Multiple Antennas: A Network View

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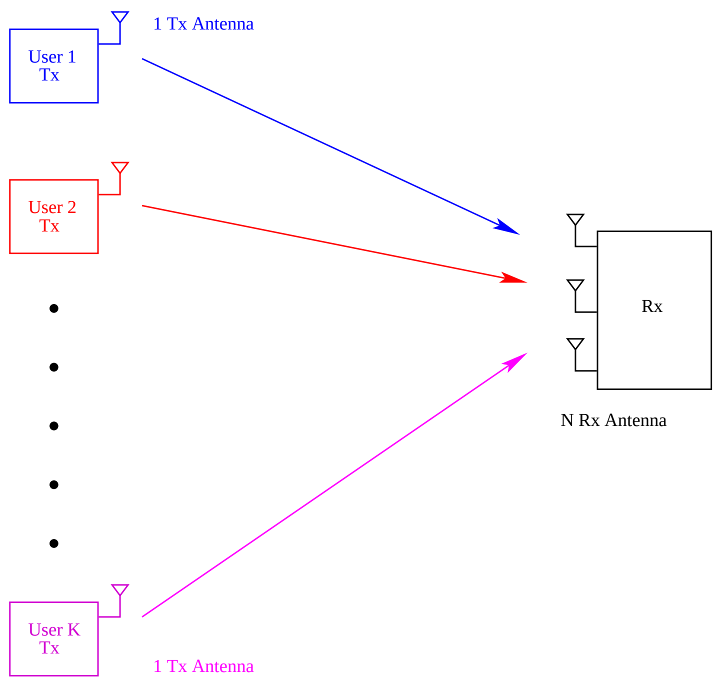
Joint work with D. Tse at UCB and L. Zheng at MIT.

MIMO in Wireless Networks

- Explosion of research in recent years
  - information theory
  - coding
  - signal processing
- Much focus on point-to-point channels
- To understand impact of multiple antennas in wireless networks, need broader view

Multiple Access Example



Question: what does adding one more antenna at each mobile buy me?

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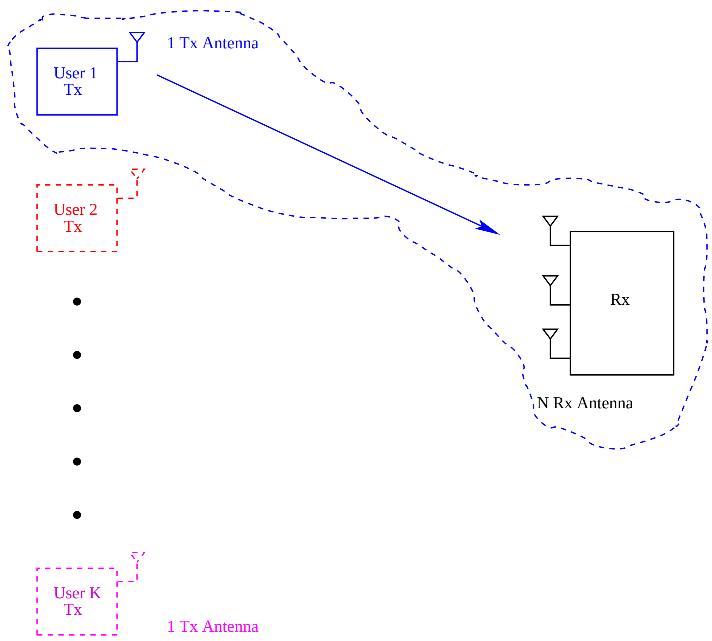
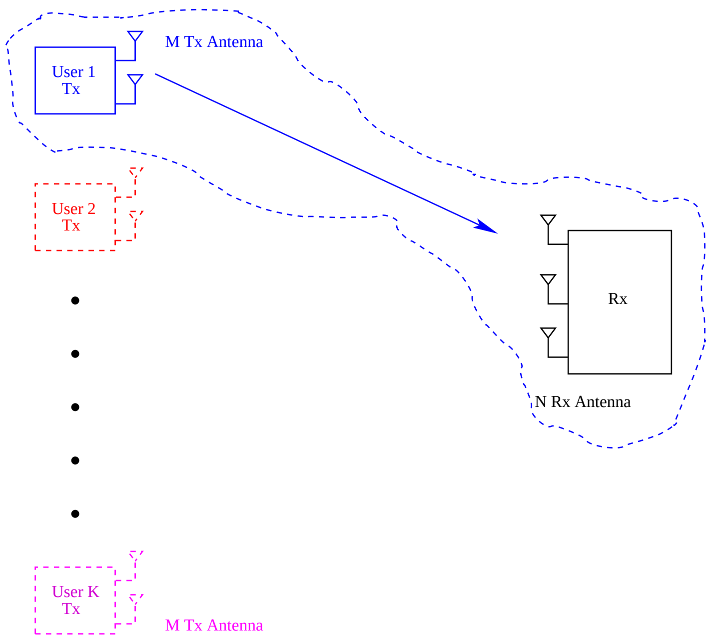


Question: what does adding one more antenna at each mobile buy me?

- Looking at each point-to-point link in isolation:

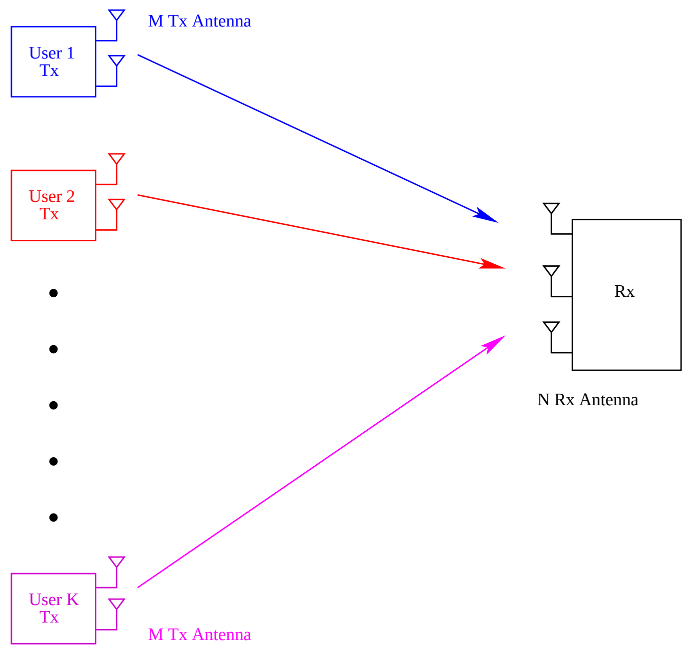
Example



Question: what does adding one more antenna at each mobile buy me?

- Looking at point-to-point link in isolation:
  - (roughly) doubles the link capacity.

Example

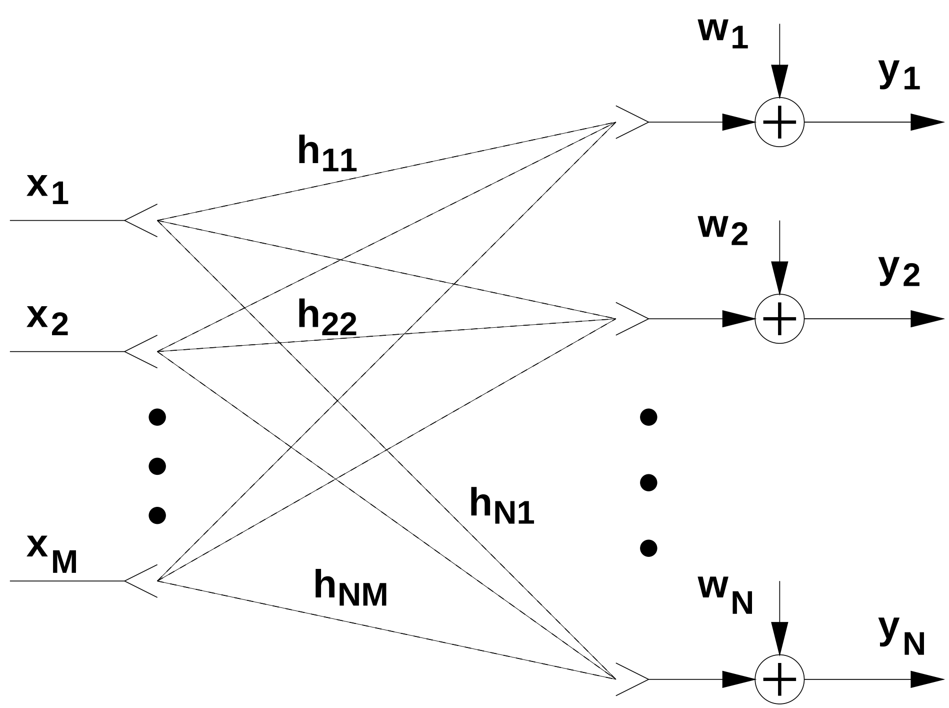


- Looking at the network:
  - number of users is greater than number of receive antennas
  - increase in overall system capacity negligible
- But does adding that antenna still buy me something?

Outline of Talk

- Review of diversity-multiplexing tradeoff in point-to-point channels.
- Extension to multiple access scenario.
- Speculation on a theory for general networks.

Point-to-Point MIMO Channel



M transmit and N receive antennas.

I.I.D. Rayleigh fading model.

Degrees of Freedom

- point-to-point link: M transmit, N receive antennas
- i.i.d. Rayleigh fading (Foschini 96):

$$C \sim \min\{M, N\} \log \mathrm{SNR} \quad \text{bits/s/Hz.}$$

- Multiple antennas provide $\min\{M, N\}$ degrees of freedom
- spatial multiplexing gain of $\min\{M, N\}$
- C is the ergodic capacity.

Diversity

- Ergodic capacity assumes infinite-depth interleaving
- Impossible in a slow fading environment
- Unreliability due to fading is a first-order issue.
- In 1 by 1 Rayleigh fading channel: very poor error probability.
- Example: for BPSK:

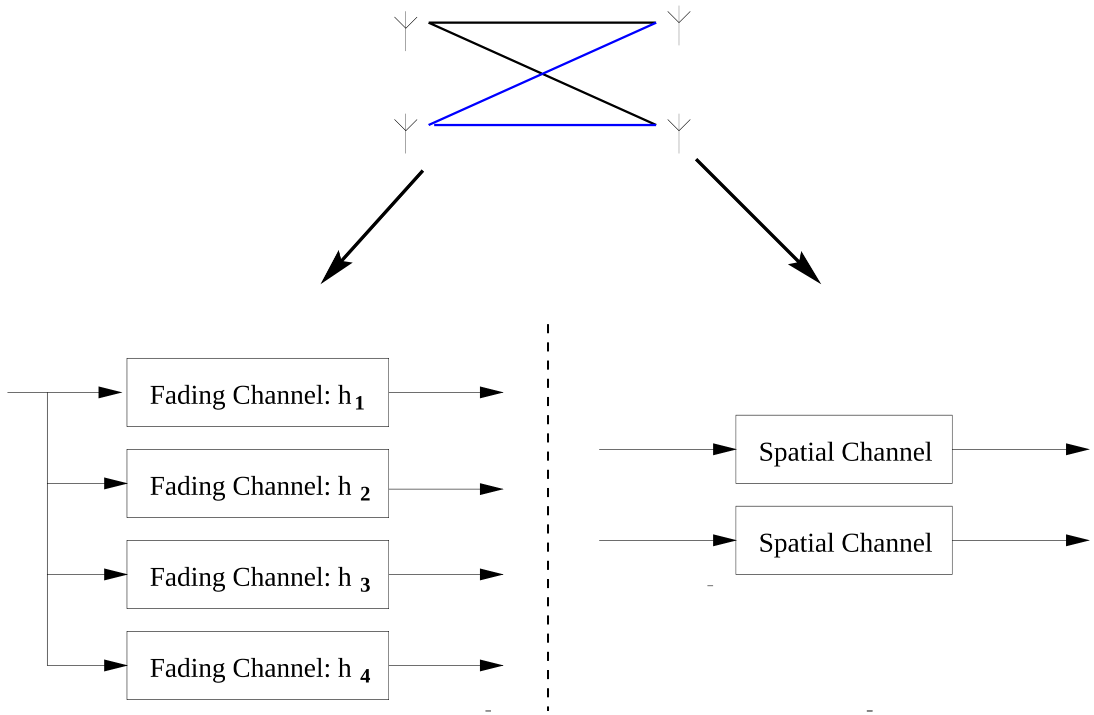
$$P_e \sim \mathrm{SNR}^{-1} \quad \text{at high SNR}$$

- In M by N channel, however,

$$P_e \sim \mathrm{SNR}^{-MN} \quad \text{at high SNR}$$

- Multiple antennas provide a maximum of MN diversity gain.

Diversity and Multiplexing



But each is only a single-dimensional view of the situation.

The right way to formulate the problem is a tradeoff between the two types of gains.

Fundamental Tradeoff

Focus on high SNR and slow fading situation.

A space-time coding scheme of block length T achieves

Spatial Multiplexing Gain r	:	if data rate $R = r \log \mathrm{SNR} \quad (bps/Hz)$
and		
Diversity Gain d	:	if error probability $P_e \sim \mathrm{SNR}^{-d}$

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Fundamental tradeoff: for any r, the maximum diversity gain achievable: $d^*_{M,N}(r)$.

$$r \longrightarrow d^*_{M,N}(r)$$

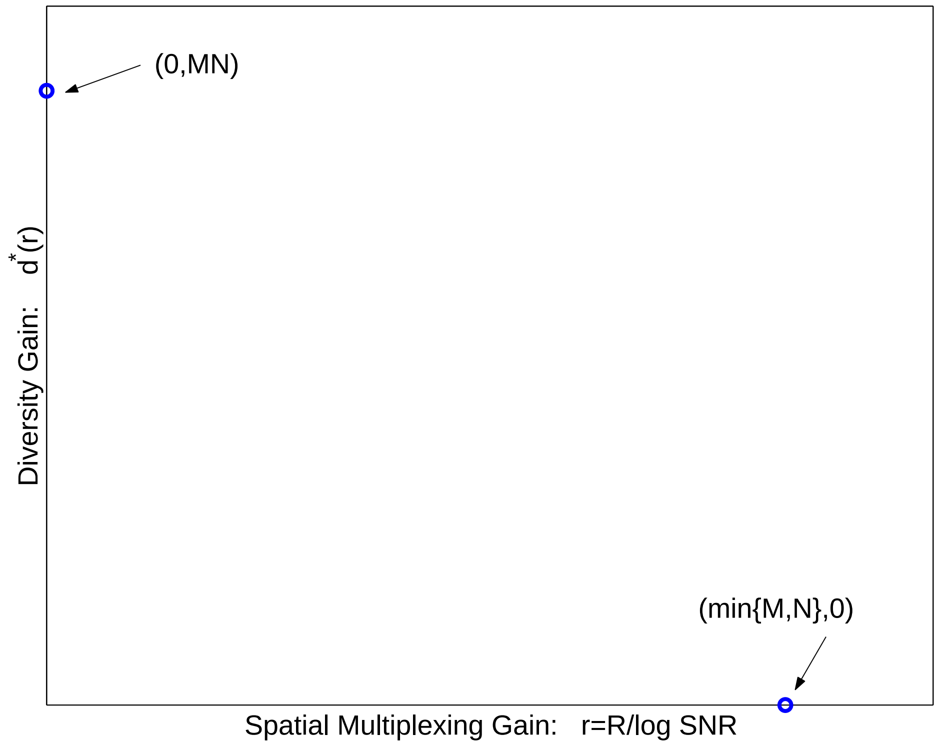
Equivalently:

$$d \longrightarrow r^*_{M,N}(d)$$

A tradeoff between data rate and error probability.

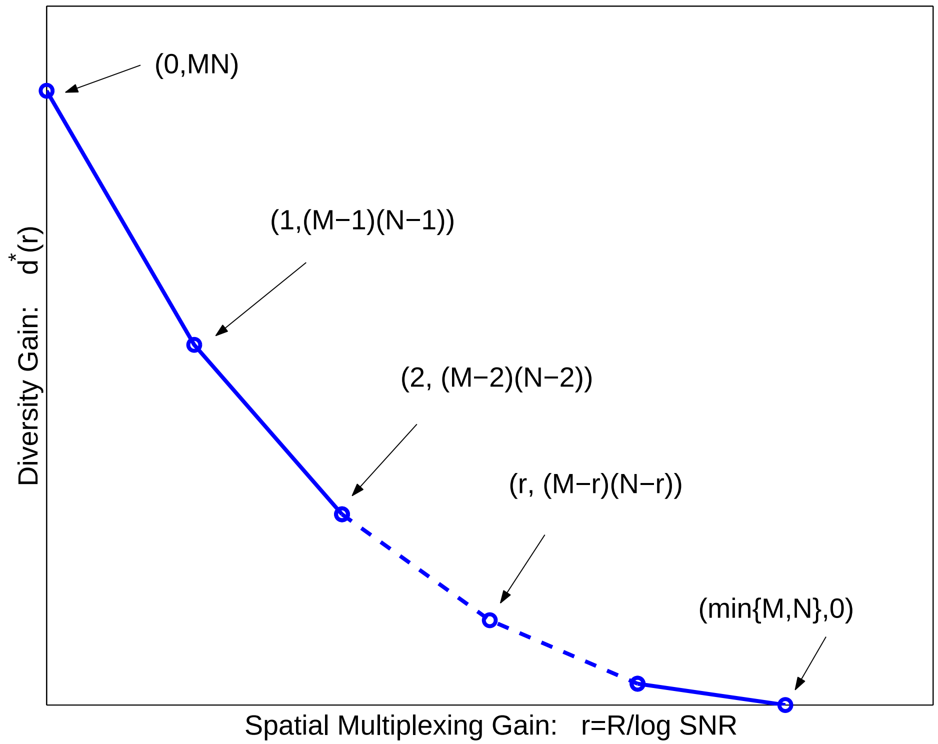
Optimal Tradeoff

(Zheng,Tse 02) If block length $T \geq M + N - 1$:



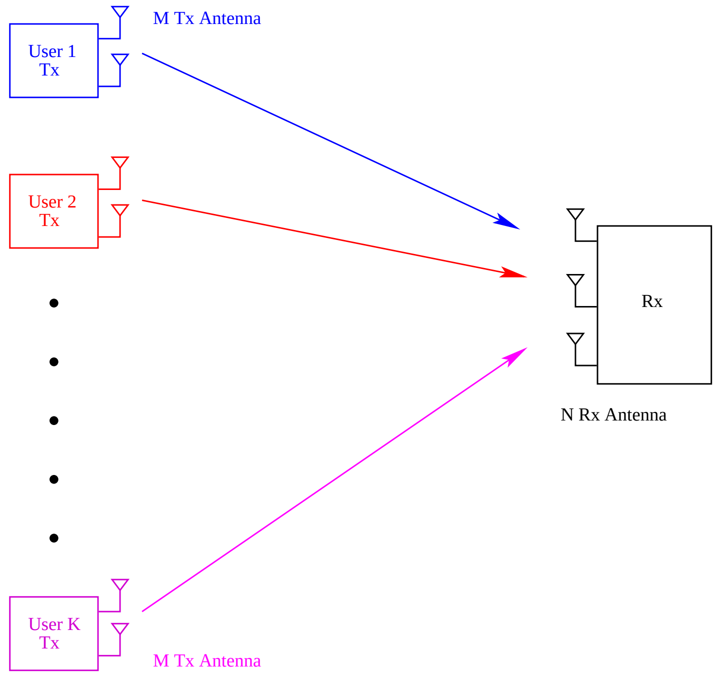
Optimal Tradeoff

(Zheng,Tse 02) If block length $T \geq M + N - 1$:



For multiplexing gain of r (r integer), best diversity gain achievable is $(M - r)(N - r)$.

Multiple Access



- For point-to-point, multiple antennas provide diversity and multiplexing gain.
- With K users, multiple antennas discriminate signals from different users too.
- i.i.d. Rayleigh fading, N receive, M transmit antennas per user.

Multiuser Diversity-Multiplexing Tradeoff

Suppose we want every user to achieve an error probability:

$$P_e \sim \mathrm{SNR}^{-d}$$

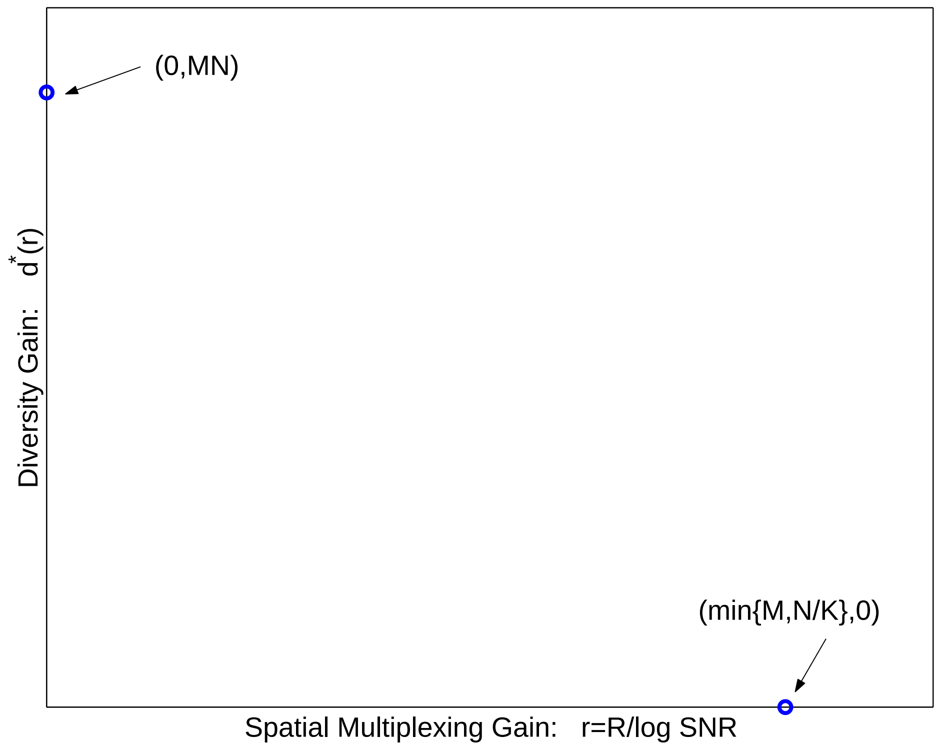
and a data rate

$$R = r \log \mathrm{SNR} \quad \text{bits/s/Hz.}$$

What is the optimal tradeoff between
d (diversity gain) and r (multiplexing gain)?

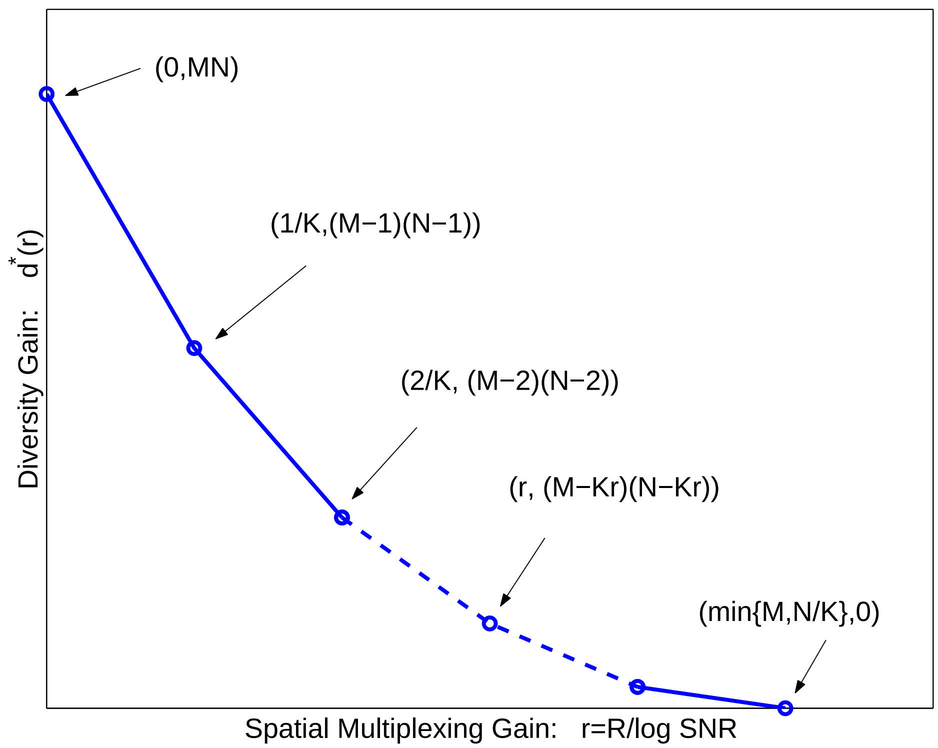
Assume a block length $T \geq KM + N - 1$.

Optimal Multiuser D-M Tradeoff



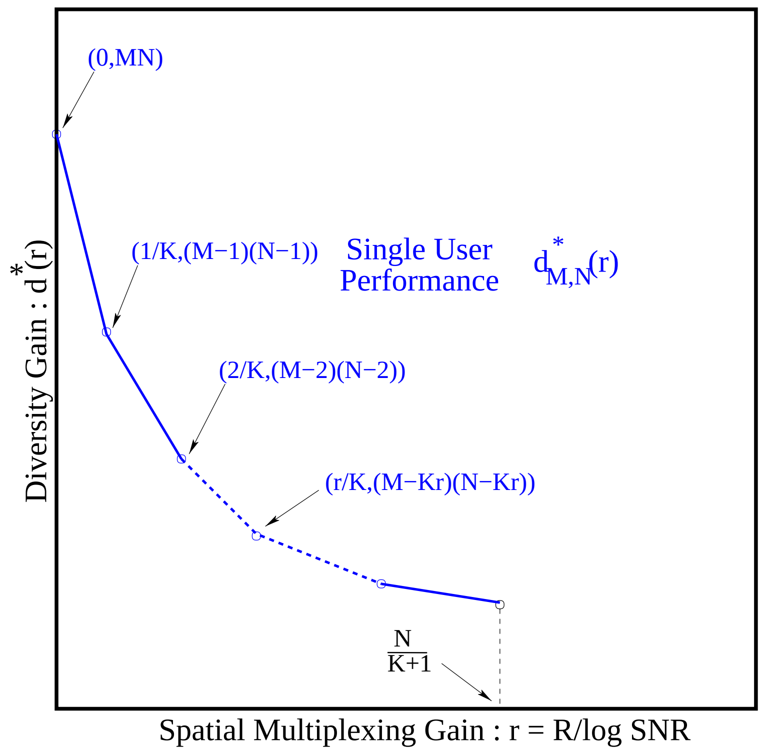
- For $r = 0$, diversity is MN
- For $r = \min\{M, \frac{N}{K}\}$, diversity is 0

Multiuser Tradeoff: $M < N/(K+1)$



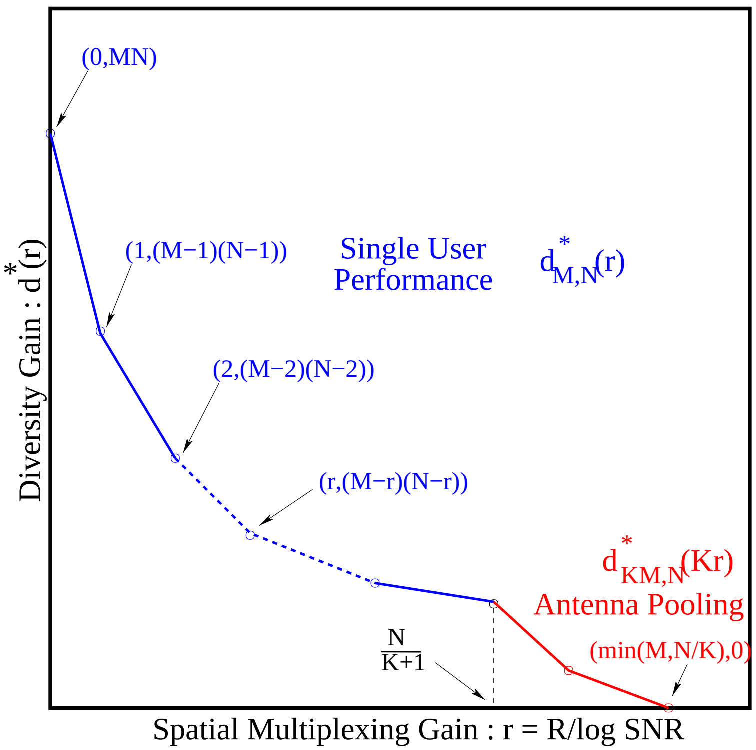
- diversity-multiplexing tradeoff of each user is $d^*_{M,N}(r)$
- as though it is the only user in the system

Multiuser Tradeoff: $M > N/(K+1)$



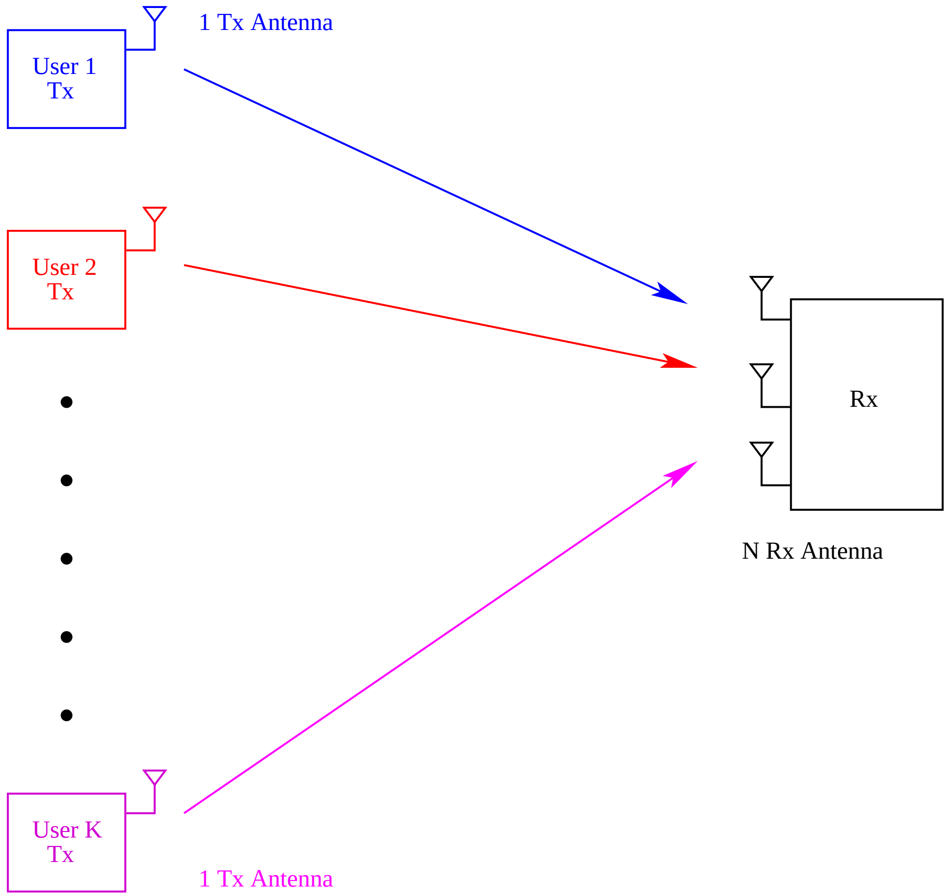
- $r \leq N/(K+1)$: Single-user tradeoff curve

Multiuser Tradeoff: $M > N/(K+1)$



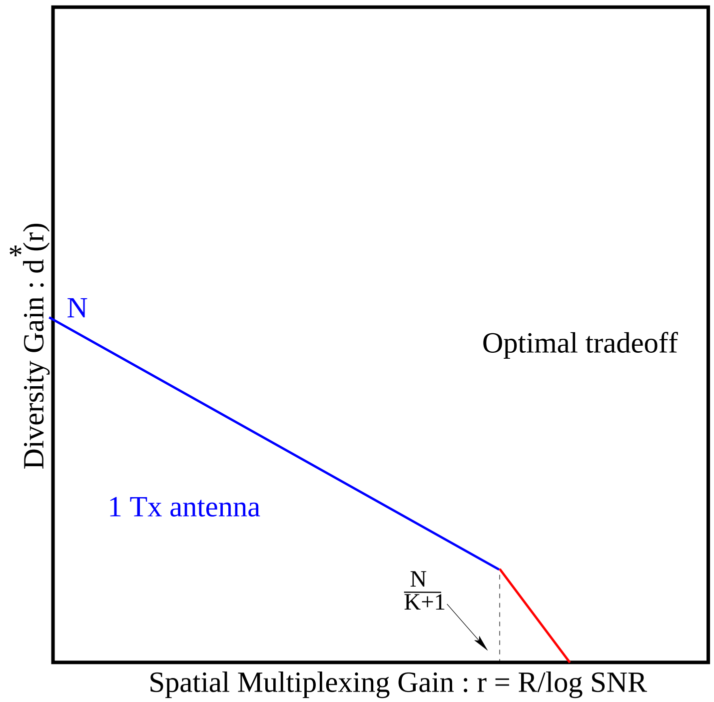
- $r \leq N/(K+1)$: Single-user tradeoff curve
- r from $N/(K+1)$ to $\min\{M, N/K\}$:
  - tradeoff as though the K users are pooled together: KM antennas and rate Kr,

Back to Motivating Example

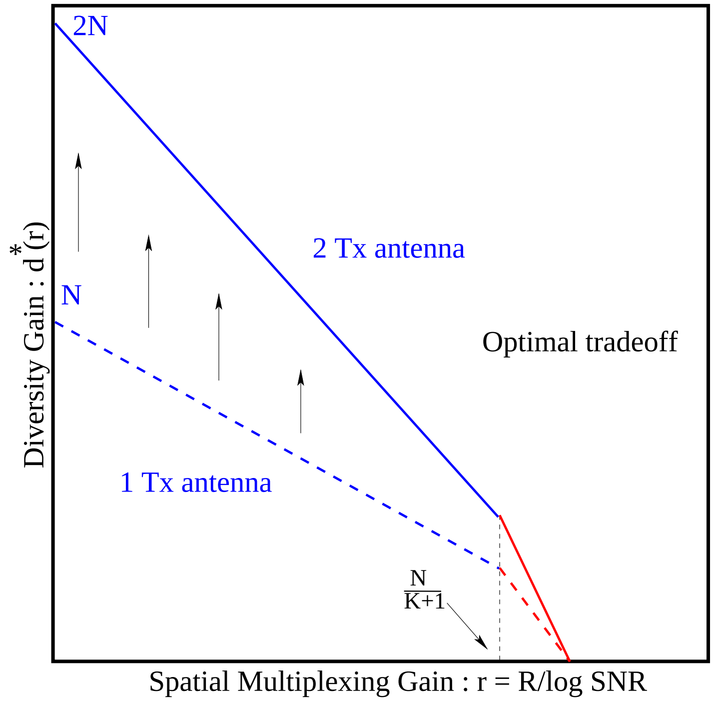


Question: what does adding one more antenna at each mobile buy me?

Scenario of 1 transmit antenna



Answer: Adding one more transmit antenna



- No increase in number of degrees of freedom
- However, increases the maximum diversity gain from N to $2N$.
- Improves diversity gain $d(r)$ for every r.

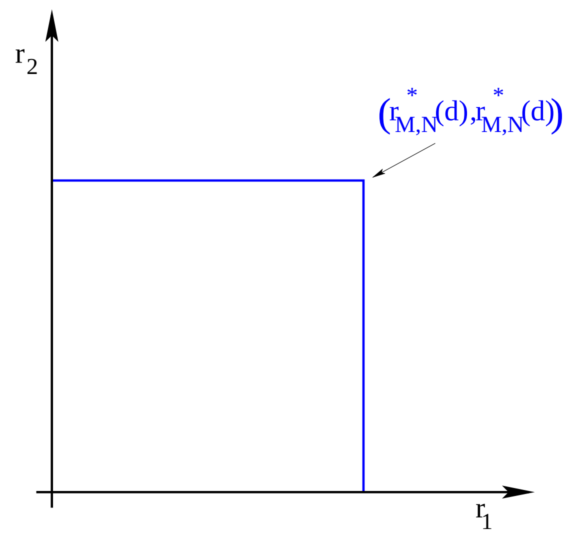
Tradeoff Between Users

- We have been looking at the symmetrical, equal rate case.
- More generally, we can ask:

What is the optimal tradeoff between the achievable multiplexing gains for a given diversity gain d?

- Given by the multiplexing gain region $\mathcal{C}(d)$ for a given d.

Cubic Region



- Multiplexing gain region $\mathcal{C}(d)$ is a cube: $r_i \leq r^*_{M,N}(d)$
- Single user performance for every user
- Require:
  - $M \leq N/(K+1)$ (large number of receive antennas), or
  - $M > N/(K+1)$ but $d \geq d^*_{KM,N}[N/(K+1)]$ (high diversity requirement)

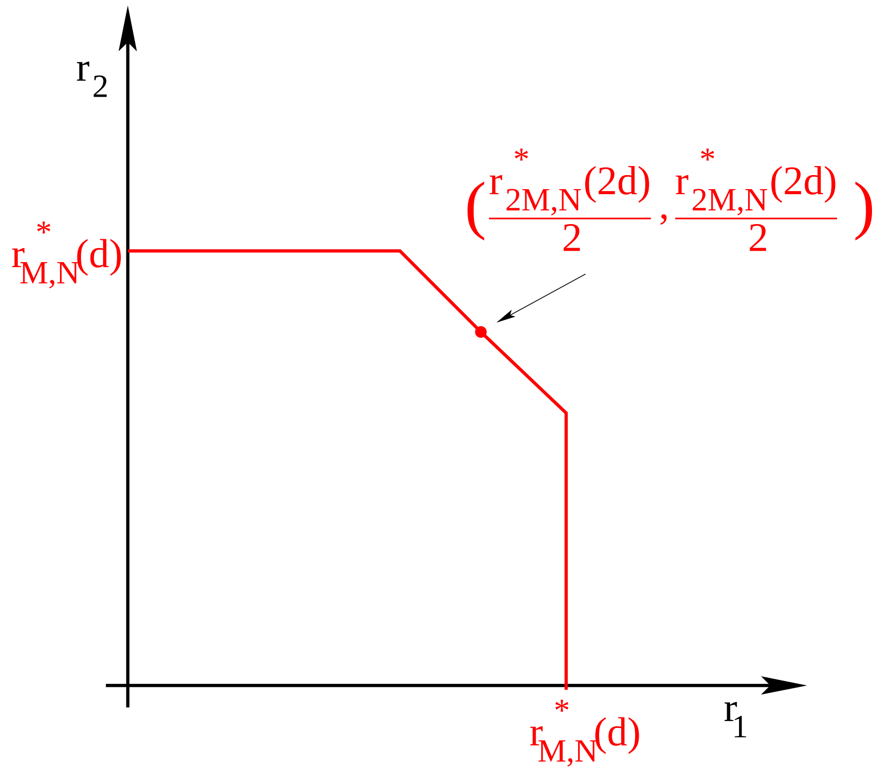
Multiplexing Region: General Case

If $d \in \left[d^*_{(k-1)M,N}[N/k], d^*_{kM,N}[N/(k+1)]\right]$:

$$\mathcal{C}(d) = \left\{(r_1, \ldots, r_K) : \sum_{i \in \mathcal{S}} r_i < r^*_{|\mathcal{S}|M,N}(d), \qquad \forall \mathcal{S} \text{ with } |\mathcal{S}| = 1 \text{ or } |\mathcal{S}| \geq k\right\}$$

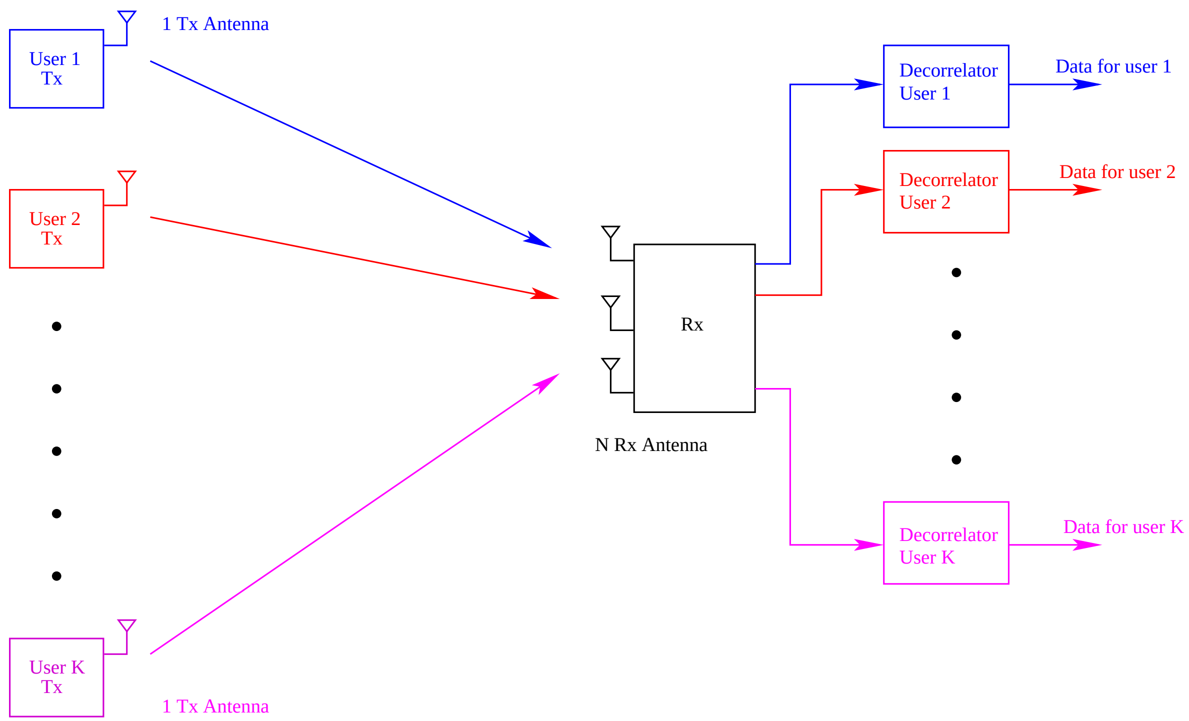
- $r^*_{|\mathcal{S}|M,N}(d)$ is point-to-point M-D tradeoff with $|\mathcal{S}|M$ Tx and N Rx antennas.
- As d decreases, more and more constraints become active
- Finally, $2^K - 1$ constraints are active: $\mathcal{C}(d)$ is a polymatroid

2-user example



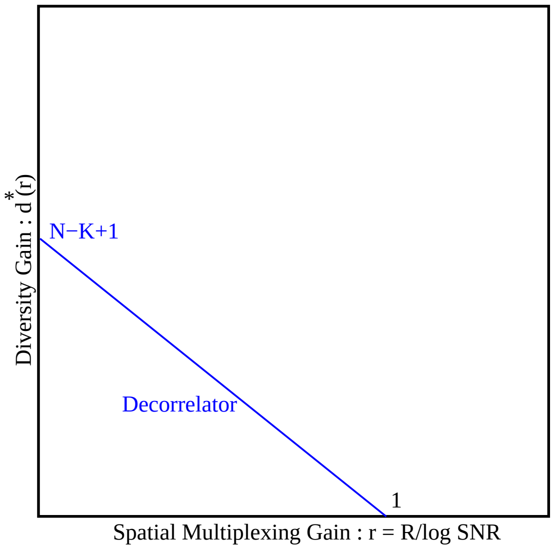
$r^*_{2M,N}(d)$ is total multiplexing gain in system with $2M$ transmit antennas pooled together.

Suboptimal Receiver: the Decorrelator/Nuller



- Consider case of $M = 1$ transmit antenna for each user
- Number of users $K < N$

Tradeoff for the Decorrelator



- Maximum diversity gain is $N - K + 1$
- "costs $K - 1$ diversity to null out $K - 1$ interferers" (Winters et al '93)

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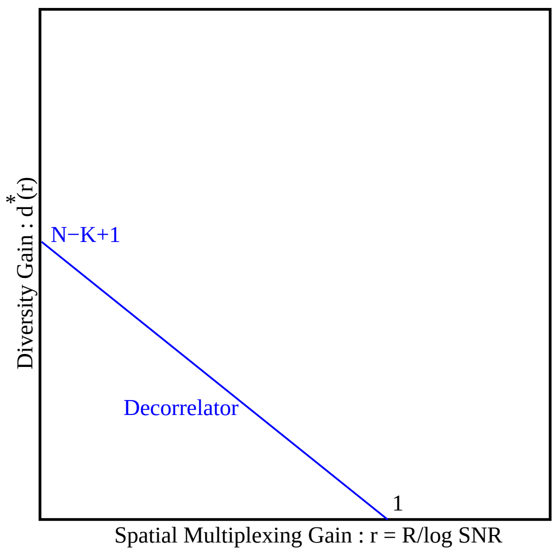

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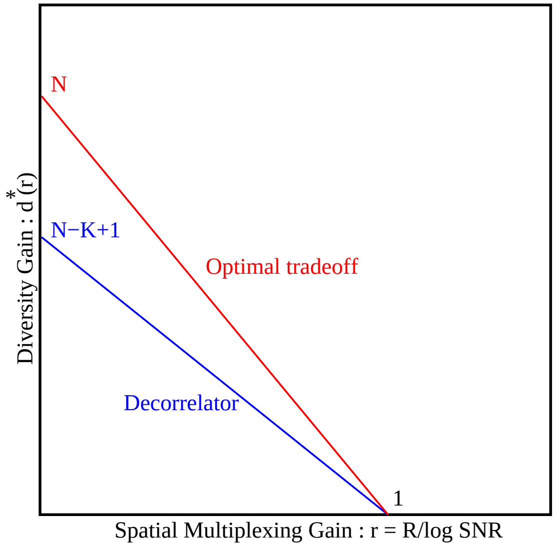
- "costs $K - 1$ diversity to null out $K - 1$ interferers" (Winters et al '93)
- Adding one receive antenna provides:
  - either more reliability per user
  - or accommodate 1 more user at the same reliability.

Tradeoff for the Decorrelator



- Optimal tradeoff curve also a straight line
  - but with a maximum diversity gain of N.
- Adding one receive antenna provides more reliability per user and accommodate 1 more user.

Multiple Antennas in General Networks

Multiple antennas serve multiple functions:

- diversity
- spatial multiplexing
- multiple access
- broadcast
- interference suppression
- cooperative relaying (distributed antennas)
- etc ....

What is the fundamental performance tradeoff in general?

Our approach may give a simple picture.