four Corners, the graph is an even cycle (reminding them, if necessary, that points are not considered a border) and we know that such graphs can be colored using two colors (reference HO#2). Ask participants, "What difference do you notice between the Kentucky graph and Tennessee graph?" Elicit "seven states around Kentucky" (or seven vertices around a central vertex) and "eight states around Tennessee" (or eight vertices around a central vertex).

Then introduce the technique of finding a *wheel* with an odd or even number of *spokes* to discover the chromatic number. Show TSP #6 which has wheels with three to eight spokes, and elicit the conclusion that wheels with an even number of spokes can be colored using three colors, although wheels with an odd number of spokes require four colors. Returning to the maps, show how for the top map, Kentucky is a *wheel* with seven, an odd number, of

spokes while for the bottom map, Tennessee is a *wheel* with eight, an even number, of *spokes*. Thus, you can color those states around Tennessee using color one for the first state, color two for the next state, etc. (For example, you can color Alabama red, and proceed counterclockwise, coloring Mississippi blue, Arkansas red, Missouri blue, Kentucky red, Virginia blue, North Carolina red, and Georgia blue.) South Carolina, which is not part of the wheel since it does not border Tennessee, can be colored the same color as Tennessee. However, you cannot color those states around Kentucky with three colors since the odd number of spokes prevents you from simply alternating colors in the "wheel" around Kentucky.

Be sure participants understand that the technique of using a wheel to discover the chromatic number is not always applicable. Direct attention back to the color balloons display on the wall. Ask participants whether this technique can be applied with the balloons: participants should conclude that there is no hub and spokes arrangement.

Inform participants that coloring graphs can be tricky business. It's not always easy to determine the fewest number of colors. Show TSP #7 and model one way to color Graph A by first noticing that there is a complete triangle graph in the upper left corner, choosing three colors to color this subgraph and then arranging these colors from left to right on the graph. Before you color the next graph, ask participants, "How many colors would it take to color Graph B? Graph C?" Elicit the answer that you would expect them to require the same number of colors because they are so similar. However, Graph B can be colored with 3 colors; indeed the coloring found above for Graph A works for Graph B as well. But, despite its similarity to Graph B, Graph C cannot be colored with three colors.

At this point, you may want to review vocabulary and techniques introduced in the workshop thus far and ask for any questions. Inform participants that they will receive a vocabulary sheet in their take home materials so they can use as a reference for what they learned at this workshop.

Activity #4 — Resolving Conflicts Using Graph Coloring — An Introduction
(Allocated time = 20 Minutes)

Show and discuss the first part of Unit 2 (the zoo problem) from COMAP's video "Geometry: New Tools for New Technologies". The seven-minute video clip is called "Connecting the Dots -Vertex Coloring." Stop the video after they have found the first coloring of the zoo graph. This is just before the narrator says "Wait a minute!" (When we have shown the entire video, participants get focused on applying Brooks' Theorem and get confused. Since this is not part of this workshop, it is best to stop the videotape.) Lead a brief discussion about the similarities between map coloring and conflict resolution problems: both can use graphs to simplify the information and help find a solution.

Place TSP#8 on the overhead and distribute HO #4. Review with participants how to construct a conflict resolution graph by inserting a few edges. Then allow them to work in their groups to complete the conflict resolution graph, as requested on HO #4. Review what the vertices represent, what the edges represent and how the graph was created. After participants are done, verify that everyone has created the conflict resolution graph on TSP #9.

Now remind them of the solution proposed by the planning team in the videotape — this is on TSP #9 where the proposed colors are indicated by letters A, B, C, and D. Summarize that you can save the zoo up to two million dollars if you can use fewer than four different habitats to place the animals in the zoo. Distribute HO #5 and ask participants to work in groups to find a solution which requires fewer than four habitats. Have them discuss in their groups why they think their solution is optimal.

Review their solutions. Explain that these type of problems are called, "Conflict Resolution Problems" and graphs can be used as a tool to help solve these hard problems. Mention that graph coloring is used to determine how to schedule meetings for student clubs so that clubs with members in common meet at different times (two clubs with a common member are "in conflict"), and how to schedule traffic lights so that two conflicting streams of traffic are not assigned "green" at the same time. Note that they have examples of several of these problems in their take-home materials, as well as the details about the zoo problem.

> *Identifying what vertices, edges, and colors represent is often quite difficult for the novice graph theory learner, so you should not be surprised if they have trouble in this area. Advise them to write out what a vertex represents, what an edge represents, and what a color represents BEFORE starting to solve these type of problems. Children also have difficulty in doing this. Adding to the difficulty with this step is that children are sometimes asked to draw edges between animals that "belong together." This is probably because in many K-4 classrooms they practice "connecting like objects." Mention that participants should be careful to watch out for such things and remind their students that "in this kind of situation, edges represent conflicts between two vertices."*

Extension Activity 1 — Using Graph Coloring to Resolve Conflicts (Allotted time = 20 minutes)

There are three activities in the TKHM materials, each of which can be used appropriately as an extension activity — these are the School Club Scheduling Problem (TKHM #17), the Errickson School Courtyard Probem (TKHM #18), and the Supply Ship Problem (TKHM #19). If you intend to use one of these, you will need to make a TSP of the appropriate page, and copies of the page for participant handouts. In each case, ask participants to draw a graph which represents the problem appropriately, color the vertices to resolve the conflicts, and then interpret the graph coloring solution in terms of the original problem.

Identifying what vertices, edges, and colors represent is often quite difficult for the novice graph theory learner, so you should not be surprised if they have trouble in this area. Advise them to write out what a vertex represents, what an edge represents, and what a color represents BEFORE starting to solve these type of problems. Children also have difficulty in doing this. Adding to the difficulty with this step is that children are sometimes asked to draw edges between animals that "belong together." This is probably because in many K-4 classrooms they practice "connecting like objects." Mention that participants should be careful to watch out for such things and remind their students that "in this kind of situation, edges represent conflicts between two vertices."

Extension Activity #2 — Exploring Graph Coloring through Children's Literature: The Patchwork Pachyderm. (This activity has not yet been reviewed!)
(Allocated time = 20 minutes)

You may use the activity "The Patchwork Pachyderm" in connection with the book Elmer by David McGee from Workshop Workshop's *Connecting Children's Literature With Discrete Mathematics Topics* K-4. For details of this activity, see LitIN #5-6, LitTSP #4-9, and LitHO #5-9. This activity provides experiences with coloring appropriate for younger students. They complete several versions of Elmer, a patchwork elephant, in as few colors as possible.

Teachers in grades K-4 often can use children's literature as a vehicle to motivate their students and enrich the curriculum. Such cross-curriculum connections can be very effective. Other books appropriate for the topics in this workshop and these grade levels include the following:

Cobb, Mary. *The Quilt-Block History of Pioneer Days*. Brookfield, CN: Millbrook Press, 1995.
Ernst, Lisa. *Sam Johnson and the Blue Ribbon Quilt*. New York: New York: Lorthrop, Lee and Shepard, 1983.
Flournoy, Valerie. *The Patchwork Quilt*. New York: Penguin Books, 1985.
Friedman, Aileen. *A Cloak for a Dreamer*. New York: Scholastic, 1995.
Hopkinson, Deborah. *Sweet Clara and the Freedom Quilt*. New York: Alfred A. Knopf, 1993.
Kinsey-Warnock, Natalie. *The Canada Geese Quilt*. New York: Cobblehill Books/Dutton, 1989.
Love, D. Anne. *Bess's Log Cabin Quilt*. New York: Holiday House, Inc., 1995.
Parton, Dolly. *Coat of Many Colors*. New York: Scholastic, Inc., 1994.
Paul, Ann Whitford. *The Seasons Sewn*. New York: Hartcourt Brace, 1996.
Paul, Ann Whitford, *Eight Hands Round*. New York: Harper Collins, 1991.
Polacco, Patricia. *The Keeping Quilt*. New York: Simon and Schuster, 1988.
Whelan, Gloria. *Bringing the Farmhouse Home*. New York: Simon and Schuster, 1992.

Young, Ed. *Seven Blind Mice*. New York: Philomel Books, 1992.

Workshop Summary

Allow some time for participant questions, then explain that this workshop has only "touched the tip of the mathematical iceberg"! Solving problems that involve map coloring, vertex coloring, or conflict resolution helps students develop problem-solving skills and therefore fosters mathematical power. This workshop provides a sampling of meaningful mathematical activities that can be used in K-4 classrooms.

Remind them that they will soon receive a packet of "Take-home Materials" (TKHM) which contains the materials used in the workshop and (in some cases) additional materials. Encourage participants to review these materials and use the activities in their classrooms. In addition, they should be encouraged to develop other activities through which they can weave graph coloring throughout their curriculum.

Workshop Evaluation

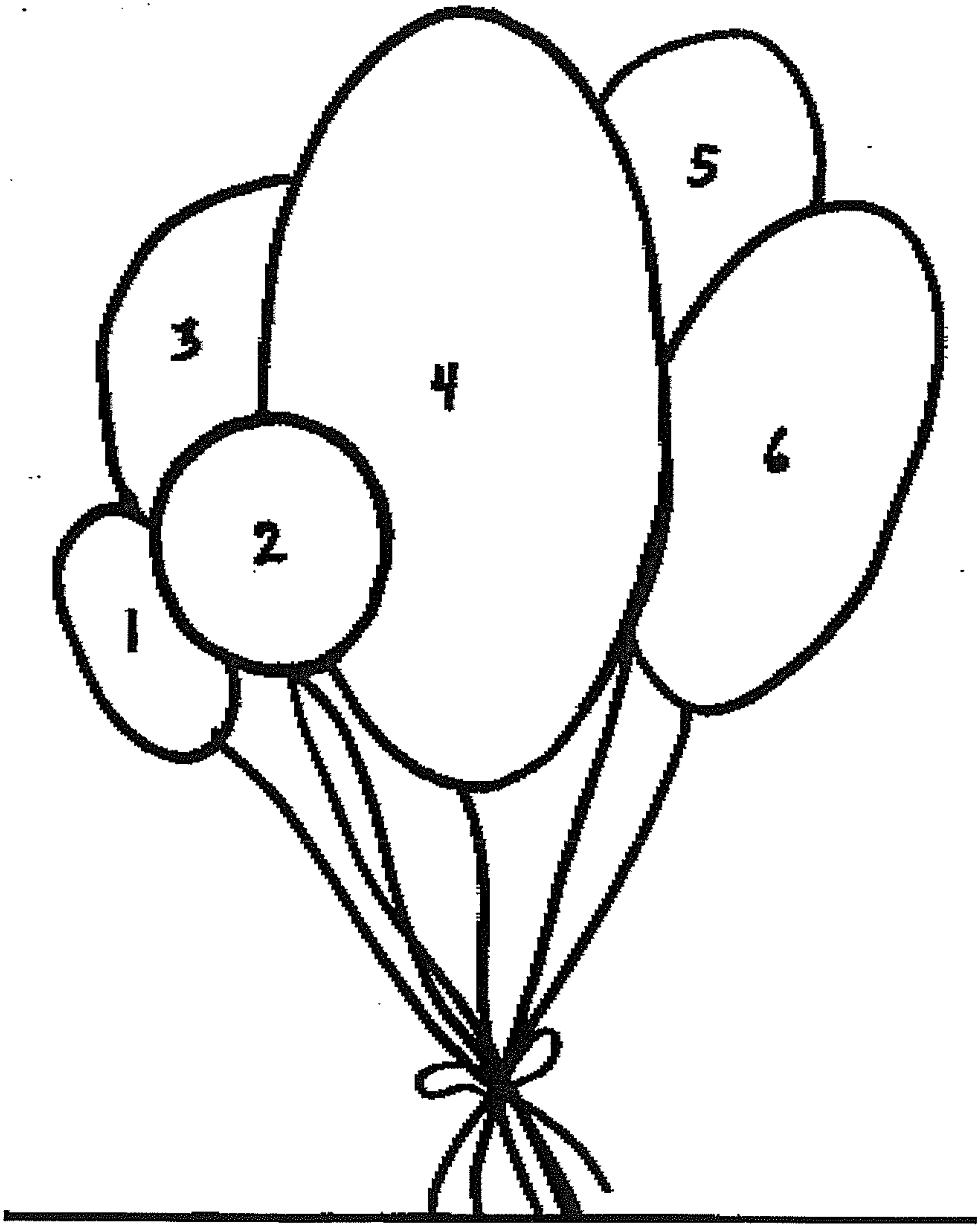
Pass out the "Participant Evaluation Forms". Encourage participants to add their constructive suggestions for future presentations of this workshop. Be sure to provide them with five minutes to complete the form.

Collect forms from participants and give them the "Take-home Materials". Draw their attention to the last page of the "Take-Home Materials", and encourage them to record on that page their impressions of the workshop and their thoughts about how they might use the workshop materials in their classrooms.

Immediately following the workshop (or at the latest that evening when you get home) please complete the "Presenter Report Form" and then the "Participant Summary Form".

Within three days of your workshop, please mail all "Participant Evaluation Forms", the Presenter Report Form", and the "Participant Summary Form" — together with an annotated copy of these Instructor's Notes and copies of any supplementary materials that you used — to the following address:

K-8 Workshop Evaluation Information
P.O. Box 10867
New Brunswick, NJ 08906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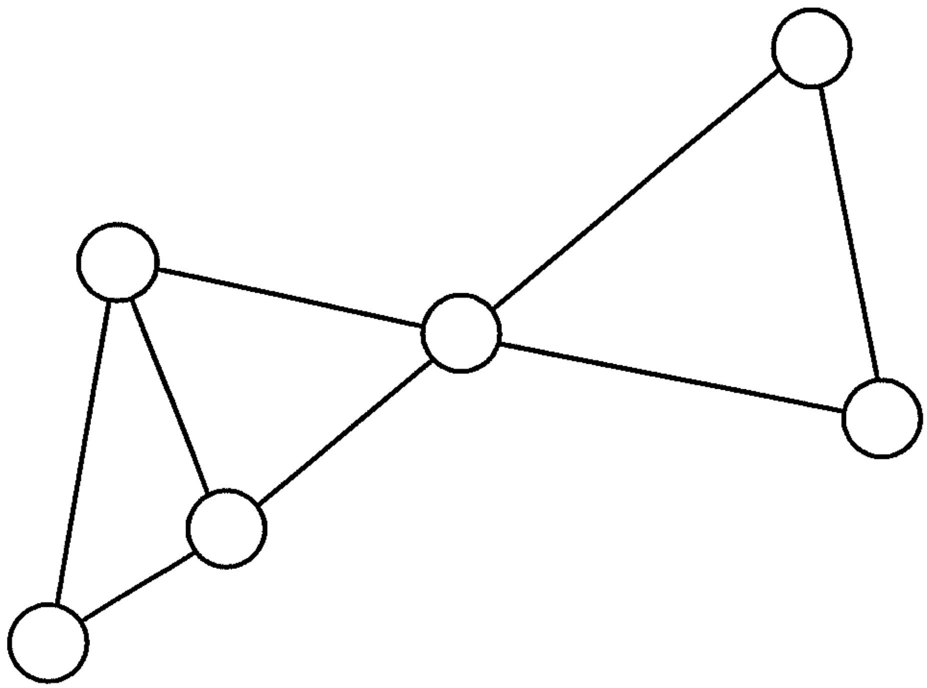
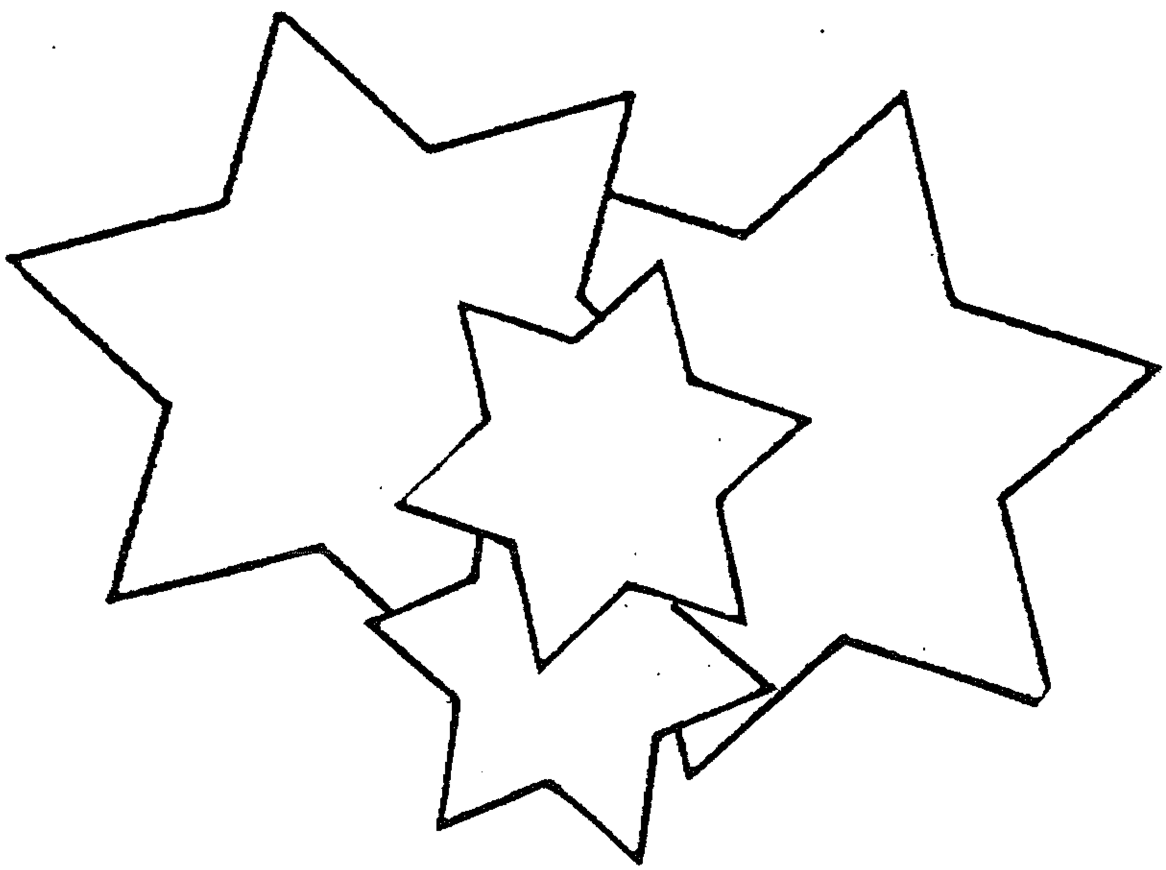


Stars

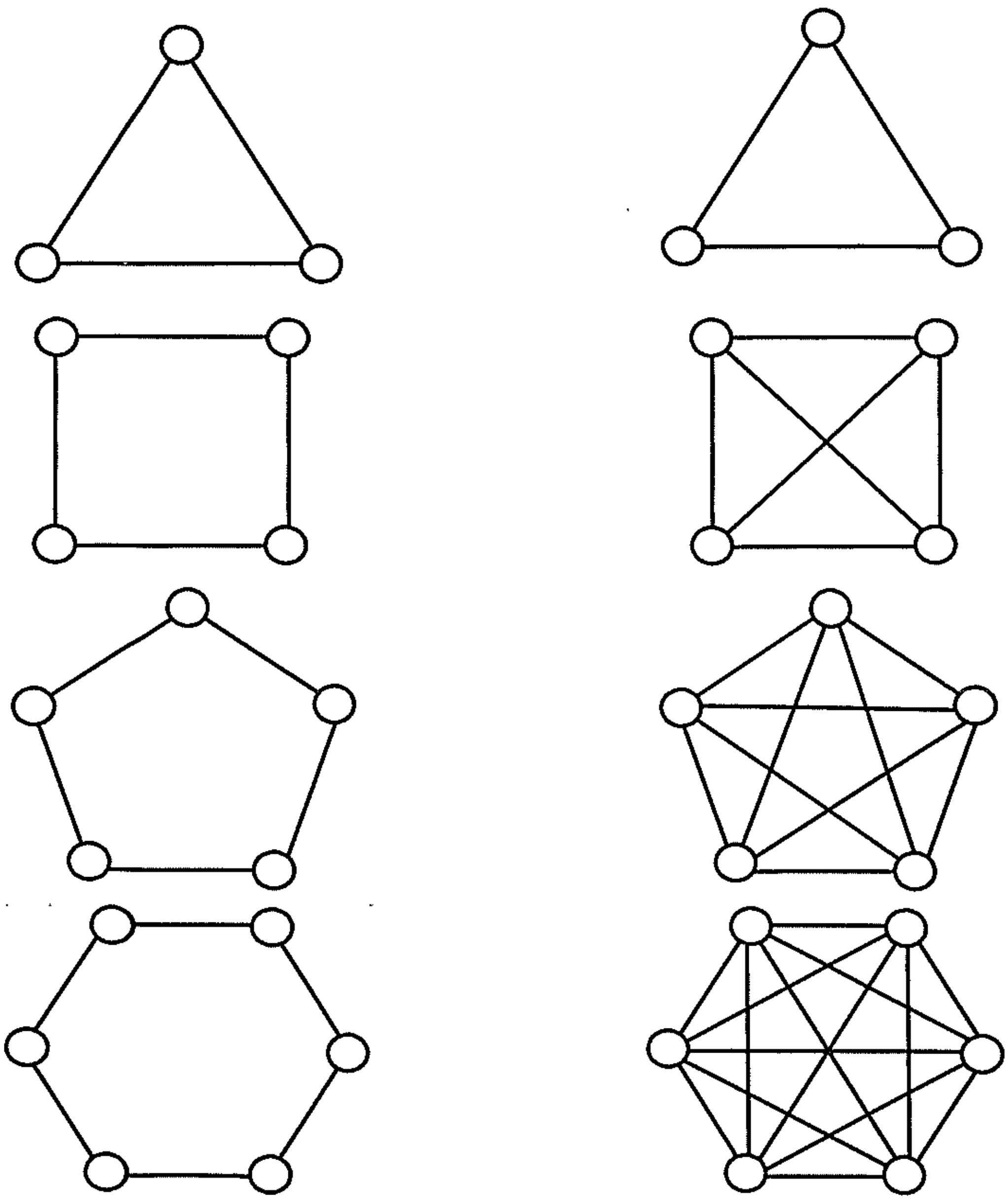
Can you draw the graph which corresponds to this figure?

How many different colors do you need?

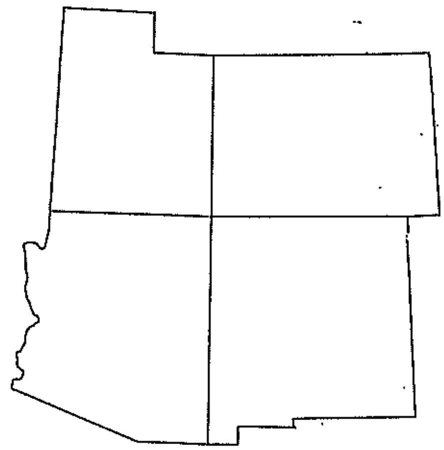


Hand-out: Vertex coloring of graphs

How many colors are needed for a vertex coloring of each of the following graphs? Do you see any patterns in your solutions?

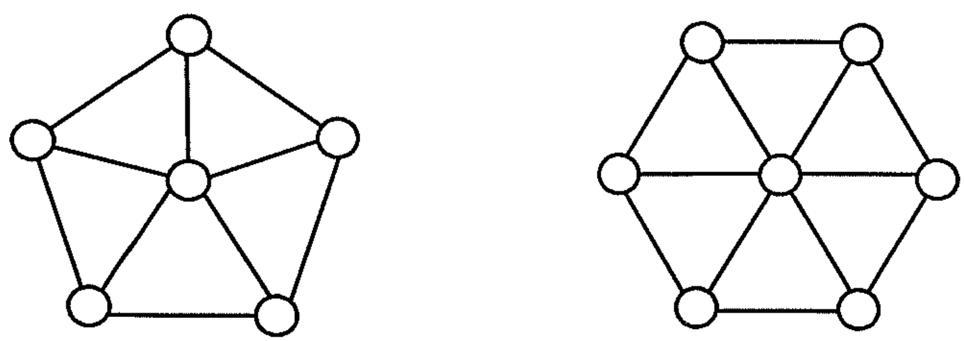


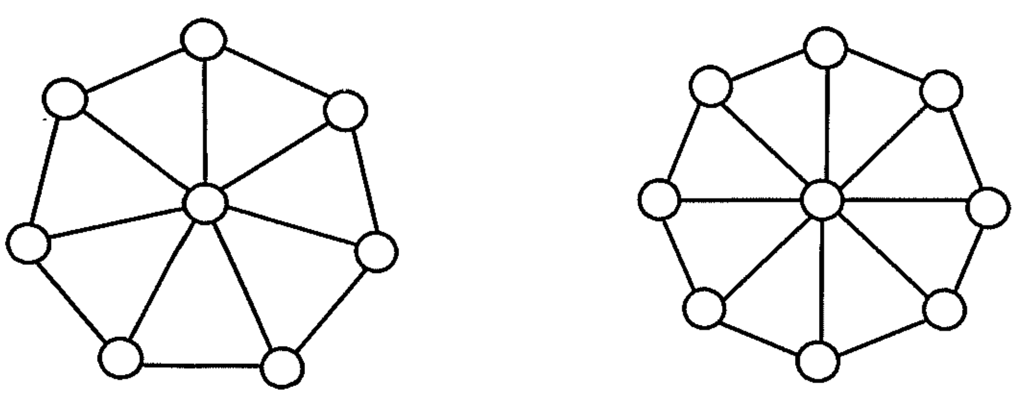
Three Sections of the USA Map



Vertex coloring of wheels

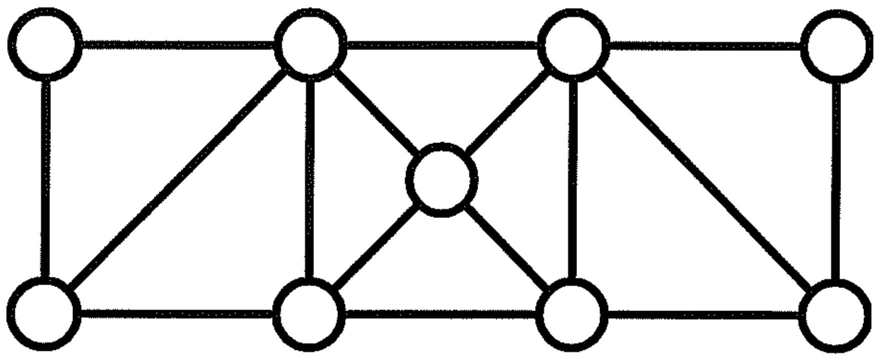
How many colors are needed for a vertex coloring of each of the following wheels? Do you see any patterns in your solutions?



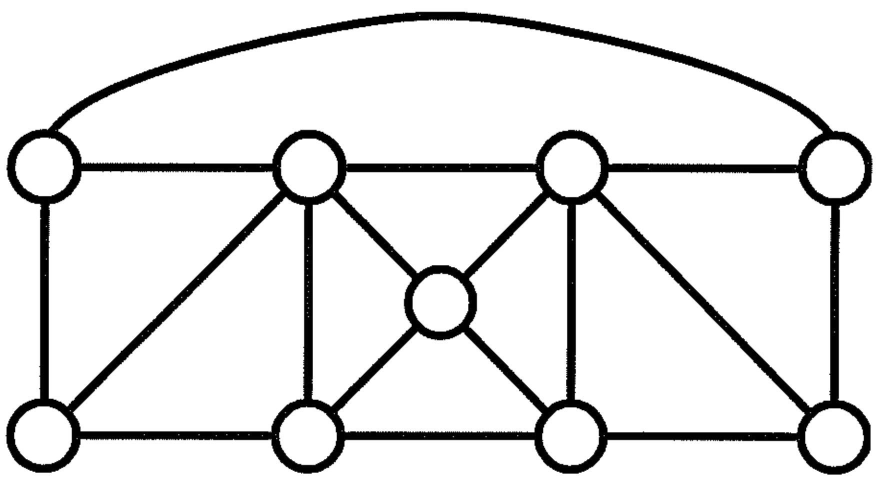


How many colors?

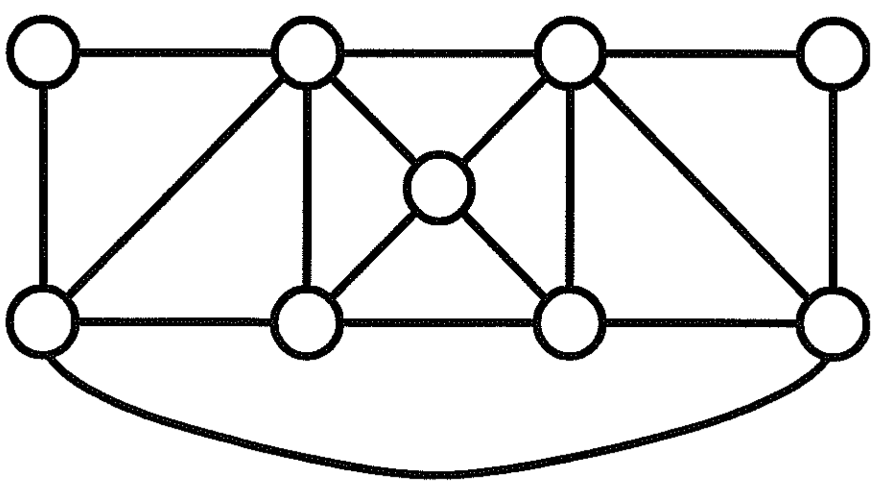
A.



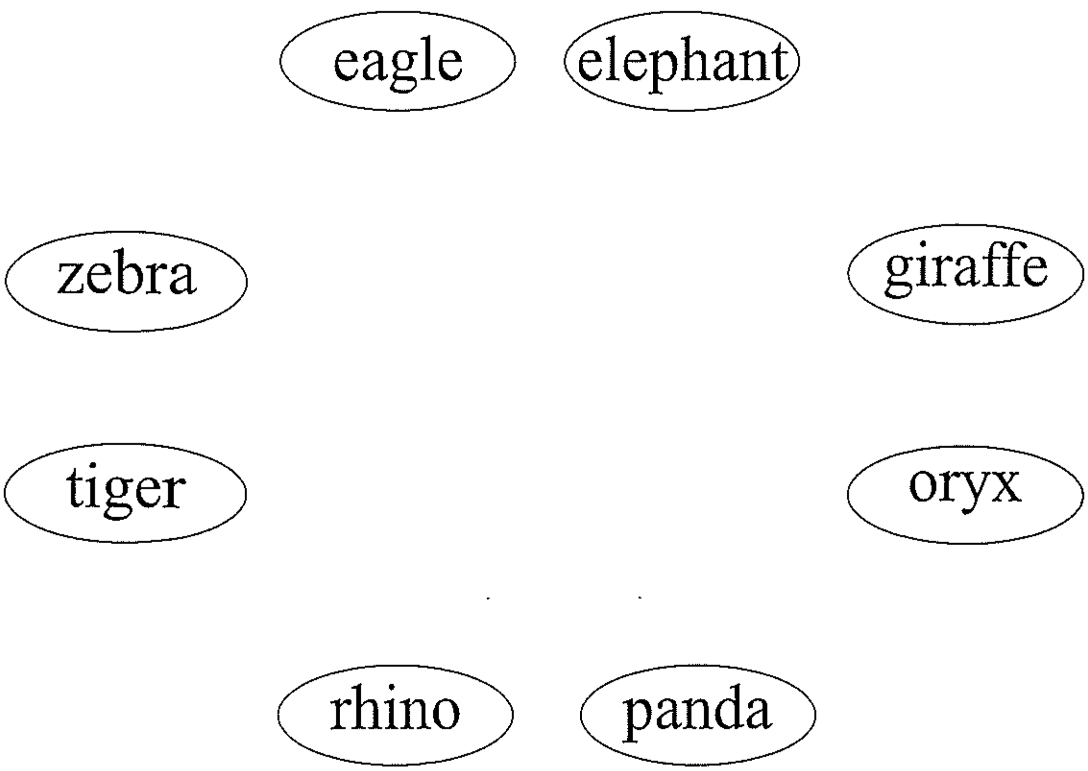
B.



C.

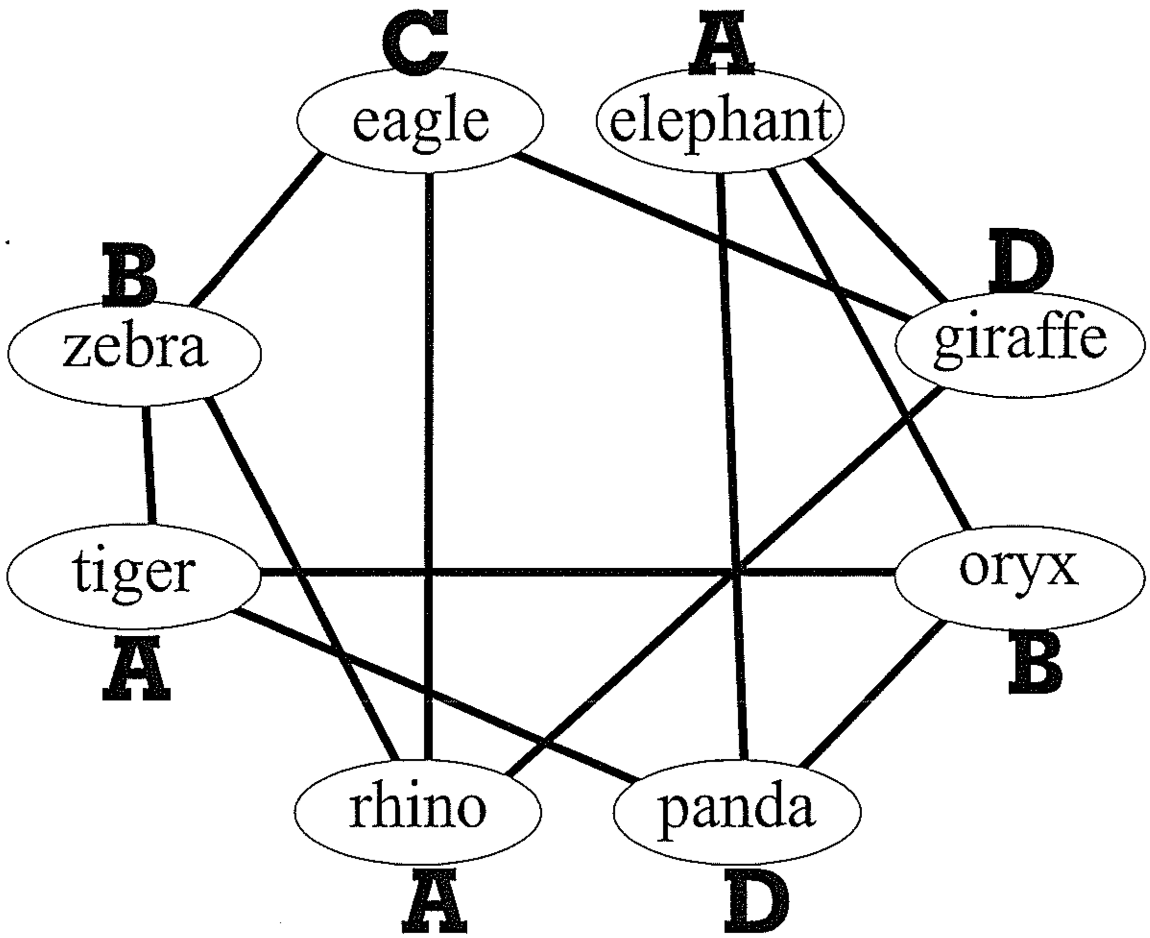


ANIMALS	CONFLICTS
Tiger	Oryx, Zebra, Panda
Oryx	Elephant, Panda, Tiger
Zebra	Tiger, Eagle, Rhino
Giraffe	Eagle, Rhino, Elephant
Elephant	Giraffe, Panda, Oryx
Panda	Tiger, Elephant, Oryx
Rhino	Giraffe, Zebra, Eagle
Eagle	Rhino, Giraffe, Zebra



In the zoo problem, a proposed solution involved four habitats, labeled A,B,C,D on the graph below.

Can you find a solution which requires fewer habitats?

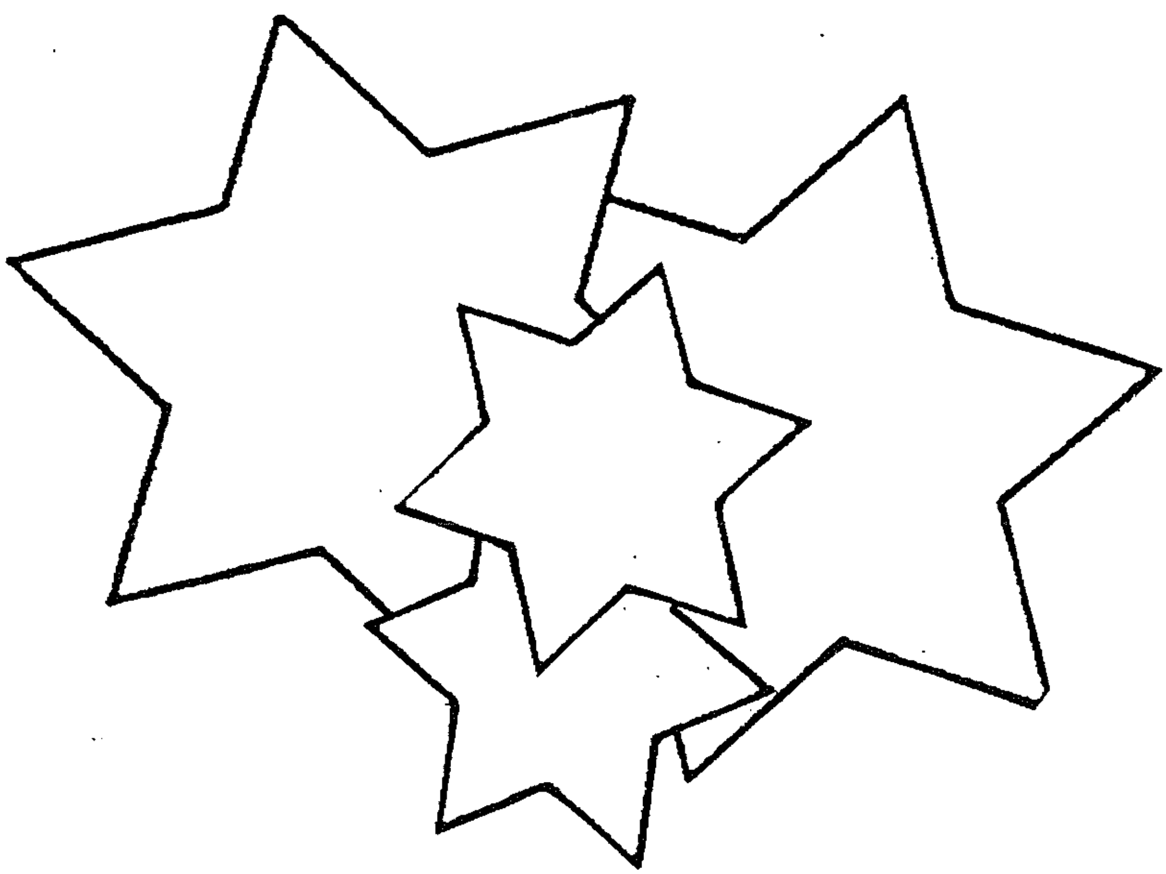


How do you know you can't use still fewer?

Stars

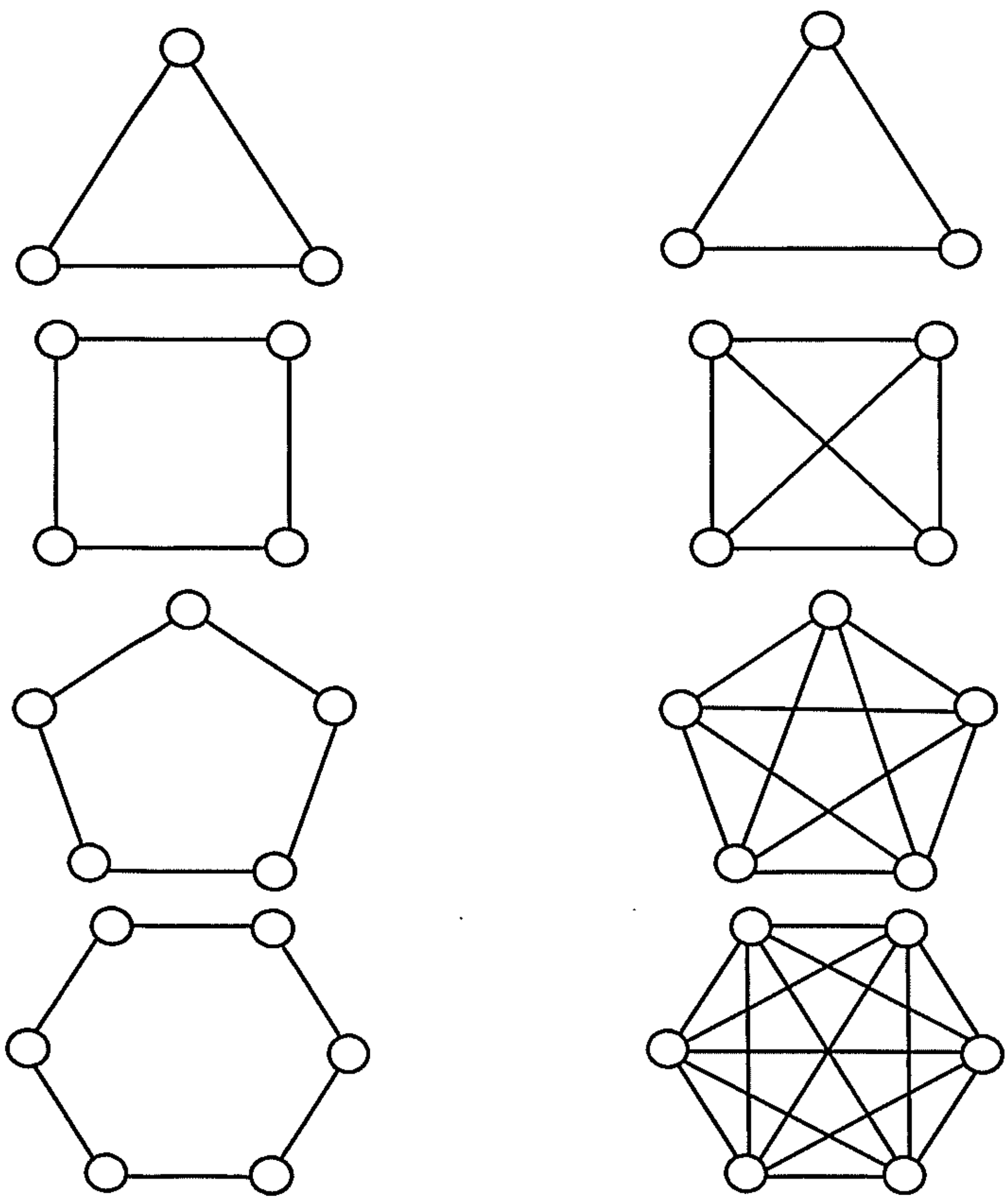
Can you draw the graph which corresponds to this figure?

How many different colors do you need?

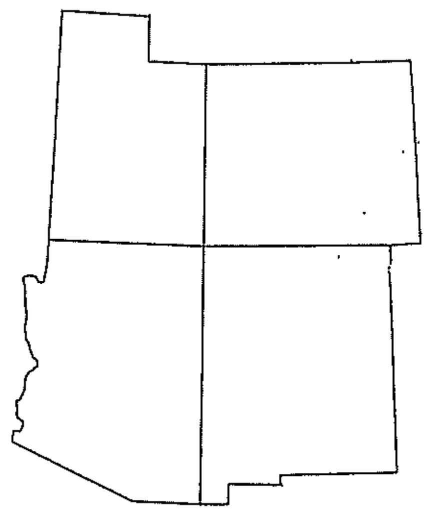


Hand-out: Vertex coloring of graphs

How many colors are needed for a vertex coloring of each of the following graphs? Do you see any patterns in your solutions?



Three Sections of the USA Map

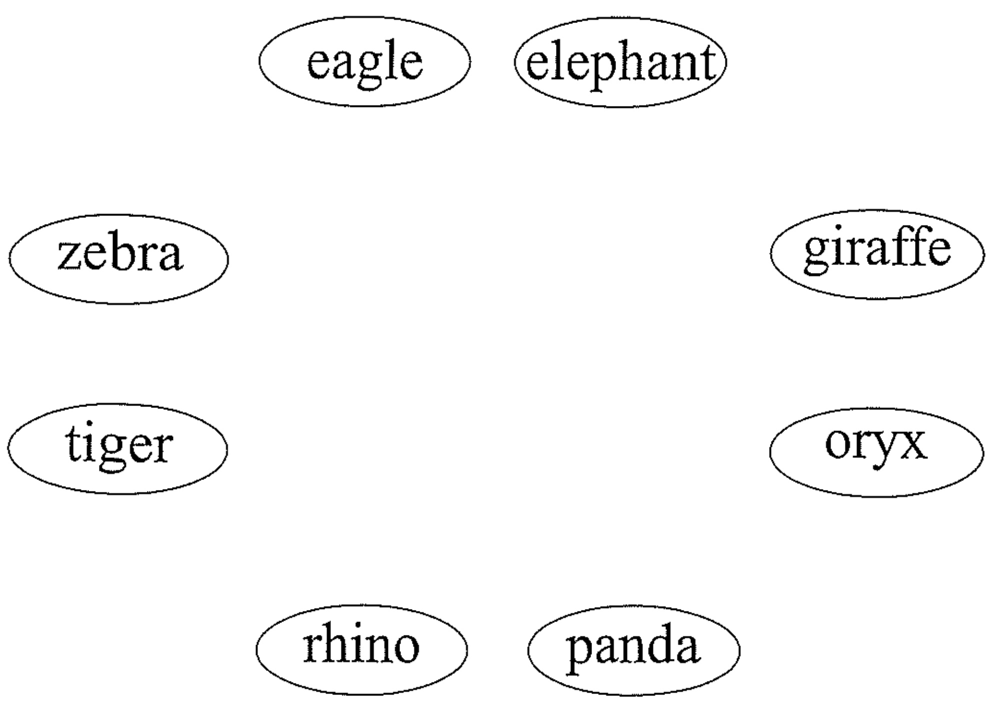


Handout: The Zoo Problem

Create the graph for the zoo problem by inserting edges between two animals if they have a conflict.

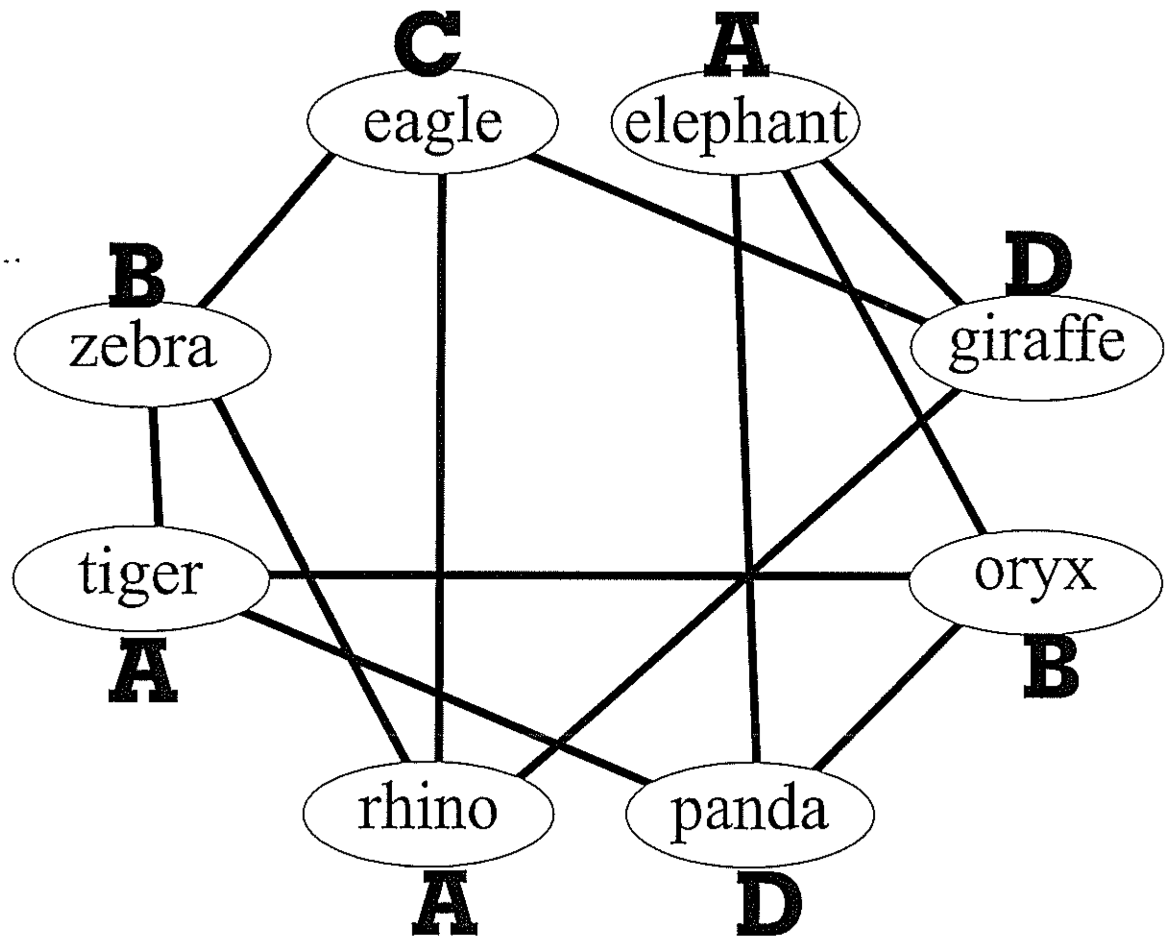
The resident zoologist has given us the following information:

ANIMALS	CONFLICTS
Tiger	Oryx, Zebra, Panda
Oryx	Elephant, Panda, Tiger
Zebra	Tiger, Eagle, Rhino
Giraffe	Eagle, Rhino, Elephant
Elephant	Giraffe, Panda, Oryx
Panda	Tiger, Elephant, Oryx
Rhino	Giraffe, Zebra, Eagle
Eagle	Rhino, Giraffe, Zebra



Handout: The Zoo Problem

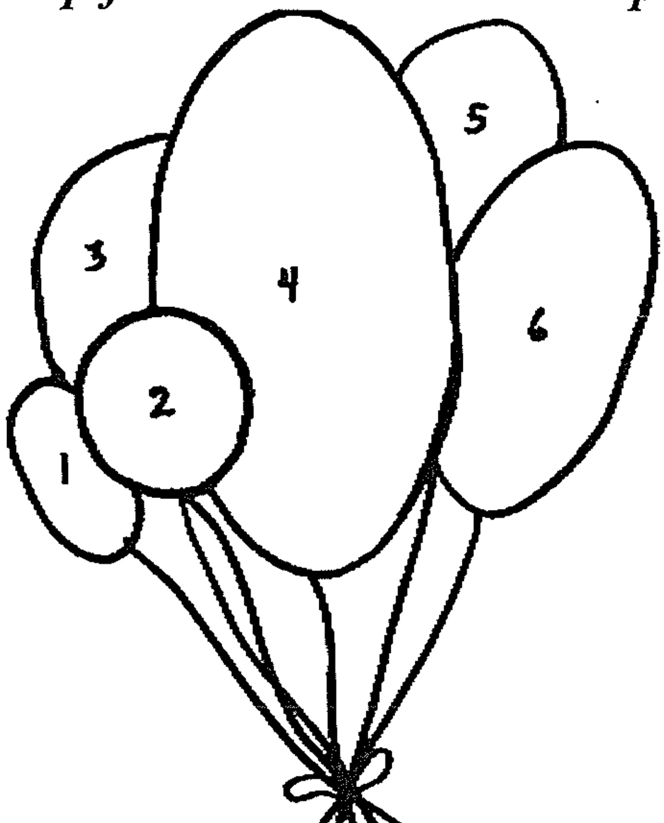
The zoo planners have proposed a solution which involves four habitats, labeled A,B,C,D on the graph below. Can you find a solution which requires fewer habitats? How do you know you can't use still fewer?



Exploring Discrete Mathematics in the Classroom

Colorful Solutions: Graph Coloring K-4

A Workshop for Teachers About Graph Coloring



Developed by Joseph G. Rosenstein and Valerie A. DeBellis

in collaboration with the following participants in the Rutgers Leadership Program in Discrete Mathematics: Pat Eisemann, Razia Hassan, and Ann Lawrence

Basic Graph Theory

Vocabulary Review from K-4 Coloring Workshop

graph — A **graph** is a collection of points (or circles) some of which are joined by lines or curves. These are called "**vertices**" and "**edges**". (The singular of "vertices is "vertex", not "vertice".) Each edge joins two different vertices. A given pair of vertices may or may not be joined by an edge.

neighbor — Vertices joined by an edge are called "**adjacent**" or "**neighbors**".

cycle — A "**cycle**" is a graph where the vertices can be arranged in a circular fashion so that each vertex is adjacent to the two vertices which come before and after it in the circle.

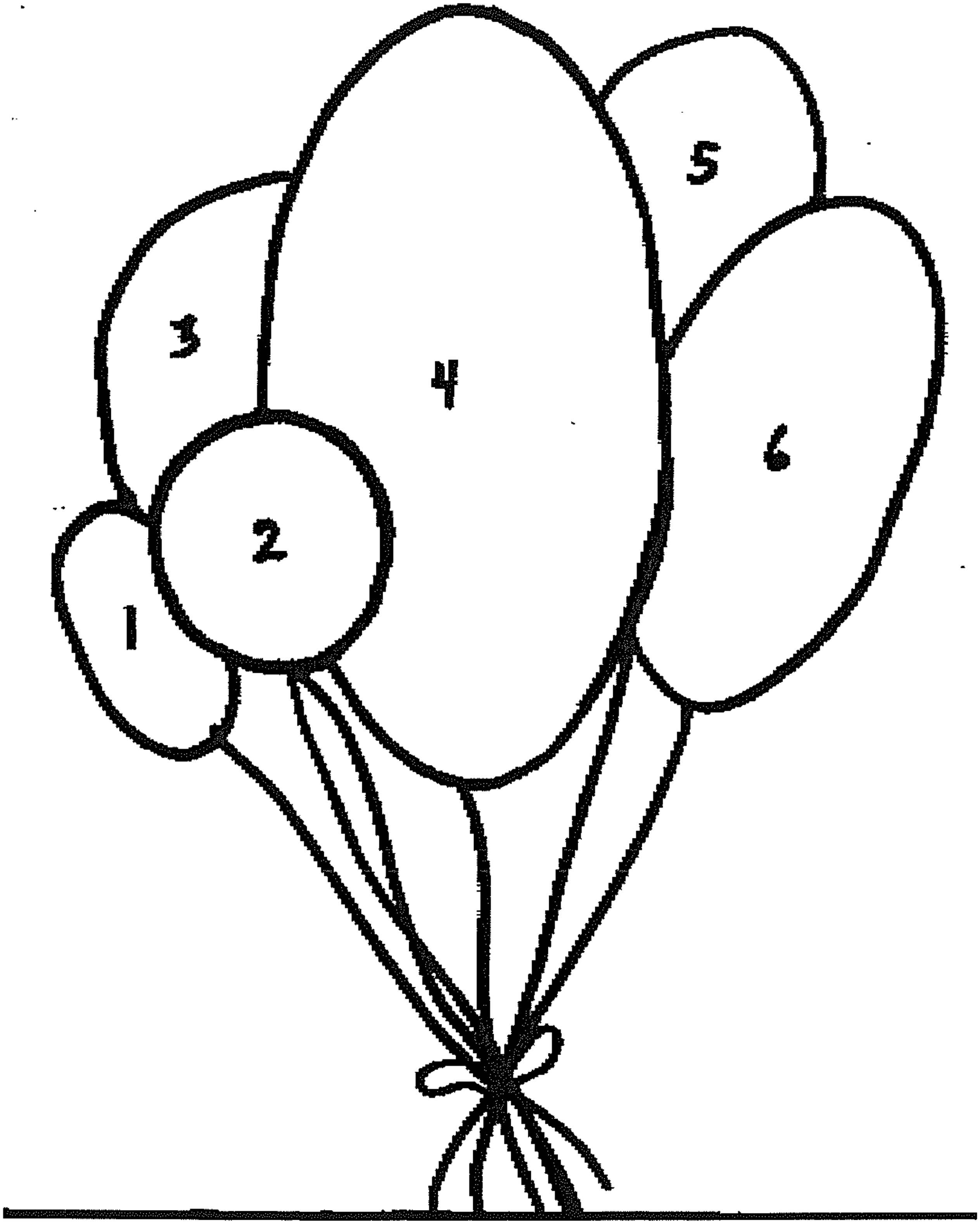
complete graph — A "**complete graph**" is a graph in which every vertex is adjacent to every other vertex.

A "**coloring of a map**" involves assigning colors to the countries of a map so that countries with a common border are assigned different colors.

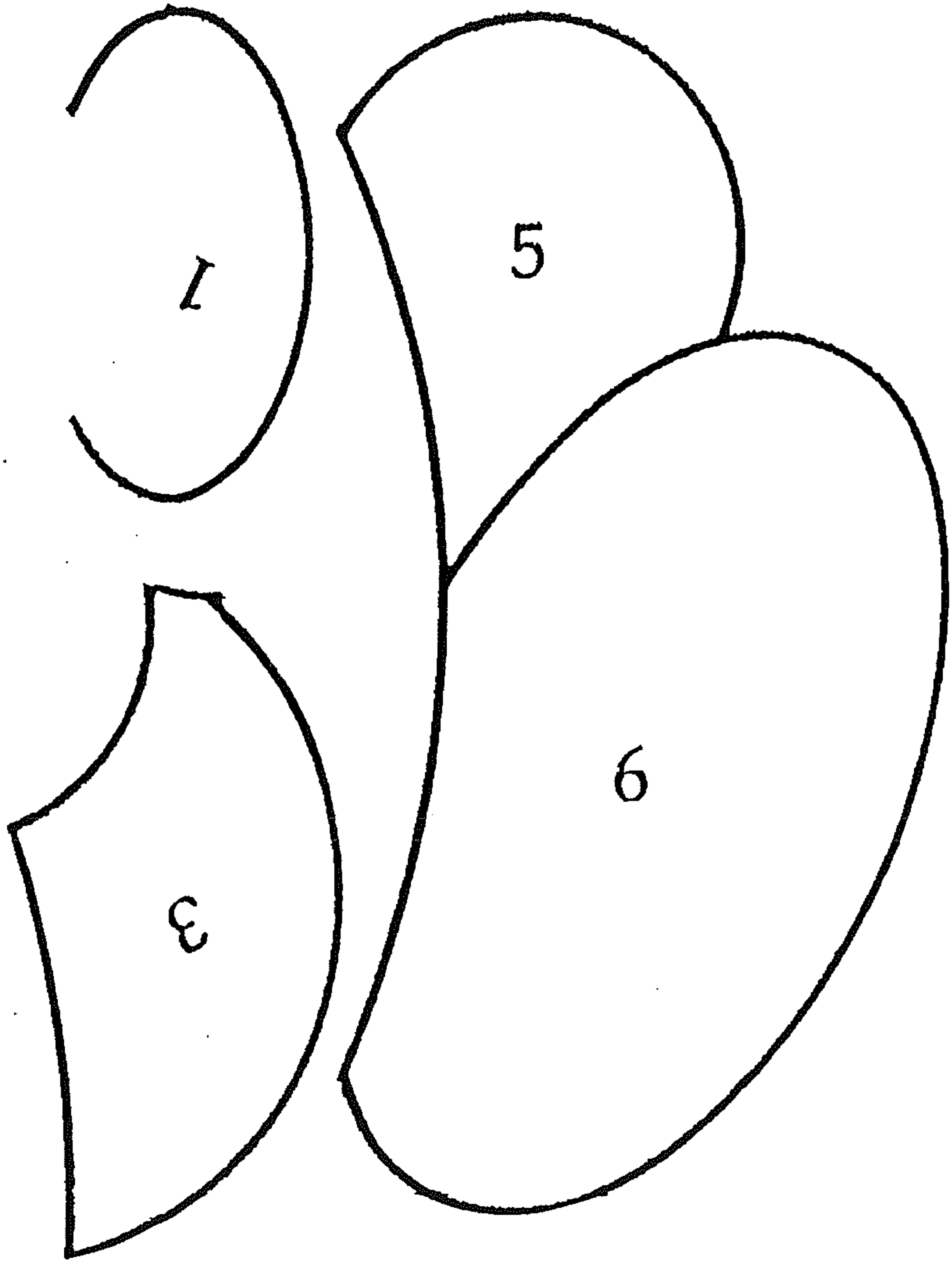
A "**coloring of a graph**" involves assigning colors to the vertices of a graph so that adjacent vertices are assigned different colors.

chromatic number — The "**chromatic number of a graph**"is the smallest number of colors that can be used for a coloring of the graph.

Vertex colorings of graphs can be used to solve a variety of problems which involve "conflict". In a situation involving maps, two countries are "in conflict" if they share a border; we resolve the conflict by assigning conflicting countries different colors. In a situation involving school club scheduling, two clubs are "in conflict" if they share a member; we resolve the conflict by assigning conflicting clubs different meeting times.



5
3
4
6
2
1


1
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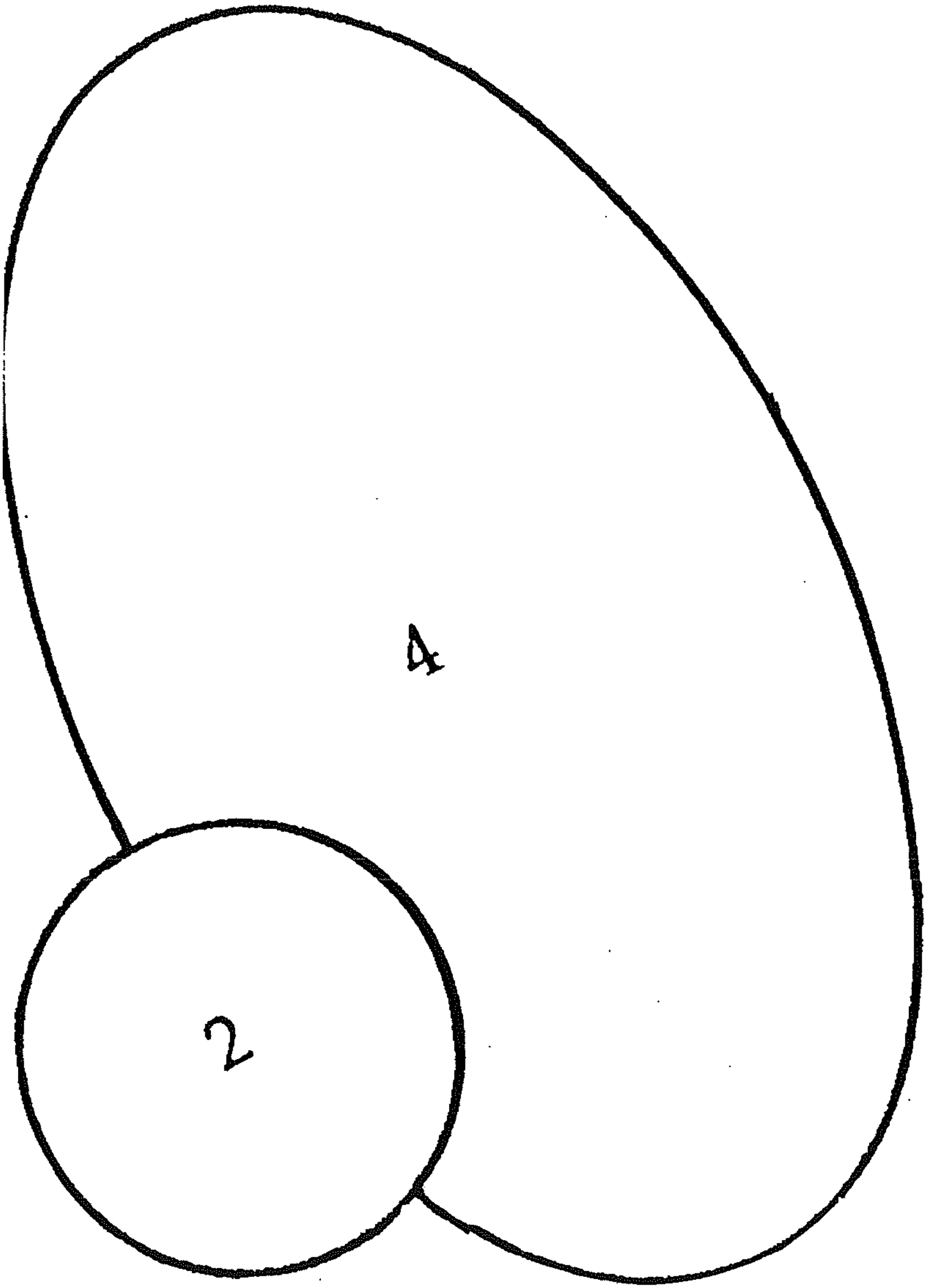
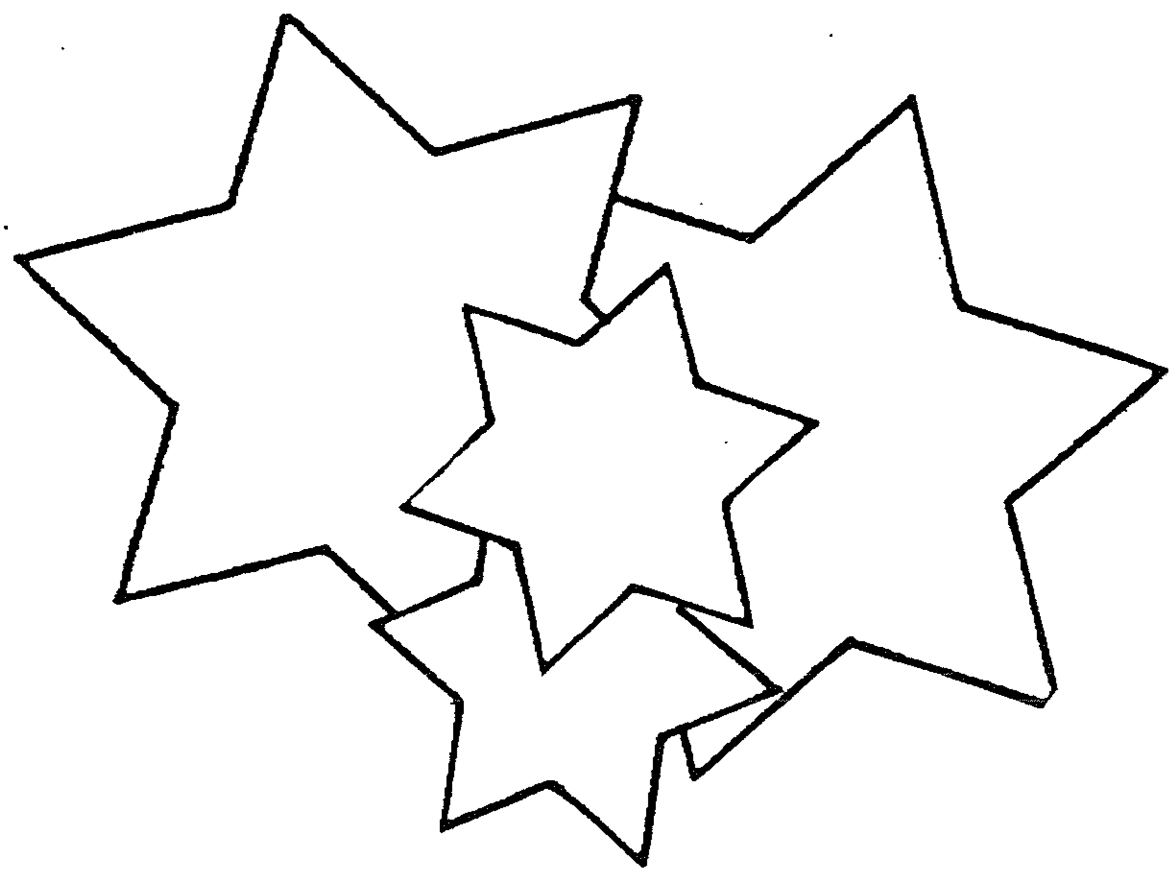
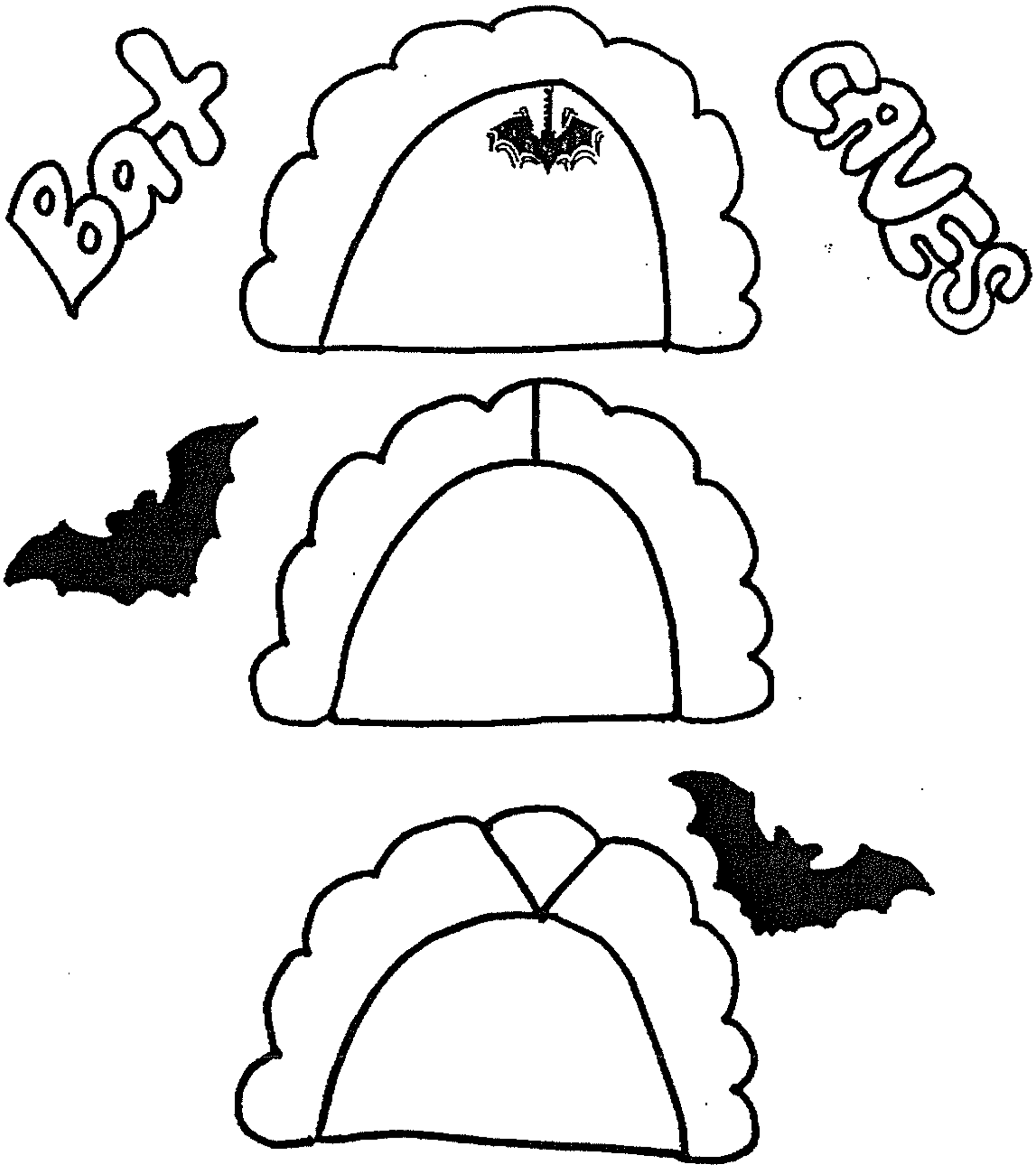

4
2

Stars

Can you draw the graph which corresponds to this figure?



How many different colors do you need?



COLOR EACH BAT CAVE USING AS FEW COLORS AS POSSIBLE.

RULE: NO TWO AREAS WHICH SHARE THE SAME BORDER (EDGE) CAN BE THE SAME COLOR.

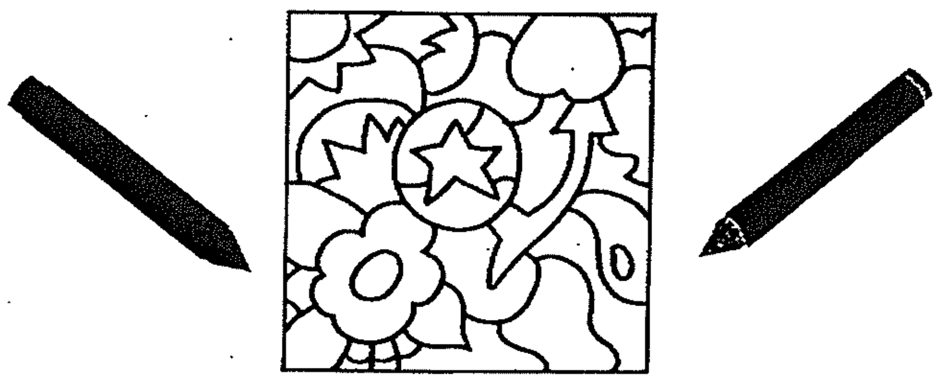
Meyers, Barbara (1997) By the Light of the Moon with Discrete Math. Ft. Worth, TX — Bat Cave

Map Games

Here are some map games that can be played by any number of people using any size sheet of paper. Get a friend and try these games using our game boards; then make some of your own and throw a party.

THE BOARD

Each game board is a mathematical map like the ones you've been working with in this chapter.



THE PLAYERS

The two-color game can be played by any even number of players divided into two teams.

The Big Rule:

- No two countries that share a border may be the same color (though two countries of the same color may touch at a point).

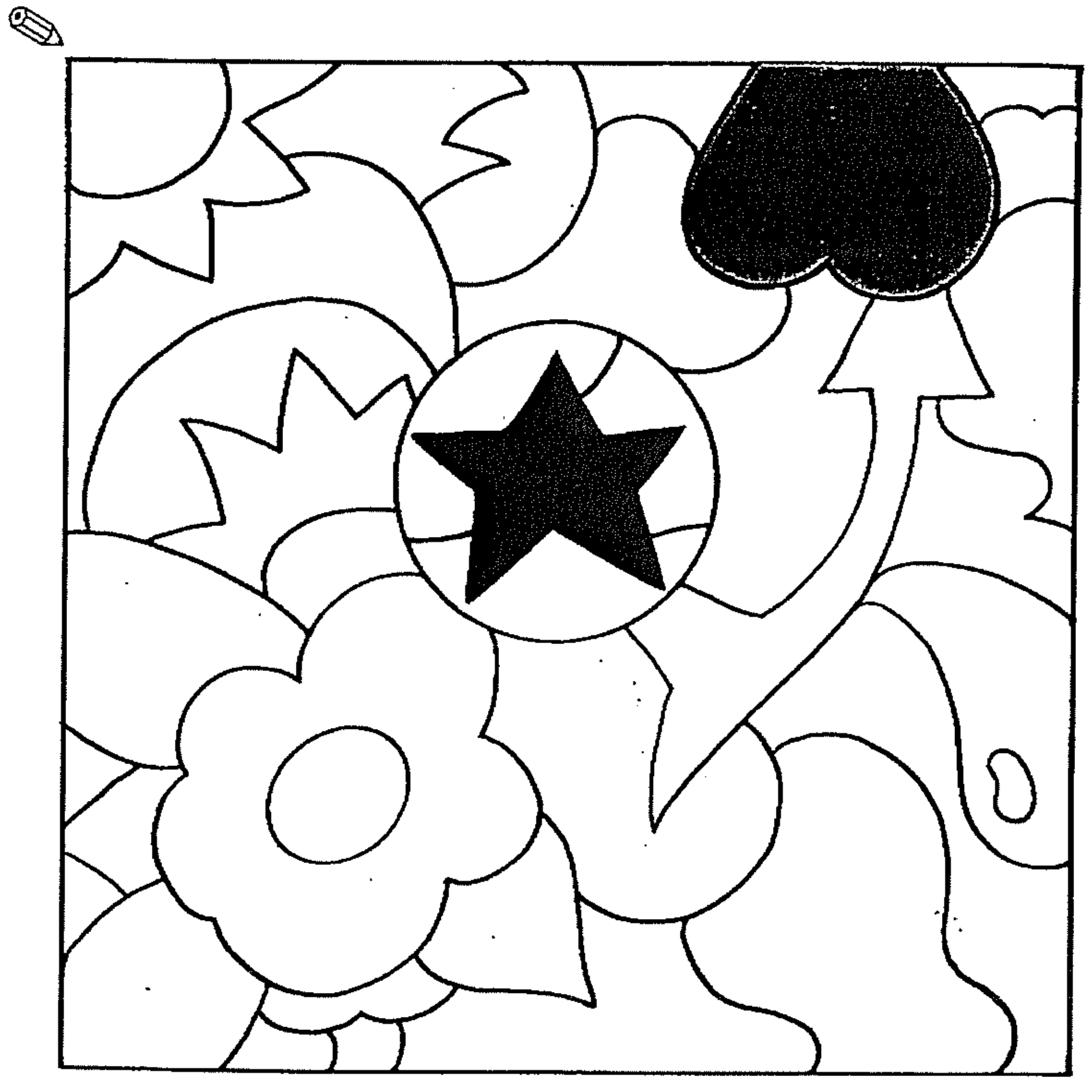
THE PLAY

1. Each team chooses a color.

2. Flip a coin to decide which team goes first.

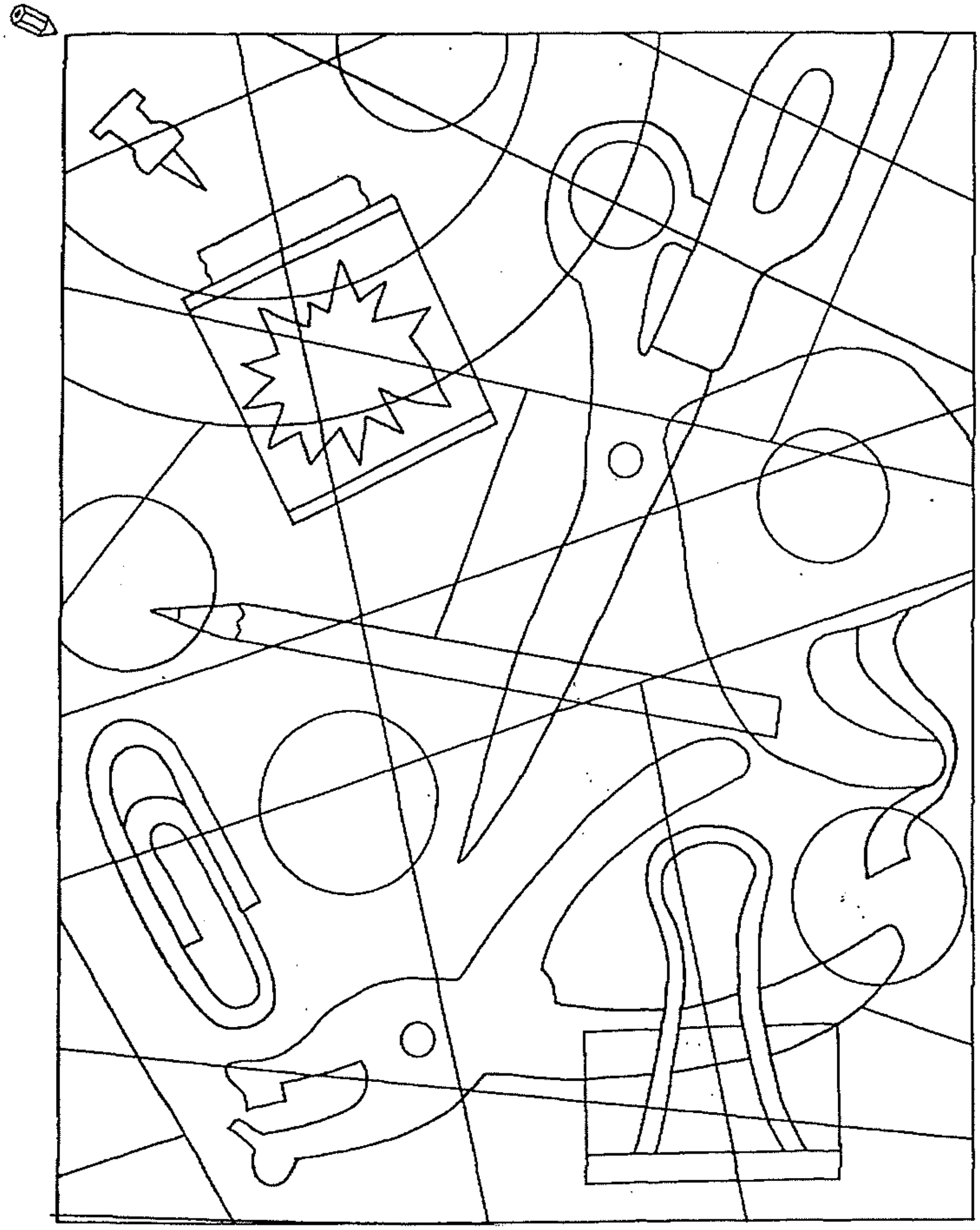
Kohl, Herbert (1995) Insides, Outsides, Loops and Lines. New York, NY: Freeman Pub.

Find a worthy opponent, and finish up this game. It's the lighter color team's turn.



Kohl, Herbert (1995) Insides, Outsides, Loops and Lines. New York, NY: Freeman Pub.

Now try this board:



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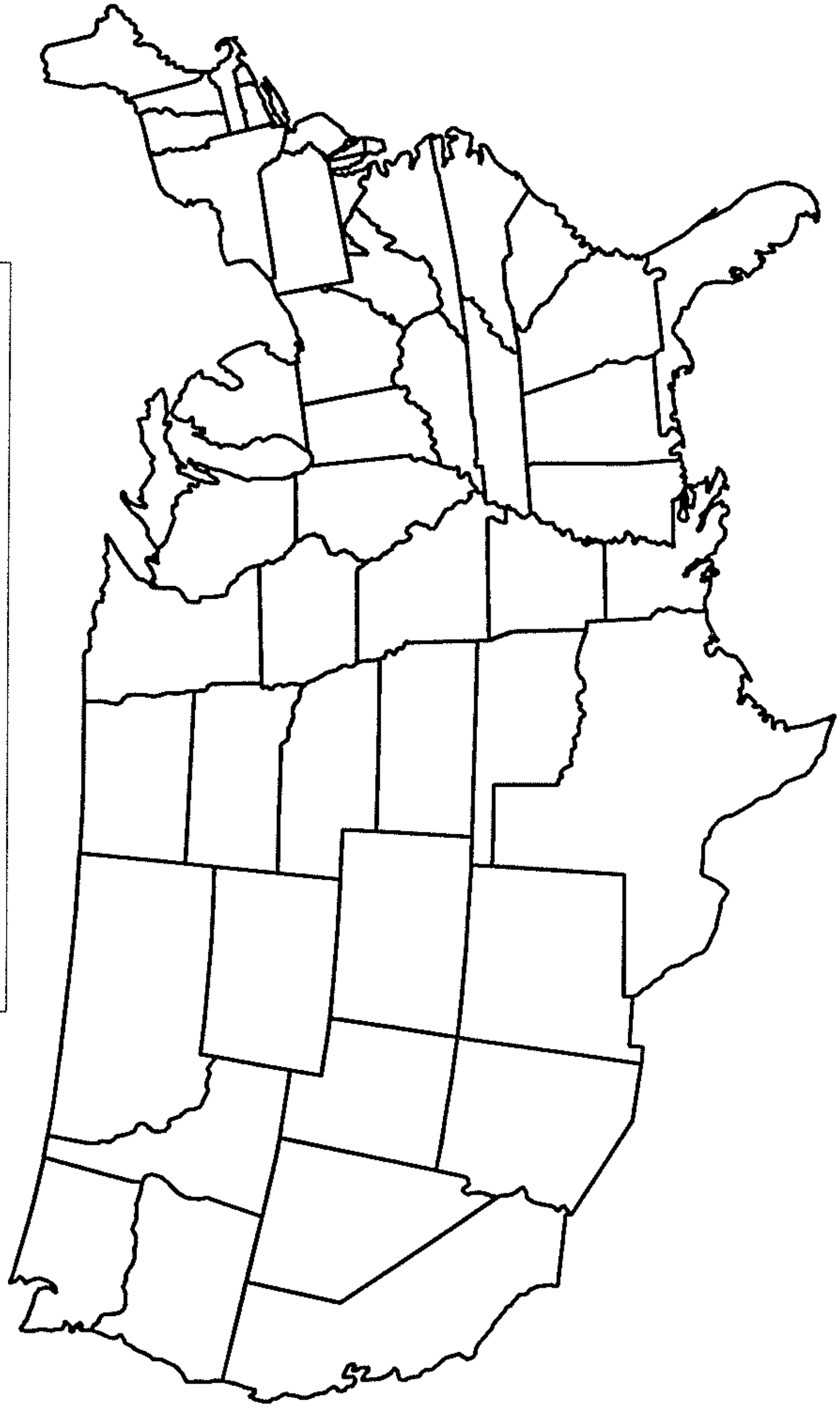
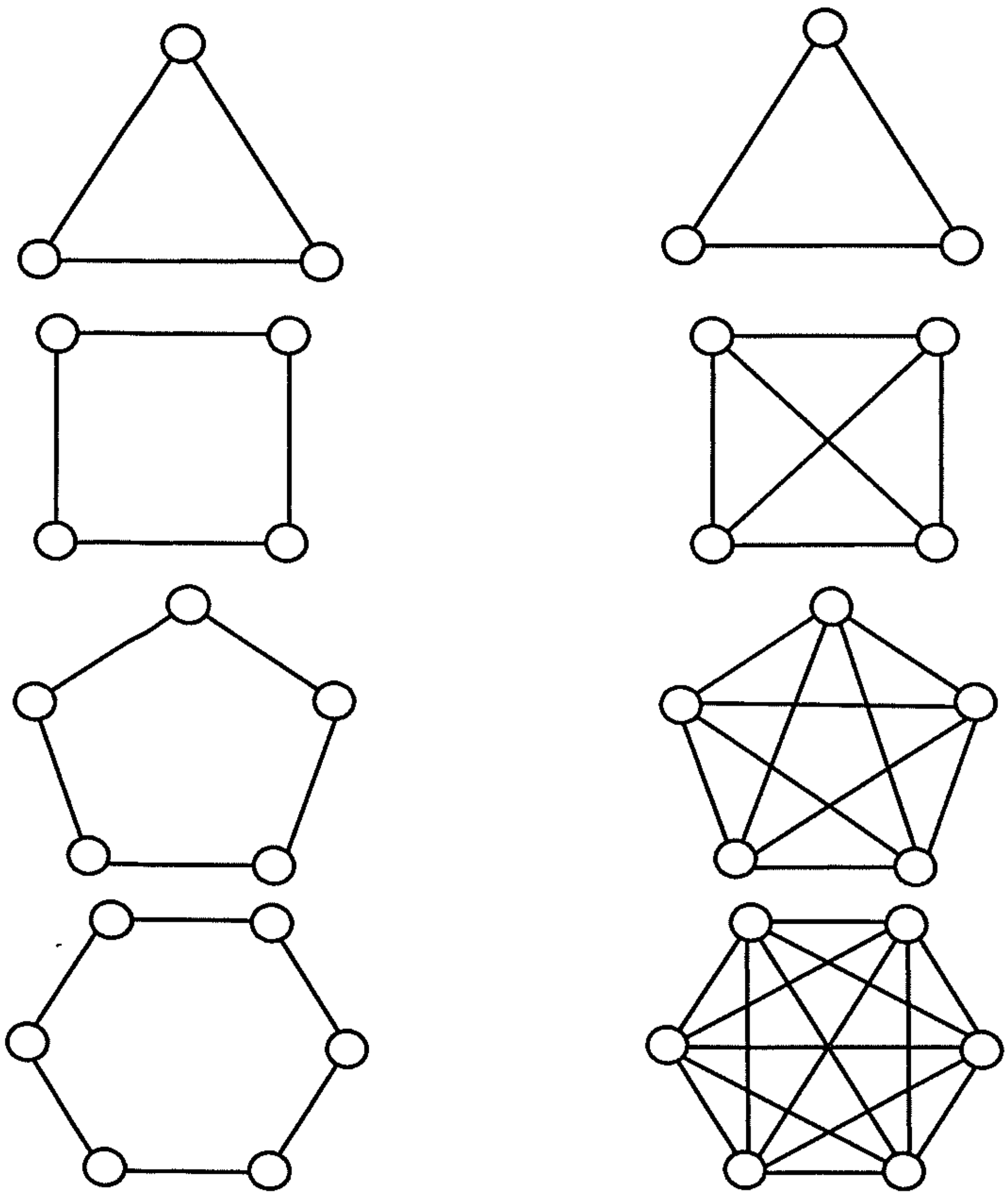
United States



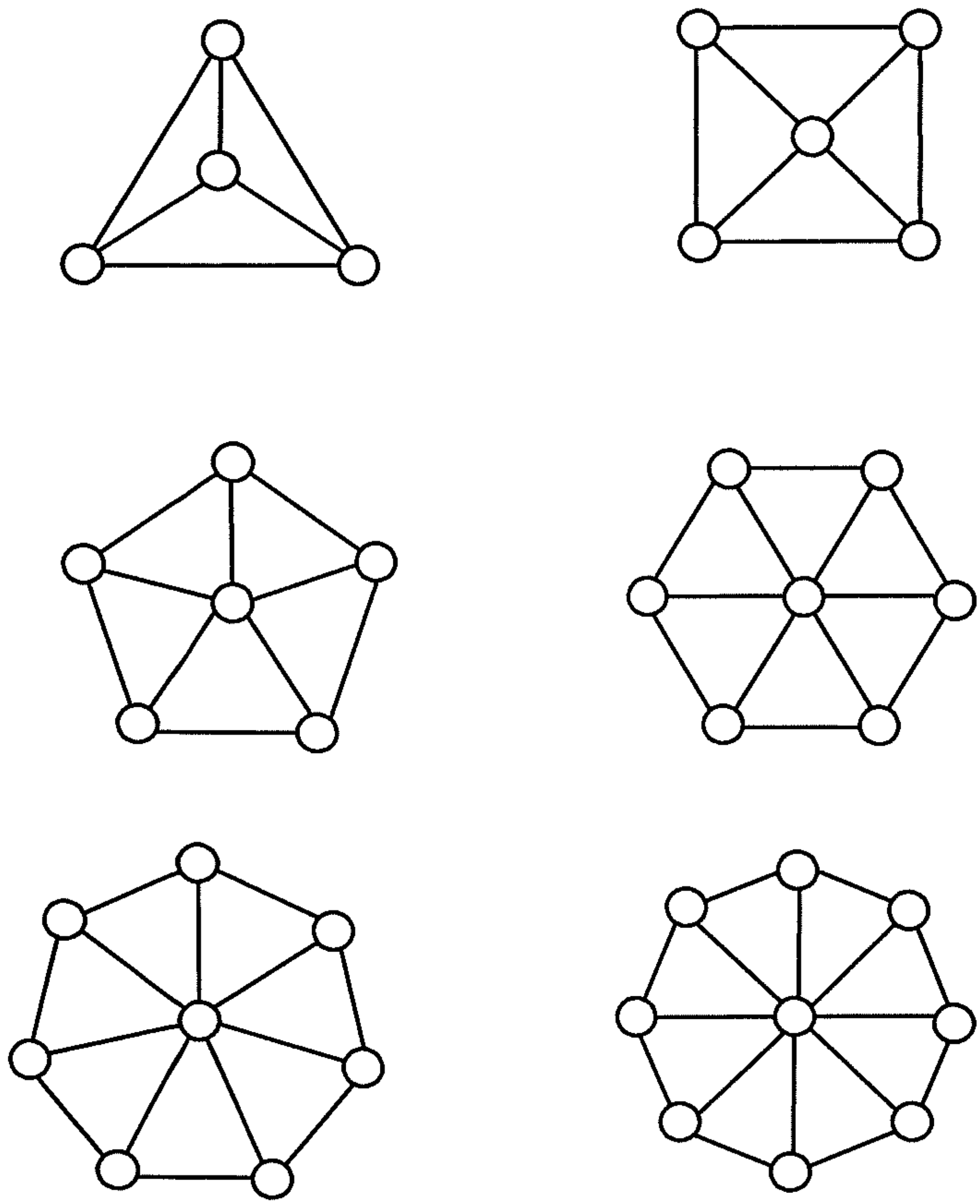
Hand-out: Vertex coloring of graphs

How many colors are needed for a vertex coloring of each of the following graphs? Do you see any patterns in your solutions?



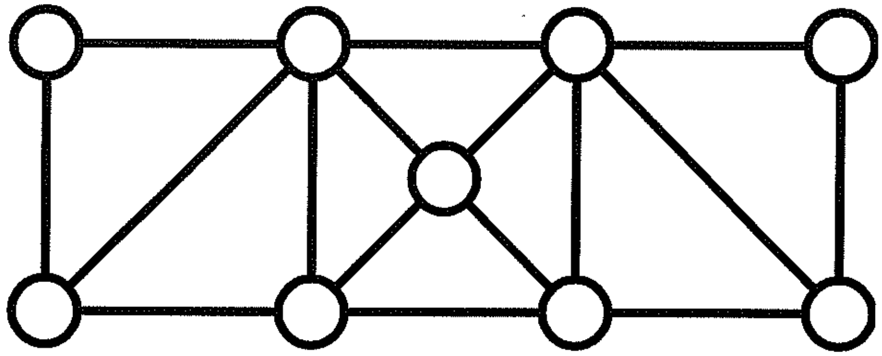
Hand-out: Vertex coloring of wheels

How many colors are needed for a vertex coloring of each of the following wheels? Do you see any patterns in your solutions?

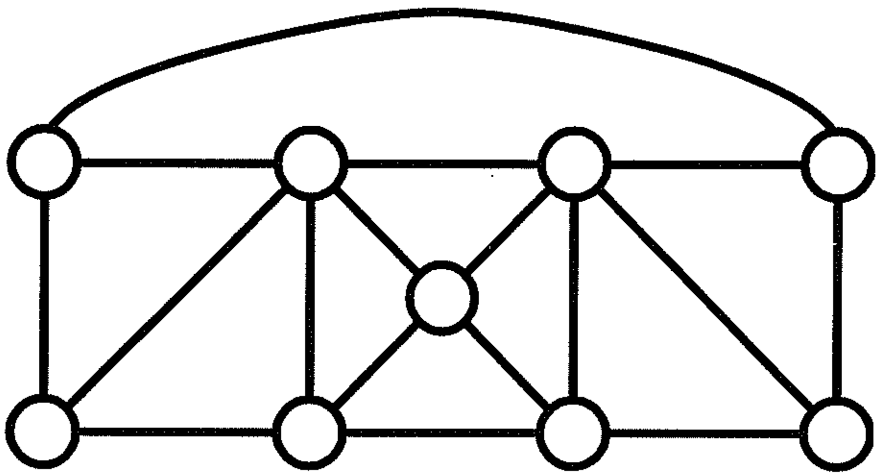


How many colors?

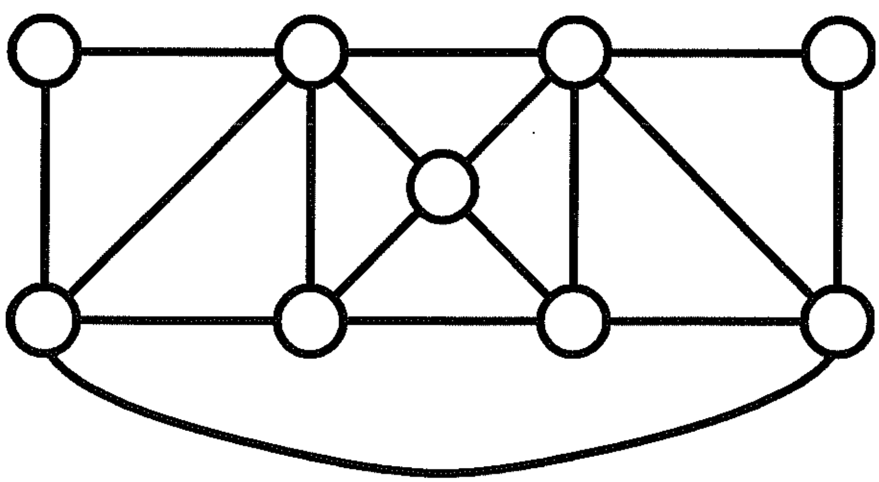
A.



B.



C.



THE ZOO PROBLEM

Taken from COMAP's - Geometry: New Tools for New Technologies

There are eight different animals in the zoo:

Amanda the Tiger
Candy the Oryx
Debbie the Zebra
George the Giraffe
Kanga the Elephant
Maribelle the Panda
Stefan the Rhino
Winifred the American Bald Eagle

One of the zoo planners thinks that he has found a way to place these 8 animals into 4 new habitats with no conflicts. With each new multi-species habitat costing as much as two million dollars, can the number of habitats be reduced?

The resident zoologist has provided the following list of the eight animals and their conflicts:

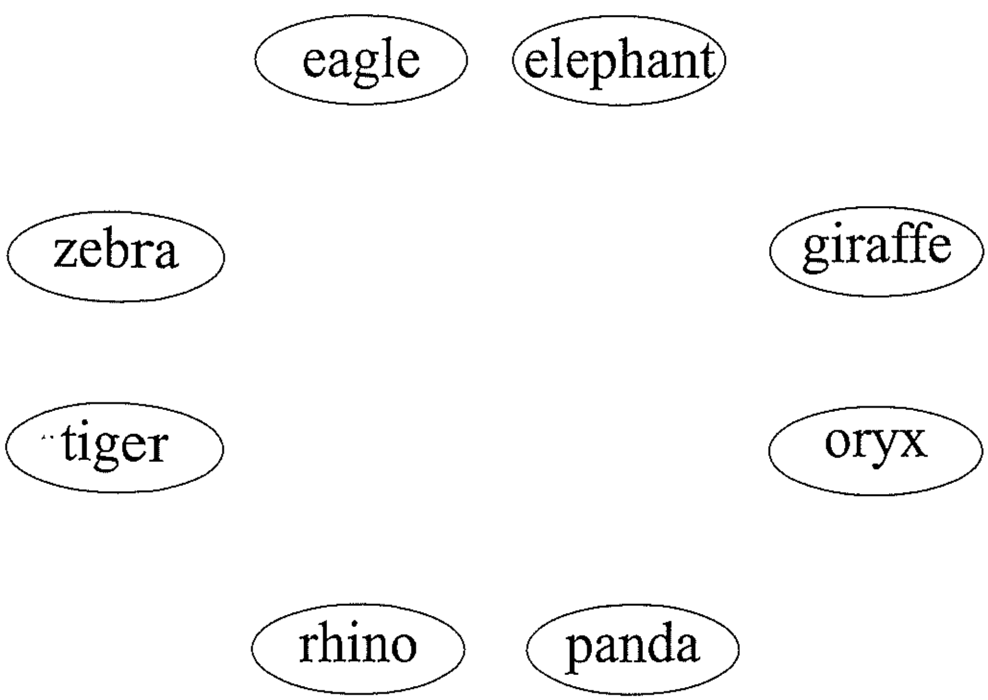
ANIMALS	CONFLICTS
Tiger	Oryx, Zebra, Panda
Oryx	Elephant, Panda, Tiger
Zebra	Tiger, Eagle, Rhino
Giraffe	Eagle, Rhino, Elephant
Elephant	Giraffe, Panda, Oryx
Panda	Tiger, Elephant, Oryx
Rhino	Giraffe, Zebra, Eagle
Eagle	Rhino, Giraffe, Zebra

Handout: The Zoo Problem

Create the graph for the zoo problem by inserting edges between two animals if they have a conflict.

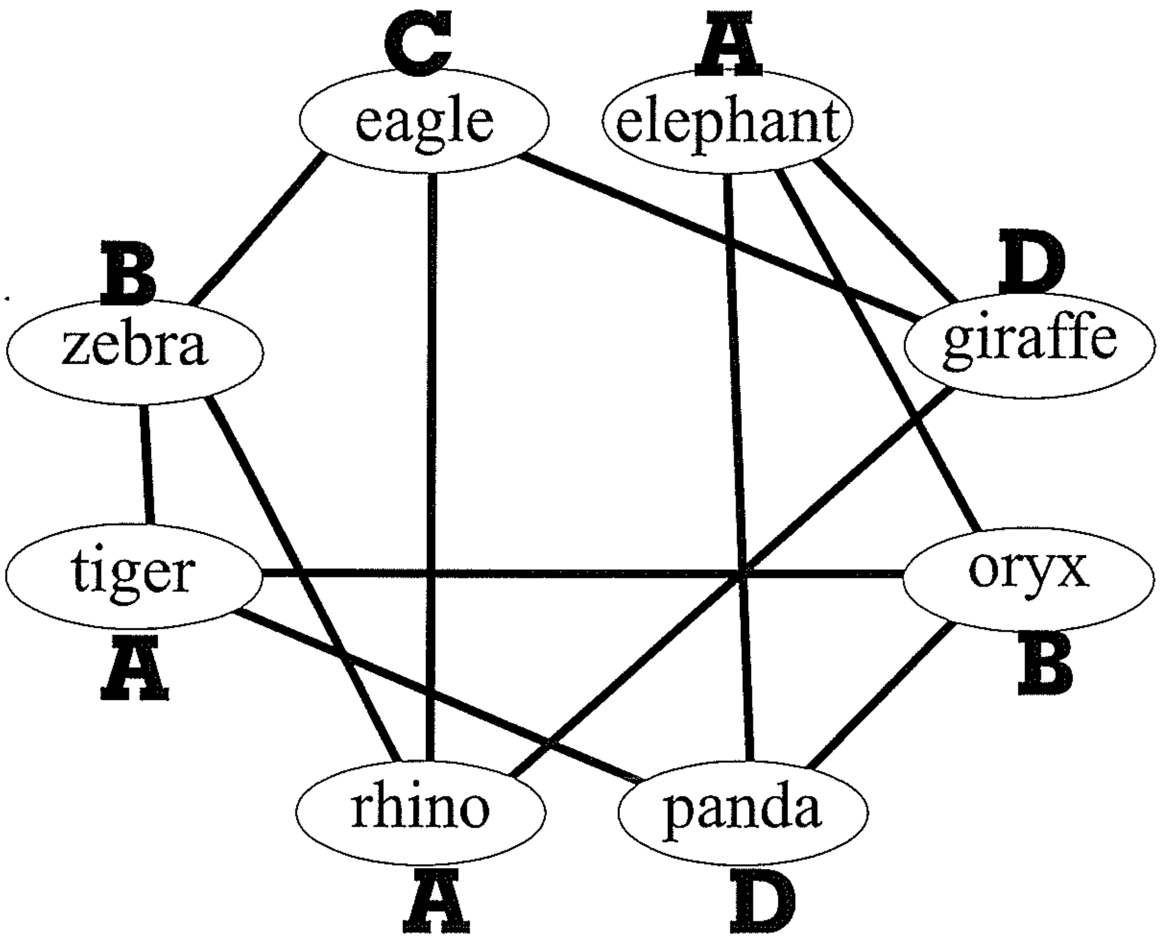
The resident zoologist has given us the following information:

ANIMALS	CONFLICTS
Tiger	Oryx, Zebra, Panda
Oryx	Elephant, Panda, Tiger
Zebra	Tiger, Eagle, Rhino
Giraffe	Eagle, Rhino, Elephant
Elephant	Giraffe, Panda, Oryx
Panda	Tiger, Elephant, Oryx
Rhino	Giraffe, Zebra, Eagle
Eagle	Rhino, Giraffe, Zebra



Handout: The Zoo Problem

The zoo planners have proposed a solution which involves four habitats, labeled A,B,C,D on the graph below. Can you find a solution which requires fewer habitats? How do you know you can't use fewer?



SCHOOL CLUB SCHEDULING CONFLICT

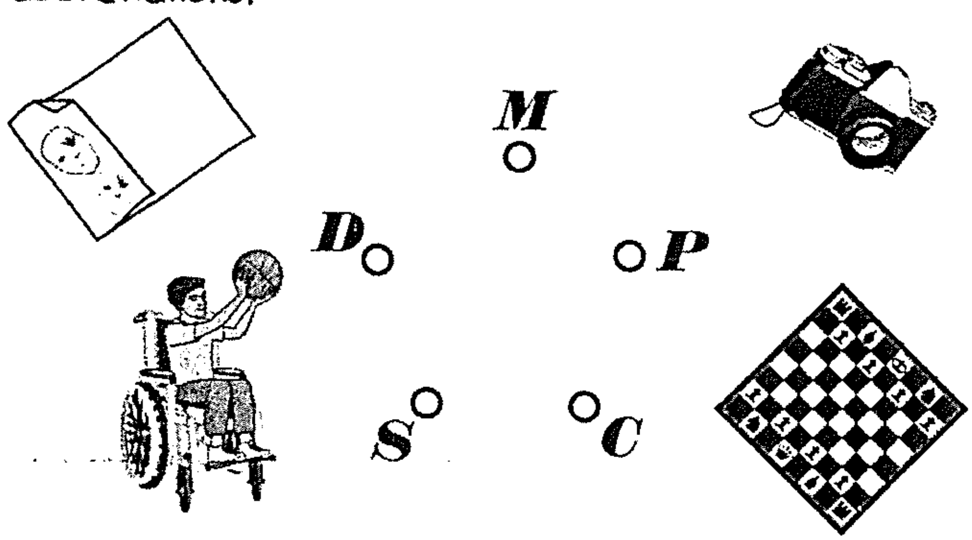
Several students in my class are involved in after school clubs. Each club can meet on Monday, Wednesday, or Friday. Is it possible to schedule the clubs so that each student is able to attend the clubs they have chosen? If so, how many of these days are necessary to schedule all the clubs?

Use the data below to construct a graph. Color the graph to solve the problem.

Amy	Photography Club, Special Olympics
Bob	Math Club, Drama Club
Chris	Chess Club, Special Olympics
David	Drama Club, Photography Club
Ed	Chess Club, Math Club

Each club will be represented by a vertex. An edge will join two vertices if the two clubs may not meet at the same time because someone is in both clubs.

For convenience, we will use the initial letter of each club as abbreviations.



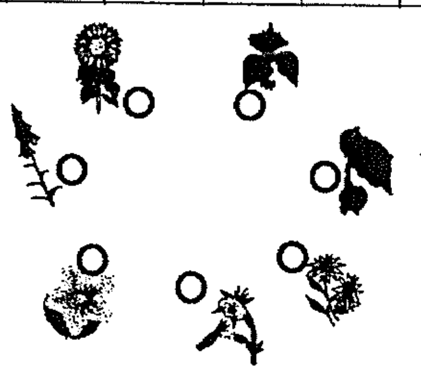
Errickson School Courtyard Problem

The courtyard at Errickson School has received some donations of plants from local nurseries. They want to cultivate as few plots as possible. There are varying amounts of sun, shade and moisture, and soil types in the courtyard.

Below is a chart showing all the donated plants. Each X represents the plants that cannot live well in the same plots because they require different growning conditions.

Create a graph which represents the data displayed in the chart. Let the vertices represent the plants. Draw edges connecting those plants which cannot be together. Color the graph to find the minimum number of plots needed.

Columbine	–	–	X	X	X	X	–
trillium	–	–	–	X	X	–	X
sunflower	X	–	–	X	X	–	–
foxglove	X	X	X	–	–	–	–
Chinese lanterns	X	X	X	–	–	X	X
brown-eyed Susan	X	–	–	–	X	–	–
balloon flower	–	X	–	–	X	–	–



THE SUPPLY SHIP PROBLEM

You are a supply engineer who is designing the inside of a cargo rocket to bring much needed supplies to the Mir space station. Below is a chart of the items being shipped. In order for you to design the inside storage spaces, you need to check the following chart to see which items can go in the same area, and which must be separated due to individual storage requirements.. Determine the fewest number of cargo areas you'll need so that you can complete the interior design.

	T	S	O	L	F	C
Tools	–	–	X	X	–	X
Space suits	–	–	X	X	–	X
Oxygen	X	X	–	X	X	X
Liquid drinks	X	X	X	–	X	–
Food packets	–	–	X	X	–	X
Chocolate	X	X	X	–	X	–