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Introductory College Mathematics for the Life Sciences: Has Anything Changed?

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Abstract

This paper focuses on issues concerning the introductory college mathematics sequence with an emphasis on students interested in the life sciences, and concentration on the time after the publication of *BIO2010* (BIO2010 in Transforming Undergraduate Education for Future Research Biologists, National Academies of Science, Medicine and Engineering, Washington, 2003). It also explores the potential uses of books targeted at introductory mathematics courses for life science majors today. As relevant background, we look at the evolution of the way that calculus has been taught over the past 50 years, including at the high school level. We also explore the implications of changes in technology and course delivery, such as online education. As we discuss different books and introductory course ideas, we focus on the needs of biology students, the inclusion of real-world problems and models, the role of technology, and the impact of data science. The paper is organized as follows: Sect. 1 provides some personal background with calculus dating back to the 1970s, and changes in calculus prior to *BIO2010*. Section 2 introduces goals for an introductory mathematics sequence and evaluates the calculus sequence in light of those goals. Sections 3–7 discuss various issues that will help to understand issues and challenges for introductory mathematics for the life sciences: Calculus in high school (Sect. 3), equity issues relative to calculus and other math topics (Sect. 4), the impact of online education (Sect. 5), math as a stumbling block for college students (Sect. 6), and the increasing importance and value of teaching data science (Sect. 7). Section 8 reviews the development of books in light of these issues and challenges. The last section (Sect. 9) summarizes conclusions.

Keywords Calculus · Life sciences · Introductory mathematics · Equity

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1 Introduction: Some Personal Background

In 1972, shortly after one of the authors, Fred Roberts, joined the Mathematics faculty at Rutgers University, he was asked by the Department Chair to meet with the life sciences faculty to discuss what they wanted their students to learn in introductory mathematics courses and to explore the possibility of designing tailor-made introductory courses on math for the life sciences. Roberts discussed calculus as well as other topics that were important for life scientists because they were being used in the field: statistics, probability, networks, discrete mathematics, linear optimization, game theory. The result of the discussions was surprising: The life scientists didn't want their students to be "short-changed" and get less of an introduction to calculus than any other students. The idea of a specialized introductory course for the biologists was put on the back burner.

As a compromise, the math department at Rutgers introduced a special variant of introductory calculus aimed at the biology students. The book *Introduction to Mathematics for Life Scientists* by Batschelet (1971) was chosen as a text and Roberts taught the course first in 1973. The book had some wonderful biological examples such as the tail-wagging dance of bees who are communicating the location of food (an example of polar coordinates) and optimization problems involving branching in the blood vascular system (to illustrate the second derivative). Roberts has used many of these examples in other courses. However, the book didn't quite match up as a calculus book. Indeed, many did not view this book as a replacement for calculus texts at all, but instead as a reference tool for those confronting mathematical techniques in their own research or while studying various research journal articles.

As a second compromise, Rutgers mathematicians, ecologists, and geneticists got together and created a novel biomathematics major in 1973, one which continues to this day.

The idea of a specialized introductory mathematics course or courses for biology students didn't go away. More generally, two other ideas continued to be discussed at various places around the country. One was that additional topics such as discrete mathematics, probability, statistics, networks, game theory, and linear programming might better aid in introducing students to mathematics than calculus alone. A second was that the basic introductory mathematics sequence might be modified in different ways depending upon the interests and potential majors of students. Both topics gained in importance because of the growth of computer science. One of the main proponents of this view was Tony Ralston. Together with Gail Young, Ralston organized a conference in 1982 at Williams College. According to the book arising from that conference: "The purpose of the conference was to discuss the restructuring of the first 2 years of college mathematics to provide some balance between the traditional calculus-linear algebra sequence and discrete mathematics" (Ralston and Young 1983). Several of the papers in that book spelled out "integrated" first 2-year curricula that included discrete math, probability/statistics, and other topics (including Alo et al. 1983; Barrett et al. 1983; Roberts 1983).

In 1982, there were still not many books one could use in modified introductory mathematics courses or in introductory math courses aimed at specific audiences, such as biology students. Between 1982 and 2010, when the National Academy of Sciences Report *BIO2010* appeared, a few books such as Claudia Neuhauser's *Calculus for Biology and Medicine* (Neuhauser 2000), Fred Adler's *Modeling the Dynamics of Life: Calculus and Probability for Life Scientists* (Adler 1998), and Mike Aitken, Bill Broadhurst, and Stephen Hladky's *Mathematics for Biological Scientists* (Aitken et al. 2009) were produced for well-defined audiences and for faculty allowed to choose their books for their own mathematics courses, generally in small colleges.

In this paper, we focus on the time after the publication of *BIO2010,* and the potential uses of books targeted at introductory courses for life science majors today. As relevant background, we will also look at the evolution of the way that calculus is taught, including at the high school level, and the implications of AP calculus for future curricular and textbook changes. We also explore the implications of new technological tools and new forms of course delivery such as online education. As we discuss different books and introductory course ideas, we will be interested in the needs of biology students; in the inclusion of real-world problems and models; and in the impact of data science.

The remainder of this paper is organized as follows. Section 2 introduces goals for an introductory mathematics sequence and evaluates the calculus sequence in light of those goals. Sections 3–7 discuss various issues that will help to understand issues and challenges for introductory mathematics for the life sciences: Calculus in high school (Sect. 3), equity issues relative to calculus and other math topics (Sect. 4), the impact of online education (Sect. 5), math as a stumbling block for college students (Sect. 6), and the increasing importance and value of teaching data science (Sect. 7). Section 8 reviews the development of books in light of these issues and challenges. The last section (Sect. 9) summarizes conclusions.

2 Goals for an Introductory Mathematics Sequence

There are multiple goals for the introductory mathematics sequence. (1) It should introduce students to what modern mathematics is about, whether or not they will go on with more advanced mathematics. (2) It should provide students with the specific tools they need to take more mathematics courses. (3) It should provide students with the tools they need and the "mathematical maturity" required to succeed in the courses they will be taking that have mathematical prerequisites, for instance courses in their major. (4) It should help the students understand and appreciate the role of mathematics and mathematical reasoning in their discipline.

Now 38 years since the 1982 Ralston-Young conference/volume, one can ask whether introductory mathematics courses and requirements have changed since the 1980s. Few question the value in having students learn mathematics at every age and at every point in their education; what seems to be controversial still to this day is what mathematics is important for whom and whether we can offer alternative courses focused on the need of the students' majors. Since the early 1980s, there have been calls for various types of mathematics curricula. Discrete Mathematics, which held a place of prominence in the 1982 proposed curricula, has come and gone in almost all introductory courses. Computer Science offers its own introductory discrete math courses. Likewise, Probability and Statistics material or courses, which held positions of importance in the 1982 conference, continue to appear, but inside Finite Math, Math for Liberal Arts, or Math for Business major courses, not as part of a standard calculus sequence. Probability and Statistics courses were developed for mathematics, engineering, and computer science majors as courses to follow calculus courses. Differential Equations as a stand-alone course has virtually disappeared with differential equations covered inside the first-year calculus sequence, regardless of intended majors. It should be mentioned that differential equations are closely related to both classical and modern biological applications, e.g., in ecology (rabbits and foxes) and in epidemiology (spread of disease, particularly relevant in light of COVID-19).

So, it does appear that calculus is still the basic introductory math sequence, with some exceptions. In some places, there are calculus options: calculus for science and engineering, calculus for business and economics, calculus for life sciences, etc. How does the modern calculus course fare in terms of the four goals above?

For goal (1), we believe that it continues to "fail" as it doesn't introduce other topics that give a broader view of mathematics: probability, statistics, discrete math, and general issues of "data science" (to which we return in Sect. 7).

For goal (2), it seems to work given the current prerequisites for more advanced math classes.

For goal (3), one needs to consider the "tools" needed for other courses. Computer science is a case in point. There is very little that is learned in a modern calculus course that is really explicitly used in courses in CS, though calculus continues to be a requirement for the CS major in most places. The requirement has something to do with required "mathematical maturity," but one could argue that such maturity could be achieved with other introductory math courses. For the life sciences, certainly topics in probability, statistics, networks, data science, etc., would seem to be needed and the calculus sequence alone is not providing them. However, as we note in Sect. 8, some books are now combining calculus with probability, statistics, and other topics.

For goal (4), we feel that calculus also falls short, unless an instructor brings modern applications from other disciplines into their calculus courses. This should not be hard to do – even the examples from Batschelet's book will play this role for the life sciences, though more modern examples are useful. Fortunately, there are now a number of calculus books that provide these kinds of examples, as we discuss in Sect. 8. Also, there are efforts to develop "small modules" with specific examples that could be brought into all or part of one day of an introductory mathematics class. The DIMACS Center at Rutgers has developed an extensive collection of such modules covering a wide variety of topics in sustainability, epidemiology, and other applications, appropriate for introductory mathematics classes even at the high school level. (See http://dimacs.rutgers.edu/archive/IMB/availmodules.html, http://dimacs.rutgers.edu/archive/PS-Future/availmodules.html, http://dimacs.rutgers.edu/archive/MPE/Sustmodule.html.) But there is a need for more such materials. The integration of the math and biology curricula would help a great deal in reinforcing the value of introductory mathematics courses, in particular calculus. Thus, more such modules could be developed for introductory life science courses, with the goal being to give examples that demonstrate the relevance of mathematics to biology. While many of the DIMACS modules are appropriate for life sciences classes as well as mathematics classes, there is a need for more. A challenge is that instructors of introductory math classes (faculty, part-time or full-time, or graduate students) themselves do not understand biology, meaning that either they don't explain things well or enthusiastically, or that they simply omit biological examples. Similarly, instructors of introductory life sciences courses may not feel comfortable with the mathematics (calculus or other topics) and so simply skip examples or, worse yet, reflect their mathematical insecurity when presenting the topics. Some sort of training courses or materials for instructors may be needed to address this challenge. Perhaps more important is the need for more math departments to promote an overall understanding of the relevance of calculus applications to other disciplines, with concomitant partnerships, communications, and concern for the needs of other departments.

Having separate calculus sequences for students with different majors in mind has advantages and disadvantages. Finding enough trained instructors to present the applied topics is a challenge. Some schools do not offer enough calculus sections to split things this way. Only the rare high school offers separate calculus sections for topics such as the life sciences, and a large number of students get their calculus in high school (Sect. 3). Not all students can be expected to know what they want to major in when choosing an introductory mathematics sequence. So, one could argue for a branch point later in the calculus sequence, e.g., by the third course, not the first.

Going beyond this simple analysis would take us too far afield. We turn, instead, to a discussion of the state of calculus in the present day, and of issues and challenges relevant to it.

3 Calculus in High School

A number of profound changes have occurred in the 38 years since the 1982 conference, changes that impact undergraduate mathematics courses and choices for all students. One has been the rapid increase in the number of students who take AP calculus in high school, either the standard first-year AB calculus or the harder BC calculus. Today 15% of high school students take AP calculus, many before the 11th grade (Sparks 2018). Many never take another mathematics course afterward in college. But now, math and science professionals and educators are beginning to question how helpful current high school calculus courses really are for advanced science fields. The ubiquitous use of data in everything from physics and finance to politics and education is helping to build momentum for a new path in high school math—one emphasizing statistics and data literacy over calculus. Some argue for statistics and data science as part of a calculus course.

Both the Common Core State Standards, on which many states' math requirements are based, and the Next Generation Science Standards call for teaching data analysis and statistics, both on their own and in the process of learning other concepts. From 2007 to 2017, 4th- and 8th-grade students' scores on the National Assessment of Educational Progress in Mathematics (see, for example, NAEP 2019) fell significantly on problems related to data analysis, statistics, and probability—a decline that helped drive overall dips on the math test in 2017. Some believe that the statistics and data analysis taught have traditionally taken a back seat to calculus in high school math. University admissions drive much of the choice of courses taken for college-bound students whether or not the student will major in mathematics, science, engineering, or liberal arts. Students who had difficulty in Geometry and Algebra 2 are still encouraged to take calculus, so it can show up on their transcript. Schools also encourage the taking of calculus as a means of assuring the public that the schools don't "track" students.

David Bressoud, a mathematics professor at Macalester College and former president of the Mathematical Association of America, examined the evolution of calculus studies and had the following to say: "until about 1980, calculus was seen as a higher education course, primarily for those interested in mathematics, physics, or other hard sciences, and only about 30,000 high school students took the course. That began to change when school reformers glommed onto calculus as an early example of a rigorous, college-preparatory course" (Bressoud et al. 2013). Calculus became a center of attention in this context during the systemic initiatives time of the 1990s and 2000s funded by large grants from the National Science Foundation to cities, states, and rural areas, then again a bit later with the mathematics science partnerships programs (Cozzens and Williams 2018). A development that some would describe as a cry for "Calculus for All" became a means of showing that all students could (and should) do high-level mathematics. To accommodate the "Calculus for All" courses in school curricula, other courses had to be taken earlier by everyone, so "Algebra for All in 8th grade" became the refrain. Today almost every middle school in the country has 100% of its students taking algebra in 8th grade. Henry Kepner, Professor of Mathematics Education at the University of Wisconsin Milwaukee and former President of the National Council of Teachers of Mathematics, together with one of the authors (Midge Cozzens), addressed the issue of algebra in middle school in a 1996 paper, "*Algebra for All in Eighth Grade, but not a Discrete Course*" (Cozzens and Kepner 1996). This paper had very little impact on a curriculum that already had begun to move Algebra I to 8th grade, Geometry to 9th grade, Algebra II to 10th grade, Precalculus to 11th grade, and Calculus to 12th grade.

It can be said of calculus and proven with respect to algebra in 8th grade that *the more that schools encouraged calculus before high school graduation, the greater the expectation that all would offer calculus.* This pressure came from parents, and district leaders—and in particular from colleges and universities. Bressoud said, "It's not just math majors or engineering majors; this has become an accepted requirement for admission to top universities. You are not going to get into Duke if you haven't taken calculus, even if you plan to major in French literature" (Bressoud et al. 2013). Today, some 800,000 students nationwide take calculus in high school; as noted above, this is about 15% of all high schoolers. Nearly 150,000 take the course before 11th grade.

A new report by the Mathematical Association of America and the National Council of Teachers of Mathematics found many students who took Advanced Placement Calculus AB still ended up retaking calculus in college—and 250,000 students end up needing to take even lower-level courses, like precalculus or algebra. In the end, the report found taking calculus in high school was associated with only a 5-percentage point increase on average in calculus scores in college—from 75% to 80%. Rather, the best predictor of earning a B or better in college calculus was a student earning no less than A in high school Algebra 1 and 2 and Geometry (NCTM/MAA 2017).

So, if high school calculus isn't the best indicator that a student is prepared for college-level math, then why teach it in every high school? To change this, we need to start with the beliefs of the parents, which in turn lead to pressures on school systems. Those beliefs come from various sources, including the Admissions Office.

Statistics and data literacy advocates hope that diversifying the field of interesting and rigorous math courses could broaden students' path to STEM and other careers. As of 2017, the U.S. Bureau of Labor Statistics estimations showed that jobs that require data literacy and statistics are among the 10 fastest-growing occupations in the country (Bureau of Labor Statistics 2017). EDC's Oceans of Data Institute is building learning progressions for statistics and data literacy at different grades. The progression would include concepts in statistics and data literacy, but also computer science—to be able to use common programming and tools used by data professionals—and more philosophical concepts, such as the ethical use of statistics and privacy protections (Gewertz 2018).

4 Equity Issues

One of the most difficult philosophical challenges for the past 30 years has been how to provide an equitable education for all students in our schools.

Calculus classes in high schools have been and remain disproportionately White and Asian, with other student groups less likely to attend schools that offer calculus or the early prerequisites (like middle school algebra) needed to gain access to the course. But what does this say about equity? In 2015–2016, Black students were 9 percentage points less likely than their White peers to attend a high school that offered calculus and half as likely to take the class if they attended a school that offered it. And if Black students did get into a class, their teachers were also less likely to be certified to teach calculus than those of White students, according to an Education Week Research Center analysis of federal civil rights data (Sparks 2018).

According to a study in the *Journal for Research in Mathematics Education,* more than half of students who take calculus in high school come from families with a household income above $100,000 a year (Gutiérrez 2008). By contrast, only 15% of middle-income students and 7% of those in the poorest 25% of families take the course. "Math is even more important to upward mobility now than it was 20 or 30 years ago, because… it's seen as related to your general ability to solve problems quickly," Uri Treisman said, adding that as a result, "there's general anxiety and panic about equity issues for anything new, even though the current [calculus] pathway is a burial ground for students of color" (Gutiérrez 2008).

Introducing a statistics/data science path to high school introductory advanced math is viewed by some as a way to make it easier for underrepresented minorities to move into STEM. (We return to data science in Sect. 7.) But how does one prevent a statistics/data sciences path from replicating the severe tracking and equity problems that have long existed in classical mathematics, in particular in calculus?

5 Online Education

College-level first-year calculus has been the bread and butter course for medium and large mathematics departments across the country, especially those with a large number of teaching assistants (TAs) and part-time faculty. It is much cheaper for a department to hire TAs and part-time faculty to teach multiple sections of calculus, especially in a lecture/recitation format, than to have their tenured faculty teach it as part of their course load.

Online education is changing things, and it is here to stay. Realizing that teaching online requires some additional teaching skills than teaching in-person classes has made mathematics departments reluctant to offer calculus online, but this is rapidly changing. Changes started out through the offering of hybrid courses, courses that combine a large lecture with recitations held online. Those books that have a mymathlab supplement to be used on a course management system are the easiest to implement for most instructors. Many of those that have used the hybrid method have now converted to a fully online calculus course. One author (Midge Cozzens) has taught calculus online for business majors every semester for 10 years at Rutgers Camden, each semester with mymathlab. The biggest advantage in teaching this course is that she can tailor the assignments to the students in the class and their interests. She received the best teacher award last spring for her teaching of this class and one of the reasons given was her ability to reach each student directly, made much easier through the online system. Online courses may also make it easier to build on a wealth of disciplinary examples (e.g., from the life sciences), including virtual laboratory exercises, that reinforce the value of mathematics in other disciplines.

6 Math is a Stumbling Block for College Students

Math is a notorious stumbling block that trips up students seeking college degrees. Every year, tens of thousands of young people fail to graduate because they can't earn enough math credits.

Precalculus and calculus lead the way at Rutgers University and many other institutions as most repeated courses, resulting from a combination of the high failure rates, with rates of DFW (grades of D, F, and W) in precalc and Calc1 typically at 50% or more, and the very large number of students taking these courses, with often more than 3000 students per semester in one of the precalc or Calc1 courses. Many students fail more than once, usually leading them to switch out of their intended major, and this disproportionately affects underrepresented minorities, economically disadvantaged, and first-generation students, who are most likely to place into precalculus (Bressoud, et al. 2013).

Two-thirds of the students at community colleges, and 4 in 10 of those at 4-year institutions, take remedial courses (Bressoud, et al. 2013). Math is a much bigger sand trap than English: Far more postsecondary students fall into remedial math than reading, and fewer move on to credit-bearing courses (Bressoud, et al. 2013). The only other solid piece of evidence that we have is that—despite the dramatic increase in the number of students who receive credit for calculus studied in high school—the number of students taking Calculus II in the Fall term has remained essentially unchanged over the past two decades: 110,000 in 1990, 106,000 in 1995, 108,000 in 2000, and 104,000 in 2005 (Gewertz 2018). The most glaring observations from this survey are how little we know about the effects of our current calculus instruction in high school and how outdated what we do know is. The most recent large-scale studies are from the Fall of 2001, back when the AP program was 60% of its current size (Sparks 2018). However, there are a few things that can be said:

1. There is no evidence that taking calculus in high school is of any benefit unless a student learns it well enough to earn college credit for it, and there is some evidence—the high percentage of students who go from calculus in high school to precalculus in college—that an introduction to calculus that builds on an inadequate foundation can be detrimental.
2. The AP Calculus program is doing what it was established to do: It identifies those students who have learned calculus well enough that they are ready to place into the next course (Bressoud et al. 2013).

7 Data Science

The ubiquitous use of data in all fields has raised the argument for including data science early in the mathematics curriculum, even in high school. As we noted in Sect. 4, some people feel that data science offers a way to introduce modern mathematics that is more equitable with regard to underrepresented minorities than calculus. Some have suggested emphasizing statistics and data literacy over calculus. Others have argued for statistics and data science as part of a calculus course.

Biologically relevant data are ubiquitous and accessible today. There is a growing trend to encourage the open sharing of research data through data repositories; and students are collecting their own data easily with portable sensors and handheld devices. A good example is a large-scale citizen science project in Georgia that engages students in collecting data with a cell phone app on cyanobacteria (blue algae) in reservoirs, lakes, and even small ponds throughout the state. Students use the data to practice the scientific process, ask questions, formulate and test hypotheses, and develop scientific skills. They submit their data to a repository, and data that indicate that a body of water is in trouble are highlighted. A handheld sensor developed at the University of Georgia is used by others to confirm (or not) the existence of high levels of bacteria (Mishra, Personal Communication). Students need an interdisciplinary approach to working with vast amounts of data: mathematics, statistics, computer science, data science, geography, and more. But how do students gain an understanding of the primary tools of data science? And how do they do that when there is not yet agreement on what those primary tools are? There are existing sources, notably a few books, that will facilitate the use of data science practices for enhancing biology education and preparing students for the workforce of the future. We discuss these in Sect. 8. These are necessary, but are they sufficient? A more fundamental question is: Are they being used and do we need different ones?

8 Books for Math and the Life Sciences

Without appropriate books, the opportunity to change the way that introductory math is presented to biology students won't change. Fortunately, there are now some very good books available. Many of them are amenable to be used online. The following discussion is summarized in Table 1.

For engineers, science, and math majors, the old calculus standbys still reign supreme according to a number of publishers (personal communications): *Calculus* by Thomas (Hass et al. 2018), a standard of math courses for over 50 years, is still one of the top options on the market. It devotes itself primarily to practical applications of the material, rather than getting bogged down in theory, making it excellent for hands-on learners as answer keys show work. James Stewart's C*alculus* (see, for example, Watson and Clegg 2019) is an excellent guide for working through math problems, as it gives you a step-by-step look into the process, rather than just providing an answer and moving on. This makes it superior for self-directed learners, as there are no gaps in the instruction. There are multiple versions of Stewart's *Calculus* (see https://www.stewartcalculus.com/) including one targeted at life science majors (see below). A third book is *Calculus, 4th edition* by Spivak (Spivak 2008), the most challenging of the three and typically favored by those wanting a more theoretical calculus book.

For mixed audiences, one of the most frequently used books is *Applied Calculus: For Business, Economics, and the Social and Life Sciences* by Hoffman (Hoffman et al. 2013). This book was created to build a foundation in basic concepts of calculus through concise instruction as well as through thorough exercise sets. Another popular book, *Calculus and Its Applications* by Goldstein, Lay, Schneider, and Asmar (Goldstein et al. 2019), does an excellent job of making the material as intuitive as possible, demonstrating how math can be accessible and useful in the real world. Applications abound for economics, business, and the life sciences. The explanations are clear and concise and homework sets can be tailored to a student's major. One author (Midge Cozzens) has taught for 10 years from various editions of this book and finds it usable for a mixed audience. Because of the proliferation of AP calculus, many math majors are placing out of the introductory calculus course,

Table 1 A summary of calculus books by audience

Specific for Bio	Math/Engineering (3 most popular)	Bus./Bio/Soc Sci (most used)
Biocalculus: calculus, Probability, and Statistics for the Life Sciences by Stewart and Day (motivates and illustrates calculus with many applications from bio, while maintaining rigor)	*Calculus* by Thomas (old standby for 50 years; emphasis on practical applications)	*Calculus and Its Applications* by Goldstein, Lay, Schneider and Asmar (numerous exercises; how can be tailored to students' majors)
Calculus for the Life Sciences by Schreiber, Smith, and Getz (basic first-year calculus motivated by real modeling applications; emphasis on numerical methods, real calculation)	*Calculus* by James Stewart (comes in many versions; includes step-by-step guide for working through math problems)	*Applied calculus: For Business, Economics, and the Social and Life Sciences* by Hoffman (basic concepts of calculus + thorough exercise sets)
Mathematics for the Life Sciences by Bodine, Gross, and Lenhart (uses large data sets to explore interface between math and bio; explores a variety of math concepts relevant to bio)	*Calculus, 4th edition* by Spivak (the most theoretical and most challenging)	
Applications of calculus to Biology and Medicine: Case Studies from Lake Victoria by Ryan and Wallace (prepares students who have had one semester of calculus to engage with research literature in math modeling in bio)		
Calculus for the Life Sciences 2nd Edition by Greenwell, Ritchey, and Lial (includes a mymathlab supplement for those teaching online or as hybrid course)		
Mathematics for Biological Scientists, by Aitken, Broadhurst, and Hladky (more than calculus. Calculus is one of four themes; probability & statistics another)		
Calculus for Biology and Medicine, 4th edition by Neuhauser and Roper (aims to motivate life science majors to study calculus; level or rigor adjustable to high levels as appropriate)		

and so it is natural to see more calculus books that assume an audience without math majors.

A number of calculus books have been developed specifically for life science majors. *Calculus for the Life Sciences* 2nd Edition by Greenwell, Ritchey, and Lial (Greenwell et al. 2015) has a mymathlab supplement for those wishing to teach the course online or as a hybrid course. Consequently, it has risen to be an extremely popular book for those teaching a separate course for biology majors. *Calculus for Biology and Medicine*, 4th edition by Neuhauser and Roper (Neuhauser and Roper 2018) was early into the market of books to motivate life and health science majors to learn calculus through relevant and strategically placed applications to their chosen fields. It presents calculus in such a way that the level of rigor can be adjusted to meet the specific needs of the audience, from a purely applied course to one that matches the rigor of the standard calculus track. It, too, has a mymathlab supplement for those teaching the course online or as a hybrid course.

Calculus for the Life Sciences by Schreiber, Smith, and Getz (Schreiber et al. 2014) presents basic first-year calculus motivated by real modeling applications. The two main goals of the text are to provide students with a thorough grounding in calculus concepts and applications, analytical techniques, and numerical methods and to have students understand how, when, and why calculus can be used to model biological phenomena. There is a Wiley online lab supplement for the book. It has been used at Berkeley for a number of years for biology majors, but once the instructor left for another college, the course and book disappeared, illustrating that it is not just the quality of a book in calculus for the life sciences that matters, but also the availability of instructors willing to teach the course (Wayne Getz, personal communication). Traditionally, calculus courses put a great deal of emphasis on calculation, but now we have machines that can do a great deal of that, which can change the emphasis and also allow students to see larger and more realistic applications. The Schreiber, et al. book puts a good deal of emphasis on numerical methods and realistic calculation,

A number of new mathematics books for life science majors that devote substantial components to topics in addition to calculus have appeared in recent years. *Mathematics for Biological Scientists*, by Aitken et al. (2009), still in its first edition, is more than 10 years old. The book's chapters are organized into four themes. The first theme covers the basic concepts of mathematics in biology, discussing the mathematics used in biological quantities, processes, and structures. The second theme, calculus, extends the language of mathematics to describe change. The third theme is probability and statistics, where the uncertainty and variation encountered in real biological data are described. The fourth theme is explored briefly in the final chapter of the book, which is to show how the 'tools' developed in the first few chapters are used within biology to develop models of biological processes.

Bodine, Lenhart, and Gross's *Mathematics for the Life Sciences* (Bodine et al. 2014) uses large data sets to explore the interplay between mathematics and biology. It provides undergraduate life science students with an overview of mathematical concepts essential for modern biology today, explicitly linking data and mathematical modeling. It uses MATLAB throughout, and MATLAB m-files with an R supplement are available online, but the book does not contain an online supplement for those teaching the course online or as a hybrid. According to the publisher it is used in some small colleges and a few large universities (personal communication with publishers).

One of the many calculus books by Stewart is *Biocalculus: Calculus, Probability, and Statistics for the Life Sciences* (Stewart and Day 2014) (and its variations). The book motivates and illustrates the topics of calculus with examples drawn from many areas of biology, including genetics, biomechanics, medicine, pharmacology, physiology, ecology, epidemiology, and evolution, to name a few, while still maintaining mathematical rigor. It is used at many medium and large universities for biology sections and where the Stewart Calculus books are used for other sections (personal communications with publishers).

Ryan and Wallace's *Applications of Calculus to Biology and Medicine: Case Studies from Lake Victoria* (Ryan and Wallace 2018) prepares students to engage with the research literature in the mathematical modeling of biological systems, assuming they have had only one semester of calculus. The text includes projects, problems, and exercises; the projects ask the students to engage with the research literature, problems ask the students to extend their understanding of the materials and exercises ask the students to check their understanding as they read the text. It is primarily used to accompany the more standard calculus books, rather than as a stand-alone text.

9 Conclusions

The standard first introductory mathematics course in college remains calculus. The calculus as traditionally taught may not be the ideal way to present modern mathematics to students in other disciplines, specifically the life sciences. In particular, given the ever-increasing importance of data science in the life sciences and virtually everywhere, we believe that this topic should be somewhere in the introductory sequence. Some have argued that data science should be a self-standing introductory course, even in high school.

Indeed, it is difficult to separate issues of introductory mathematics for college from introductory mathematics for high school, since so many students in high schools now take AP calculus. However, there is no evidence that taking calculus in high school is of any benefit unless a student learns it well enough to earn college credit for it. And it is not clear that AP calculus will make it easier for students to avoid the major stumbling block that the calculus sequence presents for college students.

Presenting introductory mathematics courses in different sections aimed at different disciplines might not be feasible in many colleges, and certainly not in many high schools. And even where feasible, there are issues of how subject-matter examples (e.g., in the life sciences) are handled when the instructors might not be trained to understand them. Some training programs are likely to be needed. To be truly effective, we believe that introducing mathematics to students interested in other disciplines such as the life sciences calls for real integration of curricula: reinforcing biology in math courses and reinforcing math in biology courses. Online teaching of calculus might help in this regard.

In contrast to earlier years, there are now a variety of books available that offer introductory calculus for life sciences, and also books that mix calculus with probability, statistics, and data science with biological applications. Design of introductory courses will surely be easier due to the existence of such books, but the key challenges of true integration of introductory mathematics and the life sciences remain.

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