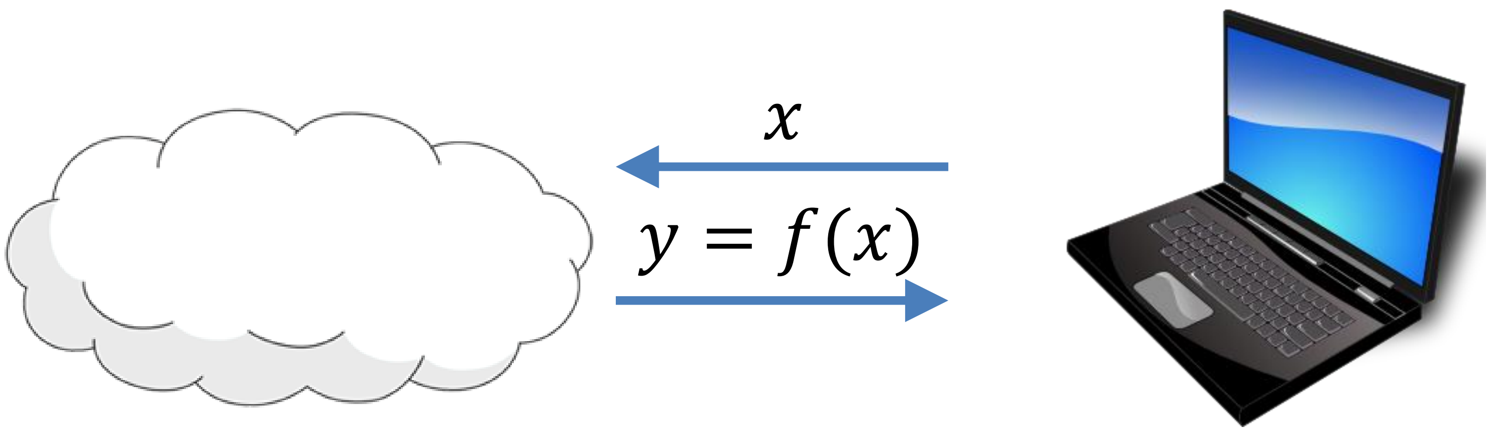
Doubly Efficient Interactive Proofs

Ron Rothblum

Massachusetts Institute of Technology

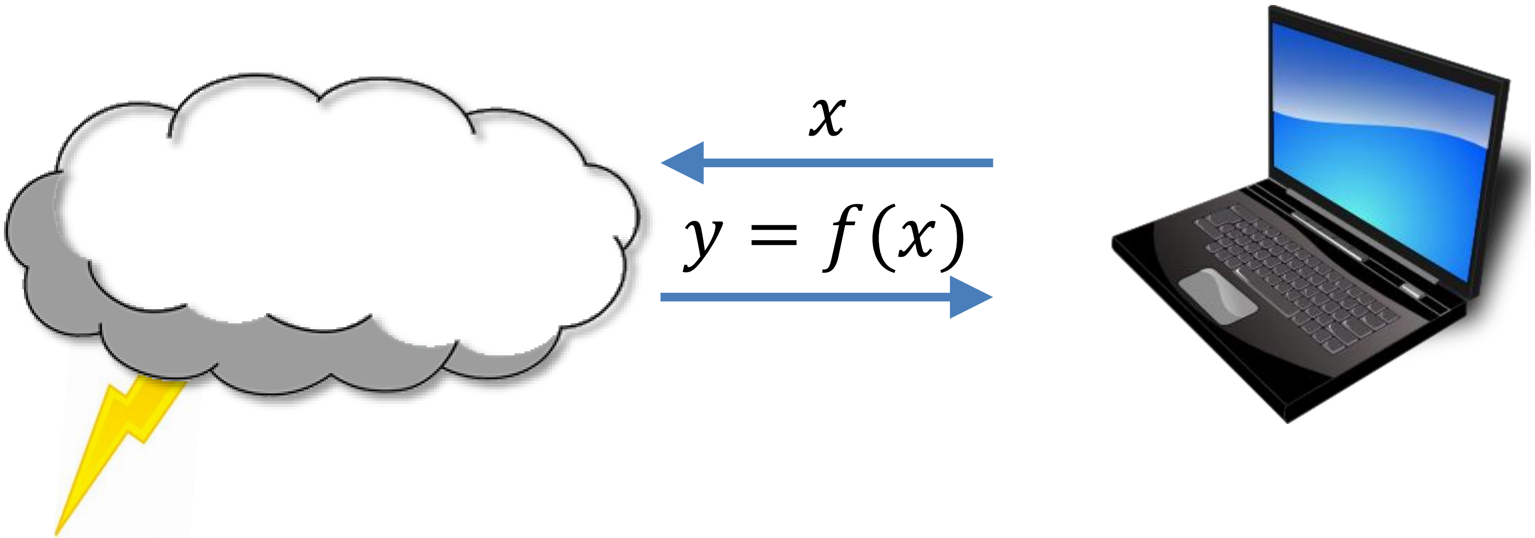
Outsourcing Computation

Weak client outsources computation to the cloud.



Outsourcing Computation

We do not want to blindly trust the cloud.



<u>Key security concern:</u>

Correctness: why should we trust the server's answer?

Interactive Proofs to the Rescue?

Interactive Proof [GMR85]: prover P tries to *interactively* convince a polynomial-time verifier V that $f(x) = y$.

$$f(x) = y \quad \Rightarrow \quad P \text{ convinces } V.$$

$$f(x) \neq y \quad \Rightarrow \quad \text{no } P^* \text{ can convince } V \text{ wp} \geq 1/2.$$

Key Problem: in classical results complexity of ***proving*** is actually exponential:

***IP=PSPACE* [LFKN90,Shamir90]:** Interactive Proofs for space S computations with $2^{\mathrm{poly}(S)}$ prover, $\mathrm{poly}(n, S)$ verification, $\mathrm{poly}(S)$ rounds.

Doubly Efficient Interactive Proof

[GKR08]

Interactive proof for $f(x) = y$ where the prover is **efficient**, and the verifier is **super efficient**.

Proportional to complexity of f

Much faster than complexity of f

Soundness holds against **<u>any</u>** (computationally unbounded) cheating prover.

Why Proof and not Arguments*?

1. Security against ***unbounded*** adversary.
   - Post-quantum secure, post post quantum secure…

2. No reliance on unproven crypto assumptions

3. Do not use any expensive crypto operations
   – Even if not currently practical, no clear bottleneck (e.g., [GKR08])…

* Disclaimer: arguments are GREAT! (e.g., [KRR14])

Doubly Efficient Interactive Proofs: The State of the Art

Logspace uniform NC

1) [GKR08]: Bounded Depth

- Any **bounded-depth** circuit.
- (Almost) linear time verifier, poly-time prover.
- Number of rounds proportional to circuit depth.

2) [RRR16]: Bounded Space

- Any **bounded-space** computation.
- (Almost) linear time verifier, poly-time prover.
- $\boldsymbol{O(1)}$ rounds.

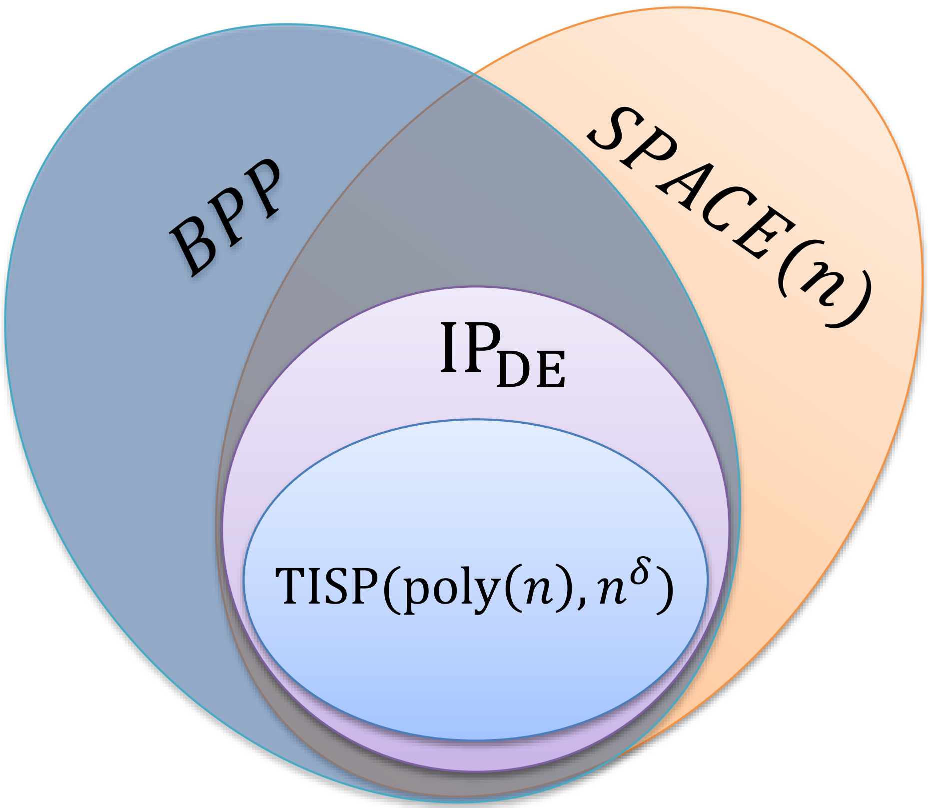
Constant-Round Doubly Efficient Interactive Proofs

Theorem [RRR16]: $\exists \delta > 0$ s.t. every language computable in $\mathrm{poly}(n)$ time and n^{δ} space has an unconditionally sound interactive proof where:

1. Verifier is (almost) linear time.
2. Prover is polynomial-time.
3. Constant number of rounds.

Tightness

Define $\mathrm{IP_{DE}}$ as class of languages having doubly efficient interactive proofs.



Roadmap: A Taste of the Proof

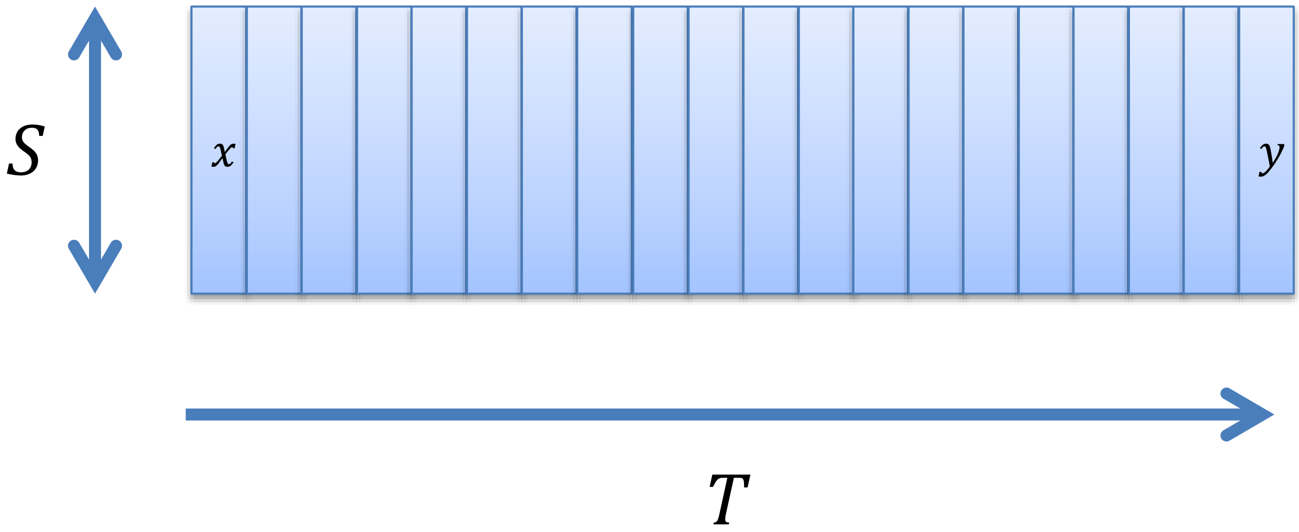
Iterative construction:

1. Start with interactive proof for short computations.
2. Build interactive proof for slightly longer computations.
3. Repeat.

Iterative Construction

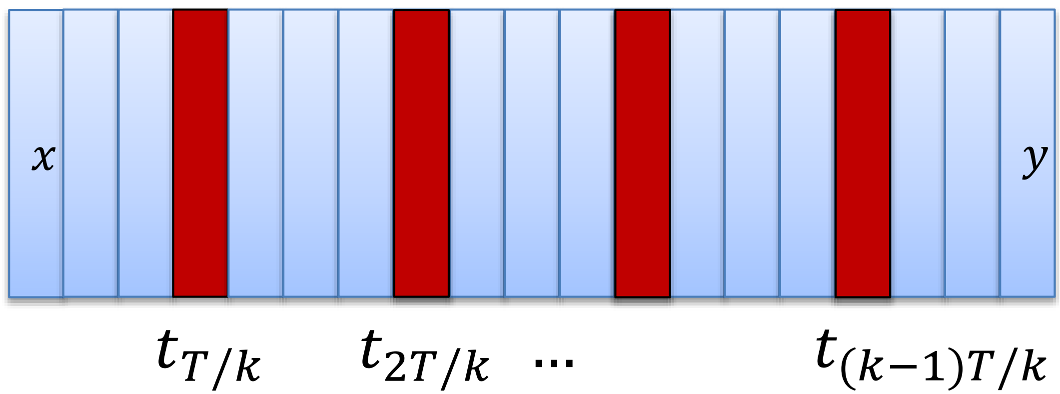
Suppose we have interactive proofs for time T/k and space S computations.

Consider a time T and space S computation.



Divide & Conquer

Divide: Prover sends Turing machine configuration in $k \ll T$ intermediate steps.

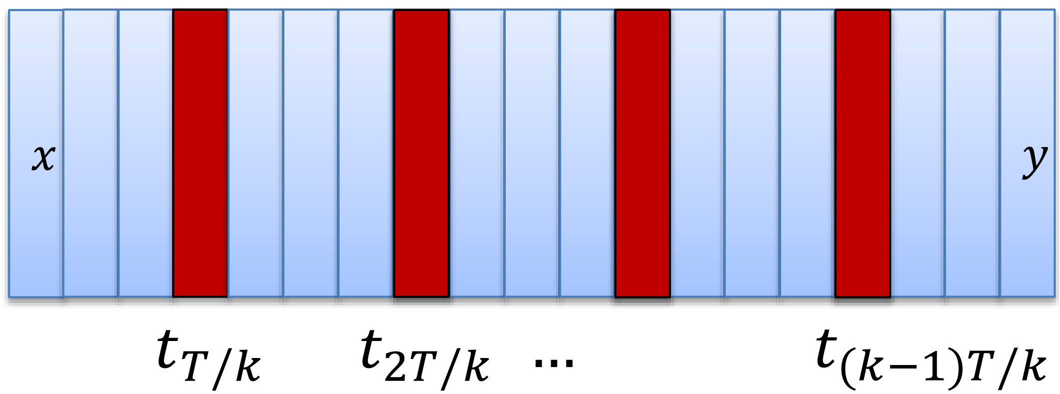


Conquer? recurse on all subcomputations.

Problem: verification blows up, no savings.

Divide & Conquer

Divide: Prover sends Turing machine configuration in $k \ll T$ intermediate steps.



Conquer? Choose a few at random and recurse.

Problem: huge soundness error.

Best of Both Worlds?

Can we **batch verify** k instances much more efficiently than k independent executions.

Goal:

- Suppose $x \in L$ can be verified in time t.
- Want to verify $x_1, \dots, x_k \in L$ in $\ll k \cdot t$ time.

Concrete Example: Batch Verification of RSA moduli

Def: integer N is an ***RSA modulos*** if it is the product of two m-bit primes $N = p \cdot q$.

The proof that N is an RSA modulos is its factorization.
Can we verify k RSA moduli more efficiently?

$\boldsymbol{P(p_1, q_1 \dots, p_k, q_k)}$ $\boldsymbol{V(N_1, \dots, N_k)}$

Warmup: Batch Verification for $\mathbf{UP}$

$\mathbf{UP} \subseteq \mathbf{NP}$ are all relations with unique accepting witnesses.

m = witness length

Theorem [RRR16]: Every $L \in \mathbf{UP}$, has an interactive proof for verifying that $x_1, \ldots, x_k \in L$ with $\boldsymbol{m \cdot \mathrm{polylog}(k) + \tilde{O}(k)}$ communication.

For batch verification of ***interactive*** proofs we introduce interactive analogs of $\mathbf{UP}$ and $\mathbf{PCP}$.

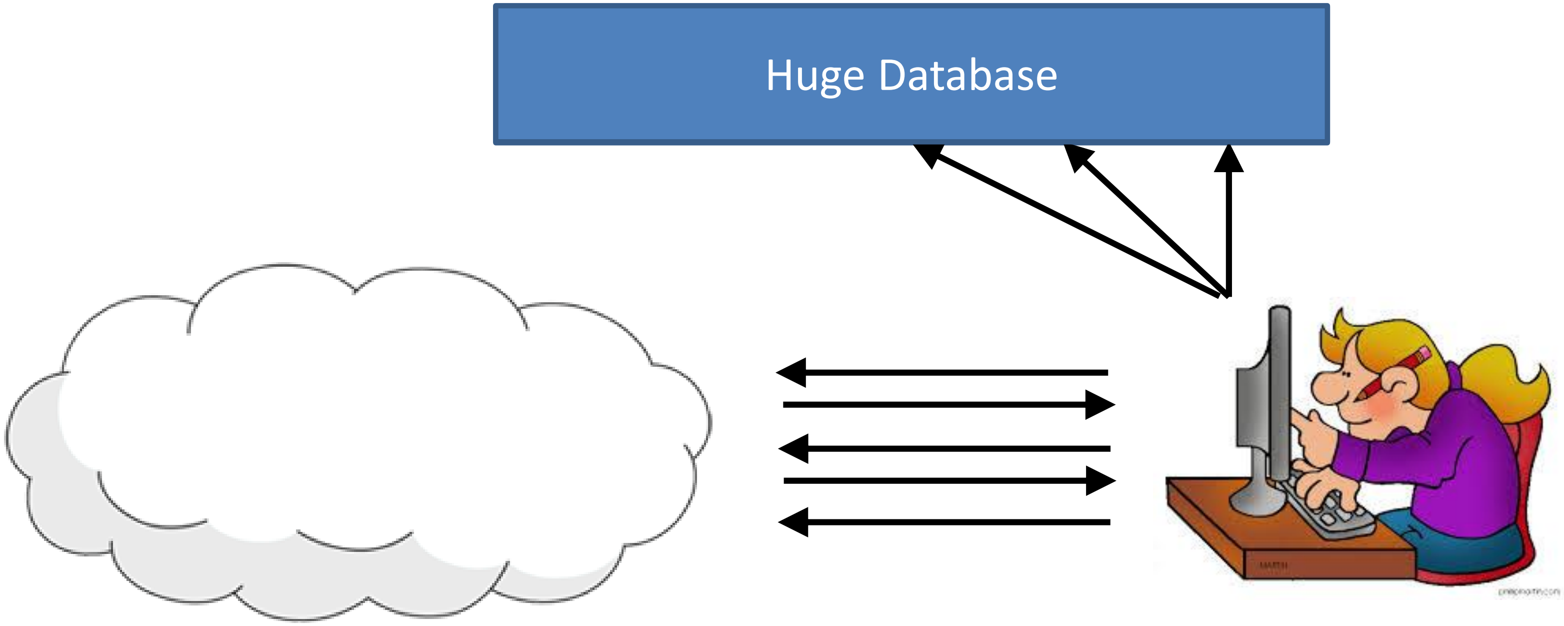
Constant-Round Doubly Efficient Interactive Proofs

Theorem **[RRR16]:** $\exists \delta > 0$ s.t. every language computable in $\mathrm{poly}(n)$ time and n^{δ} space has an unconditionally sound interactive proof where:

1. Verifier is (almost) linear time.
2. Prover is polynomial-time.
3. Constant number of rounds.

Sublinear Time Verification

Motivation: statistical analysis of vast amounts of data.

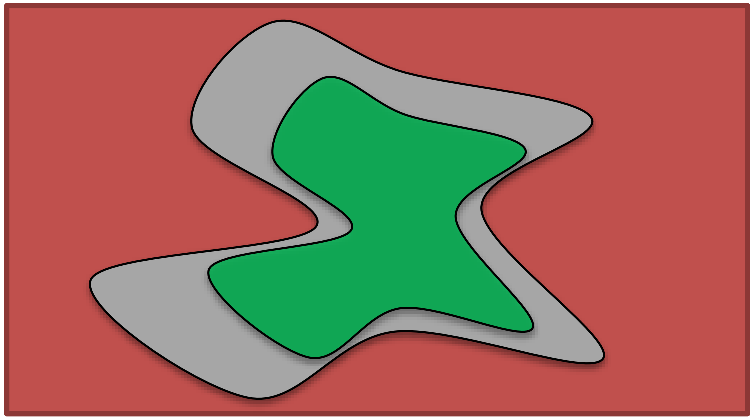


Sublinear Time Verification

Can we verify without even reading the input?

Yes! If we allow for ***approximation.***

Following **Property Testing** [GGR98]**:** only required to reject inputs that are far from the language.



Sublinear Time Verification

Revisiting classical notions of proof-systems:

NP	Gur-R13, Fischer-Goldhirsh-Lachish13, Goldreich-Gur-R15
Interactive Proof	Rothblum-Vadhan-Wigderson13, Kalai-R15, Goldreich-Gur-R15, Goldreich-Gur16, Reingold-Rothblum-R16, Gur-R17
Zero-Knowledge	Berman-R-Vaikuntanathan17
PCP/MIP	Ergun-Kumar-Rubinfeld04, Dinur-Reingold06, BenSasson-Goldreich-Harsha-Sudan-Vadhan06, Gur-Ramnarayan-R17

Open Problems

- ***Research directions:***
  - Bridge theory and practice.
  - **Sublinear** time verification.

- ***Concrete questions:***
  - IP=PSPACE with "efficient" prover.
  - Batch verification for all of NP.
  - [GR17]: Simpler and more efficient protocols (even for smaller classes).
  - Improve [RRR16] round complexity: even exponentially.