Meaningful and Meaningless Statements Using Metrics for the Border Condition

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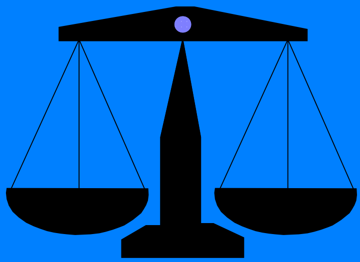

Source: US Border Patrol Strategic Plan 2012-26

Measuring the Condition of the Border

- Discussion in US: need for metrics to measure condition of the nation's border
- Major purpose: way to assess whether security at border has improved or gotten worse
- Some in Congress have asked for a single metric to measure condition at the border
- Serious CBP effort to produce a single metric called the Border Condition Index

Source: US Border Patrol Strategic Plan 2012-26

Finding a Universally-accepted Metric is Complicated

- Vastness of border
- Numerous ports of entry for legal movements of people and goods
- Variety of transport modes
- Many agencies involved, with different missions
- ***Many components of border security, including:***
  - ***Keeping "bad" things out***
  - ***Not interfering with "good" commerce***
  - ***Enhancing quality of life at the border***
- No universally accepted metrics
- Single metric may be unachievable

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What Can we Do With Metrics?

- Conveying border security is about decision making and communication of information to policy makers & public
- Metrics can help – if used properly
- Metrics can be misleading
- ***Statements using metrics can be meaningless in the precise sense of the theory of measurement***

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Conclusions we May Want to Draw about the Border

- The condition at the border has improved – *a comparative statement*
- The improvement between 2015 & 2016 was greater than between 2014 & 2015 – *a comparison of differences statement*
- The condition of the border today is 10% better than the condition last year – *a percentage change statement*

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Conclusions we May Want to Draw about the Border: Averages

- May want to average border condition at different locations (sectors, stations, zones) – *average over sectors metrics*
- May have different criteria for or aspects of border security and may want to average over them – *average over criteria metrics*
- May want to compare averages – comparative statements, comparison of difference statements, percentage change statements:
  - Average has improved
  - Improvement in average between 2015 & 2016 is greater than improvement between 2014 & 2015
  - Average is 10% better

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MEASUREMENT

- ***Measurement*** has something to do with numbers.
- Think of starting with a set *A* of objects that we want to measure.
- We shall think of a ***scale of measurement*** as a function *f* that assigns a real number *f(a)* to each element *a* of A (or some value in a set B rather than any real number)
- The representational theory of measurement gives conditions under which a function is an ***acceptable scale*** of measurement

The Theory of Uniqueness

Admissible Transformations

- An ***admissible transformation*** sends one acceptable scale into another.

Centigrade → Fahrenheit

Kilograms → Pounds

- In most cases one can think of an admissible transformation as defined on the range of a scale of measurement.
- Suppose f is an acceptable scale on A, taking values in B.
- $\varphi: f(A) \rightarrow B$ is called an ***admissible transformation of f*** if $\varphi \circ f$ is again an acceptable scale.

The Theory of Uniqueness

Admissible Transformations φ

Centigrade → Fahrenheit: $\varphi(x) = (9/5)x + 32$

Kilograms → Pounds: $\varphi(x) = 2.2x$

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The Theory of Uniqueness

•A classification of scales is obtained by studying the class of admissible transformations associated with the scale

•This defines the ***scale type*** (S.S. Stevens)

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Some Common Scale Types

Class of Adm. Transfs.	Scale Type	Example
$\varphi(x) = \alpha x,\ \alpha > 0$	*ratio*	Mass Temp. (Kelvin) Time (intervals) Length Volume Loudness (sones)?
$\varphi(x) = \alpha x+\beta,\ \alpha > 0$	*interval*	Temp (F,C) Time (calendar)

Some Common Scale Types

Class of Adm. Transfs.	Scale Type	Example
$x \geq y \leftrightarrow \varphi(x) \geq \varphi(y)$ φ strictly increasing	***ordinal***	Preference? Hardness Grades of leather, wool, etc. Subjective judgments: cough, fatigue,...
$\varphi(x) = x$	***absolute***	Counting

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Meaningful Statements

•In measurement theory, we speak of a statement as being ***meaningful*** if its truth or falsity is not an artifact of the particular scale values used.

•The following definition is due to Suppes 1959 and Suppes and Zinnes 1963.

Definition: A statement involving numerical scales is ***meaningful*** if its truth or falsity is unchanged after any (or all) of the scales is transformed (independently?) by an admissible transformation.

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Meaningful Statements

"I weigh 1000 times what that elephant weighs."

- Is this meaningful?

Meaningful Statements

"I weigh 1000 times what that elephant weighs."

- Is this meaningful?
- We have a ratio scale (weight).

(1) $f(a) = 1000f(b)$.

- This is meaningful if f is a ratio scale. For, an admissible transformation is $\varphi(x) = \alpha x$, $\alpha > 0$. We want (1) to hold iff

(2) $(\varphi \circ f)(a) = 1000(\varphi \circ f)(b)$

- But (2) becomes

(3) $\alpha f(a) = 1000\alpha f(b)$

- (1) $\leftrightarrow$ (3) since $\alpha > 0$.

Meaningful Statements

"I weigh 1000 times what that elephant weighs."

- Meaningful. It involves ratio scales.
  It is false no matter what the unit.
- ***Meaningfulness is different from truth.***
- It has to do with what kinds of assertions it makes sense to make, which assertions are not accidents of the particular choice of scale (units, zero points) in use.

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Meaningful Statements

"The average January temperature in New York City has increased by 2% since 1980."

- Is this meaningful?

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Meaningful Statements

"The average January temperature in New York City has increased by 2% since 1980."

$$f(a) = 1.02f(b)$$

- Meaningless. It could be true with Fahrenheit and false with Centigrade, or vice versa.
- Temperature defines an interval scale.
- ***Percentage change statements with ratio scales are meaningful, but with interval scales are meaningless.***

Meaningful Statements about the Border Condition

"The condition of the border has improved by 10%."

$$f(a) = 1.1f(b)$$

- Meaningful if *f* is a ratio scale, not if *f* is an interval scale.
- Can we find a metric for the border condition that defines a ratio scale?
- Yes for some components of border condition.
- ***Bad flows***: number of kilos of cocaine interdicted. Ratio scale.

Source: US Border Patrol Strategic Plan 2012-26

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Meaningful Statements about the Border Condition

"The condition of the border has improved by 10%."

- Can we find a metric for the border condition that defines a ratio scale?
- Yes for some components of border condition.
- ***Bad flows***: number of illegal aliens captured – even an absolute scale, so clearly percentage change statements are meaningful.

Source: US Border Patrol Strategic Plan 2012-26

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Meaningful Statements about the Border Condition

"The condition of the border has improved by 10%."

- Can we find one metric for "bad" flows that defines a ratio scale?
  - How to add kilos of cocaine to number of aliens to … ?
- ***Bringing in not interfering with "good" flows:***
  - Minutes of waiting time at border – ratio scale
  - Days of waiting time to get an import license – ratio scale
  - How combine into one metric?

San Ysidro Border Crossing
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Meaningful Statements about the Border Condition

"The condition of the border has improved by 10%."

- ***Bringing in quality of life at the border:***
  - Life expectancy (years) – ratio scale
  - Years of education – ratio scale
  - Length of working life – ratio scale
  - Severity of health disabilities
    - Not obvious how to measure
    - Severity of cough: scale 1 to 5 – ordinal scale
    - Piper fatigue scale: 1 to 10 – ordinal scale

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  - Utility or value of life at the border – utility functions often thought to define interval scales
  - How would you ever combine these into one metric? Even one that is an ordinal scale?

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Meaningful Statements about the Border Condition

"The improvement in the border condition between 2015 & 2016 was greater than between 2014 & 2015."

A comparison of differences statement:

$$f(a) - f(b) > f(c) - f(d)$$

- Statement invariant if change f to $\alpha f + \beta$ if $\alpha > 0$.
- So: meaningful if interval scale (or ratio scale)
- Not meaningful if ordinal scale.
- Not meaningful to say the difference in severity of health disabilities at the border between 2015 and 2016 improved over same difference between 2014 and 2015.
- Might be able to say this for utility of life at the border – utility an interval scale

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Meaningful Statements about the Border Condition

“The improvement in the border condition between 2015 & 2016 was greater than between 2014 & 2015.”

A comparison of differences statement:

$$f(a) - f(b) > f(c) - f(d)$$

- Meaningful if interval scale (or ratio scale)
- If interval scale, even meaningful to make percentage change of differences statements

“The improvement in the border condition between 2015 & 2016 was 10% that between 2014 & 2015.”

$$f(a) - f(b) = 1.1[f(c) - f(d)]$$

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Meaningful Statements about the Border Condition

- Distinctions can be subtle.
- Consider date by which achieve year's target for captured cocaine.
  - Year 1: July 19, Year 2: June 30
  - A 10% improvement from 200 days to 180 days.
  - Is this meaningful?
  - If year starts Oct. 1, not Jan. 1, then the improvement is from 292 days to 272 days, about 7%.
  - So 10% improvement is meaningless – unless specify beginning of year.
  - Time is interval scale if it is date, ratio scale if days, hours, minutes

Averaging over Sectors

The average border condition over sectors (or stations, or zones) at time *t*+1 is greater than the average value at time *t*.

- Meaningful?

- Let a_i = border condition f of sector i at time t+1, b_i = border condition of sector i at time t.

$$(1) \quad (1/n) \sum_{i=1}^{n} f(a_i) > (1/n) \sum_{i=1}^{n} f(b_i)$$

- We are comparing ***arithmetic means***.

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Averaging over Sectors

- Statement (1) is meaningful iff for all admissible transformations of scale φ, (1) holds iff

$$(2) \quad (1/n) \sum_{i=1}^{n} (\varphi \circ f)(a_i) > (1/n) \sum_{i=1}^{n} (\varphi \circ f)(b_i)$$

- **If border condition defines a ratio scale:**

- Then, $\varphi(x) = \alpha x$, $\alpha > 0$, so (2) becomes

$$(3) \quad (1/n) \sum_{i=1}^{n} \alpha f(a_i) > (1/n) \sum_{i=1}^{n} \alpha f(b_i)$$

- Then $\alpha > 0$ implies (1) $\leftrightarrow$ (3). Hence, (1) is meaningful.

Averaging over Sectors

•Note: **(1) is still meaningful if f is an interval scale.**

•For example, we could be be comparing the utility of life at the border, averaged over sectors.

•Here, $\varphi(x) = \alpha x + \beta$, $\alpha > 0$. Then (2) becomes

$$(4)\quad (1/n) \sum_{i=1}^{n} [\alpha f(a_i)+\beta] > (1/n) \sum_{i=1}^{n} [\alpha f(b_i)+\beta]$$

•This readily reduces to (1).

• **(1) is meaningless if f is just an ordinal scale.**

• ***However, if we compare medians, not arithmetic means, (1) is meaningful even for ordinal scales.***

Averaging over Sectors

- ***Thus, comparison of arithmetic means over sectors is meaningful for interval or ratio scales, meaningless for ordinal data.***
- We are skeptical if we could develop an interval scale metric for the border condition.

- Similar analysis shows that comparison of differences using arithmetic mean over sectors is meaningful for ratio and interval scales, but not ordinal scales.
- Also, percentage change statements using arithmetic mean over sectors are meaningful for ratio scales, not interval or ordinal scales.

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Averaging over Criteria

- Things can get tricky.
- Fix one sector (or union of all sectors)
- Consider different components or criteria for border security.
- Suppose:

  f_1 is metric for ability to keep bad flows out

  f_2 is metric for ability to keep good flows moving

  f_3 is metric for quality of life at the border

  And so on
- Suppose overall border metric is a weighted average:

$$M(a) = \left({}^{1}/_{n}\right) \sum_{i=1}^{n} \lambda_i f_i(a)$$

Averaging over Criteria

•Suppose we want to say that the border index M has improved from one time to another.

•Let a be one time, b be a second time.

•We want to say that $M(a) > M(b)$.

•Consider the case where all criteria are equally important, i.e., all λ_i are the same.

•Then we are saying that

$$(1) \qquad (1/n) \sum_{i=1}^{n} f_i(a) > (1/n) \sum_{i=1}^{n} f_i(b)$$

Averaging over Criteria

(1) $$({}^{1}/_{n}) \sum_{i=1}^{n} f_i(a) > ({}^{1}/_{n}) \sum_{i=1}^{n} f_i(b)$$

- If each f_i is a ratio scale, then we ask whether or not (1) is equivalent to

(2) $$({}^{1}/_{n}) \sum_{i=1}^{n} \alpha f_i(a) > ({}^{1}/_{n}) \sum_{i=1}^{n} \alpha f_i(b)$$

- This is clearly the case.
- So it seems that comparison of averages over criteria is meaningful.

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Averaging over Criteria

- However: no reason to think the $f_1, f_2, f_3, \ldots$ have the same units.
- So we want to allow independent admissible transformations of the f_i. We have to compare

$$(1) \quad (1/n) \sum_{i=1}^{n} f_i(a) > (1/n) \sum_{i=1}^{n} f_i(b)$$

$$(3) \quad (1/n) \sum_{i=1}^{n} \alpha_i f_i(a) > (1/n) \sum_{i=1}^{n} \alpha_i f_i(b)$$

- Easy to find α_i for which (1) & (3) don't both hold.
- ***So comparison of arithmetic means over criteria is not meaningful.***

Averaging over Criteria

Motivation for considering different α_i:

- $n = 2$, $f_1(a)$ = kilos of cocaine captured at time a, $f_2(a)$ = minutes of wait time at the border at time a.
- Then (1) says that the average of weight at a and time at a is greater than the average of weight and time at b.
- This could be true with one combination of weight and time scales and false with another.

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Tijuana-San Diego Border

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Averaging over Criteria

•In this context, it is safer to compare ***geometric means*** (Dalkey).

$$\sqrt[n]{\Pi f_i(a)} > \sqrt[n]{\Pi f_i(b)} \leftrightarrow \sqrt[n]{\Pi \alpha_i f_i(a)} > \sqrt[n]{\Pi \alpha_i f_i(b)}$$

all $\alpha_i > 0$.

•Thus, if each f_i is a ratio scale, even if scales for each criterion can be changed independently, ***comparison of geometric means over criteria is meaningful while comparison of arithmetic means is not.***

•But: does geometric mean have any real meaning for the border condition?

•Meaningful in measurement theory sense is not the same as meaningful in practical sense.

How Should We Average Scores?

- There are many more ways to average scores over criteria, not just (weighted) arithmetic or geometric means or medians.
- Long literature in the theory of measurement as to what averaging procedures lead to meaningful statements with averages.
- ***Message: Take great care in making statements using weighted averages of metrics for different components of the border condition.***

Source: US Border Patrol Strategic Plan 2012-26

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Applying Statistical Tests

- Even more subtle: what statistical tests may one make if we measure data on a ratio, interval, or ordinal scale?
- Foundational work of S.S. Stevens in psychology
  - Developed classification of scales
  - Provided rules for the use of statistical procedures: certain statistics are inappropriate at certain levels of measurement.
- Applications of these ideas to ***descriptive statistics*** widely accepted since the 1950s.
- Principles such as:
  - Arithmetic means are "appropriate" statistics for interval scales, medians for ordinal scales.
- Note: you can always calculate (weighted) arithmetic means. These involve averaging numbers.
- The key is what comparisons can be meaningfully made with the averages.

Applying Statistical Tests

•Stevens' ideas have come to be applied to ***inferential statistics*** -- inferences about an unknown population P.

•They have led to such principles as the following:

(1). Classical parametric tests (e.g., t-test, Pearson correlation, analysis of variance) are inappropriate for ordinal data. They should be applied only to data that define an interval or ratio scale.

(2). For ordinal scales, non-parametric tests (e.g., Mann-Whitney U, Kruskal-Wallis, Kendall's tau) can be used.

•Not everyone agrees.

•But, ***key concept: are you testing a meaningful hypothesis?***

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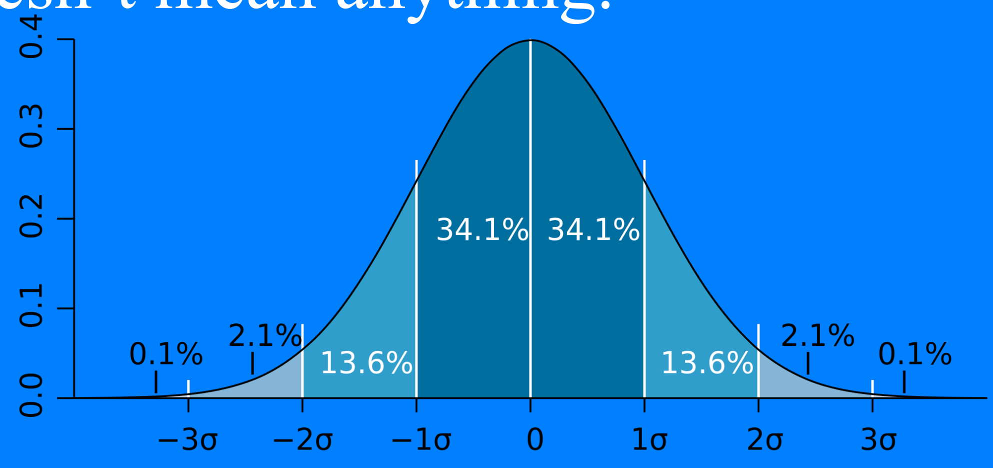
Applying Statistical Tests

- ***Key concept: are you testing a meaningful hypothesis?***
- Example Hypothesis: The arithmetic mean average (over sectors) in the quality of life at the border since last year is unchanged. $\Sigma f(a_i) - \Sigma f(b_i) = 0$.
- A meaningless hypothesis if the quality of life at the border is only an ordinal scale. (Meaningful if ratio or interval.)
- So even if a Mann-Whitney U or Kruskal-Wallis or Kendall's tau can be used, you wouldn't care because the hypothesis doesn't mean anything.



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Thank you

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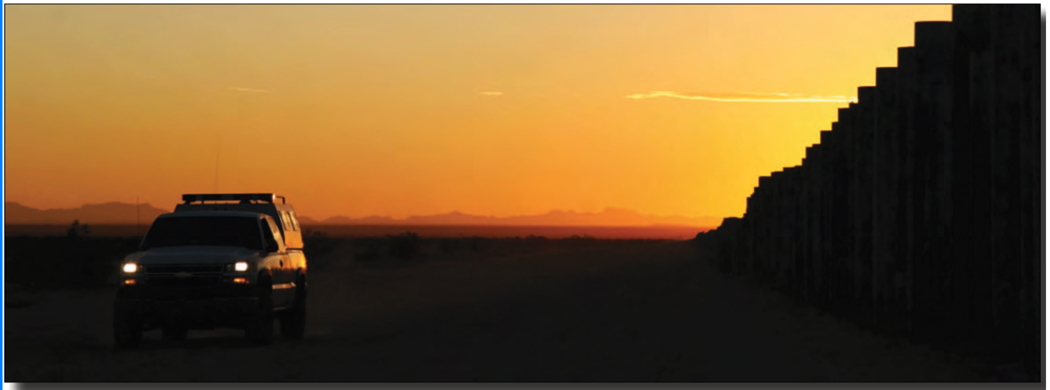
  - Brian Roberts
  - Henry Willis



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