Conference in honor of the 70th birthday of Endre Szemerédi

August 2-7, 2010, Budapest

Program of the Szemerédi 70 conference

Monday, 08.02	**Speaker**
8:00 - 10:00	*Registration*
10:00 - 10:15	*Opening (***András Hajnal***)*
10:15 - 11:00	**Noga Alon**
11:00 - 11:15	*Coffee break*
11:15 - 12:00	**József Beck**
12:00 - 14:00	*Lunch break*
14:00 - 14:45	**Béla Bollobás**
14:45 - 15:00	*Break*
15:00 - 15:45	**Mei-Chu Chang**
15:45 - 16:00	*Coffee break*
16:00 - 16:45	**Van Vu**
18:00 - 20:30	*Wine and cheese reception**

* details on the ticket

Tuesday, 08.03	**Speaker**
10:00 - 10:45	**Zoltán Füredi**
10:45 - 11:00	*Coffee break*
11:00 - 11:45	**Ben Green**
12:00 - 14:00	*Lunch break*
14:00 - 14:45	**Jeff Kahn**
14:45 - 15:00	*Coffee break*
15:00 - 15:45	**Gil Kalai**
17:00 - 19:00	*Vernissage (Art exhibition opening)* **

** details in the catalogue

Wednesday, 08.04	**Speaker**
10:00 - 10:45	**László Lovász**
10:45 - 11:00	*Coffee break*
11:00 - 11:45	**Jiří Matoušek**
12:00 - 14:00	*Lunch break*
14:00 - 14:45	**Jaroslav Nešetril**
14:45 - 15:00	*Break*
15:00 - 15:45	**János Pach**
15:45 - 16:00	*Coffee break*
16:00 - 16:45	**Vojtěch Rödl**
19:00 - 22:00	*Boat trip/Banquet* *

* details on the ticket

Thursday, 08.05	**Speaker**
10:00 - 10:45	**János Pintz**
10:45 - 11:00	*Coffee break*
11:00 - 11:45	**Miklós Simonovits**
12:00 - 14:00	*Lunch break*
14:00 - 14:45	**József Solymosi**
14:45 - 15:00	*Break*
15:00 - 15:45	**Joel Spencer**
15:45 - 16:00	*Coffee break*
16:00 - 16:45	**Imre Ruzsa**

Friday, 08.06	**Speaker**
10:00 - 10:45	**Balázs Szegedy**
10:45 - 11:00	*Coffee break*
11:00 - 11:45	**Terence Tao**
12:00 - 14:00	*Lunch break*
14:00 - 14:45	**Tom Trotter**
14:45 - 15:00	*Coffee break*
15:00 - 15:45	**Avi Wigderson**

Abstracts*

Noga Alon, Tel Aviv University (Israel)

Universality, tolerance, chaos and order

What is the minimum possible number of edges in a graph that contains a copy of every graph on n vertices with maximum degree at most k? This question, as well as several related variants, received a considerable amount of attention during the last decade. I will describe the known results focusing on the main ideas in the proofs, discuss the remaining open problems, explain the meaning of the title of the lecture, and mention a recent application in the investigation of the complexity of subgraph containment problems.
Based on joint results with Asodi, Capalbo, Kohayakawa, Marx, Rödl, Ruciński and Szemerédi.

József Beck, Rutgers University (USA)

Super-uniformity

We start with an unexpected new result about the continuous Kronecker–Weyl equidistribution theorem. Given any (arbitrarily complex!) Lebesgue measurable set in the unit square, a typical torus line is astonishingly uniform with respect to this given set: the difference between the actual time in the set and the expected time in the set (=area times the total time) is at most square-root logarithmic in terms of the total time. Since square-root logarithm is almost constant, we obtain the counter-intuitive result that the "error term is basically independent of the complexity (=ugliness) of the test set". This fact (i.e., super-uniformity) almost(!) contradicts the basic philosophy of uniform distribution. The proof is a combination of number theory and Fourier analysis.
As a corollary, we obtain that the typical billiard path in any rectangle is unexpectedly uniform. The so-called deterministic(!) Bernoulli model of the kinetic theory of gases represents the particles as point-billiards in a box container with elastic collision against the wall of the container. By using basically the same proof technique (a combination of number theory and Fourier analysis), we can show that this deterministic model exhibits a lot of randomness (in the form a central limit theorem for the global behavior, and a Poisson limit theorem for the local behavior).

*in alphabetical order of the speakers' surnames

Béla Bollobás, University of Cambridge (UK)

Percolation on Self-Dual Polygon Configurations

In this talk I shall sketch some new results obtained recently with Oliver Riordan of Oxford.
Recently, Scullard and Ziff noticed that a broad class of planar percolation models are self-dual under a simple condition which, in a parametrized version of such a model, reduces to a single equation. They stated that the solution of the resulting equation gave the critical point. However, just as in the classical case of bond percolation on the square lattice, noticing self-duality is simply the starting point: the mathematical difficulty is precisely showing that self-duality implies criticality. Riordan and I have managed to overcome this difficulty: we have shown that for a generalization of the models considered by Scullard and Ziff self-duality indeed implies criticality.

Mei-Chu Chang, University of California Riverside (USA)

On multiplicative character sums

We present various new estimates on incomplete character sums over finite fields and for certain composite moduli. In particular, we consider polynomials in several variables that factor completely over an extension field and certain mixed character sums. An analogue of Vinogradov's exponential bound and an extension of Postnikov's method are given. Several of these issues go back to Burgess' work.

Zoltán Füredi, University of Illinois Urbana-Champaign and Rényi Institute (USA and Hungary)

A new proof of the stability of extremal graphs: Simonovits' stability from Szemerédi's regularity

After a short review of Turán type problems we present a concise, contemporary proof (i.e., one using Szemerédi's regularity lemma) for the following classical stability result of Simonovits: If an n-vertex F-free graph G is almost extremal, $\chi(F) = p + 1$, then the structure of G is close to a p-partite Turán graph.
More precisely, for every graph F and $\varepsilon > 0$ there exists a $\delta > 0$ and a bound n_0 (depending on F and ε) such that if $n > n_0$ and

$$e(G) > (1 - \frac{1}{p})\binom{n}{2} - \delta n^2$$

then one can change (add and delete) at most εn^2 edges of G in order to obtain a complete p-partite graph.

Ben Green, University of Cambridge (UK)

An arithmetic regularity lemma

Szemerédi's regularity lemma may be viewed as a kind of structure theorem for all (dense) graphs: every such graph may be decomposed into a "low-complexity" piece and a "pseudorandom" piece. I will talk about a similar decomposition for subsets of $\{1, \dots, N\}$. The "low-complexity" piece is something called a nilsequence, and "pseudorandomness" is measured in terms of the Gowers norms. I will attempt to motivate these concepts and explain the statement of the regularity lemma. I will also indicate applications, one of which is a proof of Szemerédi's theorem on arithmetic progressions. Another resolves a conjecture of Bergelson, Host and Kra: If $A \subseteq \{1, \dots, N\}$ has density α, there is some $d \neq 0$ such that A contains at least $(\alpha^4 - o(1))N$ four-term progressions with common difference d. It is known by an example of Ruzsa that such a statement fails for five-term and longer progressions.

Jeff Kahn, Rutgers University (USA)

Counting (more or less) with entropy and sampling

We discuss a few results of varying vintage, mostly involving approaches to approximate counting questions based on entropy and/or simple random sampling.

Gil Kalai, Hebrew University of Jerusalem and Yale University (Israel and USA)

Helly type theorems, combinatorics, and topology

Helly's theorem from 1912 asserts that for a finite family of convex sets in a d-dimensional Euclidean space, if every $d+1$ of the sets have a point in common then all of the sets have a point in common. This theorem found applications in many areas of mathematics and led to numerous generalizations. Helly's theorem is closely related to two other fundamental theorems in convexity: Radon's theorem asserts that a set of $d+2$ points in d-dimensional real space can be divided into two disjoint sets whose convex hulls have non empty intersection. Caratheodory's theorem asserts that if S is a set in d-dimensional real space and x belongs to its convex hull then x already belongs to the convex hull of at most $d+1$ points in S.
We will discuss several developments around Helly's theorem among them:
1) Various quantitative versions of Helly's theorem and mainly the fractional Helly's theorem: For a family of convex sets in R^d if a large fraction of $(d+1)$-tuples are intersecting then a large fraction of all sets have a point in common.
2) Alon-Kleitman (p,q) theorems. (If from every p sets q have a point in common what can you deduce?)
3) Amenta's theorem (a result about families of unions of convex sets)
4) Tverberg's theorem (a far-reaching extension of Radon's theorem)
An important question is to understand the combinatorics and topology of Helly-type theorems. I will describe recent results and conjectures in this direction. The talk will represents an ongoing research project with Roy Meshulam, and I will mention recent works by several other researchers. I will also mention surprising relation with Gyárfás type questions.

László Lovász, Eötvös University (Hungary)

Regularity Lemma and the dimension of graphs

Szemerédi's Regularity Lemma, first obtained in the context of his theorem on arithmetic progressions in dense seuqences, has become one of the most important and most powerful tools in graph theory. Weaker versions with better bounds (Frieze and Kannan) and stronger versions (Alon, Fisher, Krivelevich and Szegedy) have been proved and used. However, the significance of it goes way beyond graph theory: it can be viewed as statement in approximation theory, as a compactness result concerning the completion of the space of finite graphs, as a statement in information theory, etc. It serves as the archetypal example of the dichotomy between structure and randomness (Tao).
This lecture will try to give a taste of these rich connections, and then focus on one of these, where the regularity lemma appears as a result about dimensionality. We define metric space on the nodes of graph, by representing each node by the corresponding row vector in the square of the adjacency matrix. Then a (weak) regularity partition corresponds to partitioning this metric space into sets of small diameter.
If we apply this to graphons (limit objects of growing graph sequences), we see that the Regularity Lemma says that these spaces may be infinite dimensional, but "just barely". Our main result is that if we exclude a bipartite graph as an induced subgraph (in the bipartite sense), then this space will be finite dimensional.
This is joint work with Balázs Szegedy.

Jiří Matoušek, Charles University (Czech Republic)

The number of unit distances is almost linear for most norms

We prove that there exists a norm in the plane under which no n-point set determines more than $O(n \log n \log\log n)$ unit distances. Actually, most norms have this property, in the sense that their complement is a meager set in the metric space of all norms (with the metric given by the Hausdorff distance of the unit balls).

Jaroslav Nešetřil, Charles University (Czech Republic)

Extremal problems for sparse graphs

The recently defined dichotomy for sparse graphs - nowhere dense vs somewhere dense - has algorithmic, structural and extremal theory consequences. We survey some of these, particularly those related to counting of subgraphs. This is a joint work with Patrice Ossona de Mendez.

János Pach, EPFL and Rényi Institute (Switzerland and Hungary)

Geometric Expanders

The *overlap number* of a finite $(d+1)$-uniform hypergraph H is defined as the largest constant $c(H) \in (0, 1]$ such that no matter how we map the vertices of H into $\mathbf{R^d}$, there is a point covered by at least a $c(H)$-fraction of the simplices induced by the images of its hyperedges. We answer a question of Gromov by constructing a sequence $\{H_n\}_{n=1}^{\infty}$ of arbitrarily large $(d+1)$-uniform hypergraphs with bounded degree such that their overlap numbers are bounded from below by a positive constant $c = c(d)$. We also show that, in every dimension d, the best value of the constant $c = c(d)$ that can be achieved by such a construction is asymptotically equal to the limit of the overlap numbers of the complete $(d+1)$-uniform hypergraphs with n vertices, as $n \to \infty$.
For the proof of the latter statement, we establish the following geometric partitioning result of independent interest. For any d and any $\epsilon > 0$, there exists $K = K(\epsilon, d) \geq d+1$ satisfying the following condition. For any $k \geq K$, for any point $q \in \mathbb{R}^d$ and for any finite Borel measure μ on $\mathbf{R}^d$ with respect to which every hyperplane has measure 0, there is a partition $\mathbf{R}^d = A_1 \cup \ldots \cup A_k$ into k measurable parts of equal measure such that all but at most an ϵ-fraction of the $(d+1)$-tuples $A_{i_1}, \ldots, A_{i_{d+1}}$ have the property that either all simplices with one vertex in each A_{i_j} contain q or none of these simplices contain q. Joint work with Jacob Fox, Mikhail Gromov, Vincent Lafforgue and Assaf Naor.

János Pintz, Rényi Institute (Hungary)

Are there arbitrarily long arithmetic progressions in the sequence of twin primes?

The speaker plans to sketch some ideas which lead to the following, weaker and conditional Theorem. If the level of distribution of primes exceeds 1/2 then there exists a bounded even number d (different from zero) with the property that there are arbitrarily long arithmetic progressions of primes p such that $p + d$ is also a prime for each element of the progression. The bound for d depends on the level of distribution of primes. If we suppose the Elliott-Halberstam conjecture (that is a level equal to 1, or even at least 0.971) then the theorem is true with a d not exceeding 16. Several other results will be mentioned, among them a condition (requiring a hypothetical distribution level at least 3/4 for the primes and some other sequences involving the Liouville function and/or the primes) which implies an affirmative answer for the question in the title. Also some unconditional results of similar type will be mentioned for primes, products of two primes and some other numbers with special multiplicative constraints like $d(n) = d(n+1)$.

Vojtěch Rödl, Emory University (USA)

On generalized Ramsey numbers

Old and new joint results with Endre in Ramsey theory will be presented. We discuss bounds on the size Ramsey number of bounded degree graphs. In particular, we show that there are graphs of maximum degree 3 whose size Ramsey number is not linear in the number of vertices and discuss a general upper bound.

Imre Z. Ruzsa, Rényi Institute (Hungary)

Towards a noncommutative Plünnecke-type inequality

We relax the assumption of commutativity in certain Plünnecke-type inequalities.

Miklós Simonovits, Rényi Institute (Hungary)

Regularity Lemmas and Extremal Graph Theory

Extremal graph theory is one of the oldest areas of Graph Theory. In the 1960's it started evolving into a large, deep, connected theory. As soon as Szemerédi proved his Regularity Lemma, several aspects of the theory changed completely. Several deep results of extremal graph theory became accessible only through the application of this central result. Also, large part of Ramsey Theory is strongly connected to Extremal graph theory. Application of the Regularity Lemma in these areas was also crucial. The first difficult result of Ramsey-Turán theory was also proved using (an earlier version of) the Regularity Lemma, by Szemerédi. The lecture will give a survey of this area, though not trying to be complete, yet covering several historically important old and also several new results.

József Solymosi, University of British Columbia (Canada)

Many collinear k-tuples in pointsets with no $k+1$ points collinear

The following problem originates from an old paper of Erdős he wrote to Hungarian high-school students in 1962 (KÖMAL). He asked the same question again in the same journal in 1980 from where I learned it.
For a finite set S of points in the plane, let $t_k(S)$ denote the number of lines passing through exactly k points of S. Let $r_k(n) = \max t_k(S)$, where the maximum is over all sets S of n points in the plane with no $k+1$ collinear points. It is well known that $r_3(n) = n^2/6 - \Theta(n)$. Erdős conjectured that $r_k(n) = o(n^2)$ for all $k \geq 4$. The best known lower bounds for $r_k(n)$ were improved for all $k \geq 5$ to $r_k(n) = \Omega(n^{1+\log 2/\log k})$ by Noam D. Elkies in 2006.
In this talk we show that $r_k(n) \geq n^{2-\varepsilon}$ where ε goes to zero as n goes to infinity.
Joint work with Milos Stojakovic.

Joel Spencer, New York University (USA)

Quasirandom Multitype Graphs

One of the central ideas of quasirandomness is that if a graph G has (in the appropriate asymptotic sense) the same subgraph counts as $G(n,p)$ ($p \in (0,1)$ constant) for all H up to four vertices then it will have the same counts for all H up to any bounded number of vertices. In a multitype random graph there are a finite number of vertex classes C_i, each having a given positive proportion of the vertices, with each C_i and each $C_i \times C_j$ being random, though with possibly different p's. The celebrated Szemerédi Regularity Lemma can be interpreted as saying that every large graph is quite close to a multitype random graph. Here we give an alternate argument for a result of Lovász and Sós that the above quasirandomness result, with 4 replaced by an appropriate constant, holds with $G(n,p)$ replaced by an arbitrary multitype random graph. We start with an easy fact: If $a_1, \ldots, a_s$ are distinct reals and X is a random variable with $E[\prod_i (X-a_i)^2] = 0$ then X takes on only the values $a_1, \ldots, a_s$.

Balázs Szegedy, University of Toronto (Canada)

Limits of functions on abelian groups and higher order Fourier analysis

For every natural number k we study an interesting limit notion for functions on abelian groups which is related to the hypergraph limit theory. The limit object is a measurable function on a so-called "k-step compact nilspace". A compact nilspace is an algebraic structure which generalizes abelian groups. Finite dimensional versions are basically nilmanifolds. One step nilspaces are abelian groups. It turns out that k-step nilspaces are forming a category together with functions that we call nilmorphisms. We give a structure theorem for Gower's norms in terms of nilmorphisms. Ordinary Fourier analysis can be described as a subject which deals with continuous homomorphisms of a compact abelian groups into the circle group. In our interpretation the k-th order Fourier analysis is a subject which deals with nilmorphisms of compact k-step nilspaces into bounded dimensional k-step nilspaces. We prove a Szemeredi type regularity lemma for functions on abelian groups. (Part of the results were obtained jointly with O. Antolin Camarena)

Terry Tao, University of California Los Angeles (USA)

The inverse conjecture for the Gowers uniformity norms

The Gowers uniformity norms $U^{k+1}[N]$, introduced by Gowers in 2001, is very useful when counting the number of linear patterns (such as arithmetic progressions of length $k+2$) in a set of integers. The inverse conjecture gives a necessary and sufficient condition for the Gowers norm of a bounded function to be large; roughly speaking, a function $n \mapsto f(n)$ has large $U^{k+1}[N]$ norm if and only if it correlates with a bounded complexity nilsequence $n \mapsto F(g(n)\Gamma)$ on a nilmanifold G/Γ of step (or degree) k. This generalises the Fourier-analytic case k=1, in which a function has large $U^2[N]$ norm if and only if it correlates with a Fourier phase $n \mapsto e(\xi n)$; this case is ultimately behind the Fourier-analytic proofs of results such as Roth's theorem, and the inverse conjecture for higher k can similarly be used to give a proof of Szemerédi's theorem.
In this talk we outline a proof of the general case of the inverse conjecture, in joint work with Ben Green and Tamar Ziegler. The arguments are based on those of Gowers, with the main new difficulty being that of "integrating" a family of degree $k-1$ nilsequences to obtain a single degree k nilsequence as an "antiderivative".

William T. Trotter, Georgia Institute of Technology (USA)

Online Linear Discrepancy of Partially Ordered Sets

The linear discrepancy of a poset $\mathbf{P}$ is the least k for which there is a linear extension L of $\mathbf{P}$ such that if x and y are incomparable in $\mathbf{P}$, then $|h_L(x) - h_L(y)| \leq k$, where $h_L(x)$ is the height of x in L. In this paper, we consider linear discrepancy in an online setting and devise an online algorithm that constructs a linear extension L of a poset P so that $|h_L(x) - h_L(y)| \leq 3k - 1$, when the linear discrepancy of P is k. This inequality is best possible, even for the class of interval orders. Furthermore, if the poset P is a semiorder, then the inequality is improved to $|h_L(x) - h_L(y)| \leq 2k$. Again, this result is best possible. This is joint work with Mitchel T. Keller and Noah Streib.

Van H. Vu, Rutgers University (USA)

Inverse Littlewood-Offord theory

We are going to give a survey on recent developments of the inverse Littlewood-Offord theory (introduced by Tao and the speaker 5 years ago). Our motivation is the following basic question:
Let x_i be iid random variables and a_i be real coefficients. Let S be the random sum $S := \sum_{i=1}^{n} a_i x_i$. When does S have lot of mass on one point or a short interval ?
This can be seen as the inverse version of the classical Littlewood-Offord-Erdos problem posed in the 1940s. We will discuss several applications.

Avi Wigderson, IAS, Princeton (USA)

Szemerédi & TCS

I plan to survey some of Endre Szemerédi's fundamental contributions to Theoretical Computer Science, and for each describe subsequent work and open problems.