Title: Future Science: How fuel efficient can cars possibly be?

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Module Summary: We discuss the need to quantify energy consumption in any discussion of sustainability. We then model the energy dynamics of the car, in order to determine how much energy is necessarily required to power automobile transport, independent of future advancements in technology. Finally, we use our model to answer questions about how we can modify our behavior to improve sustainability.

Primary Reference: *Sustainable Energy Without the Hot Air* by David McKay

Target Audience: Advanced high school students or college students taking Precalculus, Environmental Science, Public Policy, or Introductory Physics.

Prerequisites: Comfort with algebraic manipulation and units of measurement

Mathematical Fields: Precalculus, Algebra

Goals and Objectives: Students will: Use elementary mathematics to develop a physical model for the energy expenditure of a car. Use this model to infer physical limitations of cars and answer questions pertaining to environmental policy. Estimate physical quantities using order of magnitude estimates. Apply the conservation of energy to a real world setting.

Preparation: Instructor should bring a container of butter with nutritional info. Students should check odometers before class in order to estimate their average daily driving, as well as look up the weight of their car. For students who don't drive, or who don't manage to look up these stats, we've provided average national figures when necessary.

THE MODULE:

PART 1

Today, the transportation sector accounts for 29% of the total US energy consumption, a figure hasn't changed much over the last 60 years (http://www.eia.gov/totalenergy/data/annual/pdf/sec2_6.pdf). Roughly two thirds of that (so nearly 20% of total energy use) goes into automobile use (http://www.need.org/needpdf/infobook_activities/IntInfo/ConsI.pdf). The automobile is an indispensable part of American life, and perhaps of your life as well.

Say you are on a government committee to map out the future of the automobile industry. As our reliance on fossil fuels increases, it is all the more important that we understand how technological innovations can reduce our dependence on these depleting resources. So you are faced with a key question: what will the energy consumption of a car look like in the future? How can current automobile designs be improved to increase energy efficiency? And how far can we take this? Is there a limit to the most efficient possible car?

These are science questions, and to answer them we need to develop a mathematical model for the energy needs of a car. And in answering these questions, we can begin to address further policy questions that go along with it. Should we change speed limits to increase energy?

efficiency, and if so, by how much? What directions are the most promising for funding research in energy efficient automobiles? How can math help us answer these questions?

Activity: At what speed do you typically drive in the city? How about on interstate highways?

If your primary concern is fuel efficiency, at what speed should you drive if you:

Are visiting you friend who lives in the same city? Assume you encounter traffic lights and stop signs.

Are visiting a friend who lives 40 miles away, connected by an interstate highway?

Discuss these questions and write down your answers--we'll come back to them later.

In order to discuss energy consumption and sustainability, we need to first decide on a way to quantify energy and power.

The SI unit for *energy* is the **joule** (J), which is the amount of energy it takes to apply a force of one newton over a distance of one meter - not the most intuitive measure! To give you a sense of this unit, imagine raising a 1 kilogram weight (2.2 lbs) by 10 centimeters - you will have expended about a joule of energy in the process.

A joule is a small amount of energy when compared to our daily energy requirements. For example, in 2003, Americans used 896,930,000 joules of energy per person per day (Source: World Resources Institute).

The **watt** (W) is the SI unit for *power*, which is defined as the rate at which energy is used.

power = energy / time

So one watt equals one joule per second. If your light bulb is rated 40W, that means whenever it is switched on, it's consuming 40 joules of electrical energy in a second.

A useful analogy for the relation between energy and power is that of water volume and the flow rate of water. A drinking fountain may flow at one liter per minute, in which case you'd have to run it for a full minute to fill your one liter water bottle. On the other hand, you could fill it in six seconds with a hose that flows at 10 liters per minute. In both cases the volume of water delivered is equal to the flow rate multiplied by the time.

volume = flow rate x time

energy = power x time

We can liken energy to water-volume, and power to water-flow. When a 1000W toaster is switched on, it consumes energy at a rate of 1000 joules per second. That's a much more powerful flow than our light bulb. In fact, if our 40W bulb is a kitchen faucet (which flows at 8 gallons/minute), then the toaster is a pressurized fire-hose (flowing at 200 gallons/minute)!

Exercise: What is the power consumption of an average 2003 American, in Watts?

Exercise: A sedentary male in the age range 19-30 needs approximately 2,400 food Calories per day to stay healthy. For sedentary females in the same age range, the number is

2,000 food Calories per day (Source: US Department of Health and Human Services http://bcccp.ncdhhs.gov/linksandresources/EducationMaterialsandResources/nutrition.pdf).

Estimated Calories Needed by Gender, Age, and Activity Level [a]				
Gender	Age (Years)	Sedentary[b]	Moderately Active[c]	Active[d]
Child	2 - 3	1,000	1,000 - 1,400[e]	1,000 - 1,400[e]
Female	4 - 8	1,200	1,400 - 1,600	1,400 - 1,800
	9 - 13	1,600	1,600 - 2,000	1,800 - 2,200
	14 - 18	1,800	2,000	2,400
	19 - 30	2,000	2,000 - 2,200	2,400
	31 - 50	1,800	2,000	2,400
	51+	1,600	1,800	2,000 - 2,200
Male	4 - 8	1,400	1,400 - 1,600	1,600 - 2,000
	9 - 13	1,800	1,800 - 2,200	2,000 - 2,600
	14 - 18	2,200	2,400 - 2,800	2,800 - 3,200
	19 - 30	2,400	2,600 - 2,800	3,000
	31 - 50	2,200	2,400 - 2,600	2,800 - 3,000
	51+	2,000	2,200 - 2,400	2,400 - 2,800

Look up your daily energy requirement from the above table. Assuming we only consider the energy that you get from food, what is your power consumption? In other words, what amount of power does your food supply? You will need to use that 1 food Calorie = 4184 Joules.

As you can see, Joules and Watts are not human-sized units. A more convenient unit of energy that is commonly used to measure daily usages is the **kilowatt-hour (kWh)**. Don't get confused by the wording, even though it contains 'kilowatt', *this is a unit of energy*. In defining this unit, we're using that fact that **energy = power x time.** So, we can express a unit of energy as a unit of power (a kilowatt = 1000 W) multiplied by a unit of time (1 hour). If you look at your utilities bill, you will find your monthly usage listed in kilowatt-hours.

Exercise: How many **Joules** of energy are there in a **kilowatt-hour**?

Objects in our lives that guzzle energy usually guzzle a small number of kilowatt-hours in a day. That suggests a new convenient unit for power, the **kilowatt-hour/day** (kWh/d). This is an energy divided by a time (1kWh/1 day), so it is a unit of power.

Exercise: How many watts make up one kilowatt-hour/day? How much power does a 40W light bulb consume, when expressed in kWh/d?

Most human activities use a small number of kilowatt-hours of energy. We'll now calculate how much energy *you* use when driving your car, assuming you do in fact drive. In case you don't, you can pretend you drive 40 miles per day--the approximate average for college-aged people (http://www.fhwa.dot.gov/ohim/onh00/bar8.htm) .

The amount of energy your car consumes each day is equal to:

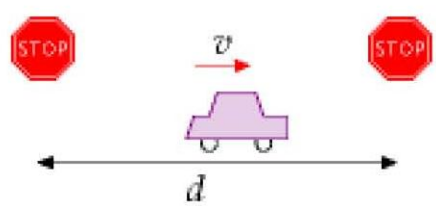
(distance travelled per day / distance per gallon of fuel) × (energy per gallon of fuel)

Exercise: Estimate the quantities in the above expression. For the fuel, you can use this butter package for reference, since butter is a hydrocarbon. The estimate you make should come out close to the actual energy density of gasoline. You'll need to know that one food Calorie is about 4.2 kilojoules (4184 Joules), and that one gallon is 256 tablespoons.

PART 2

Activity: We saw above that the average energy consumption of your car per day is on the order of 50 kWh. In what ways does this energy end up being consumed? We often hear of more "energy-efficient" cars. Does energy dissipation depend on properties of the car? Which properties are these? Write these down and discuss. We will revisit these questions and answers.

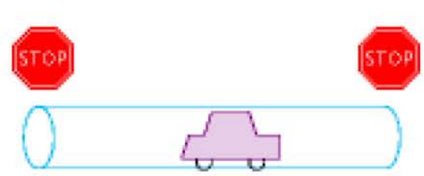
We'll begin by building a model of the energy consumption of a car. Assume that the driver starts from rest, accelerates to a cruising speed of v miles per hour, and then maintains this speed v over a distance of d miles. This could be the distance between traffic lights or stop signs.



Any car, by virtue of its motion, has a certain amount of energy associated with it. This is the minimum amount of energy that it took to bring it to this speed, and also the amount of energy you would need to spend in slowing it down.

Question: When moving at a speed of v, what kind of energy does a car possess due to its motion? How can this be expressed numerically?

So back to the driver, she is cruising at a speed of v, possesses kinetic energy and after covering a distance d, she slams the brakes coming to a stop. This converts all the kinetic energy into heating up the brakes (and some energy into the screeching noise).



Braking is not the only way for energy-loss to occur. Another way that a moving car consumes energy is by making air swirl around it as it moves. When moving, a car pushes air out of its way, creating a tube of swirling air behind it, which moves at a speed close to v. Since this tube of air is moving, it possesses kinetic energy as well.

To summarize, the main ways in which energy are dissipated from the car are by slowing down the car using brakes, and pushing the air out of the way as the car moves forward (air resistance).

Let's take a closer look at these two kinds of energy.

1) **Braking energy.** If the mass of the car is $\mathbf{m_c}$ and it moves a distance **d** at speed **v** before coming to a stop, then the rate at which energy is poured into the brakes is given by

(kinetic energy of the moving car) / (time between stops)

Question: Express the rate at which energy is poured into the brakes in terms of the mass of the car, the speed, and the distance between stops.

2) **Air Resistance.** This is energy it takes to bring a stationary tube of air up to a speed v (or, if you prefer, the kinetic energy of the tube of air). This can be written as ½ $m_{air}v^2$, where m_{air} is the mass of the air that is being accelerated.

How do we compute the mass of the moving tunnel of air? If we know the density of air, we can express mass in terms of volume.

mass = density x volume

Question: What is the mass of the tube of air swept out in a given period of time t? Express your answer in terms of the cross sectional area A, the speed v and the time period t. You may assume that the air is moving at the same speed as the car.

Question: Now that we have an expression for the mass of the air tube, **derive an expression for the rate at which energy is spent in moving the tube of air** (express your answer in terms of the density of air is ϱ (the Greek letter rho), the cross sectional area of the car A, and the speed of the car v).

Discussion: What is a good approximation for the cross-sectional area of the swirling tube? How would it change for a more streamlined car?

So, the total rate of energy consumed by the car = **power going into the brakes + power in pushing air out of the way**. = **½ $m_c v^3/d$ + ½ $\varrho A v^3$.**

Question: Both these rates of energy consumption scale as v^3. How does a driver who halves her speed change the energy consumption of the car? If she drives the same distance, how much longer will the journey take? And how will the energy consumption change?

PART 3

Our model takes into account two primary ways energy is dissipated while driving, namely braking and air resistance.

The rate of energy spent in braking is **$\frac{1}{2} m_c v^3/d$**

The rate of energy spent bringing air to the speed of the car is **$\frac{1}{2} \varrho A v^3$**

If we want to improve our car's efficiency, then a natural starting point is to determine which of the two energy dissipations is bigger.

Activity: Determine whether more power goes into the brakes or into pushing the tube of air. Your answer should depend on whether the mass of your car is greater or lesser than another quantity. What is the physical meaning of this other quantity?

To get started, it might help to consider the ratio of the two terms i.e.

power spent in braking / power spent in pushing air

and determine if it is more or less than 1.

What does your car mass need to be for energy dissipation to be dominated by braking? What does it need to be for energy dissipation to be dominated by air resistance?

Answer: The ratio simplifies to $(m_c/d) / (\varrho A)$. It is bigger than 1 if $m_c > \varrho Ad$. Since Ad is the volume of the tube of air swept out as the car travels d feet, ϱAd is the mass of that same tube. Thus energy dissipation is dominated by braking when the mass of the car is greater than the mass of the tube of displaced air. And otherwise energy dissipation is dominated by the loss due to air resistance (also known as drag).

You should have found that:

if $m_c > m_{air}$ driving is braking dominated
if $m_c < m_{air}$ driving is drag dominated

In other words, the dominant form of energy loss depends on the type of driving we are doing. In particular, it depends on the whether the mass of car is greater or lesser than the mass of the tube of air displaced by the car (starting from the time it accelerates to the moment it brakes to a halt).

Let's focus on the mass of the tube of air, $m_{air} = \varrho Ad$. This depends on a few factors. First, there is d, the distance between stops of the car. We'll talk about this in a moment. Then there's the cross sectional area A. In reality, this is composed of two terms

effective cross sectional area A = drag coefficient x cross sectional area of car

$(A = c_d A_{car})$

The drag coefficient is a fraction that has to do with how streamlined your car is. More streamlined shapes have smaller drag coefficients. For example, swimmers form a stream line with their arms and minimize their drag coefficient when diving into water.

Exercise: Determine the distance d at which both terms are equal. You can also think of this as the distance that separates brake-dominated driving from drag-dominated driving. If you know the weight of your car, you can use that. Otherwise you can approximate it by 1000 kg. You will need to estimate the cross sectional area of your car, in order to calculate the volume of the tube of displaced air. A reasonable estimate might be 3 squared meters . Finally, you'll need the drag coefficient, for which you can use ⅓.

Exercise: Using this threshold distance **d**, argue that in "stop-and-go" traffic, such as city driving, braking is the primary source of energy loss, whereas highway driving is dominated by air resistance.

Exercise: How can you improve your car's efficiency in city driving? Consider the braking term of the energy loss formula and describe two ways of improving efficiency. How about highway driving? Use the air drag term to come up with three ways of improving efficiency.

So far we've neglected one significant source of energy loss: engine inefficiency. A typical petrol engine uses only 25% of fuel energy towards powering the car--the rest serves only to heat up the engine. So to get a realistic estimate of the total power of a car, we need to scale our formula by a factor of 4:

$$\textbf{total power of car} = 4\,(\tfrac{1}{2}\, m_c v^3/d + \tfrac{1}{2}\,\rho A v^3)$$

Activity: Plug in plausible numbers to the above formula to determine the amount of power a typical driver might expend while driving. Let's first consider highway driving at a speed of 70 miles per hour. You can assume the braking distance is large enough to render the first term insignificant. (70 miles per hour = 110 km/h = 31 m/s and $A = c_d A_{car} = 1\ m^2$)

If you were to drive at this speed for one hour, how much energy would your car expend (in kWh)? What if you drove at half this speed?

Question: Using the model we've built so far, argue whether the energy savings from a small electric car will be more substantial in city driving or in highway driving. Support your argument using one of the equations derived above.

The contents of this teaching module are based on *Chapter 3* and *Technical Chapter A* of David McKay's excellent book *Sustainable Energy Without the Hot Air*, available free online at http://www.withouthotair.com