Toward the Optimal Bit Aspect Ratio in Magnetic Recording

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- Approach for LDPC Codes
- Optimal Code Rates for LDPC Codes on the Lorentzian Channel
- On the Optimal Bit Aspect Ratio
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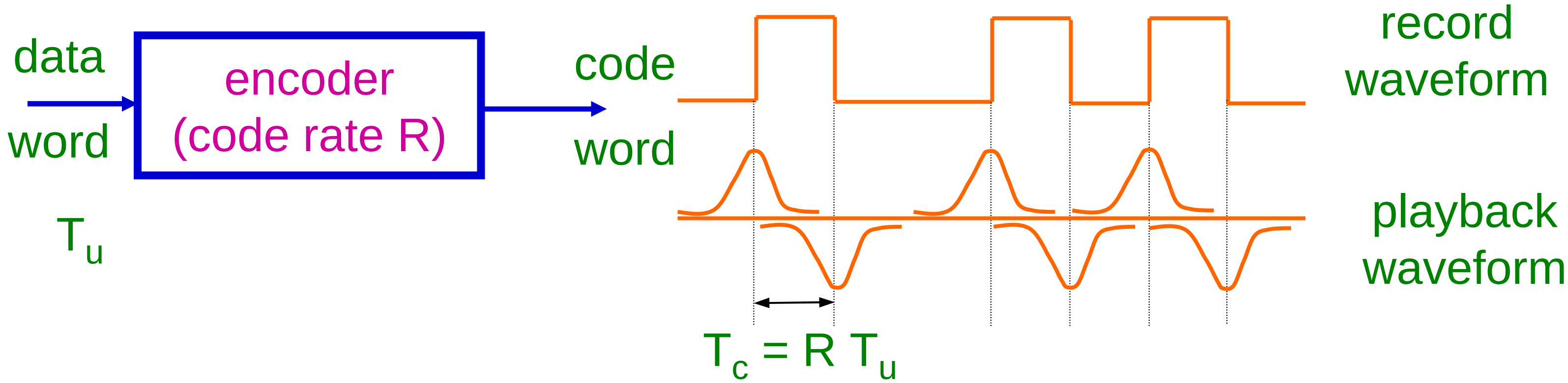
Background

⌘ Coding on a magnetic recording channel: Lorentzian model

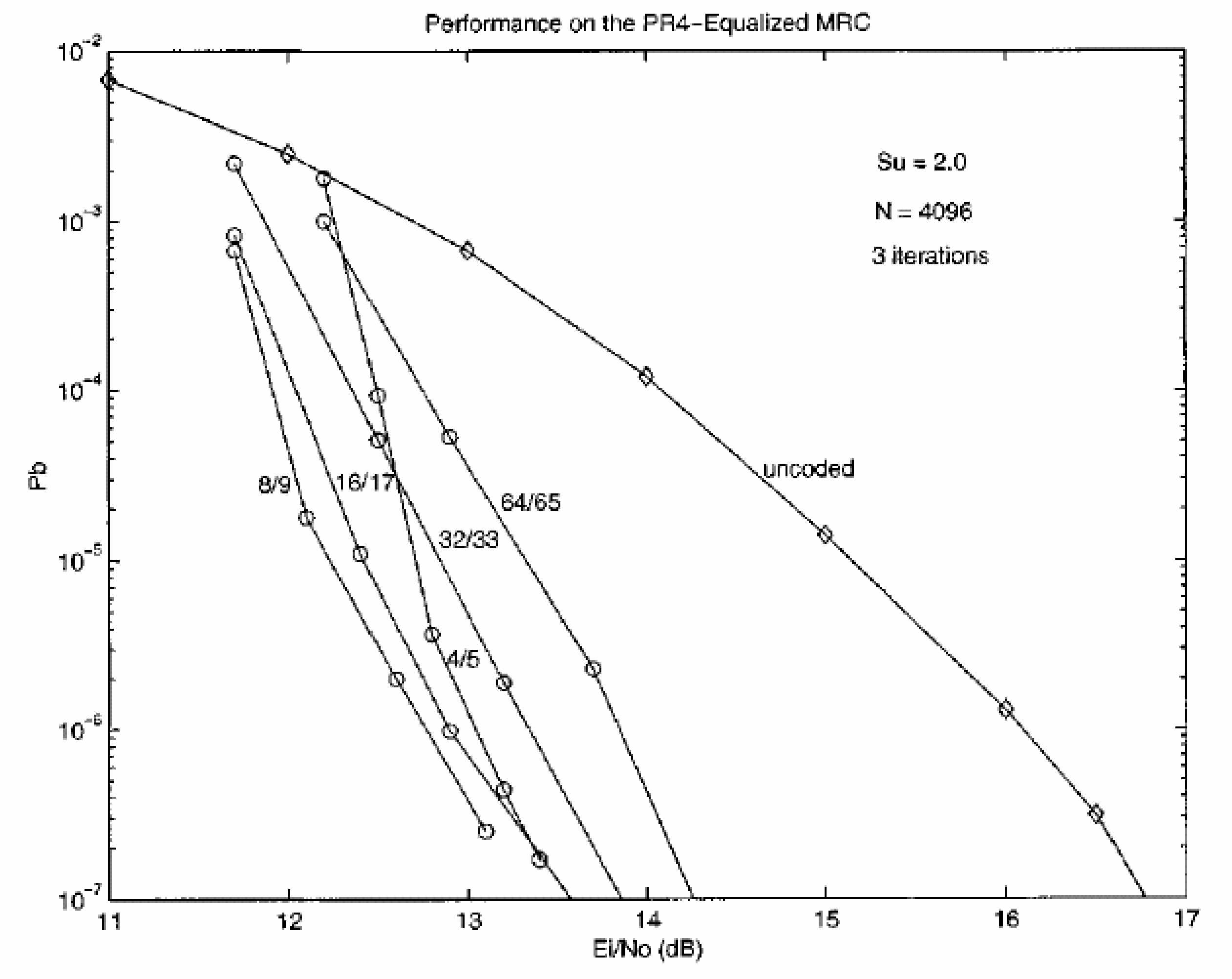


⌘ due to ISI, the code rate loss is R^2 -- on the AWGN channel it is R

⌘ on the AWGN channel, performance improves with decreasing code rate; on ISI channels such as the Lorentzian, it does not

Background (cont'd)

- In [Ryan, Trans. Magn., Nov. 2000] we examined optimal code rates empirically for specific parallel and serial turbo codes



Performance of various PCCC's on the PR4-equalized Lorentzian channel with user density $S_u = 2.0$.

$S_u = PW_{50}/T_u$

Channel Model

- Lorentzian model (in AWGN)

$$r(t) = \sum_k \frac{1}{2} a_k \, s(t - kT_c) \quad + \quad w(t)$$

where s(t) = $h(t) - h(t-Tc)$ is the dibit is AWGN with spectral density $N_0/2$ and $h(t)$ is the Lorentzian pulse

$$h(t) = \sqrt{\frac{4E_i}{\pi \, pw_{50}}} \, \frac{1}{1 + \left(2t / pw_{50}\right)^2}$$

- E_i = the energy per isolated Lorentzian pulse $h(t)$ and pw_{50} is its width measured at half its height

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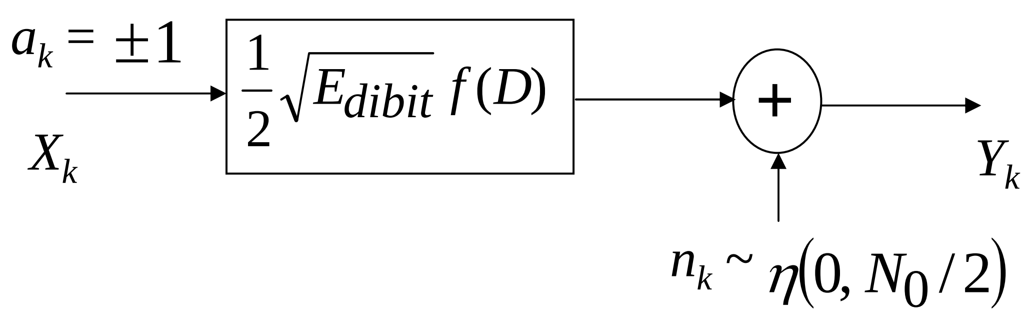
Channel Model (cont'd)

- applying a whitened matched filter to r(t) leads to the discrete-time equivalent model depicted below



- where
  - E_{dibit} is the energy in s(t),
  - f(D) is the minimum phase factor in the T_c-sampled auto-correlation function of s(t), $R_s(D)$
  - $\sum_k f_k^2 = 1$

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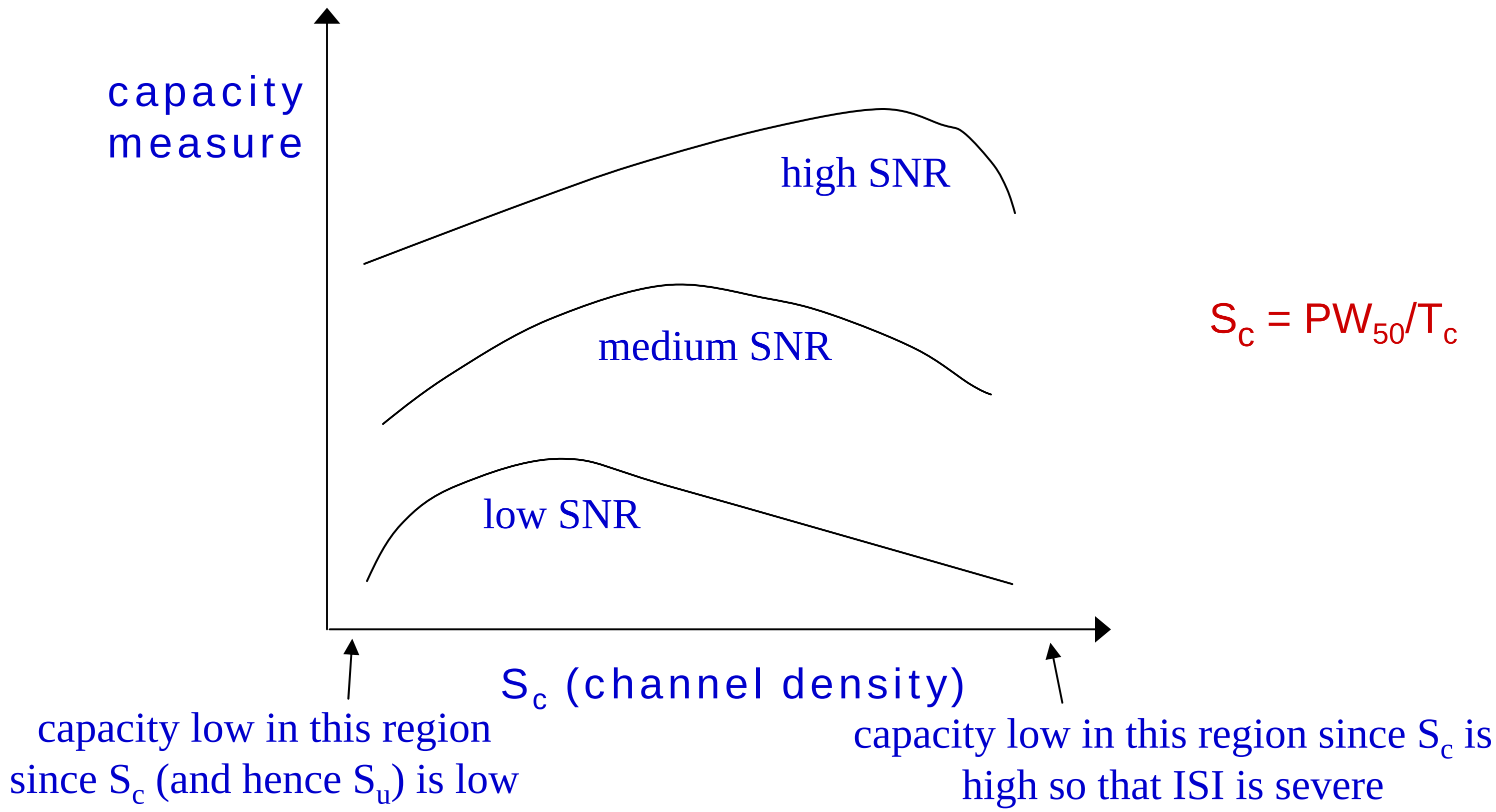
Approach for Shannon Codes

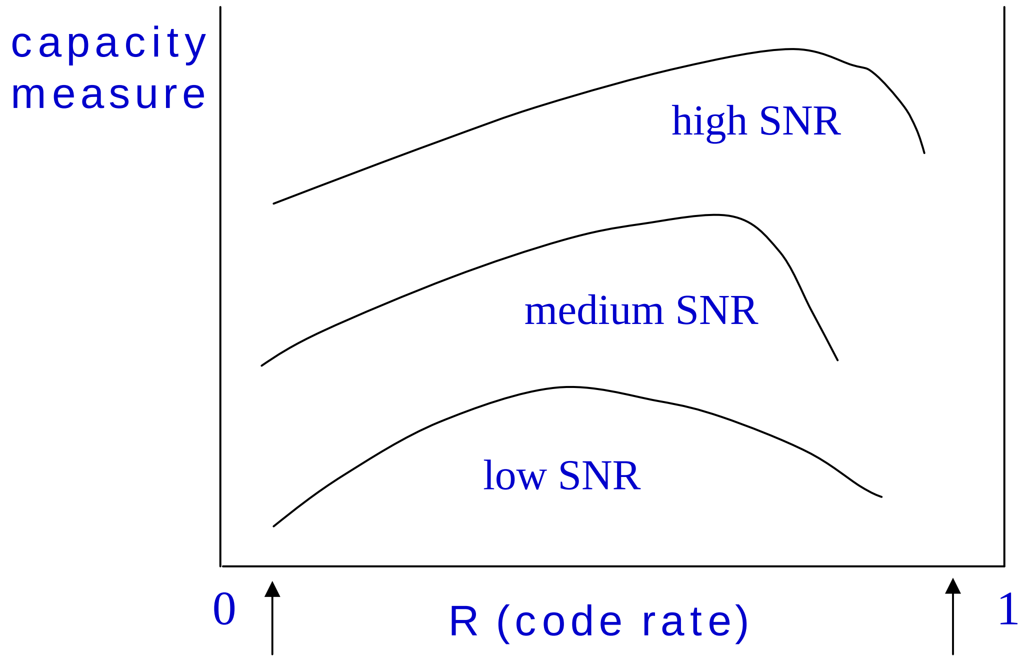
- Our goal is to determine optimal code rates for this channel for both Shannon codes and LDPC codes.



$S_c = PW_{50}/T_c$

Approach for Shannon Codes (cont'd)

- possibly better is data such as that in the figure below



coding overhead high
➔ large Sc and ISI becomes too severe to overcome with coding

coding overhead low
➔ to provide sufficient coding gain, Su and Sc must be reduced

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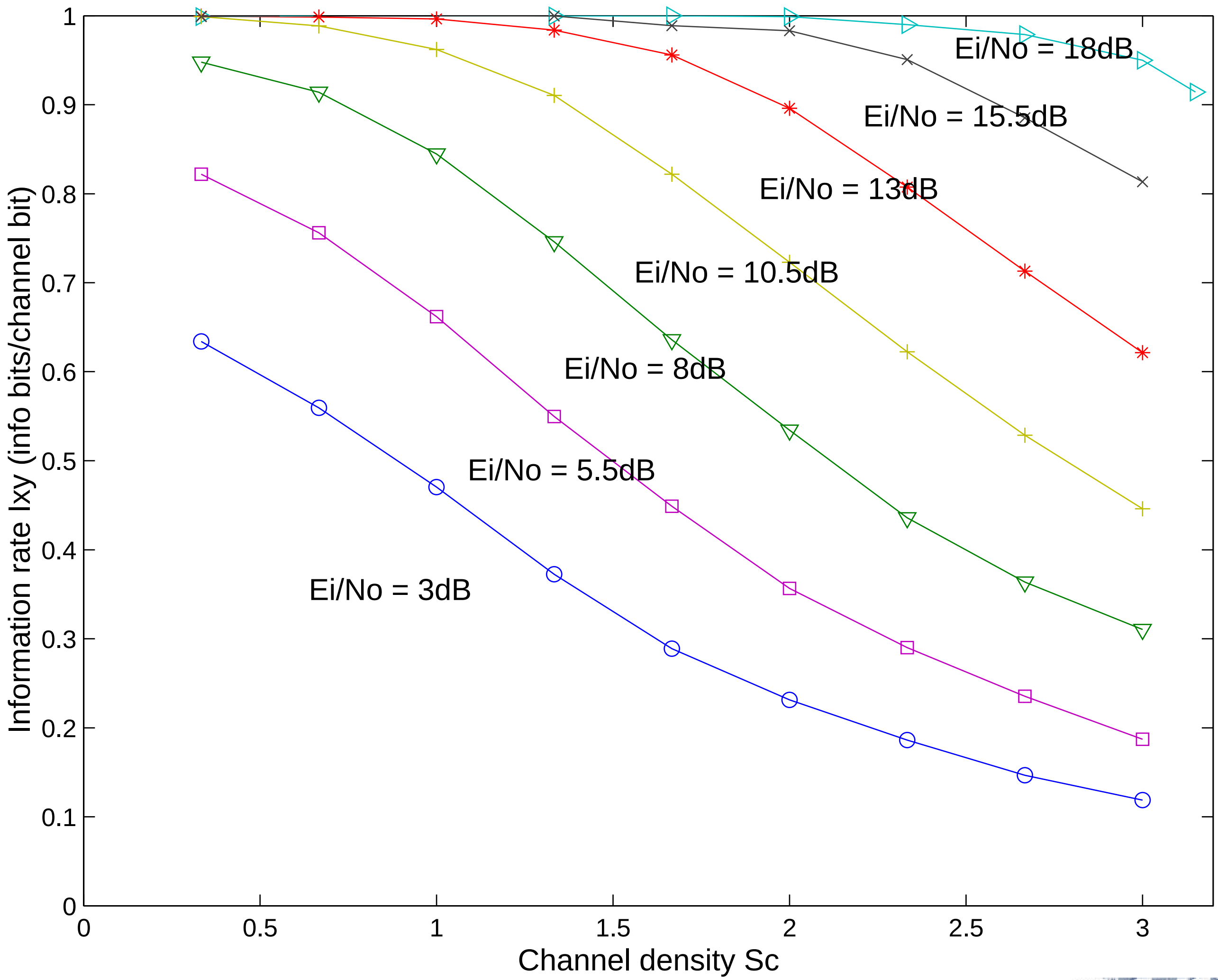
Approach for Shannon Codes (cont'd)

- can now use the result of Arnold-Loeliger (ICC'01) (also, Pfister-Siegel, GC'01) to compute the achievable information rate of the binary-input ISI channel $\frac{1}{2}\sqrt{E_{dibit}}\, f(D)$ assuming iid inputs
- Note by computing the information rate for $\frac{1}{2}\sqrt{E_{dibit}}\, f(D)$, we do not assume PR equalization. Rather, optimal (ML) detection is assumed.
- Note also that we use E_i / N_0 as our SNR measure

Results for Shannon Codes



Information rate of Lorentzian channel versus channel density S_c.

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Results for Shannon Codes (cont'd)

- Note I_{xy} is in units of *information bits/ channel bit*
- we would like a capacity measure relative to a physical parameter of the channel, such as *info bits/ inch* (along a track)
- *info bits/pw*$_{50}$ is particularly convenient:
  - note $S_c = pw_{50}/T_c$ may be regarded as *channel bits/ pw*$_{50}$
  - (Example: $S_c = 3$ ➔ 3 *channel bits/ pw*$_{50}$)
- define a new information rate

$$I'_{xy}\left(\frac{\text{info bits}}{pw_{50}}\right) = I_{xy}\left(\frac{\text{info bits}}{\text{channel bit}}\right)\cdot S_c\left(\frac{\text{channel bits}}{pw_{50}}\right)$$

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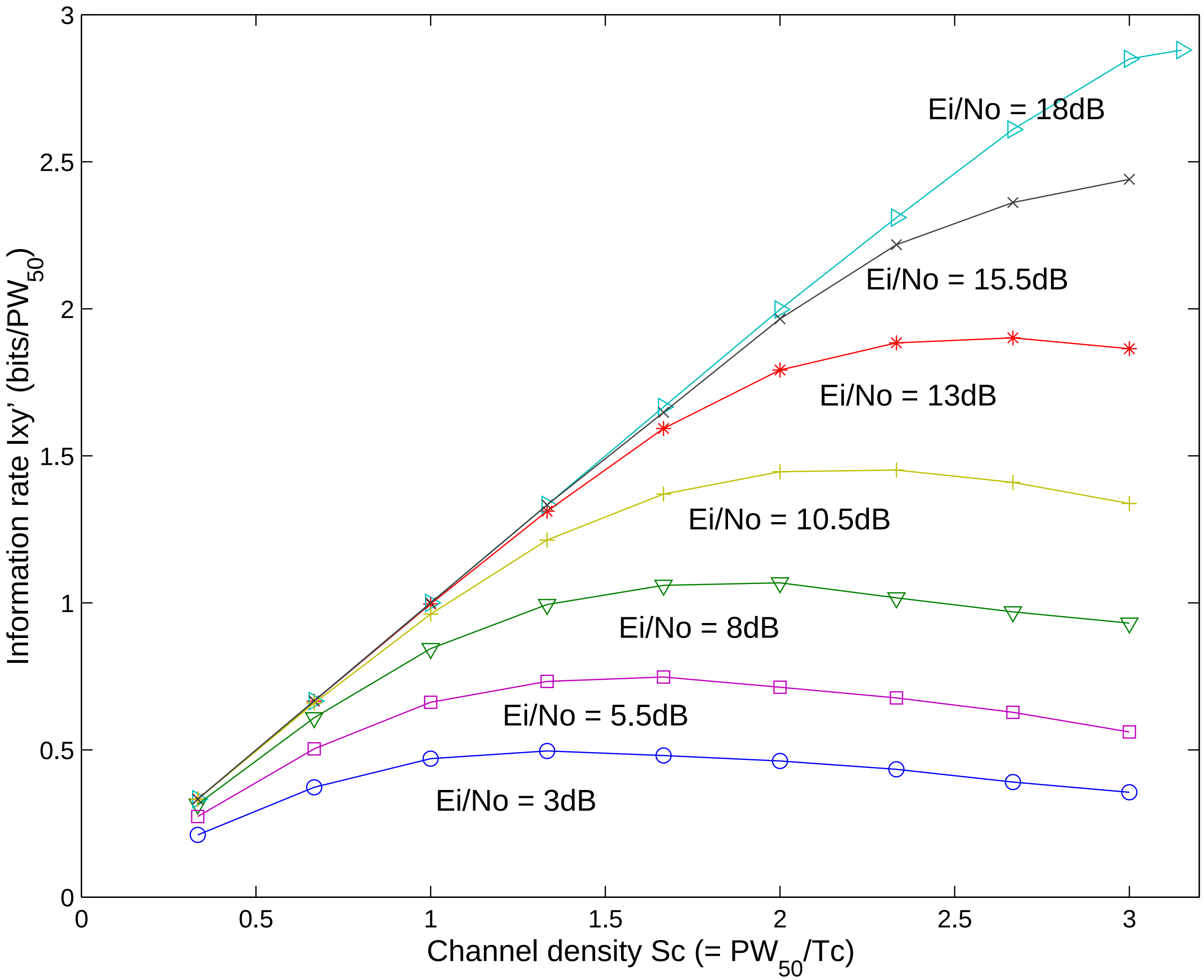
Results for Shannon Codes (cont'd)



Normalized Information rate of Lorentzian channel versus channel density S_c.

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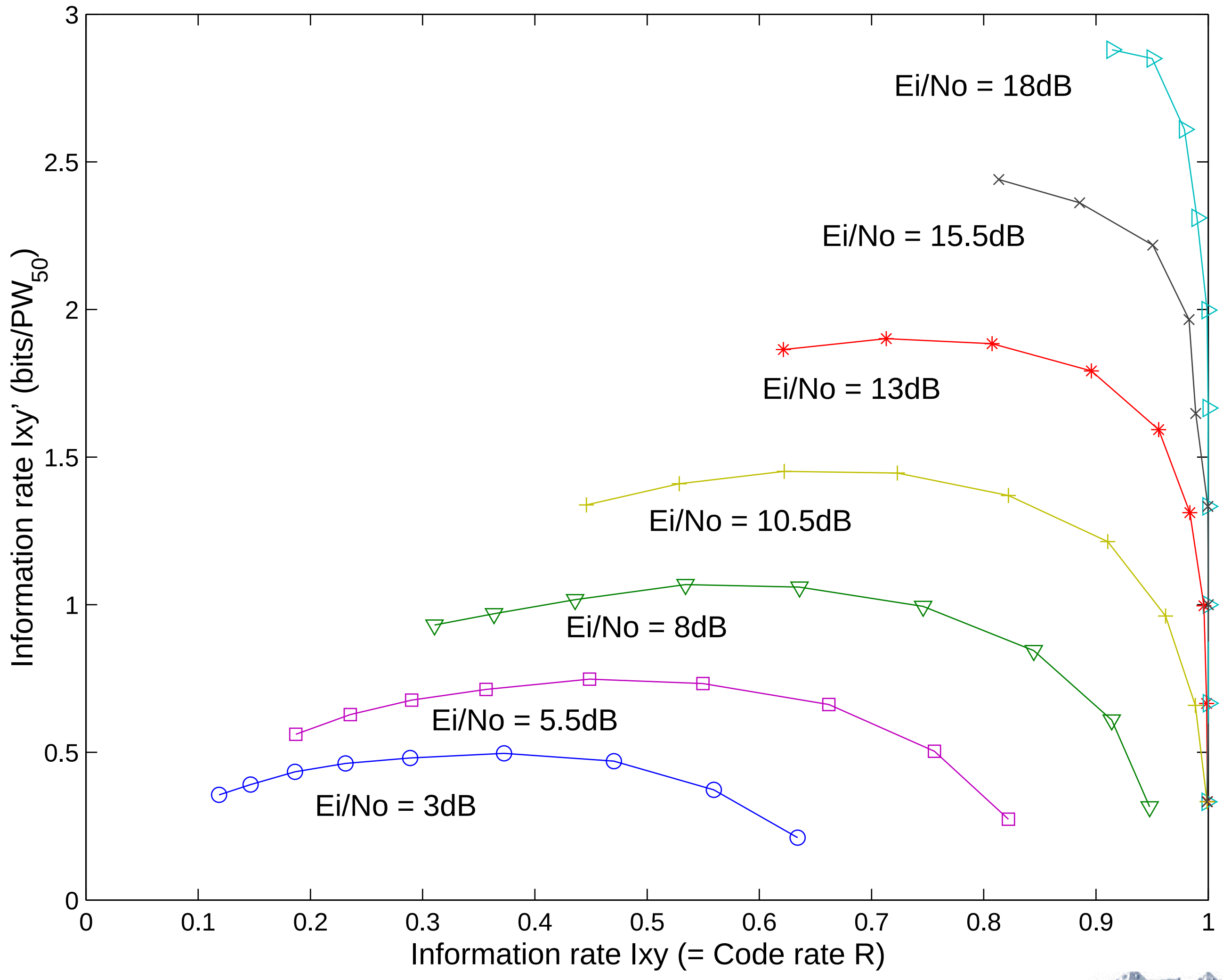
Results for Shannon Codes (cont'd)



Information rate of Lorentzian channel versus code rate R.

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Results for Shannon Codes (cont'd)

EXAMINATION OF THE DEVIATION OF $I(X;Y)$ WITH κ_{max}

S_c	κ_{max}	L	E_i/N_0 (dB)	$I(X;Y)$	$\%\Delta_{max}$
3	18	15	3	0.1368	3.04
	19			0.1328	
	23			0.1353	
	19		8	0.3057	4.75
	25			0.3025	
	30			0.3169	
	18		13	0.6264	1.53
	25			0.6211	
	30			0.6306	

Examination of deviation of I(X;Y) with $R_s(D)$ truncation parameter κ_{max}

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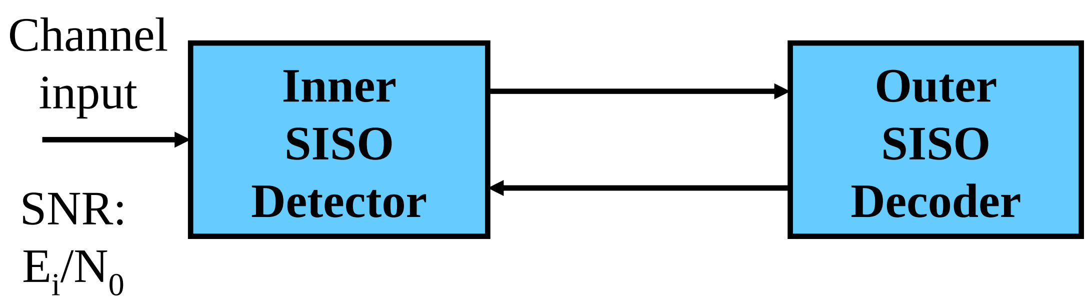
Approach for LDPC Codes

- Extrinsic information transfer (EXIT) chart
  - provides a simple way of determining the capacity limit (or decoding threshold) for a specific coding scheme.
  - describes the flow of extrinsic information through SISO processors (detectors/decoders) operating cooperatively and iteratively.



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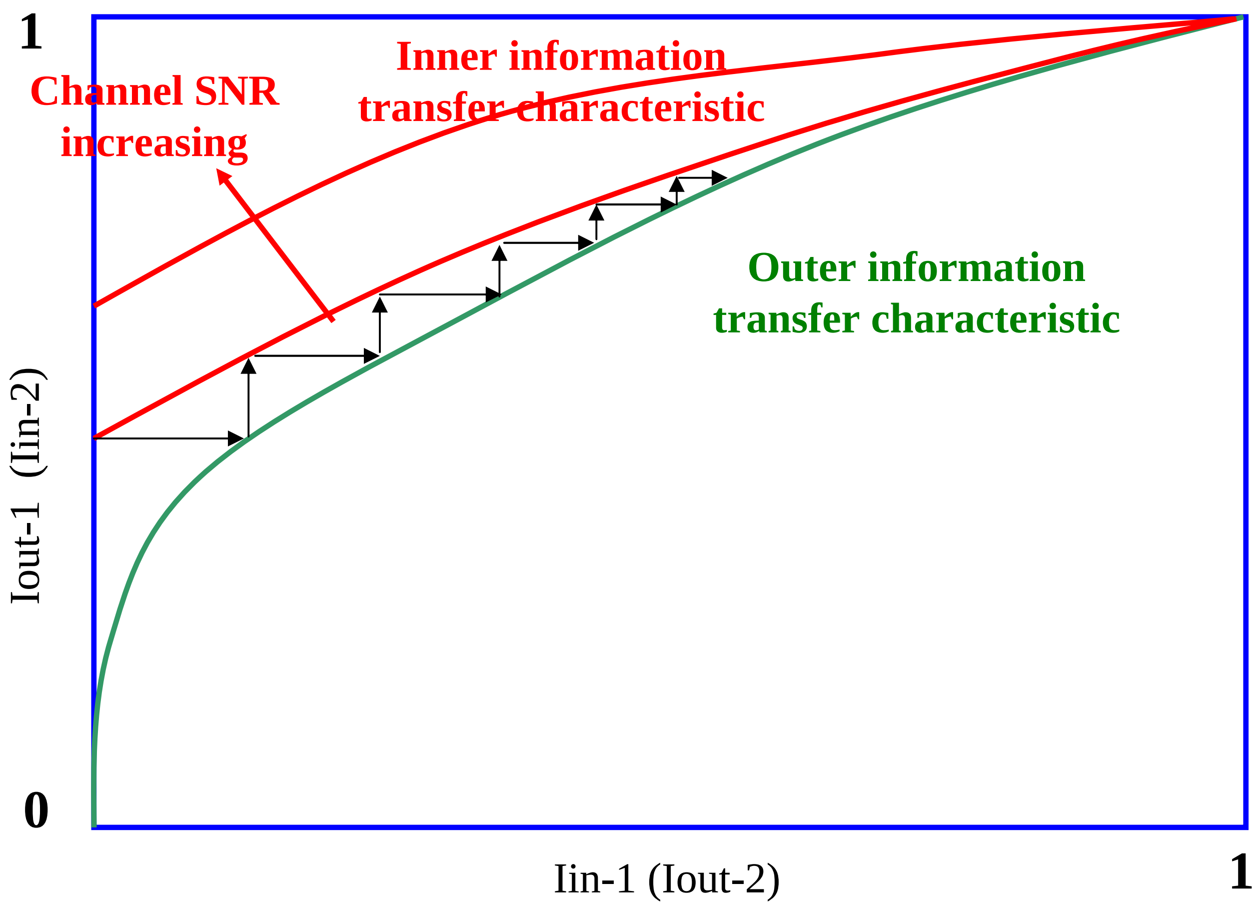
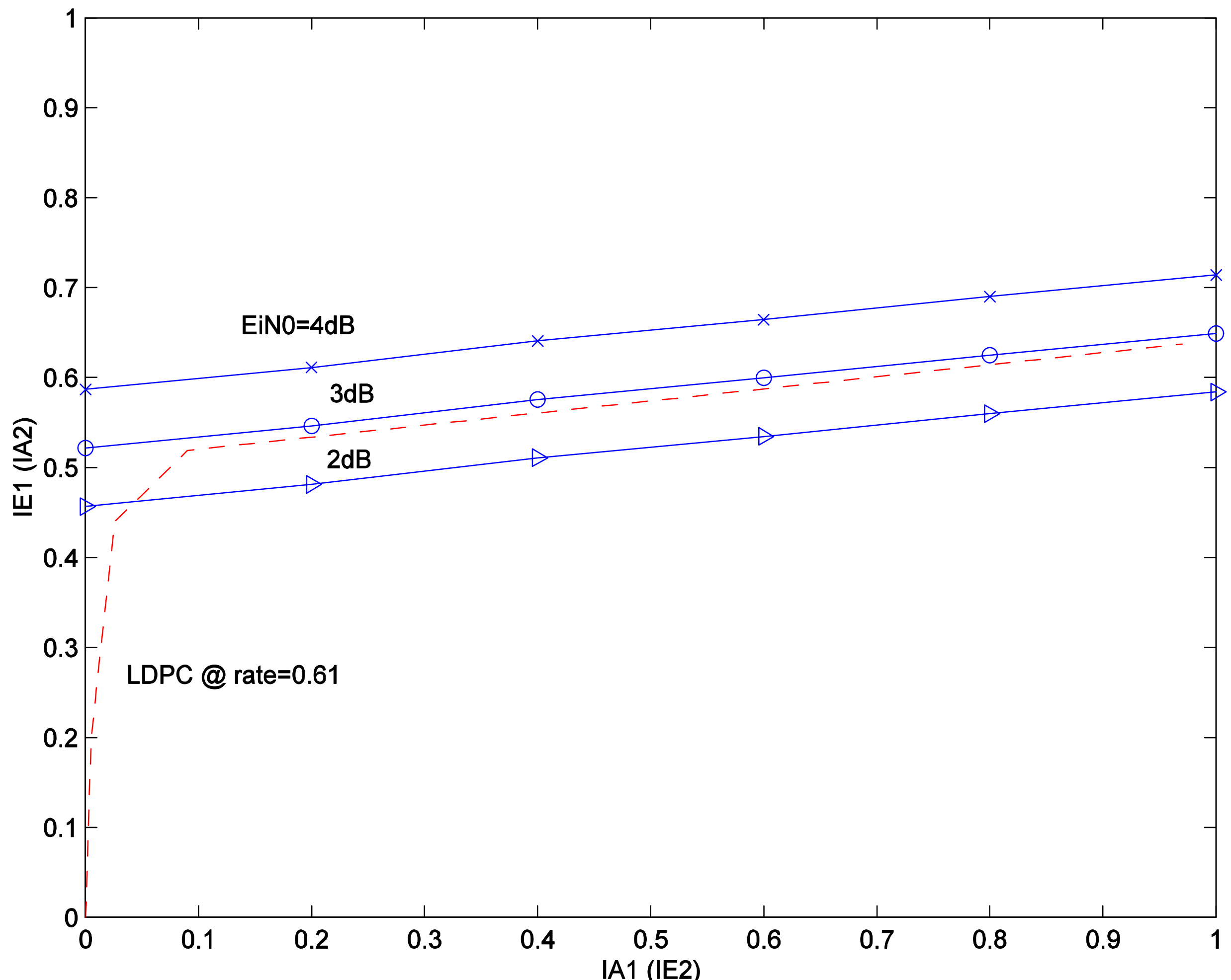
Approach for LDPC Codes (cont'd)

- possibly better is data such as that in the figure below



Approach for LDPC Codes (cont'd)

EXIT chart for channel density Sc=1/3 and LDPC code rate 0.61



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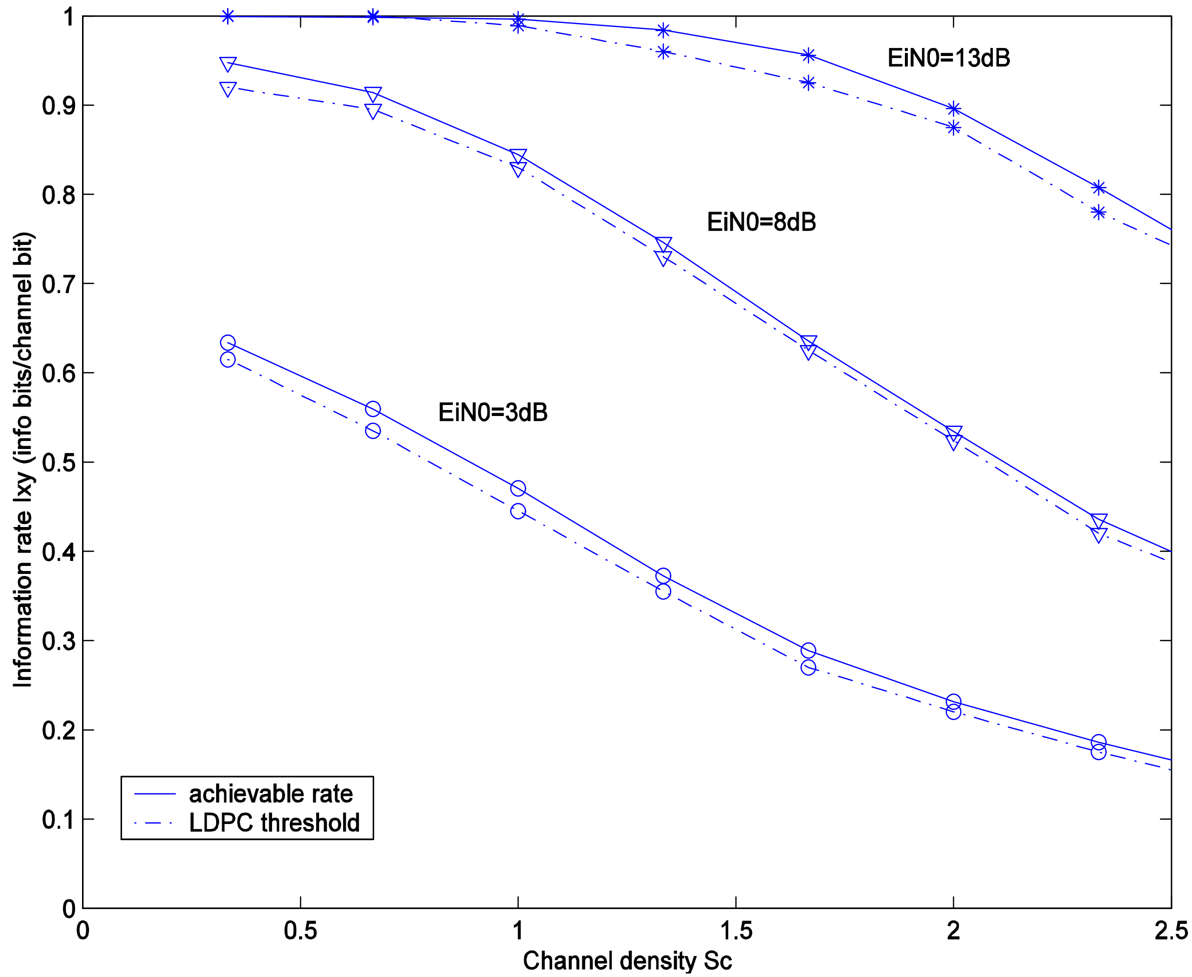
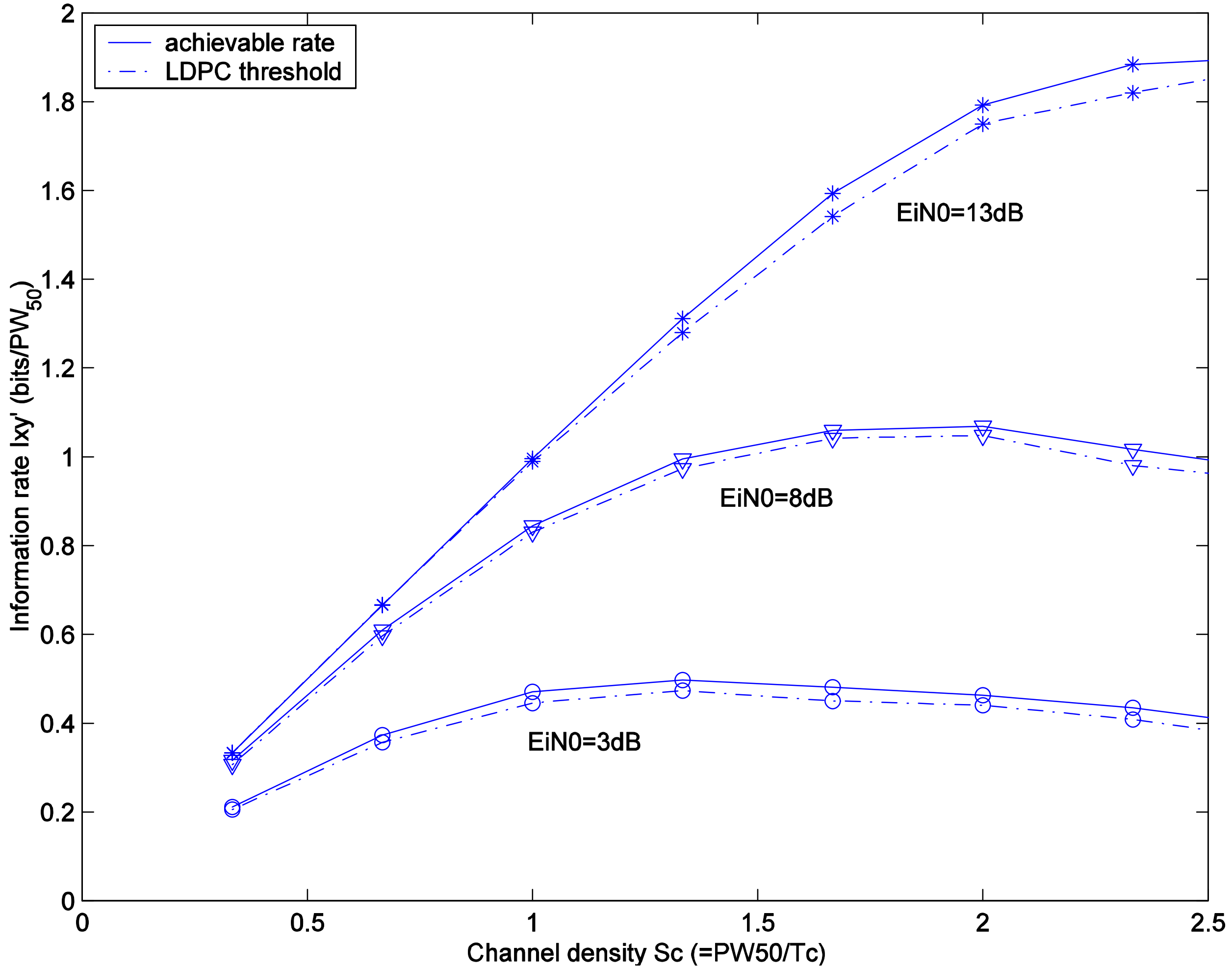
Results for LDPC Codes

Information rate I(X;Y) for Lorentzian channel versus channel density - Shannon codes and LDPC codes.



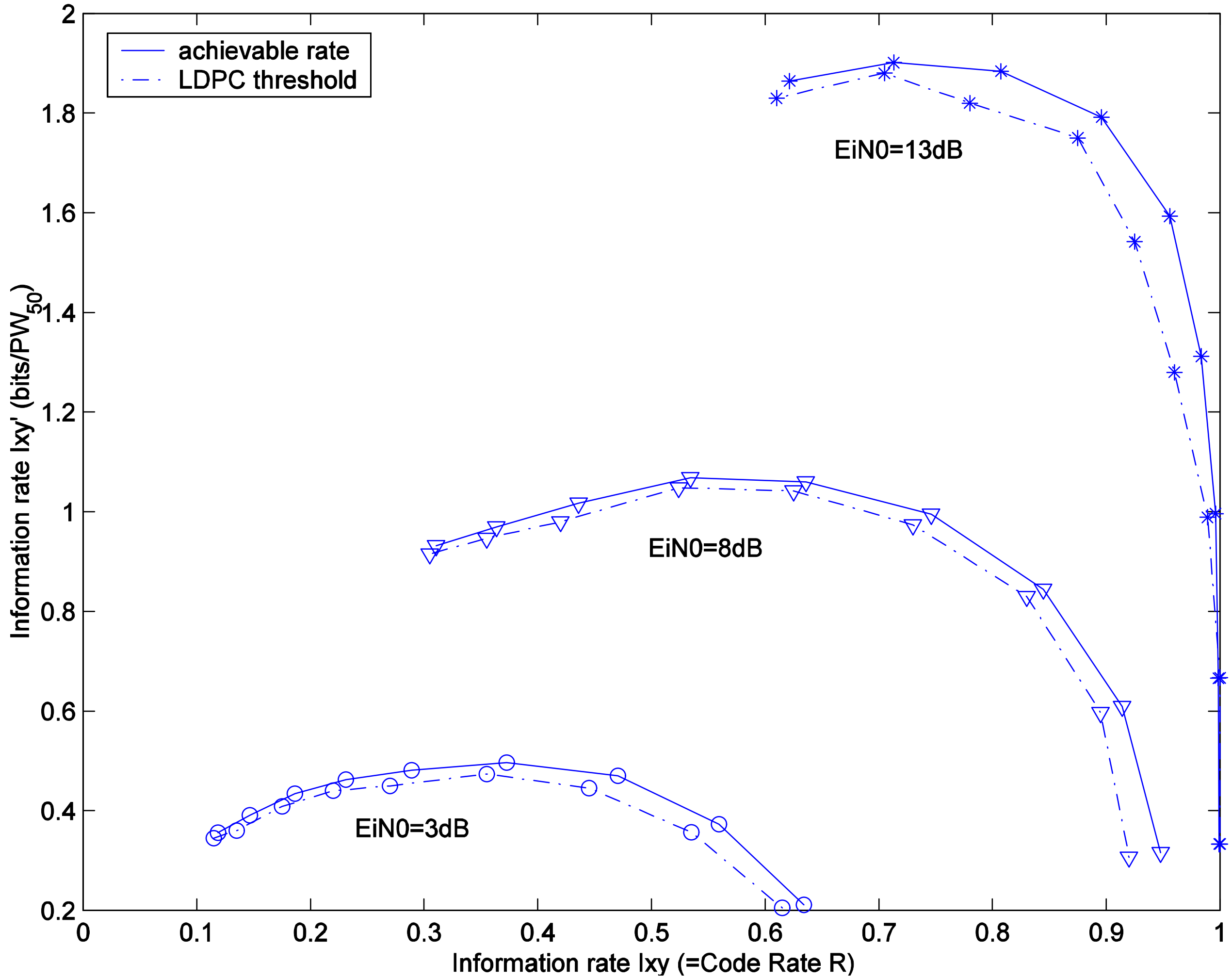
Results for LDPC Codes (cont'd)

Scaled Information rate I'(X;Y) for Lorentzian channel versus channel density - Shannon codes and LDPC codes.



Results for LDPC Codes (cont'd)

Scaled Information rate I'(X;Y) for Lorentzian channel versus code rate - Shannon codes and LDPC codes.



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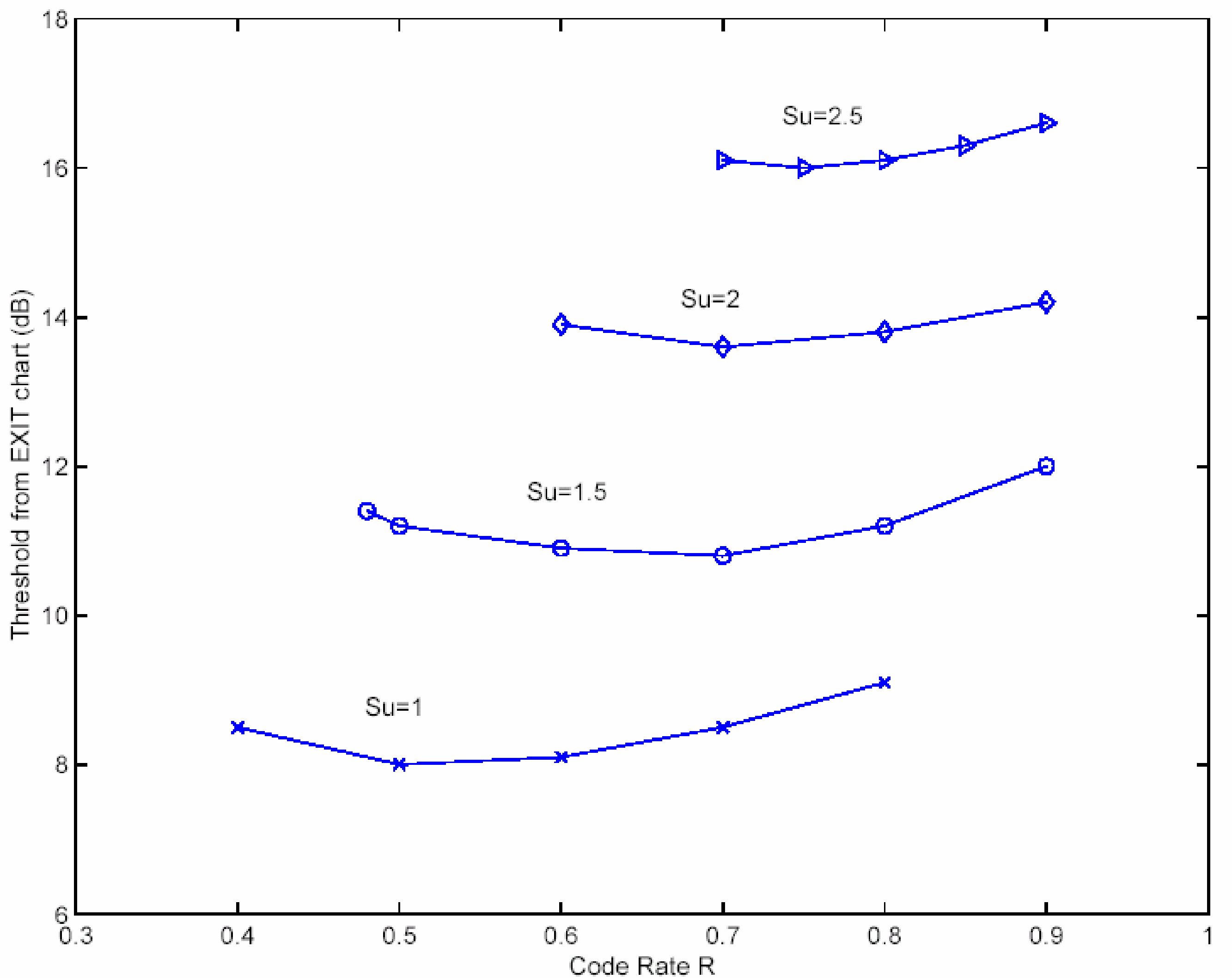
Results for LDPC Codes (cont'd)

Decoding threshold vs. code rate for various user densities



On the Optimal Bit Aspect Ratio

- The information-theoretic areal density may be computed via

$$I_{areal}\ (\text{bits/nm}^2) = I'_{xy}\ (\text{bits/PW}_{50}) \,/\, \left[L_{50}\ (\text{nm/PW}_{50}) \times TW^{-1}\ (\text{tracks/nm})\right]$$

where L_{50} is the length of PW_{50} in nm and TW is the track width.

- It is well-known that the SNR along a track is proportional to the bit-length^2 under the Lorentzian model (Bergmans, Immink)

- One may argue that at the optimal track density (which maximizes areal density), SNR will be proportional to the bit-width^2 as well (let bit-width = TW):

$$SNR = \alpha TW^2$$

Optimal Bit Aspect Ratio (cont'd)

- Combining these two equations yields

$$I_{areal} = \sqrt{\alpha} \cdot I'_{xy} / \left(L_{50} \sqrt{SNR} \right)$$

- Since α and L_{50} are constants dependent on a specific hard disk drive, we define a normalized areal density measure

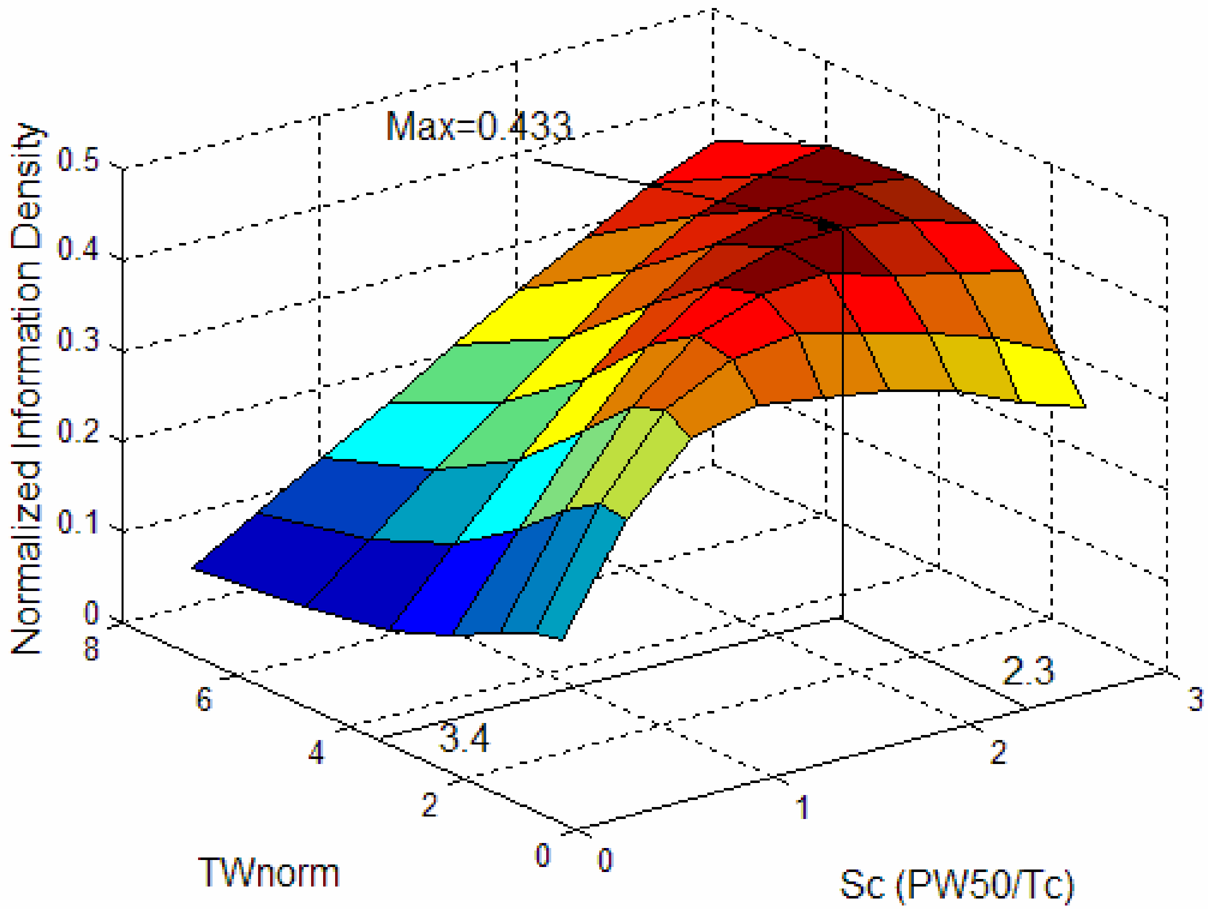
$$I_{areal,norm} = I_{areal} / \left(\sqrt{\alpha} / L_{50} \right) = I'_{xy} / \sqrt{SNR}$$

- We may plot $I_{areal,norm}$ as a function of S_c (since I'_{xy} is a function of S_c) and the normalized track width $\sqrt{\alpha}\,\mathrm{TW}$ (since $\sqrt{SNR}$ in the previous equation may be replaced by $\sqrt{\alpha}\,\mathrm{TW}$).

Optimal Bit Aspect Ratio (cont'd)

$I_{areal,norm}$ is maximized at $TW_{norm} = 3.4$ and $S_c = 2.3$.

We could convert $I_{areal,norm,max} =$ 0.433 to a density in bits/in^2 by scaling this value by the factor $\sqrt{\alpha}/L_{50}$, if known.



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Optimal Bit Aspect Ratio (cont'd)

- Even in the absence of knowledge of a density measure in bits/in^2, this analysis yields the following operating values at the optimum:
  - SNR: E_i/N_0 = 10.5 dB
  - Code rate: R = 0.62
  - Channel density: S_c = 2.35
  - User density: S_u = 1.45
- For comparison, today's (approximate) values:
  - SNR: E_i/N_0 = 18 dB
  - Code rate: R = 0.95
  - Channel density: S_c = 3.0
  - User density: S_u = 2.85

Conclusion

- These results serve as a guide to choosing the optimal operating parameters (linear density, bit aspect ratio, code rate, etc.).
- This work can be extended to include media noise and/or perpendicular recording.
- It can also be extended to codes which do not have iid inputs (e.g., Markovian codes).
- One of the implications is that work toward increased areal densities should target bit-width, not bit-length, leading to new challenges in track servo design.