Asymptotic Capacity Bounds for Magnetic Recording

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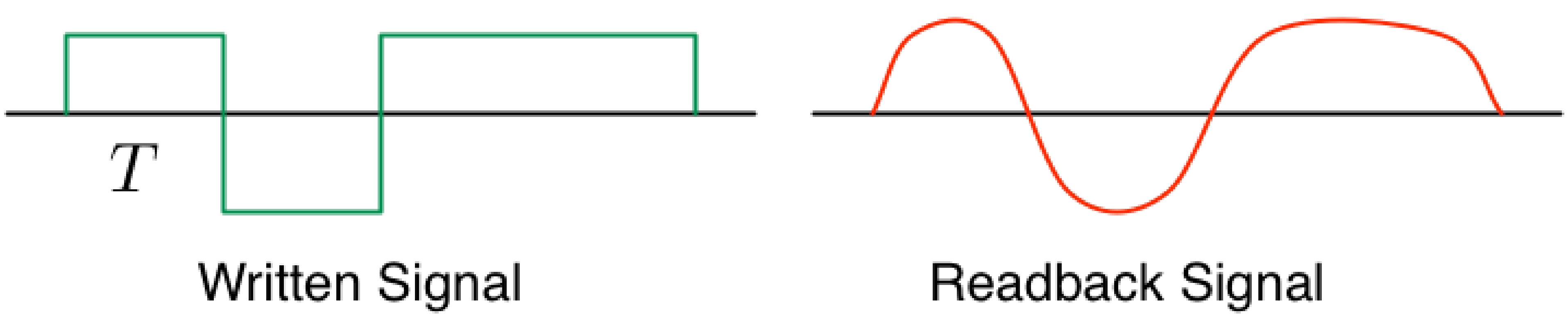
Outline

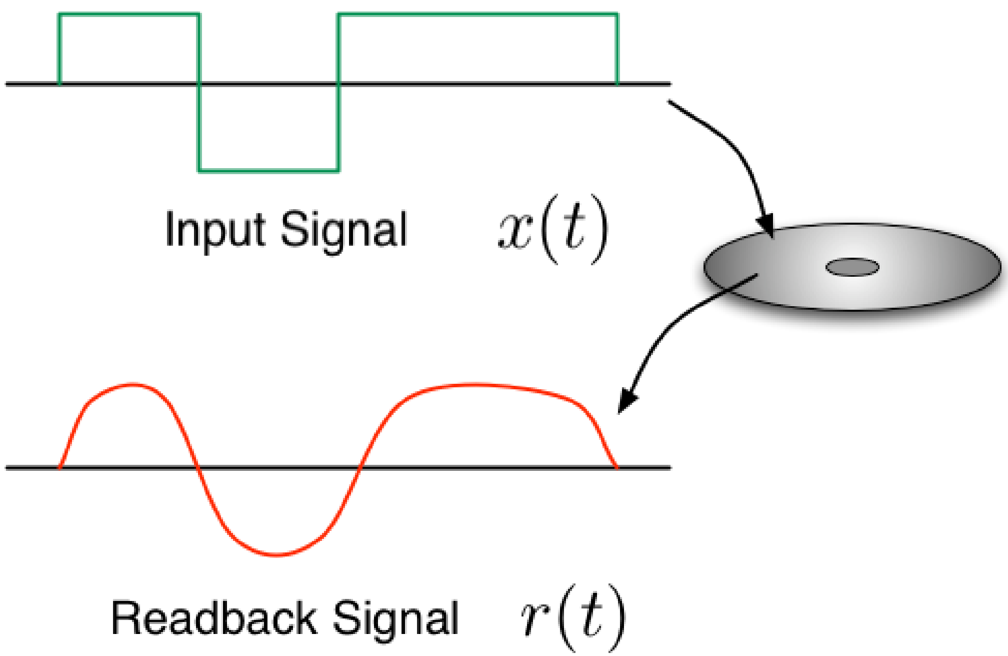
- Problem Statement
- Signal and Noise Models for Magnetic Recording
- Capacity Bounds
  - Additive noise dominated channels
  - Jitter noise dominated channels

Problem Statement

- How does the capacity of the system scale with the bit-width T?
- What is the optimum bit width from a purely information theoretic viewpoint?



Signal and Noise Models



- Signal: the magnetization in the medium is either ± 1
- Noise: electronic noise and transition noise.

Signal and Noise Models

$$x(t) = \sum_k a_k p(t - kT), \quad p(t) = u(t) - u(t - T)$$

$$r(t) = \sum_k \frac{a_k - a_{k-1}}{2} h_T(t - kT - \Delta t_k) + w(t)$$

$a_k \in \{+1, -1\}$

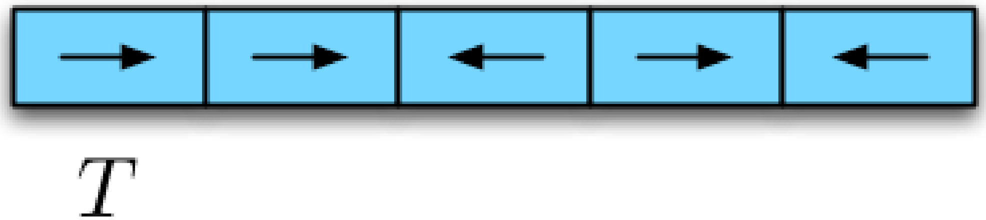
Δt_k : transition noise

$w(t)$: electronic noise–AWGN with PSD $N_0/2$

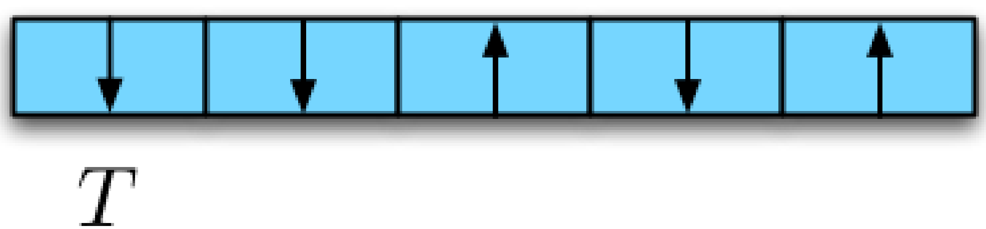
$h_T(t)$: transition ($-1 \rightarrow +1$) response

Recording Modes

Magnetization profile for longitudinal recording



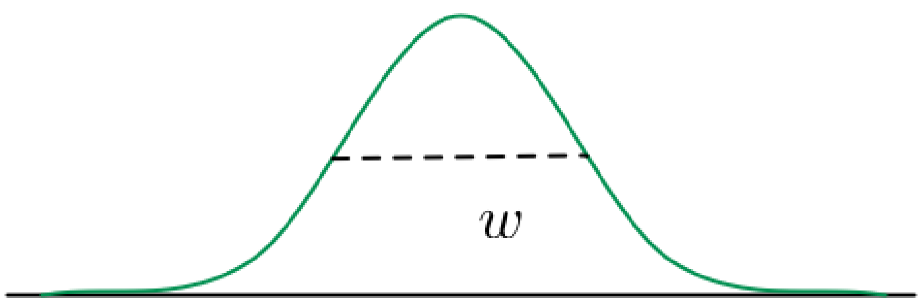
Magnetization profile for perpendicular recording



Transition Response

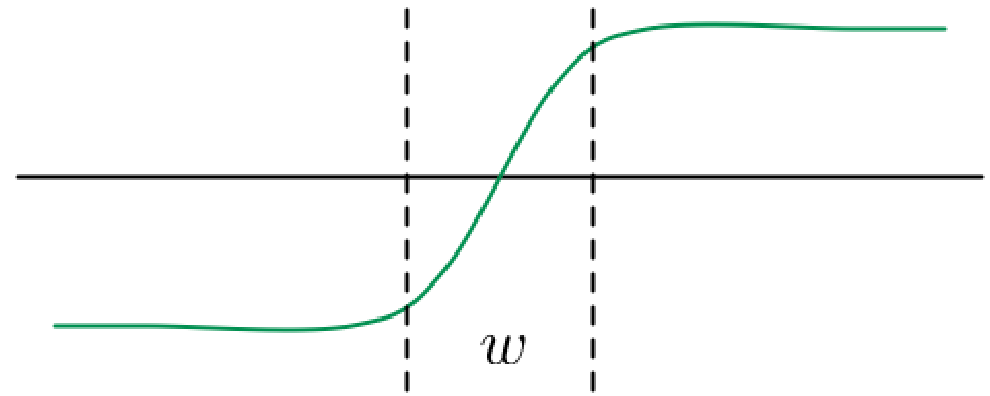
Longitudinal recording

$$h_T(t) = \frac{A}{1 + (2t/w)^2}$$



Perpendicular recording

$$h_T(t) = A \,\mathrm{erf}\left(\frac{2\sqrt{\log 2}\,t}{w}\right)$$



The pulse width is sometimes also written as PW50.

Capacity

- The fundamental quantity of interest is the areal capacity of the magnetic medium.
  - In the Shannon sense, this is the number of bits per unit area that can be stored and recovered reliably.
- Assuming a fixed track width, we can focus on the linear density of bits or linear capacity:

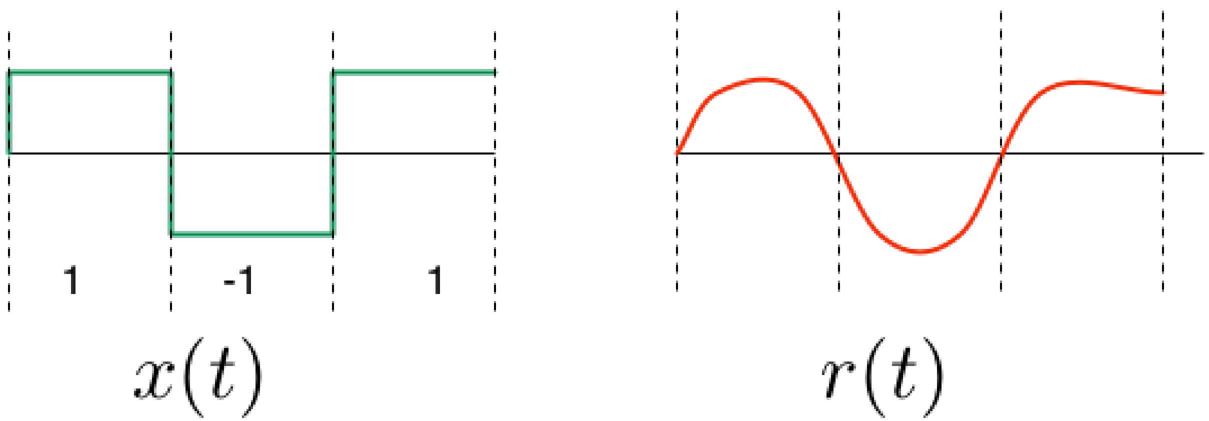
$$C = \lim_{\tau \to \infty} \frac{1}{\tau} \max(\{x(t), t \in [0, \tau]\}; \{r(t), t \in [0, \tau]\})$$

Problem Statement

- How is the optimal bit width T for maximum capacity per unit length?
- Misconceptions: Very small bit-widths are bad because...
  - There is higher transition noise.
  - There is longer ISI and lower SNR per bit.
- Truth: We can overcome all these problems with clever coding.

Thought Experiment

Experiment 1

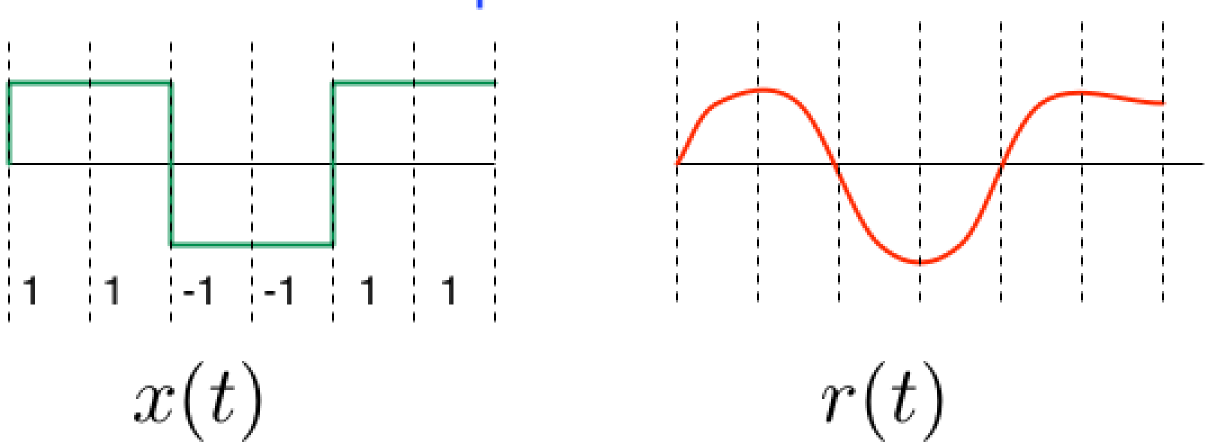


No code

$T = 1$

$R = 1$

Experiment 2

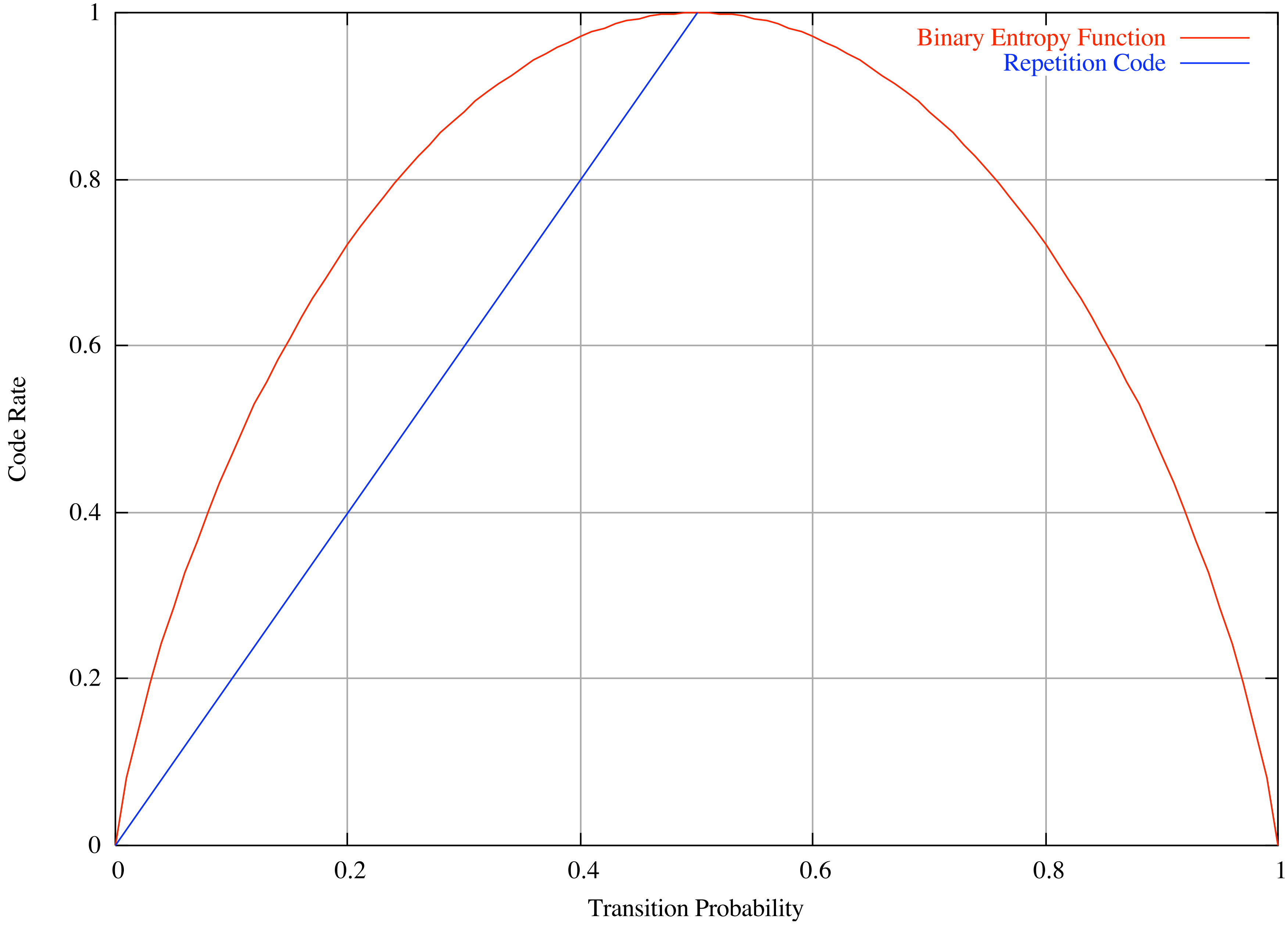


Repetition code

$T = 0.5$

$R = 0.5$

If we use better code, case 2 will outperform case 1.



Achievable region of code rate and transition probability.

A Note about the SNR

We define: $$\mathrm{SNR} = \frac{E_i w^2}{N_0}$$

$$\text{where } E_i = \int_{\mathbb{R}} \left| \frac{dh_T(t)}{dt} \right|^2 dt$$

- Reasons for this definition:
  - It is finite for both perpendicular and longitudinal modes.
  - It is a function of the head and medium alone. It is independent of T or the code we use.

Electronic Noise Dominated Channel

$$x(t) = \sum_k a_k p(t - kT), \quad p(t) = u(t) - u(t - T)$$

$$r(t) = \sum_k (a_k - a_{k-1}) h_T(t - kT) + w(t)$$

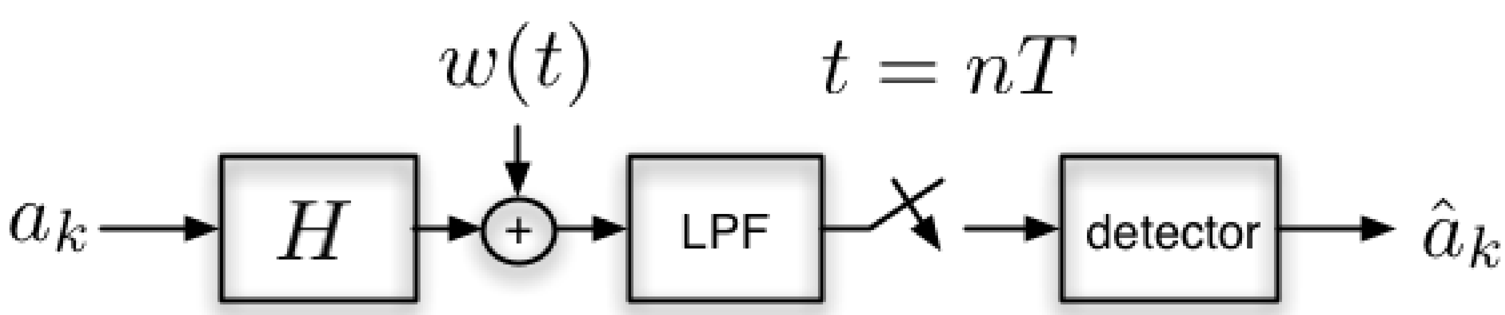
$$= \sum_k a_k h_D(t - kT) + w(t)$$

$$h_D(t) = h_T(t) - h_T(t - T) : \text{dibit response}$$

$$w(t) : \text{electronic noise–AWGN with PSD } N_0/2$$

Sufficient statistics

- To be exact, we need to pass $r(t)$ through a matched filter and sample at baud rate.
- If $T \ll w$, it is almost optimal to use a simple low-pass filter and sample a baud rate:



Discrete-time Model

We obtain an ISI channel with binary inputs:

$$r_n = \sum_k a_k \tilde{h}_{n-k} + w_n$$

$$w_n \sim N(0, N_0/2T), \text{ i.i.d.}$$

where, $\tilde{h}_n \simeq h_D(nT)$

Let the above ISI channel have capacity $C_B(T)$.
Then, the linear capacity is

$$K_B(T) = \frac{1}{T} C_B(T)$$

Simple Upper Bound

Suppose we relax the input constraint:

$$\{a_k = \pm 1\} \rightarrow \{a_k \in \mathbb{R},\ \mathrm{E}\, a_k^2 \leq 1\}$$

We obtain a Gaussian channel, whose capacity is computed using the "water-filling" method.

$$C_B(T) \leq C_G(T)$$

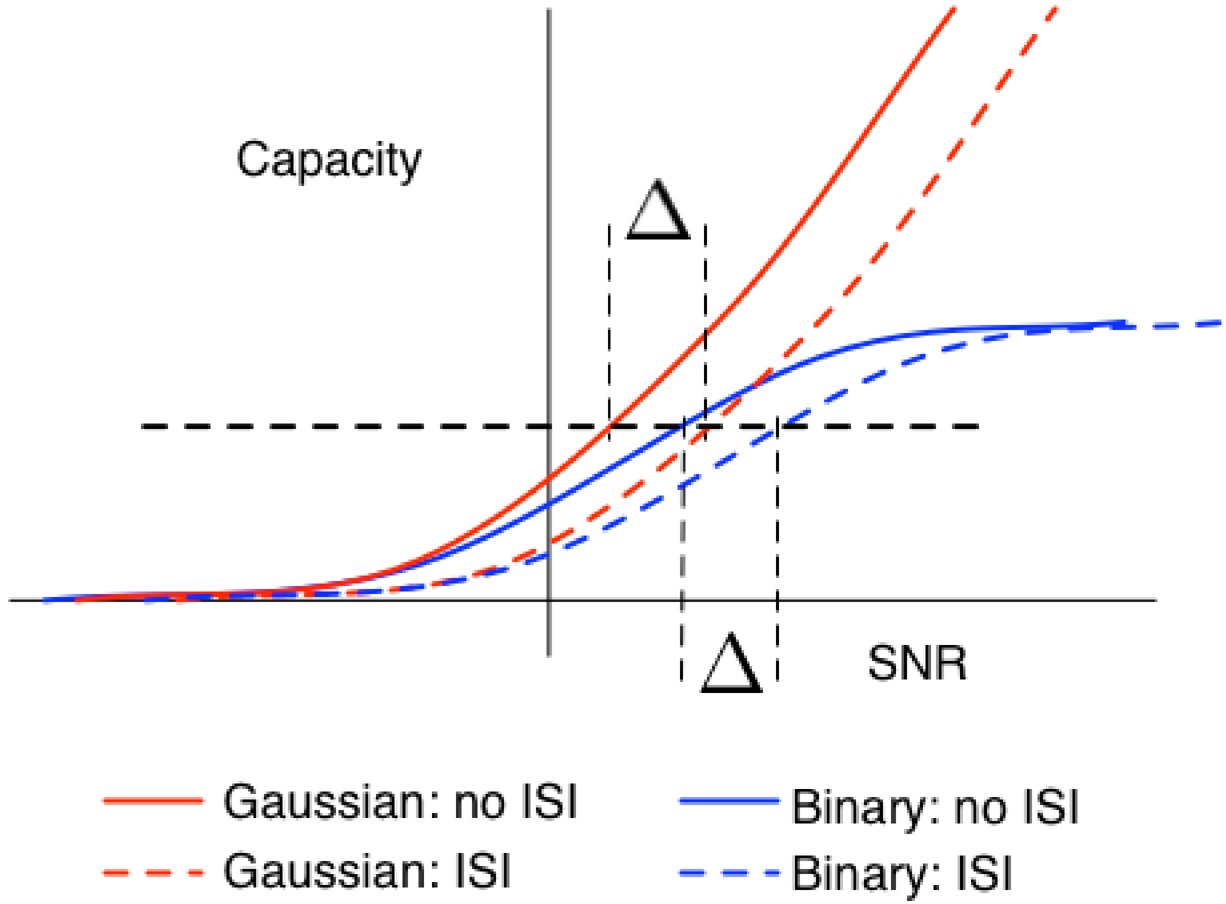
$$C_G(T) = \max_{S_x[\cdot]} \frac{1}{2} \int_{-0.5}^{0.5} \log\left(1 + \frac{S_x[\nu]|\tilde{H}[\nu]|^2}{N_0/2T}\right)$$

$$\text{such that} \int_{-0.5}^{0.5} S_x[\nu] d\nu = 1$$

Lower Bounds

- Several researchers have computed information rates for binary ISI channels:
  - Monte-Carlo method: compute the quantity $I(x^N; r^N)/N$ for a very long sequence of length N with the input generated using an appropriate Markov model.
  - This method becomes unreliable or computationally intense for high recording densities.

Shamai-Laroia Conjecture



Along any horizontal line, the SNR "loss" due to ISI is the same for both Gaussian and binary signalling.

Shamai-Laroia Conjecture

Suppose that the input to the binary ISI channel is independent and uniformly (IUD) distributed

$$C_B(T) \geq C_{BPSK}(2^{I_G(T)} - 1)$$

where $I_G(T)$ is the i.i.d Gaussian information rate.

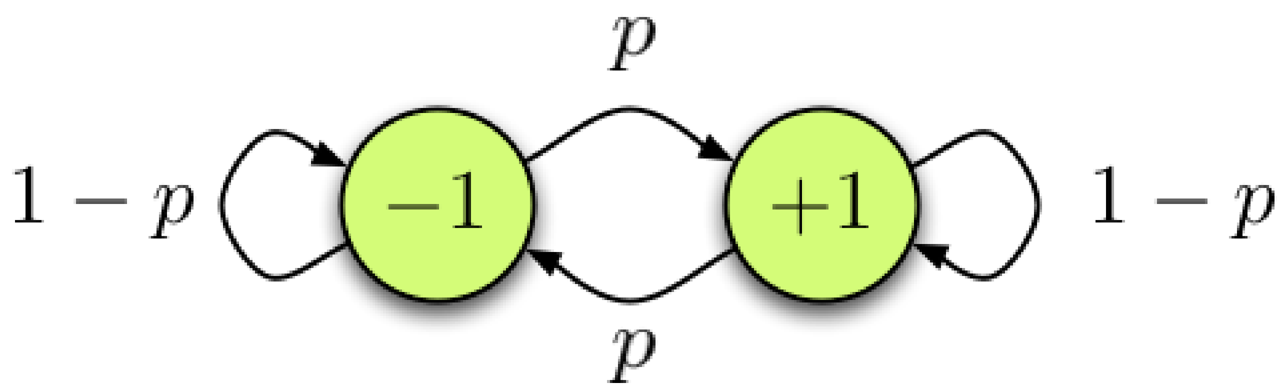
Remark: This bound is also found to be surprisingly tight for non IUD Markov sources up to memory 6 and we have

$$C_B(T) \geq C_{BPSK}(2^{C_G(T)} - 1)$$

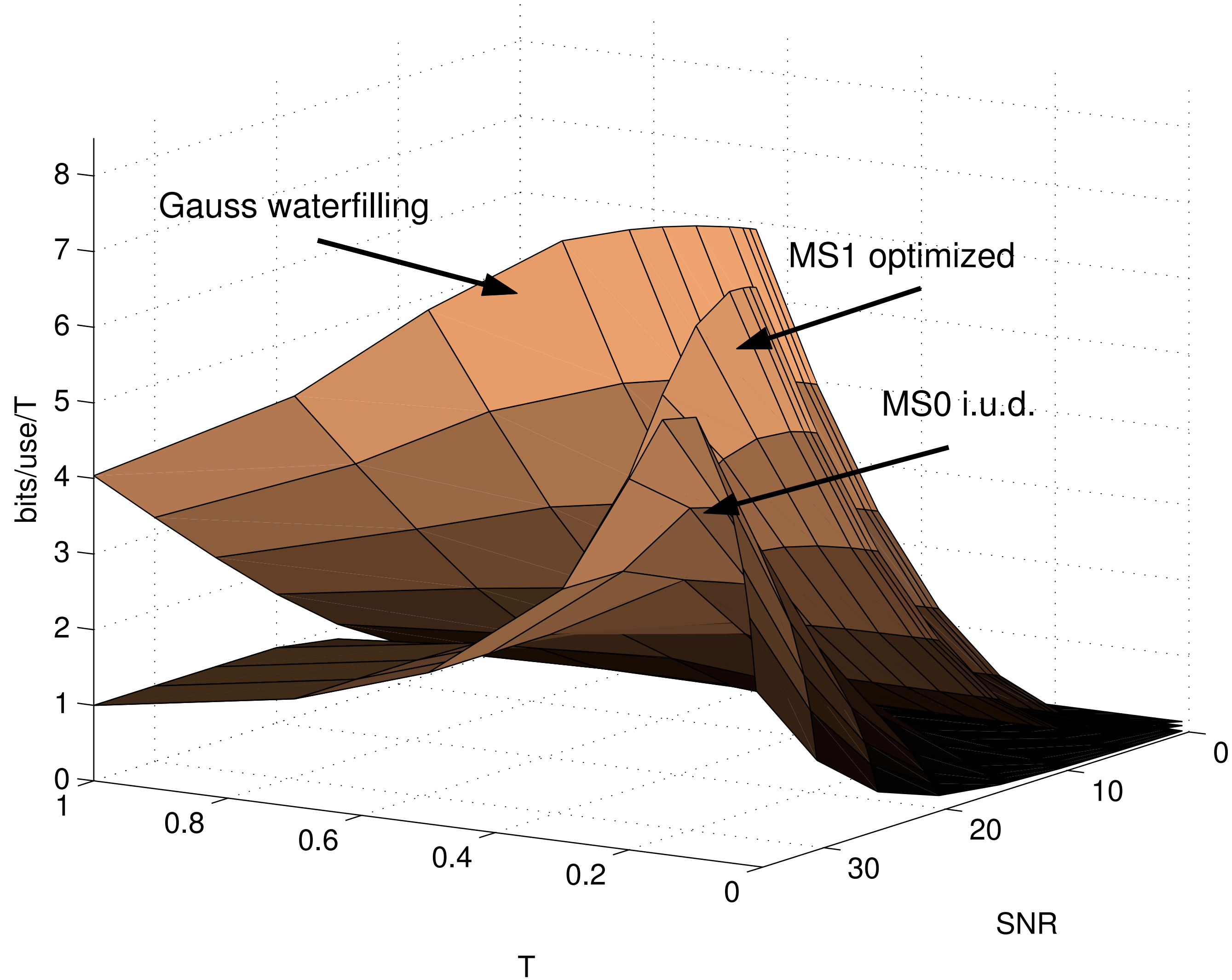
Optimal Input Distribution

- For Gaussian inputs, the optimal input spectrum is not flat. It becomes highly non i.i.d as $T \to 0$
- For binary inputs, we could mimic the Gaussian water-filling spectrum, but it is neither optimal nor achievable in general.
- For the binary case, we optimize over first-order Markov sources to get a lower bound.

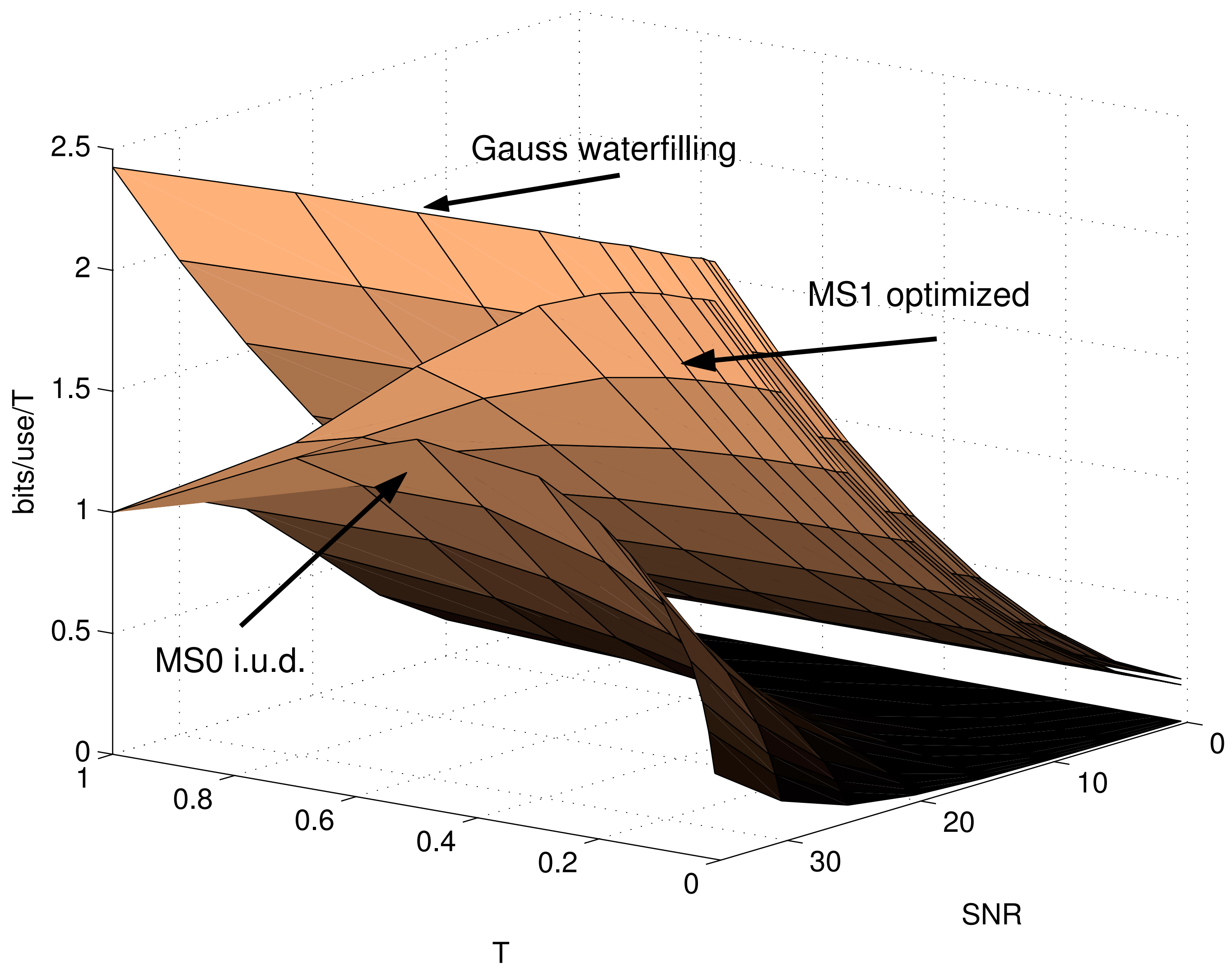
Markov Source Model



- The first order model is specified by one probability parameter p
- For each T we pick the optimal probability to maximize the Shamai-Laroia bound.
- As $T \to 0$ we find that $p \to 0$



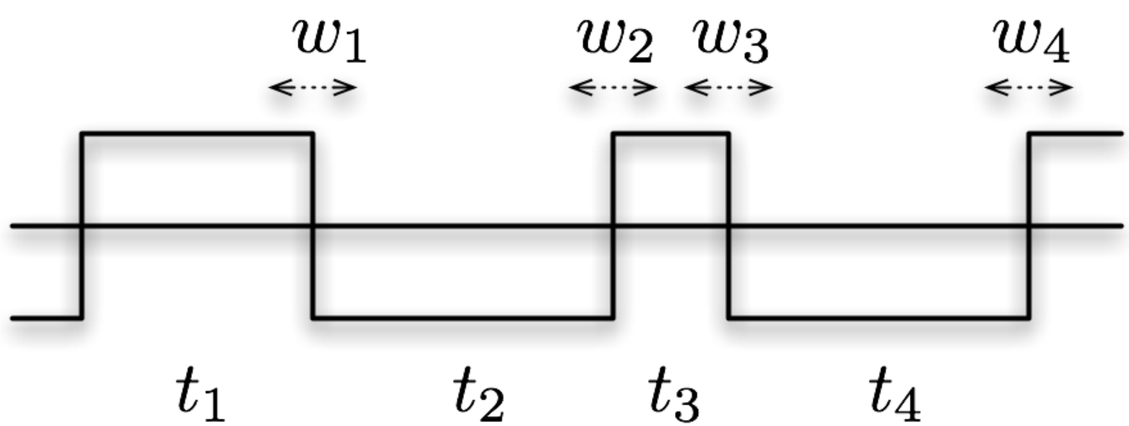
Linear capacity bounds for longitudinal recording



Linear capacity bounds for perpendicular recording

Jitter Noise Dominated Channel

Most general method: pulse-width modulation



$$r(t) = \sum_n (-1)^n h_T(t - \sigma_n - w_n)$$

$$\sigma_n = \sum_{k=1}^{n} t_k, \qquad w_n \sim N(0, \sigma_j^2)$$

Capacity

Since there is no additive noise, we can use a zero forcing equalizer to estimate $\hat{\sigma}_n = \sigma_n + w_n$

Thus,

$$\begin{aligned} \hat{t}_n &= \hat{\sigma}_n - \hat{\sigma}_{n-1} \\ &= t_n + w_n - w_{n-1} \end{aligned}$$

These are sufficient statistics and the channel is ISI!

Linear capacity: $C = \max_{p(T^N)} \frac{I(\hat{T}^N; T^N)}{\mathrm{E} \sum_n T_n}$

Lower Bound on Capacity

We can show that the following is a lower bound on the linear capacity:

$$K \geq \frac{2\sqrt{2}-1}{2} K_0$$

where K_0 is the capacity of the ISI-free channel:

$$\hat{t}_n = t_n + w_n$$

i.e.,

$$K_0 = \max_{p(T)} \frac{I(\hat{T};T)}{\mathrm{E}\,T}$$

Remark: This constant is not important. The bound behaves like $1/\sigma_j$.

Summary

- From a purely information theoretic viewpoint there are considerable gains at we higher recording densities.
    - This ignores timing recovery and other implementation issues.
- In today's recording medium transition noise is a dominant noise source and $\sigma_j \simeq 0.1T$
    - We should lower T by a factor of 10 to see potential gains.