Behavioral Social Choice Theory

DIMACS Tutorial

Social Choice & Computer Science

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Multi-Year Interdisciplinary Effort

- Collaborators:
  Adams (& Karcher), Grofman, Kantor,
  Kim, Marley, Tsetlin
- Past NSF SBR 9730076, Duke B-School
- Past UIUC Research Board
- Book forthcoming with
  Cambridge University Press

Criteria for a Unified Theory of Decision Making

(Inspired by Luce and Suppes, Handbook of Math Psych,1965)

- ✓ Treat individual & group decision making in a unified way
- ✓ Reconcile normative & descriptive work
- ✓ Integrate & compare competing normative benchmarks
- ✓ Reconcile theory & data
- ✓ Encompass & integrate multiple choice, rating and ranking paradigms
- ✓ Integrate & compare multiple representations of preference, utilities & choices
- ▪ Develop dynamic models as extensions of static models
- ✓ Systematically incorporate statistics as a scientific decision making apparatus

Today:

- Statistical Sampling and Inference
- Why no Cycles? (General Value Restriction)
- Behavioral Social Choice Analysis of STV

Majority rule:

Majority Winner

- Candidate who is ranked ahead of any other candidate by more than 50%
- Candidate who beats any other candidate in pairwise competition

Condorcet Paradox
a.k.a. *Majority Cycles*

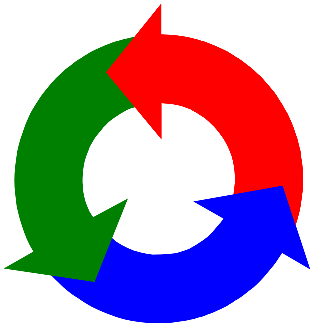
ABC	**1 person**
BCA	**1 person**
CAB	**1 person**

Democratic Decision Making at Risk!?!

A is majority preferred to B

B is majority preferred to C

C is majority preferred to A



State of the Art: Shepsle et al. 1997

Probability of a Cycle: Pr(m, n)
Based on Sampling from a Uniform Distribution on Linear Orders
("Impartial Culture")*

	number of voters (n)						
number of alternatives (m)	3	5	7	9	11		limit
3	.056	.069	.075	.078	.080		.088
4	.111	.139	.150	.156	.160		.176
5	.160	.200	.215				.251
6	.202						.315
limit	≈1.00	≈1.00	≈1.00	≈1.00	≈1.00		≈1.00

*Source: Riker (1982: 122) as reproduced in Shepsle and Bonchek (1997: Table 4.1, 54)

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6	.202						.315
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5	.160	.200	.215				.251
6	.202						.315
limit	≈1.00	≈1.00	≈1.00	≈1.00	≈1.00		≈1.00

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Shepsle & Bonchek (1997)

"In general, then, we cannot rely on the method of majority rule to produce a coherent sense of what the group 'wants', especially if there are no institutional mechanisms for keeping participation restricted (thereby keeping *n* small) or weeding out some of the alternatives (thereby keeping *m* small)."

Drawing Random Samples from Realistic Distributions

What happens if we interview
20 randomly drawn voters from the 1996 ANES?

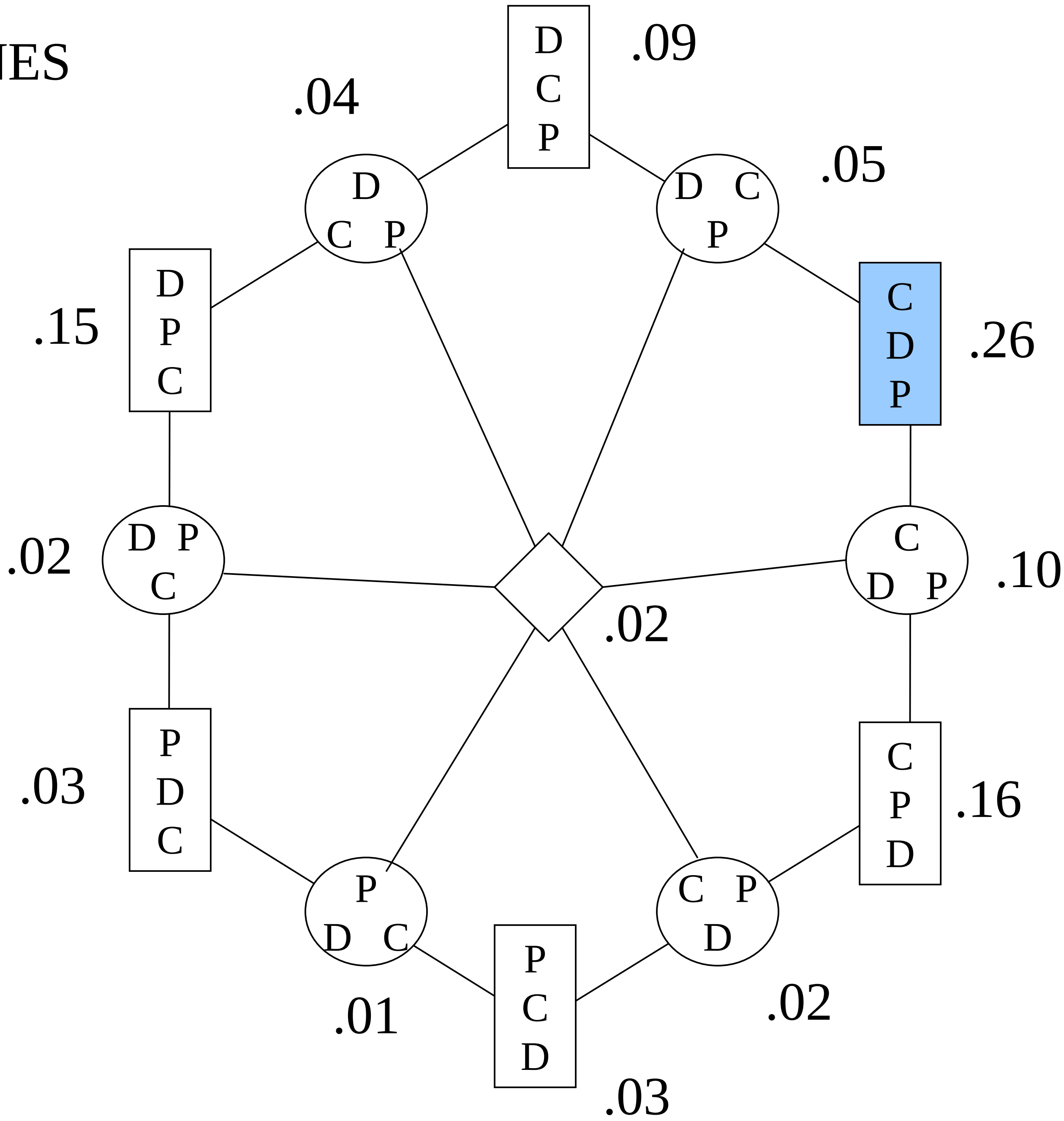
Do they display cyclical majorities?

Do they display the correct majority preference order?

For a while I assume that
Individual Preferences
are WEAK ORDERS
over three choice alternatives

There are **13** possible **weak orders**
There are **27** different
possible **majority** preference **relations**

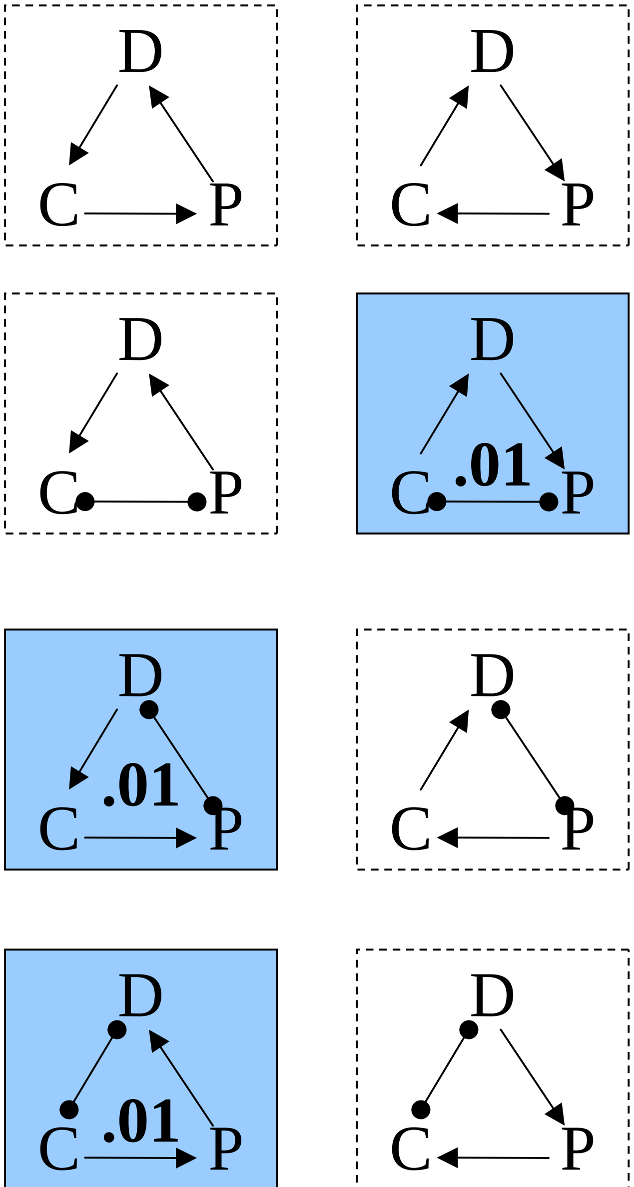
1996 ANES

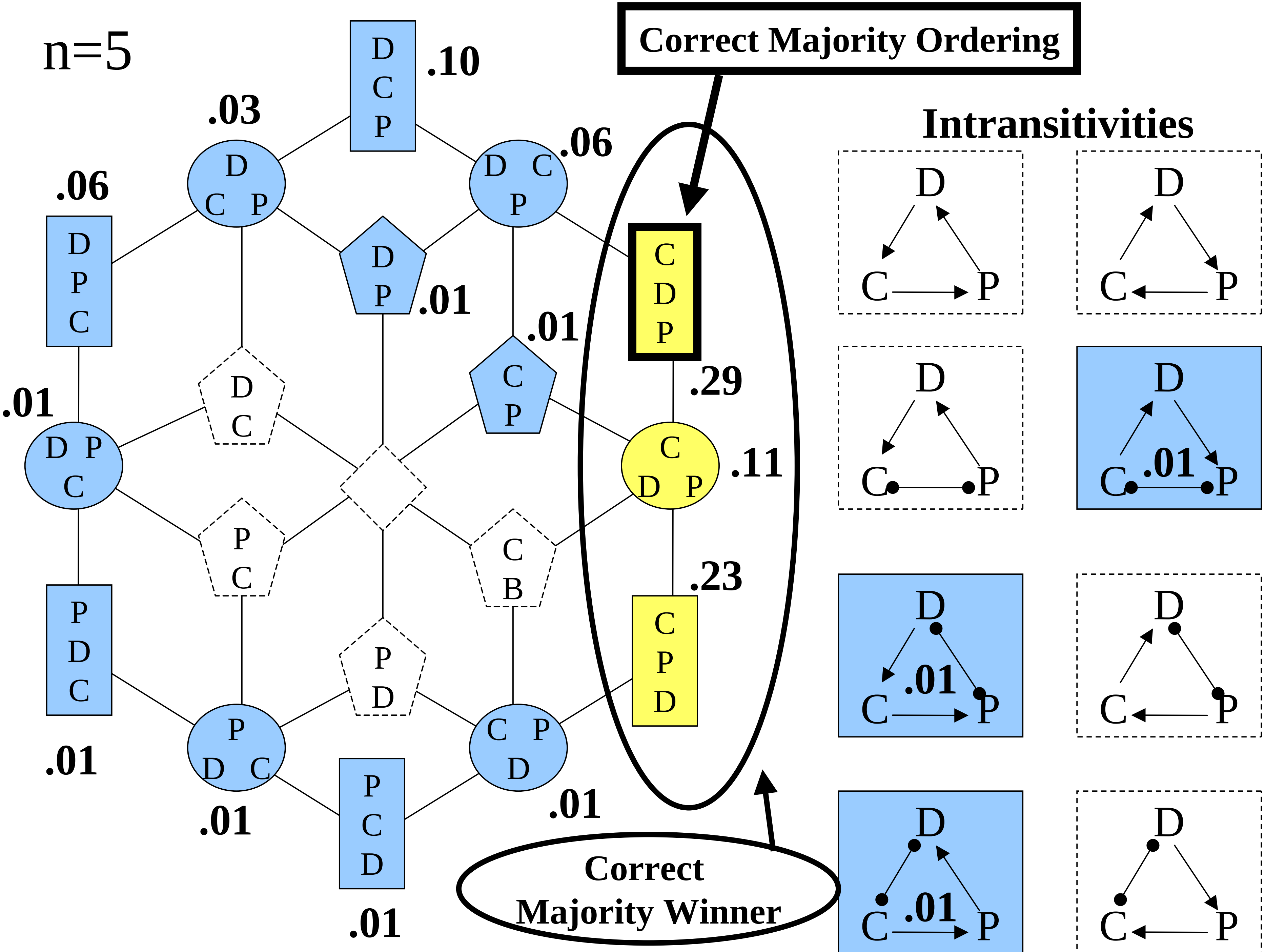


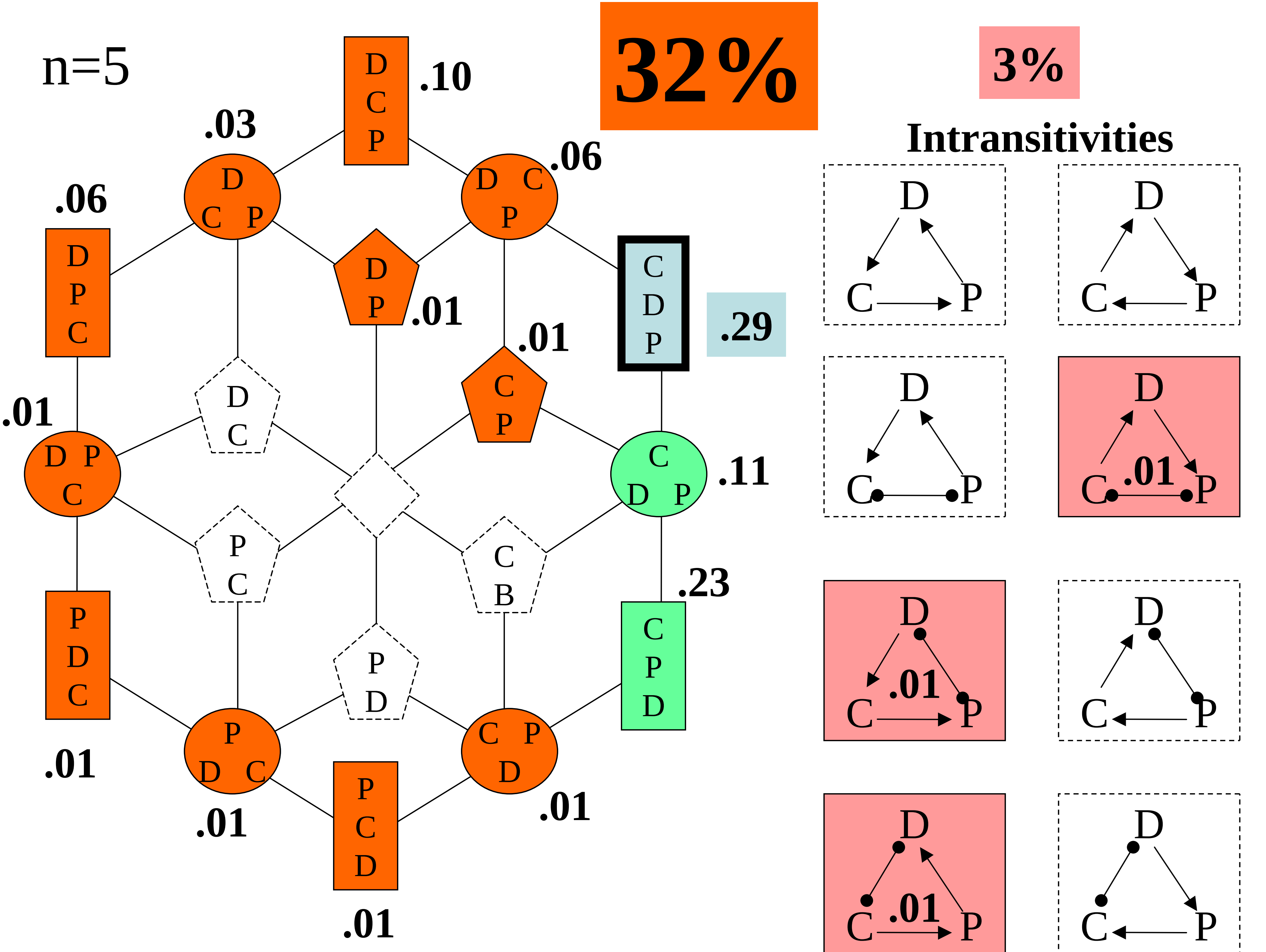
D C P
.09
.04
D C P
D C P
.05
D P C
.15
C D P
.26
D P C
.02
C D P
.10
.02
P D C
.03
C P D
.16
P D C
.01
P C D
.03
C P D
.02

n=5

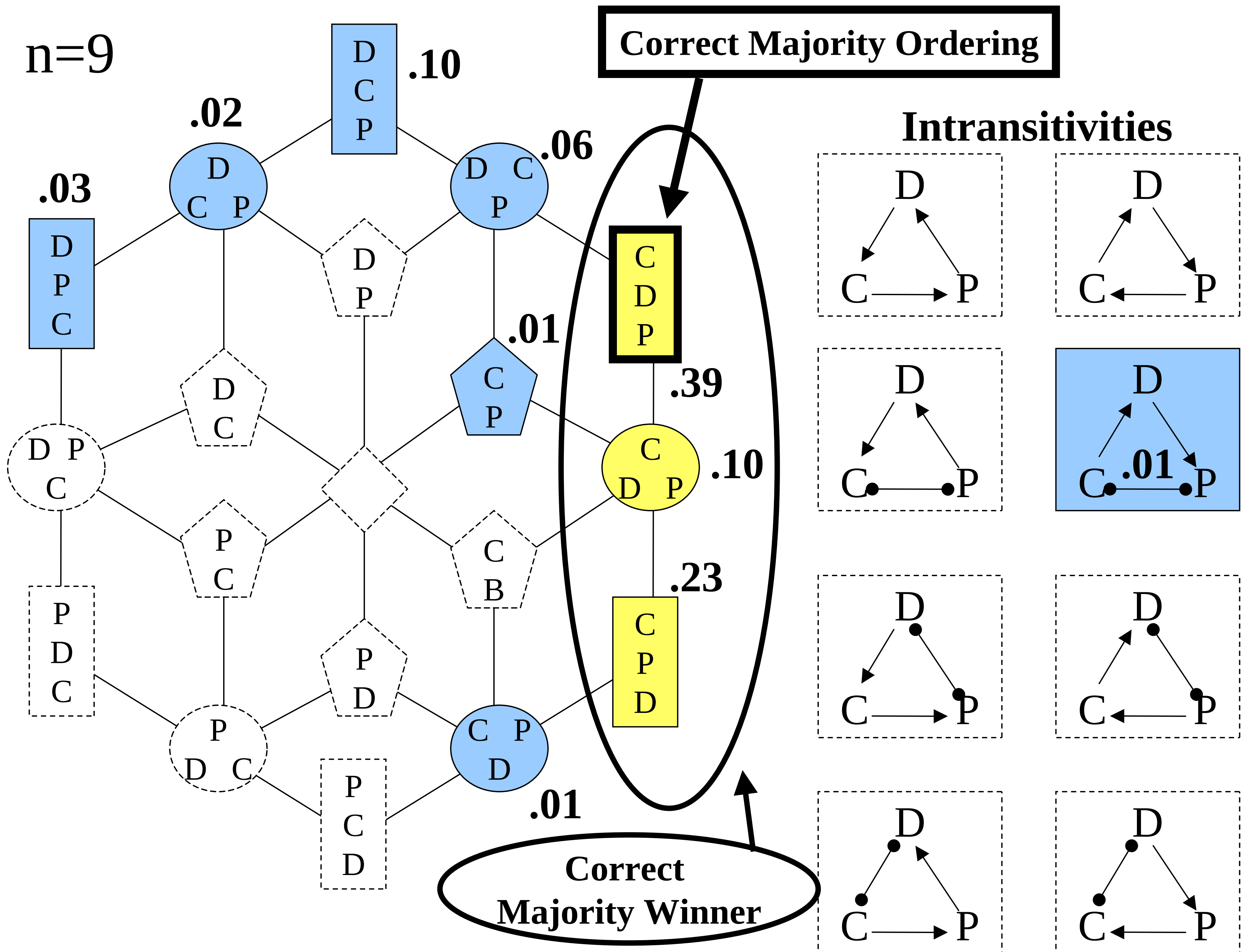
Intransitivities



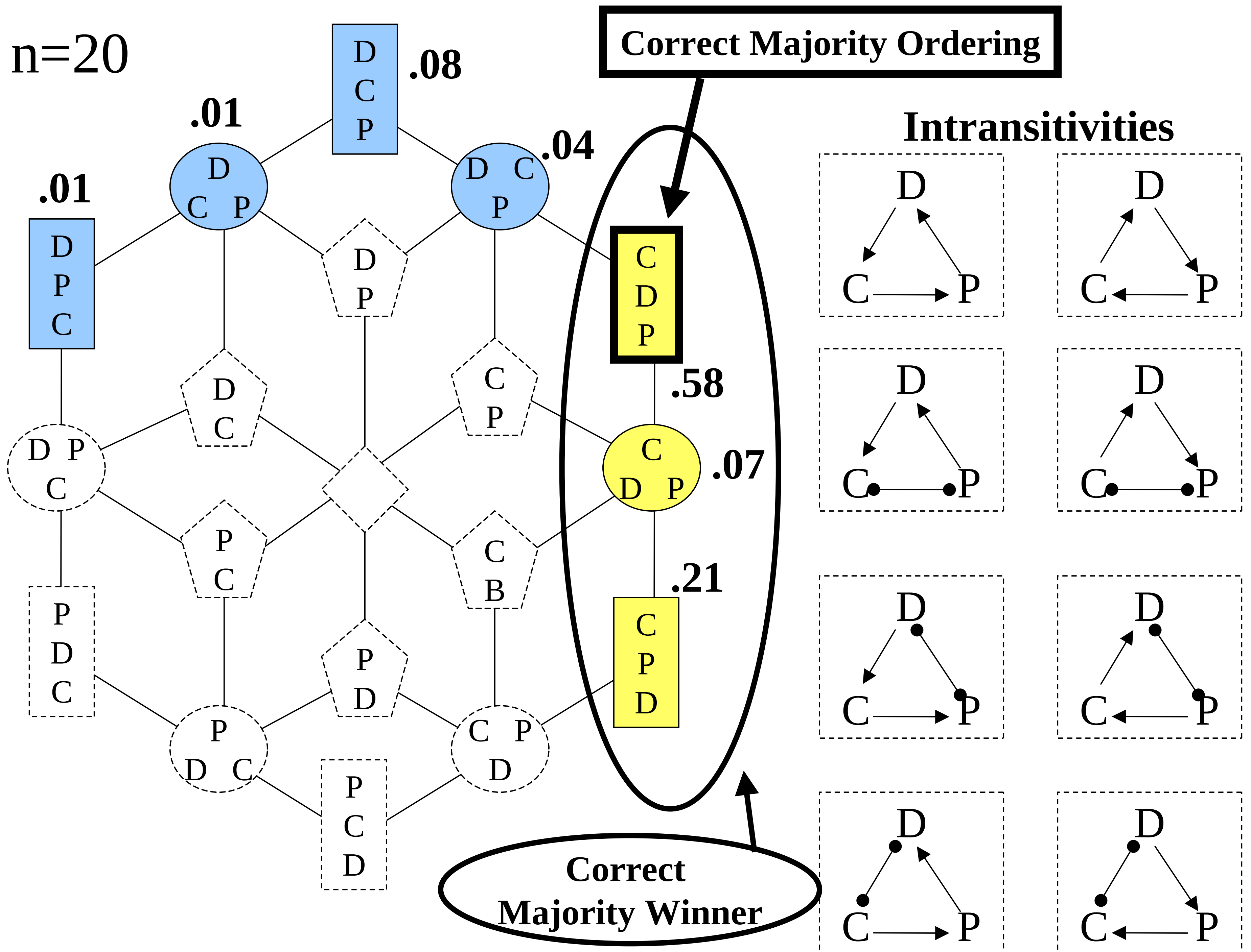

n=5
Correct Majority Ordering
Intransitivities
D C P .10
D C P .03
D C P .06
D P C .06
D P .01
C P .01
C D P .29
C D P .11
C P D .23
D P C .01
D C
P C
C B
P D
P D C .01
P D C .01
C P D .01
P C D .01
.01
.01
.01
Correct Majority Winner
D C P
D C P
D C P
D C P .01
D C P .01
D C P
D C P .01
D C P

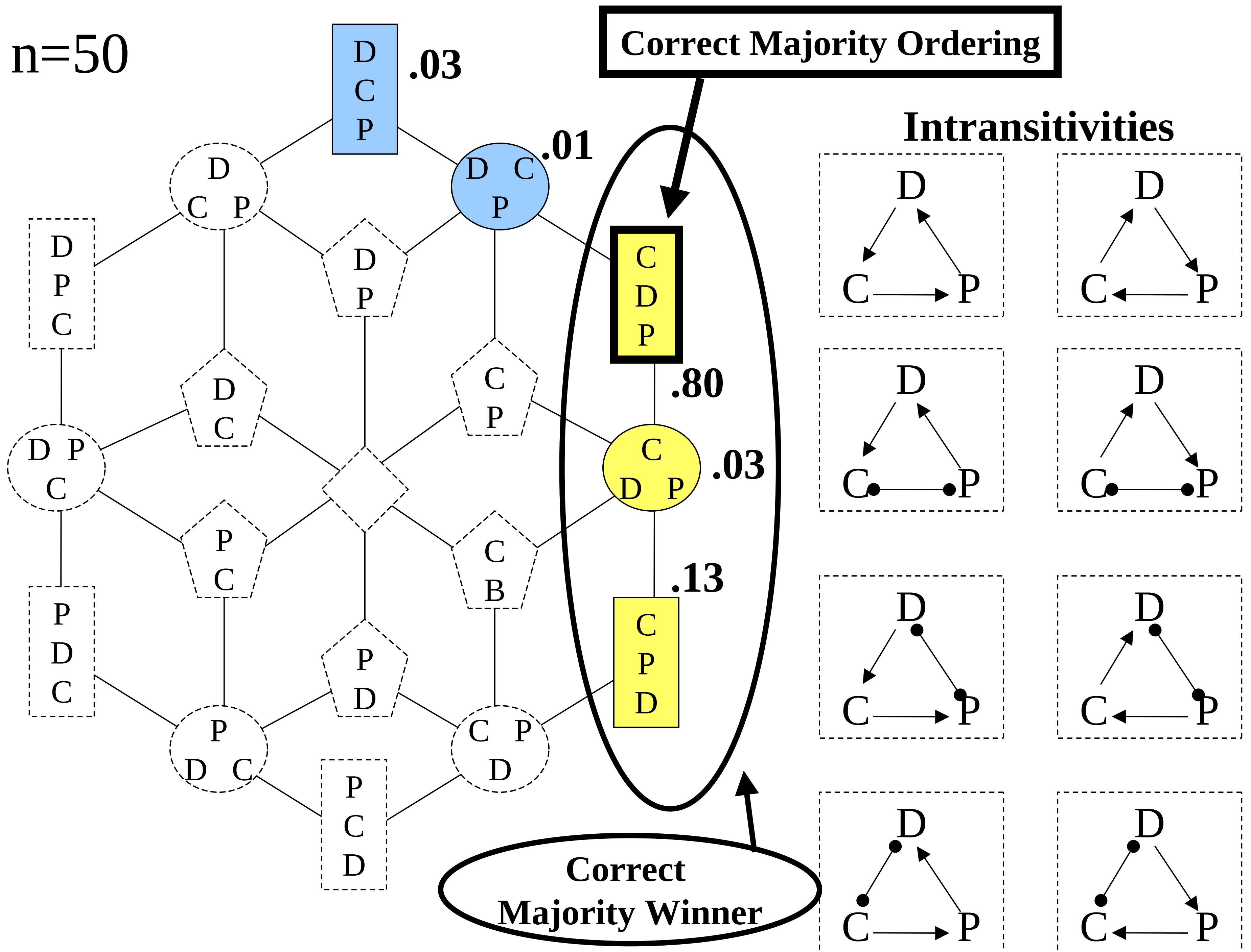


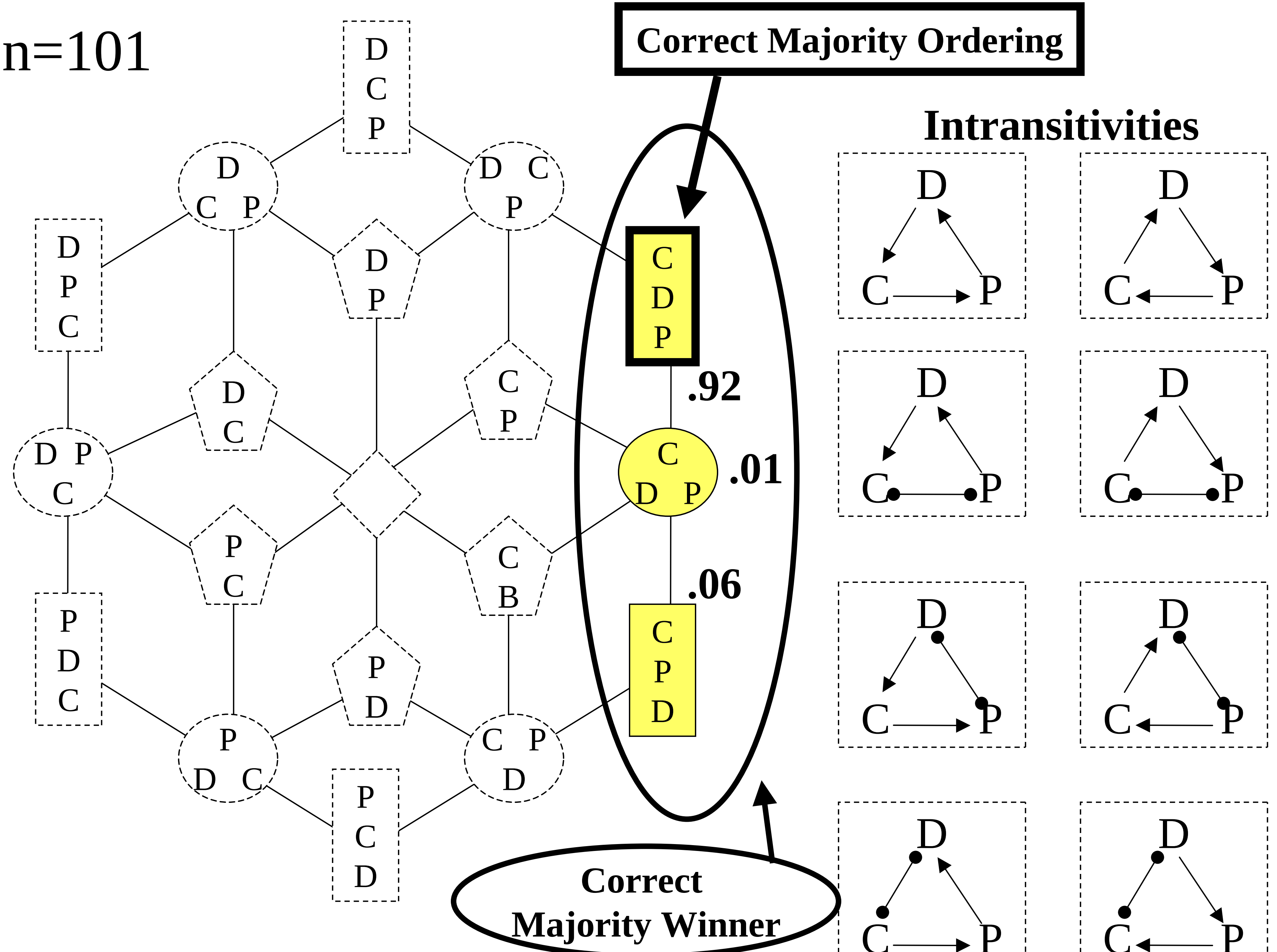
n=5
32%
3%
Intransitivities
D C P
.10
.03
.06
.06
.01
.01
.29
.01
.11
.23
.01
.01
.01
.01
.01
.01
.01



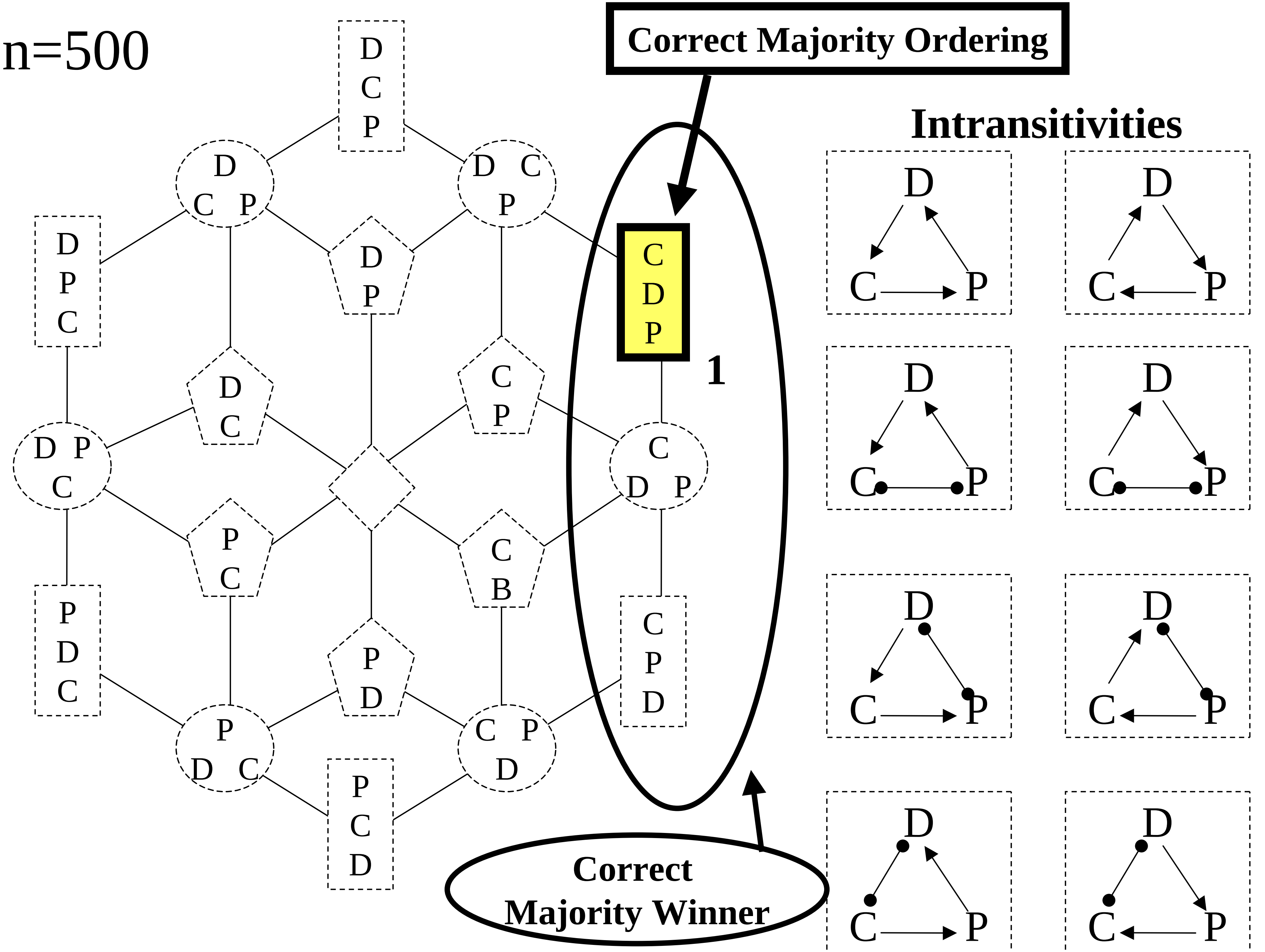
n=9
Correct Majority Ordering
Intransitivities
.10
.02
.06
.03
.01
.39
.10
.23
.01
.01
Correct
Majority Winner

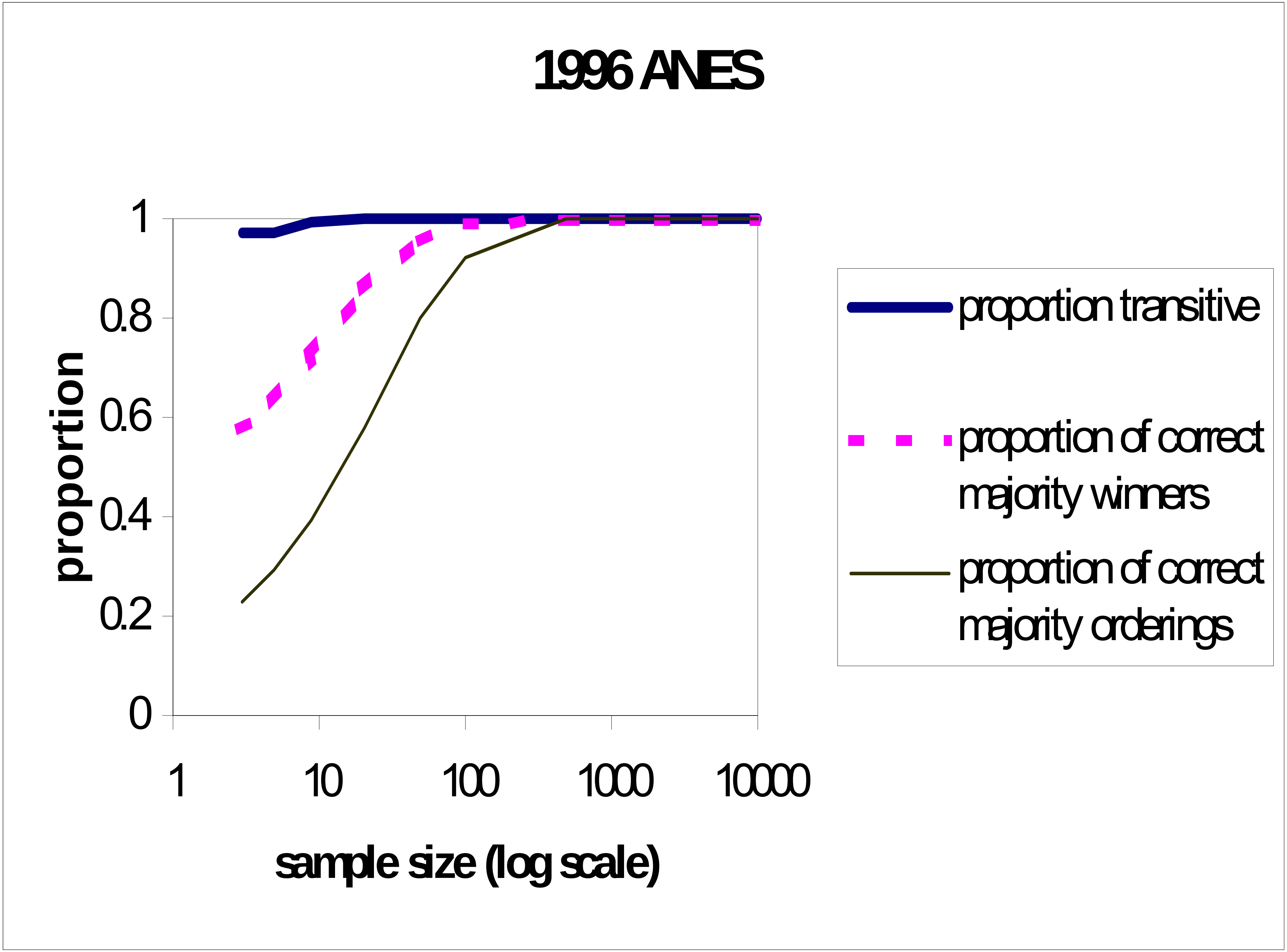

n=20
Correct Majority Ordering
Intransitivities
.08
.01
.04
.01
.58
.07
.21
Correct
Majority Winner
D
C
P
B


n=50
Correct Majority Ordering
Intransitivities
Correct Majority Winner
.03
.01
.80
.03
.13
D C P
D C P
C D P
C D P
C P D
D P C
P D C
P C D
D C
D P
C P
P C
C B
P D
D C P
D P C
P D C
C P D

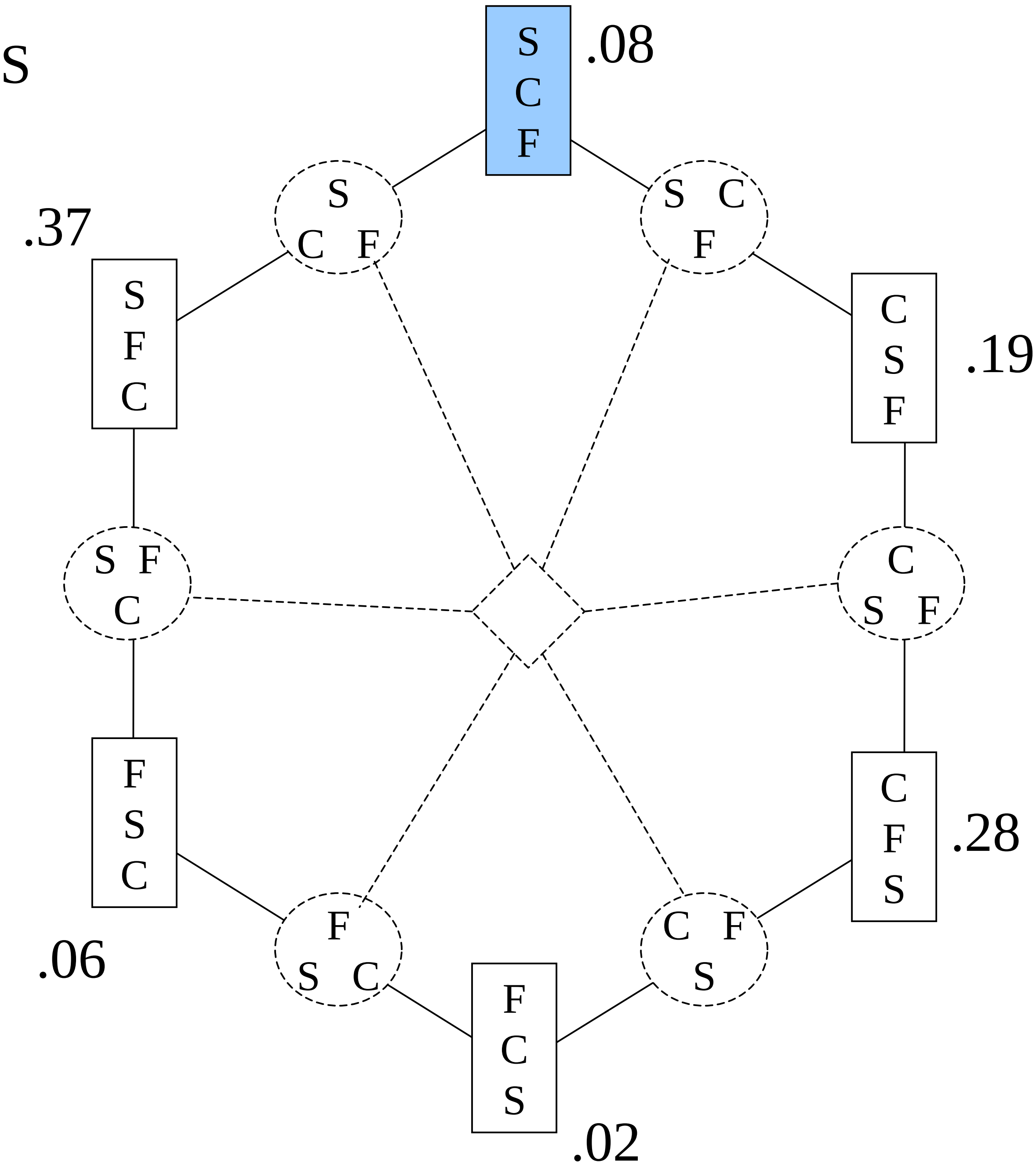


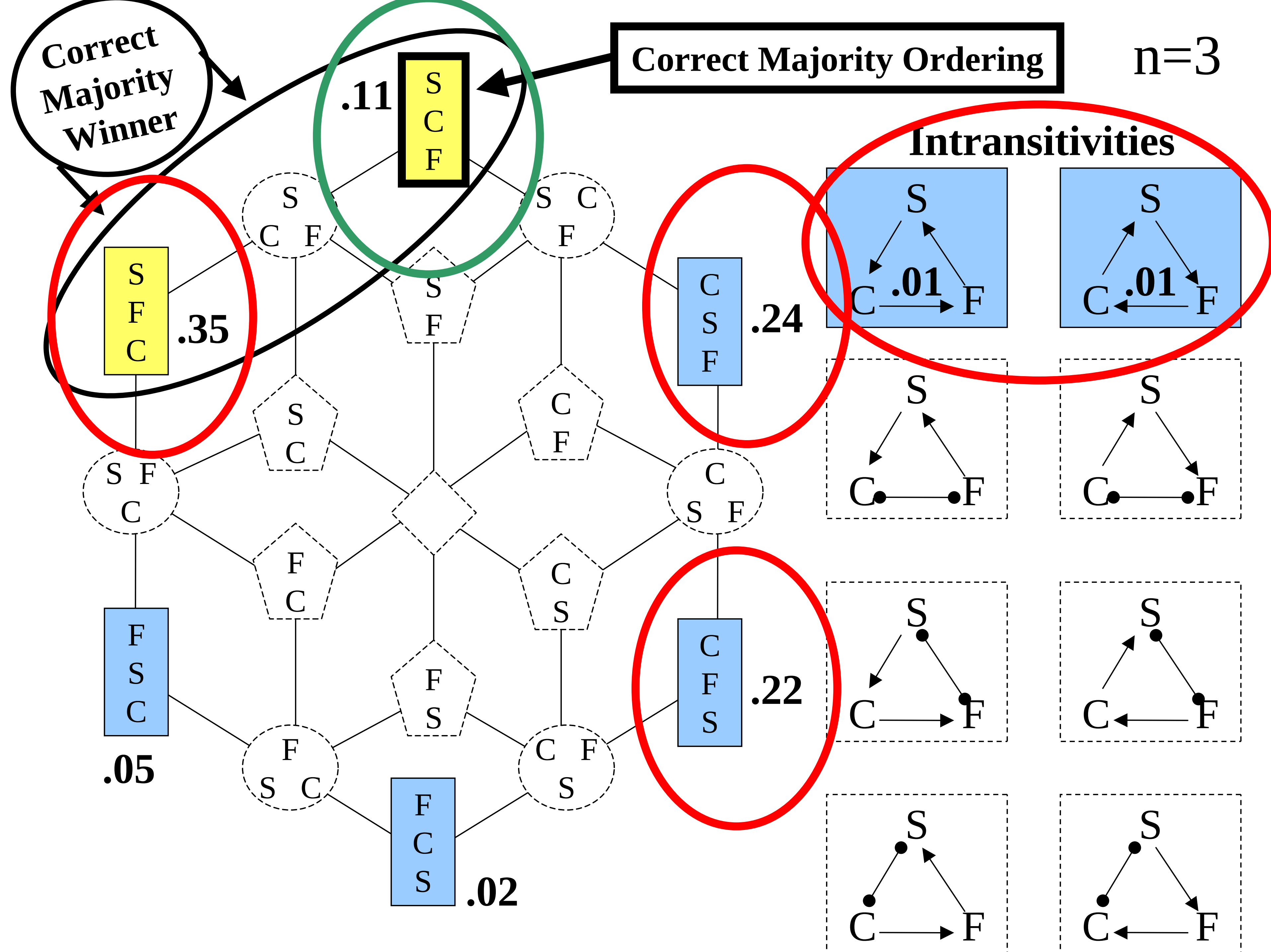
n=101
Correct Majority Ordering
Intransitivities
D
C
P
D
C P
D C
P
D
P
C
D
P
D
C
C
P
C
D
P
.92
D P
C
C
D P
.01
P
C
C
B
.06
C
P
D
P
D
C
P
D
P
D C
C P
D
P
C
D
Correct
Majority Winner


n=500
Correct Majority Ordering
Intransitivities
D C P
D C P
D C P
D P
D P C
D C
C P
D P C
C D P
1
C D P
P C
C B
P D C
P C
P D
P D C
C P D
C P D
P C D
Correct Majority Winner
D C P
D C P
D C P
D C P
D C P
D C P
D C P
D C P

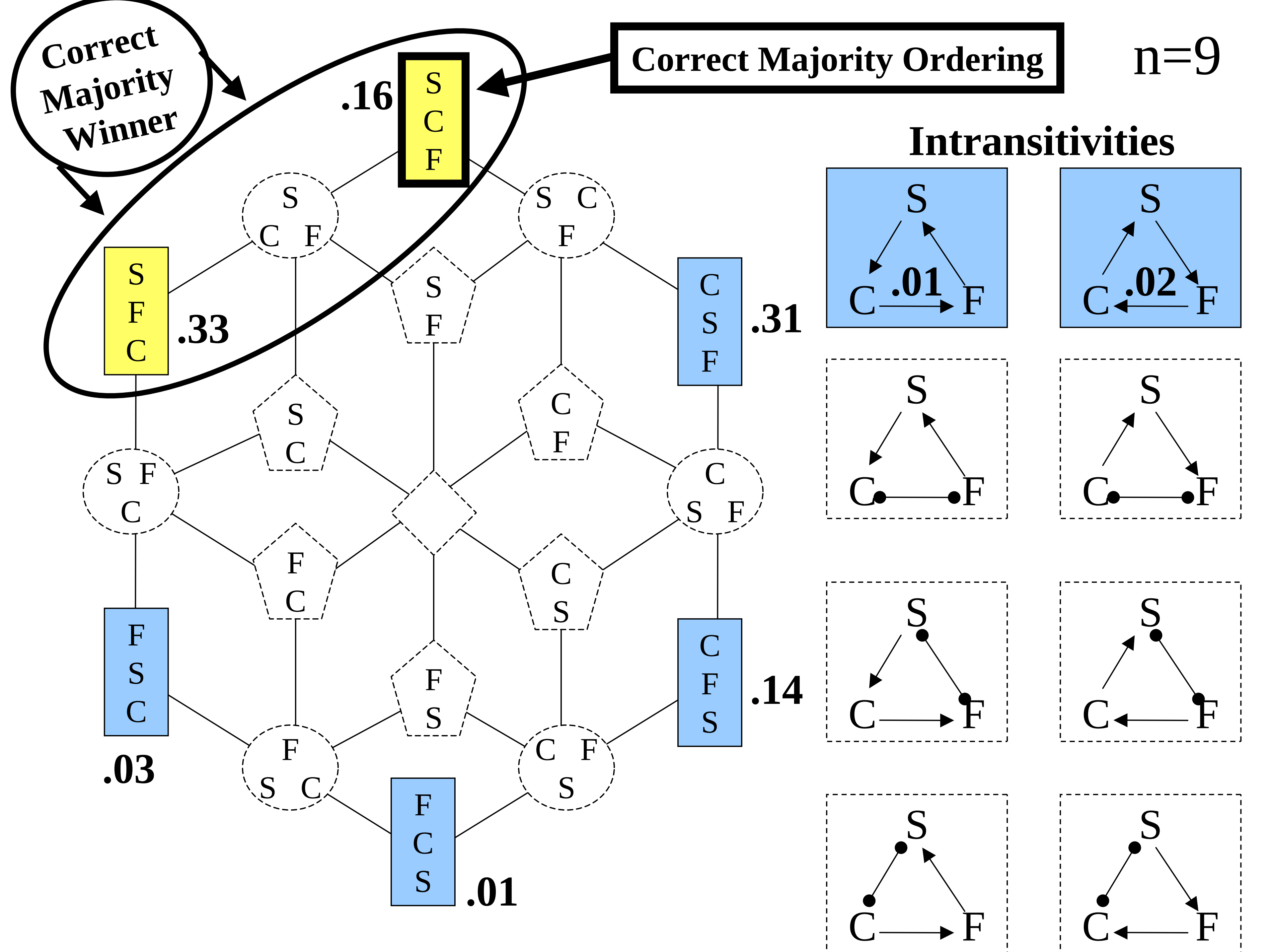

1996 ANES
proportion
1
0.8
0.6
0.4
0.2
0
1
10
100
1000
10000
sample size (log scale)
proportion transitive
proportion of correct majority winners
proportion of correct majority orderings

1976 GNES

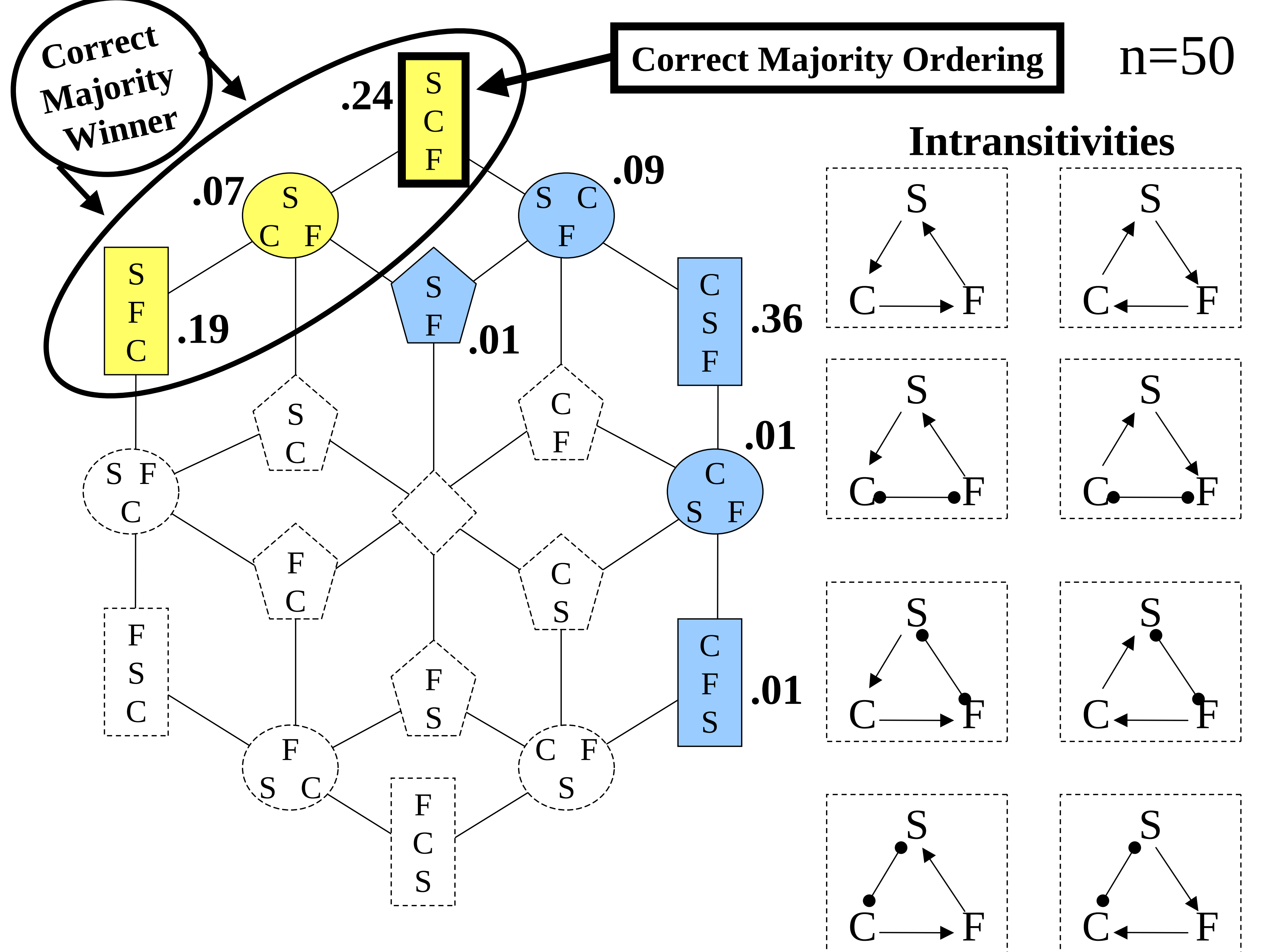




Correct Majority Winner
Correct Majority Ordering
n=3
Intransitivities
.11
S C F
.35
S F C
.24
C S F
.22
C F S
.05
F S C
.02
F C S
.01
.01



Correct Majority Winner
Correct Majority Ordering
n=9
Intransitivities
.16
S C F
S F C
.33
S C F
S C F
S F
S C
C F
S F C
C S F
.31
C S F
F C
C S
F S C
.03
F S
F S C
C F S
F C S
.01
C F S
.14
.01
.02
S C F



Correct Majority Winner
Correct Majority Ordering
n=50
Intransitivities
.24
S C F
.07
S C F
.09
S C F
.19
S F C
.01
S F
.36
C S F
.01
C S F
.01
C F S
S C
C F
S F C
F C
C S
F S C
F S
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C F S
F C S
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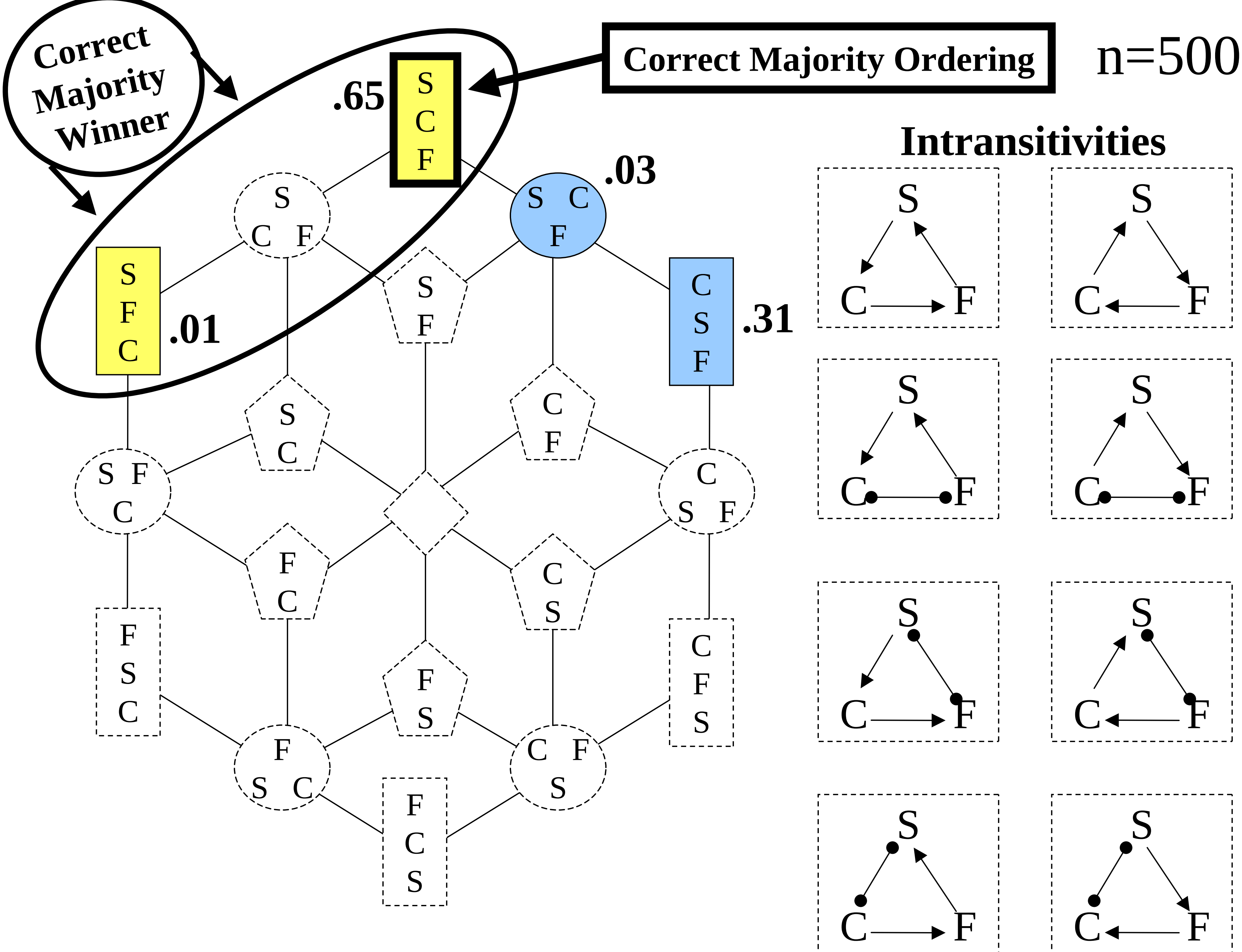
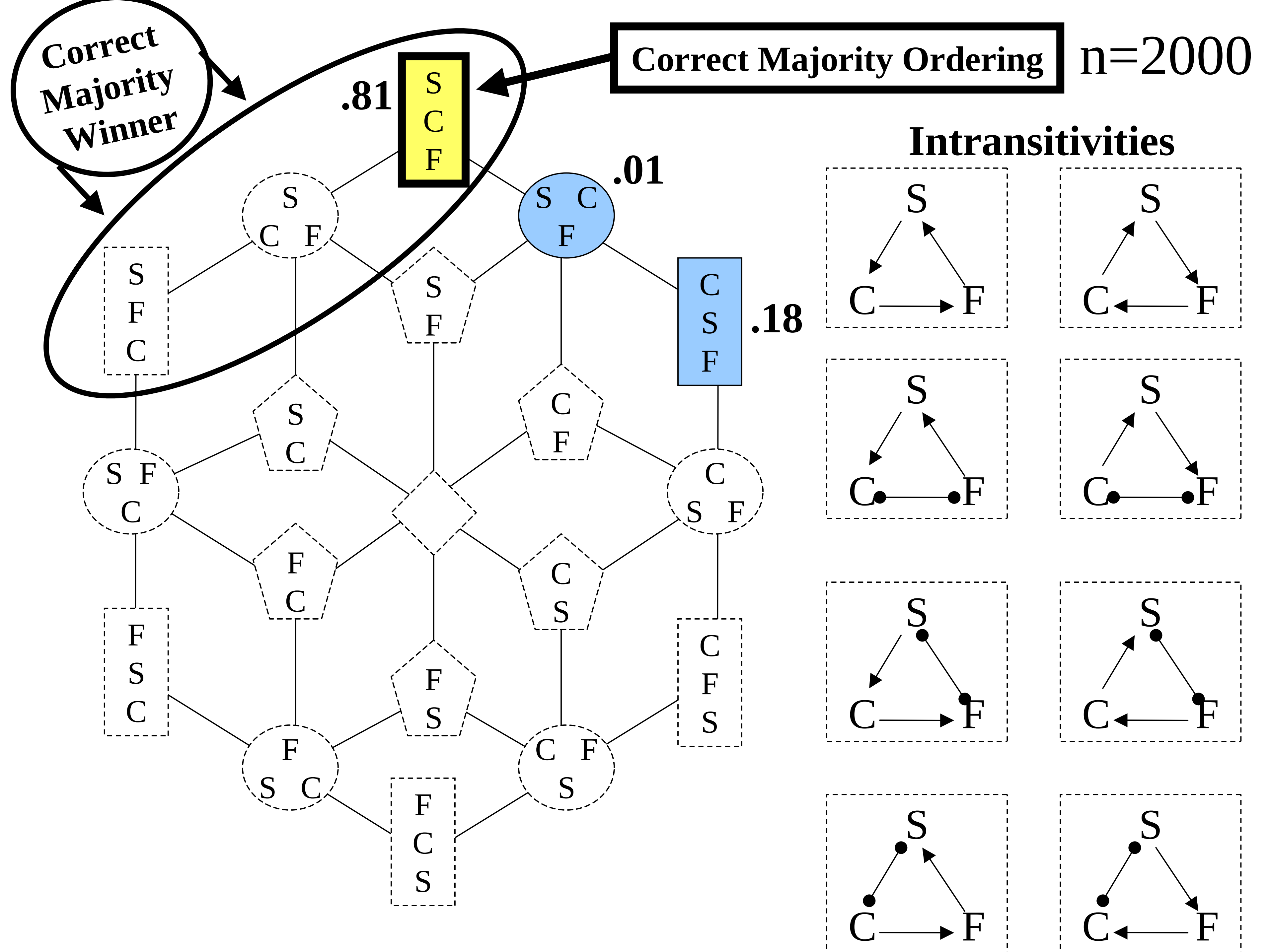
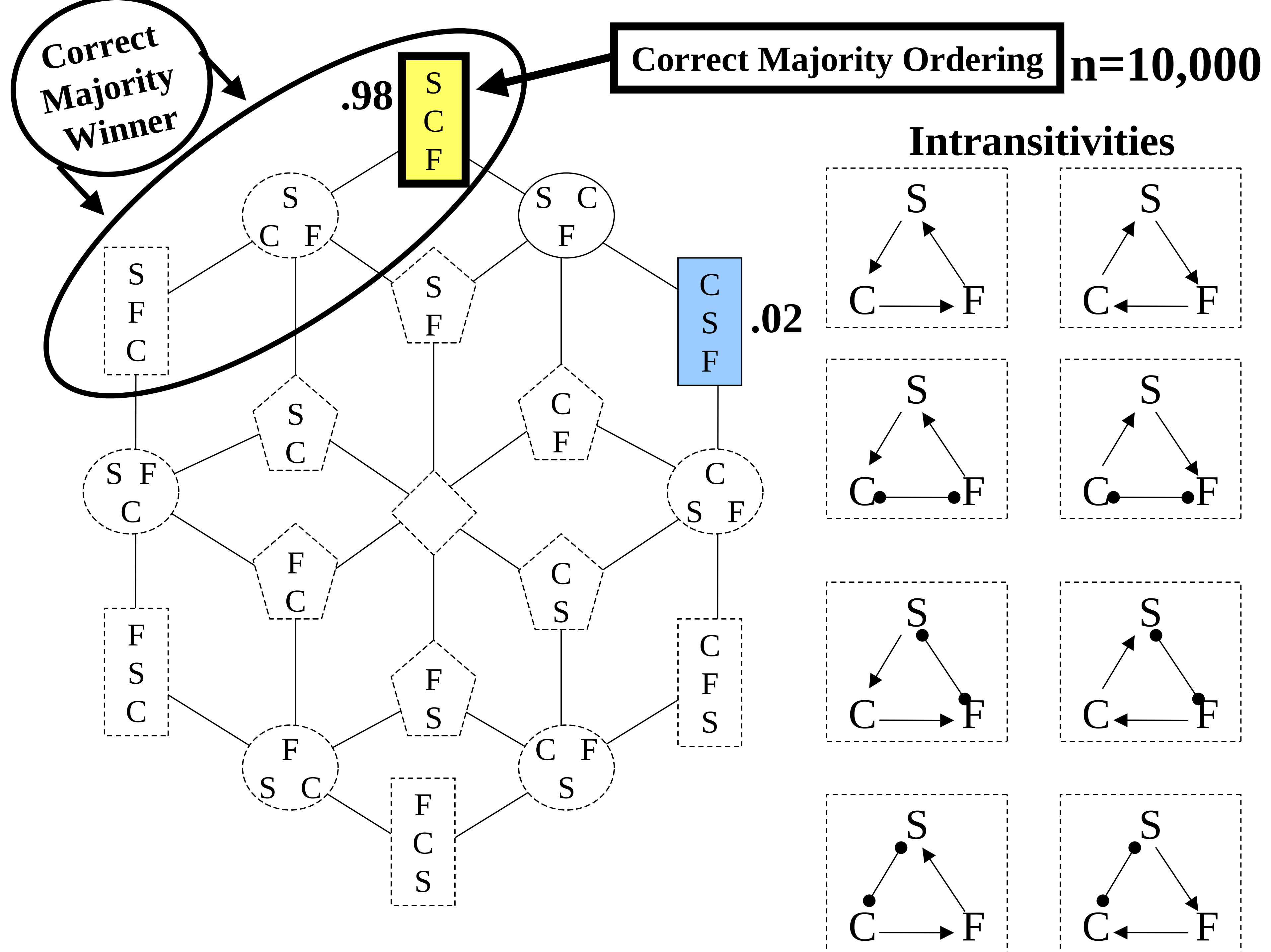
S C F
S C F
S C F
S C F
S C F
S C F
S C F



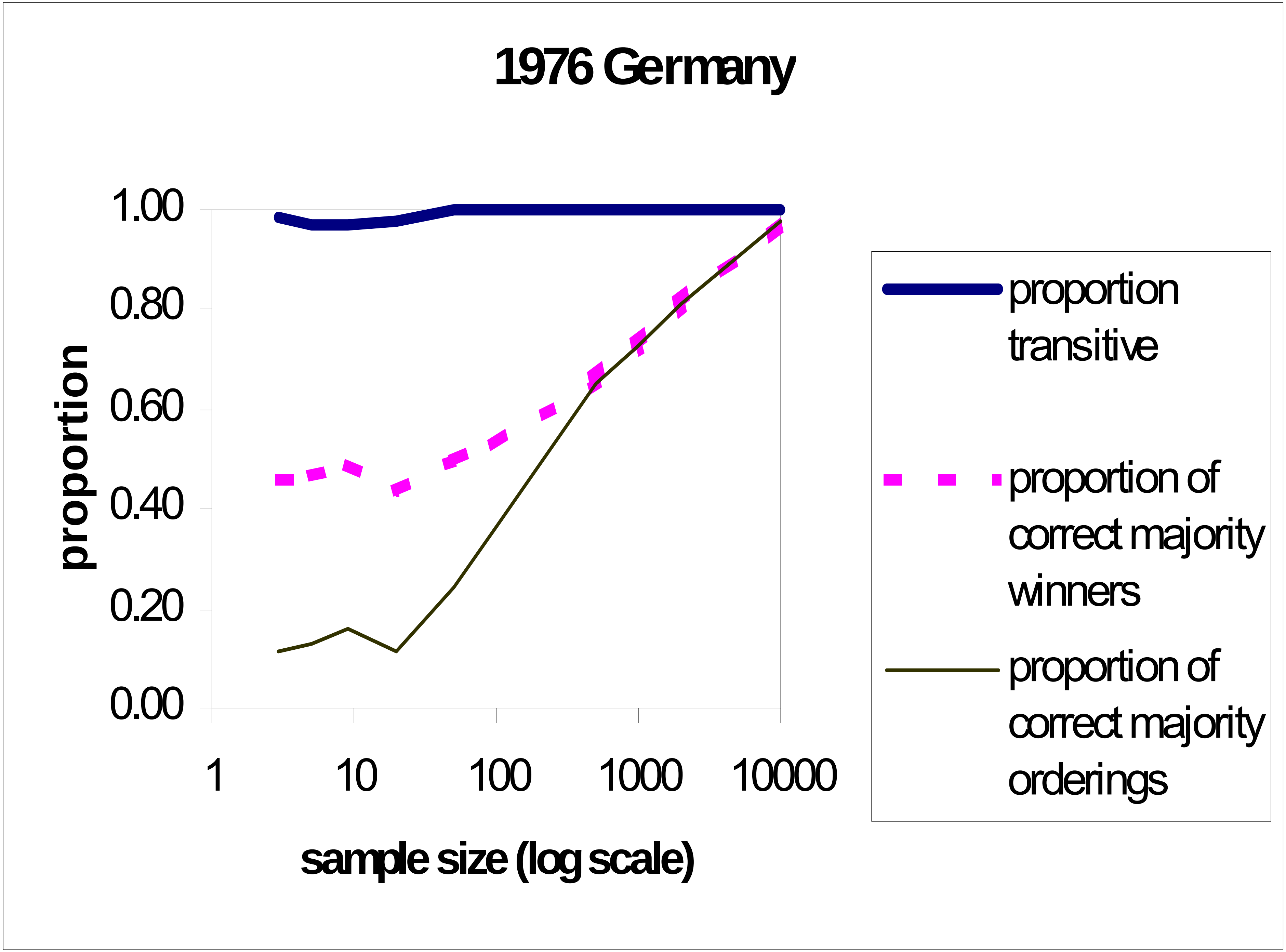
Correct Majority Winner
Correct Majority Ordering
n=500
.65
S C F
.03
S C F
.01
S F C
.31
C S F
S C F
S F
S F C
S C
C F
C S F
F C
C S
F S C
C F S
F S
F S C
C F S
F C S
Intransitivities
S C F



Correct Majority Winner
Correct Majority Ordering
n=2000
.81
S C F
.01
S C F
.18
C S F
S C F
S F C
S F
S C
C F
S F C
C S F
F C
C S
F S C
F S
C F S
F S C
C F S
F C S
Intransitivities
S C F

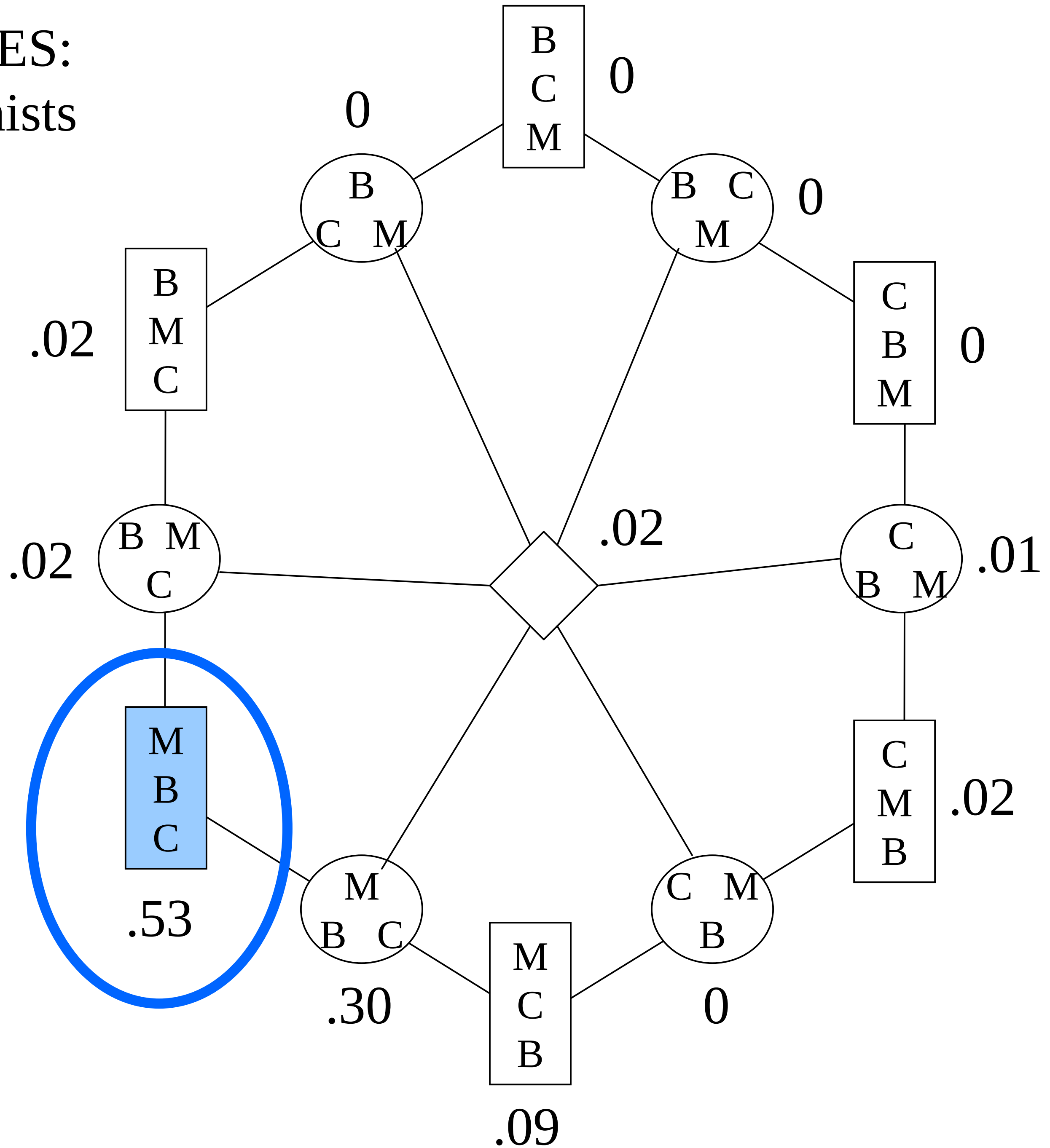


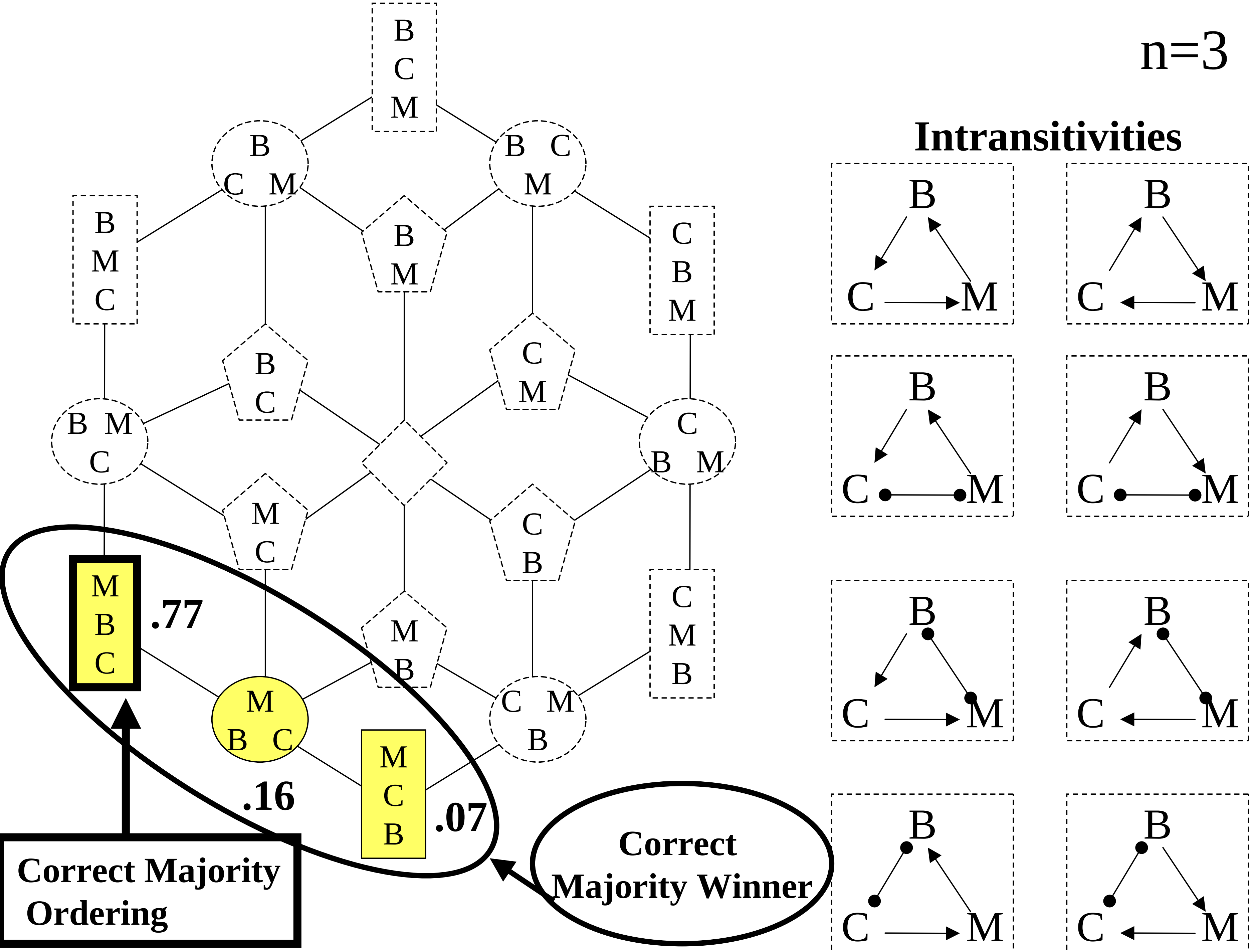
Correct Majority Winner
Correct Majority Ordering
n=10,000
.98
S C F
.02
C S F
Intransitivities



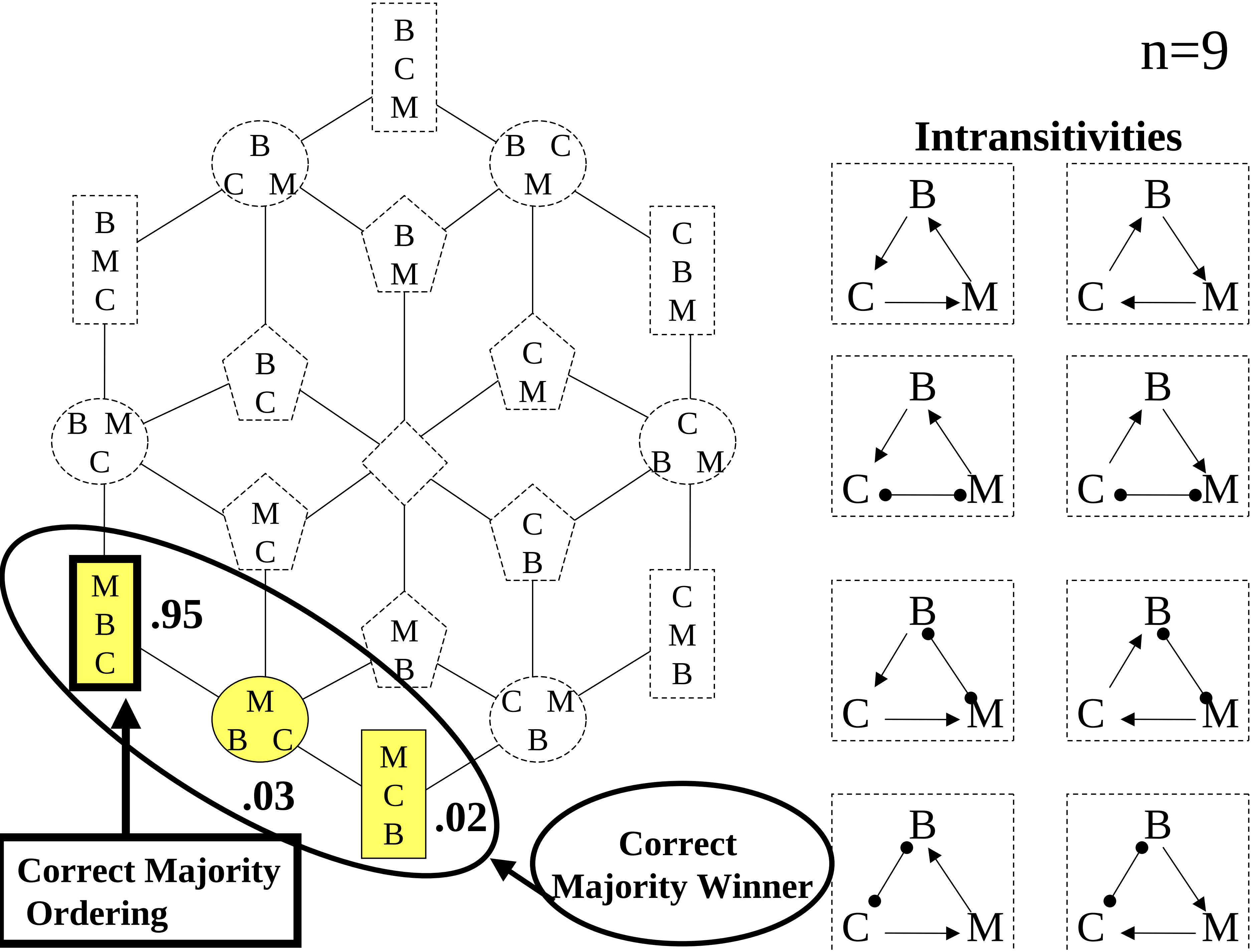
1976 Germany
proportion
1.00
0.80
0.60
0.40
0.20
0.00
1
10
100
1000
10000
sample size (log scale)
proportion transitive
proportion of correct majority winners
proportion of correct majority orderings

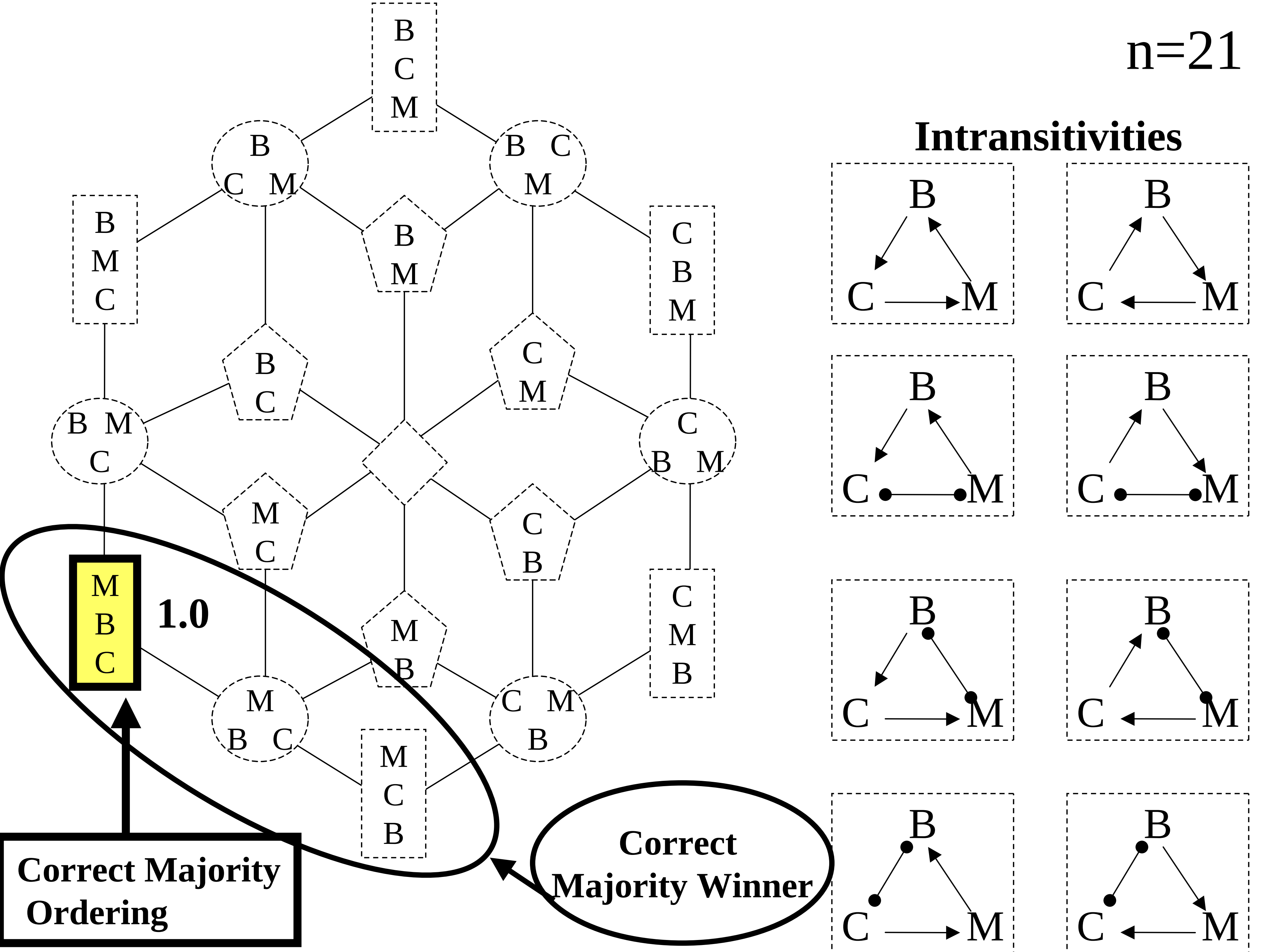
1988 FNES:
Communists



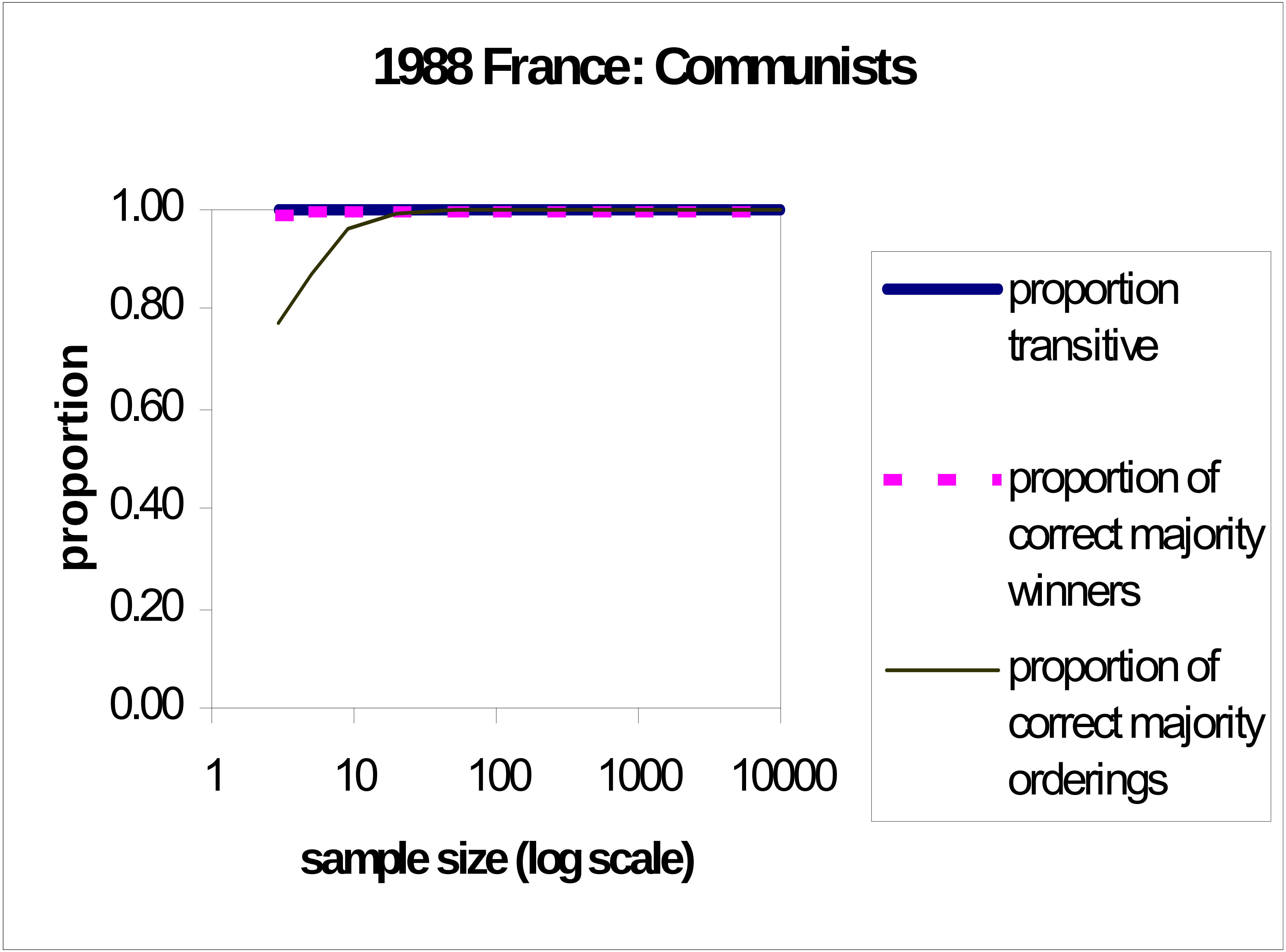


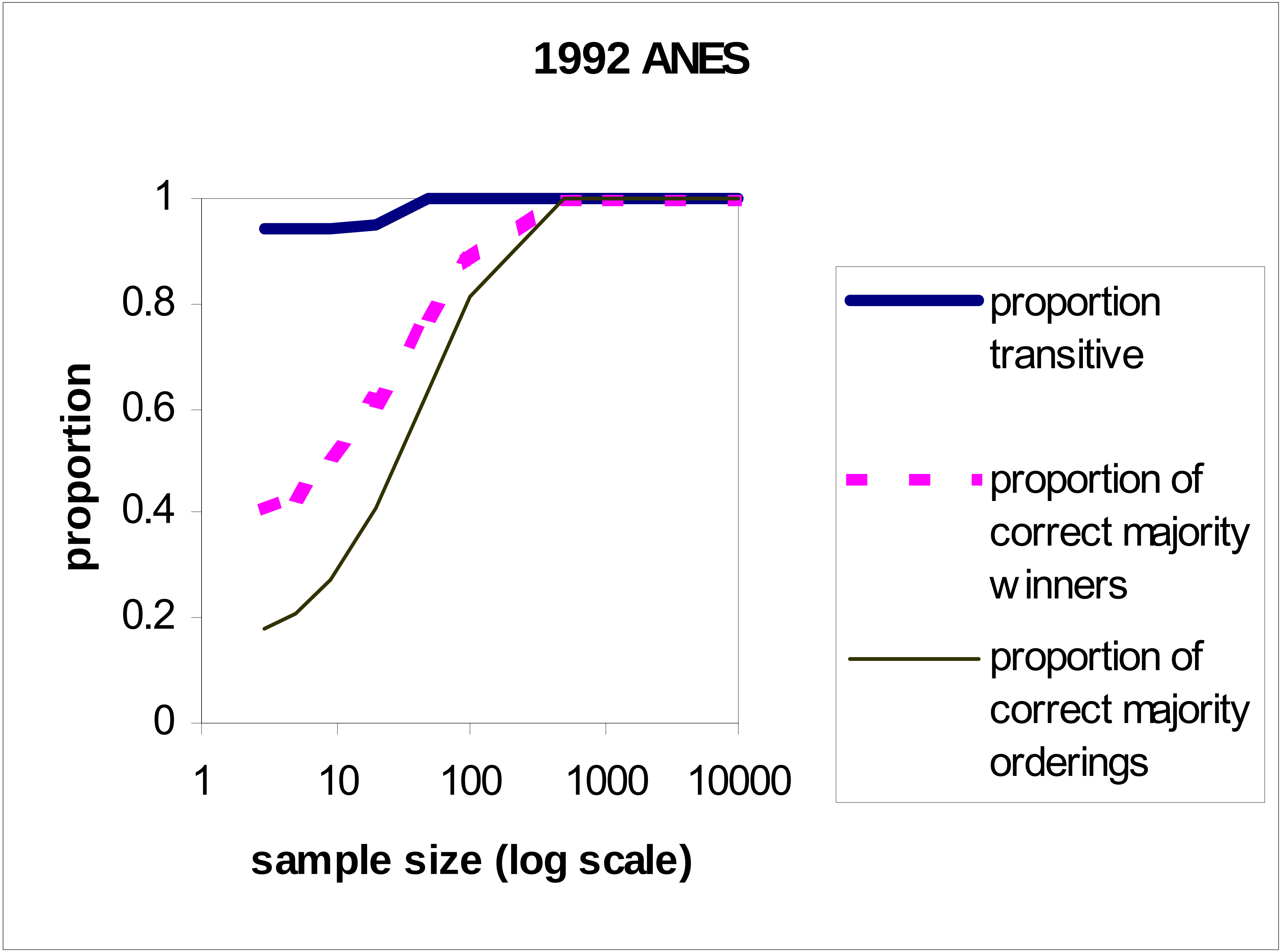
n=3
Intransitivities
.77
.16
.07
Correct Majority Ordering
Correct Majority Winner


n=9
Intransitivities
.95
.03
.02
Correct Majority Ordering
Correct Majority Winner

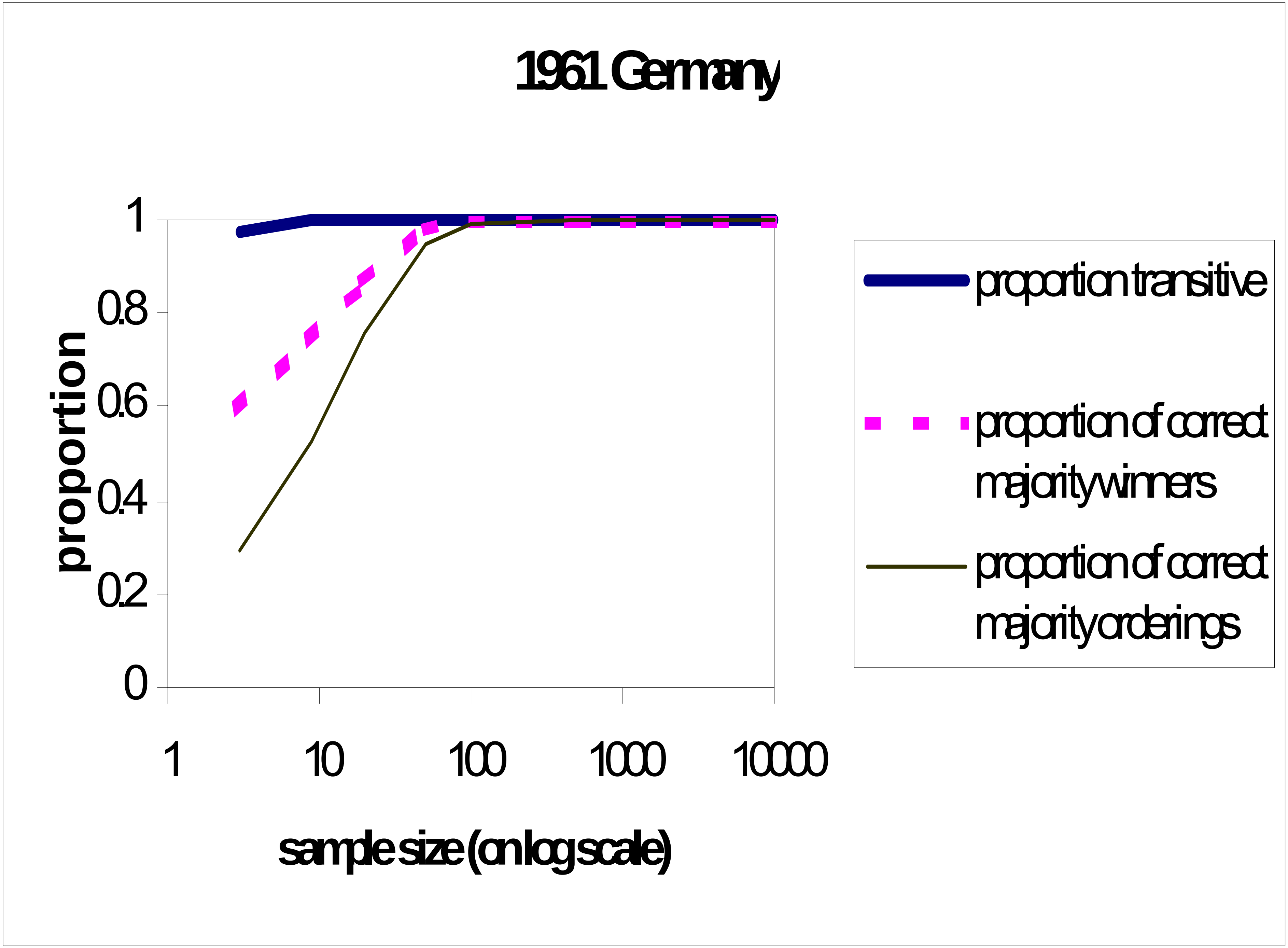


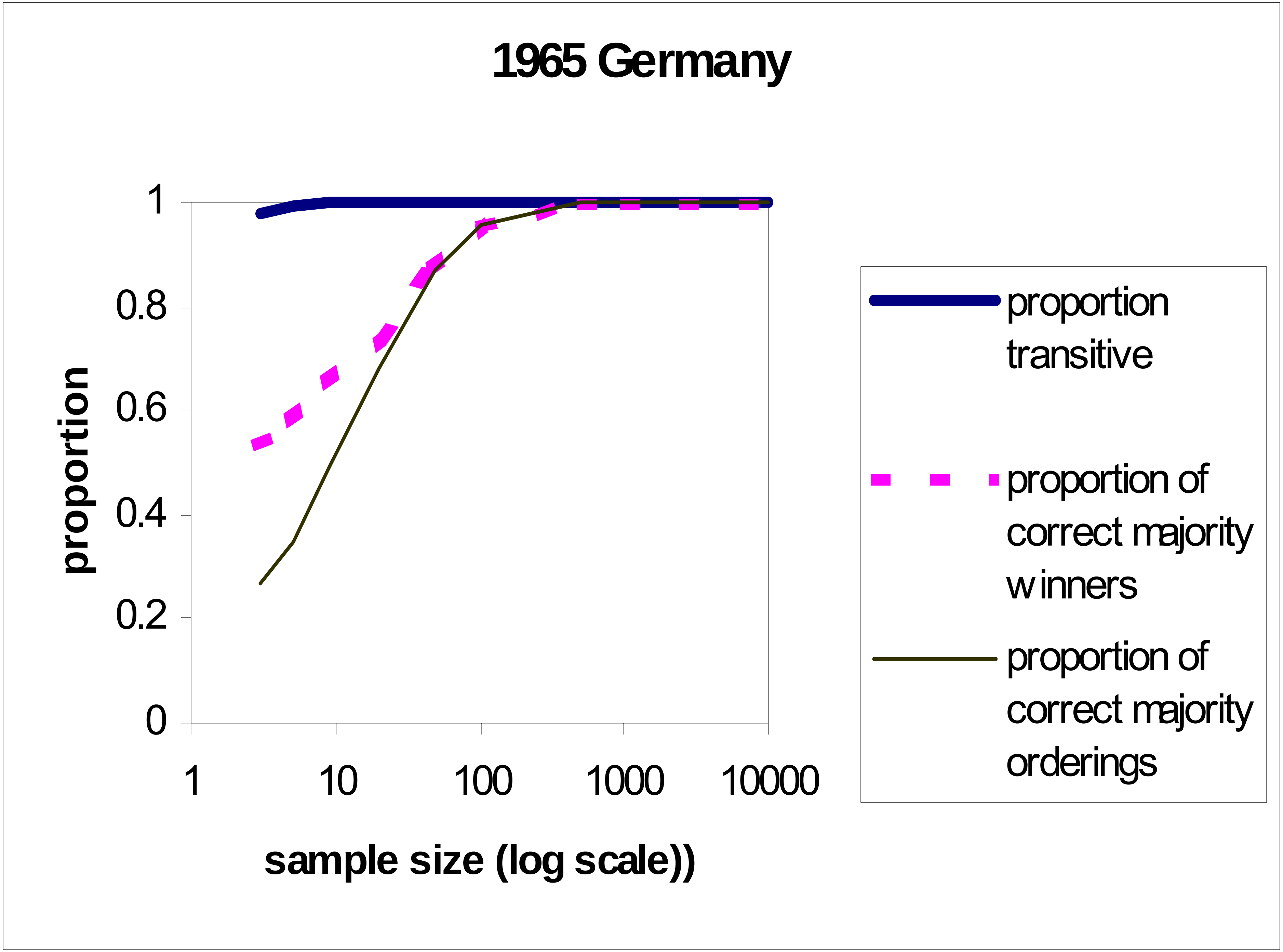
n=21
Intransitivities
1.0
Correct Majority Ordering
Correct Majority Winner


1988 France: Communists
proportion
1.00
0.80
0.60
0.40
0.20
0.00
1
10
100
1000
10000
sample size (log scale)
proportion transitive
proportion of correct majority winners
proportion of correct majority orderings

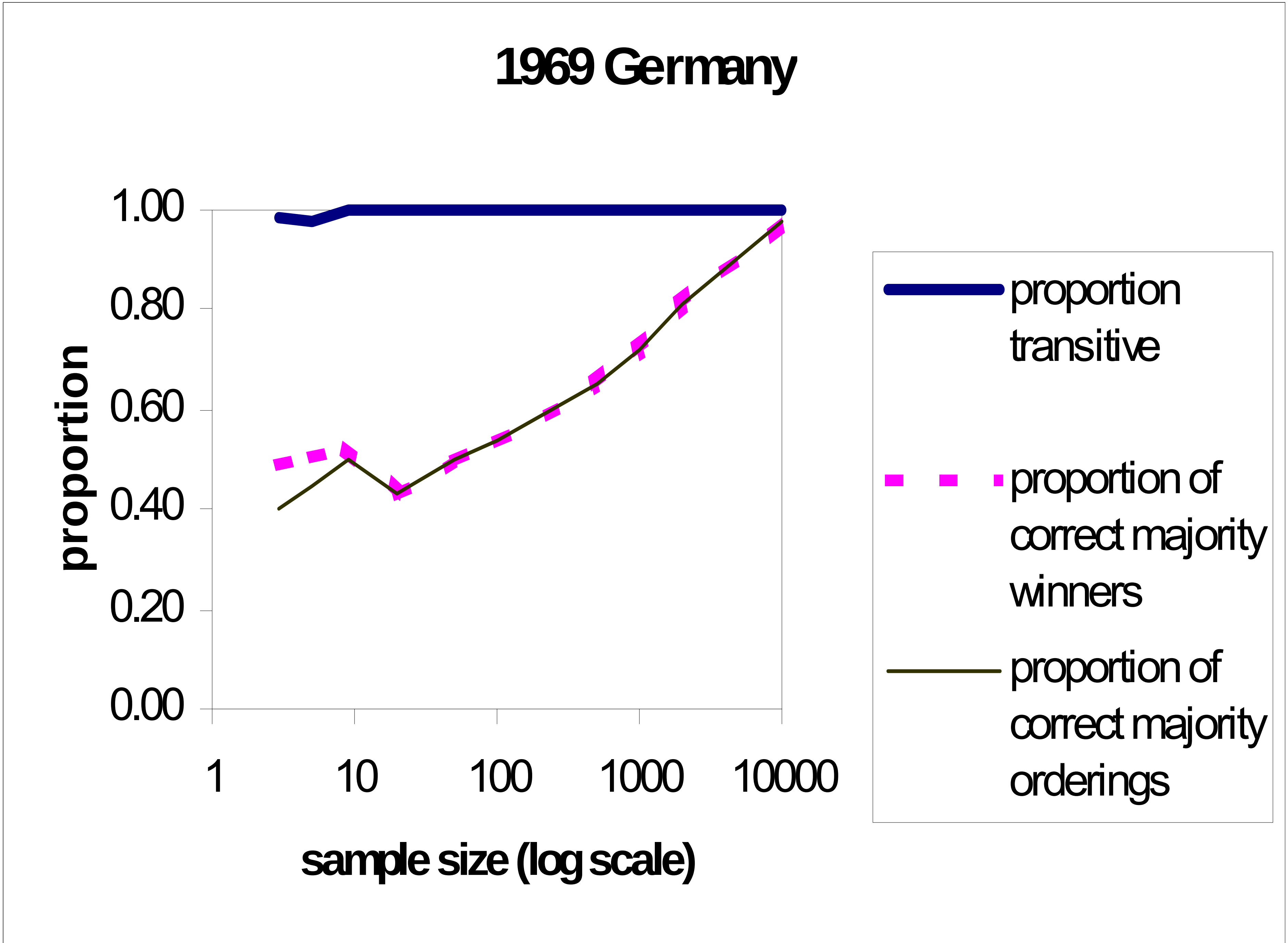


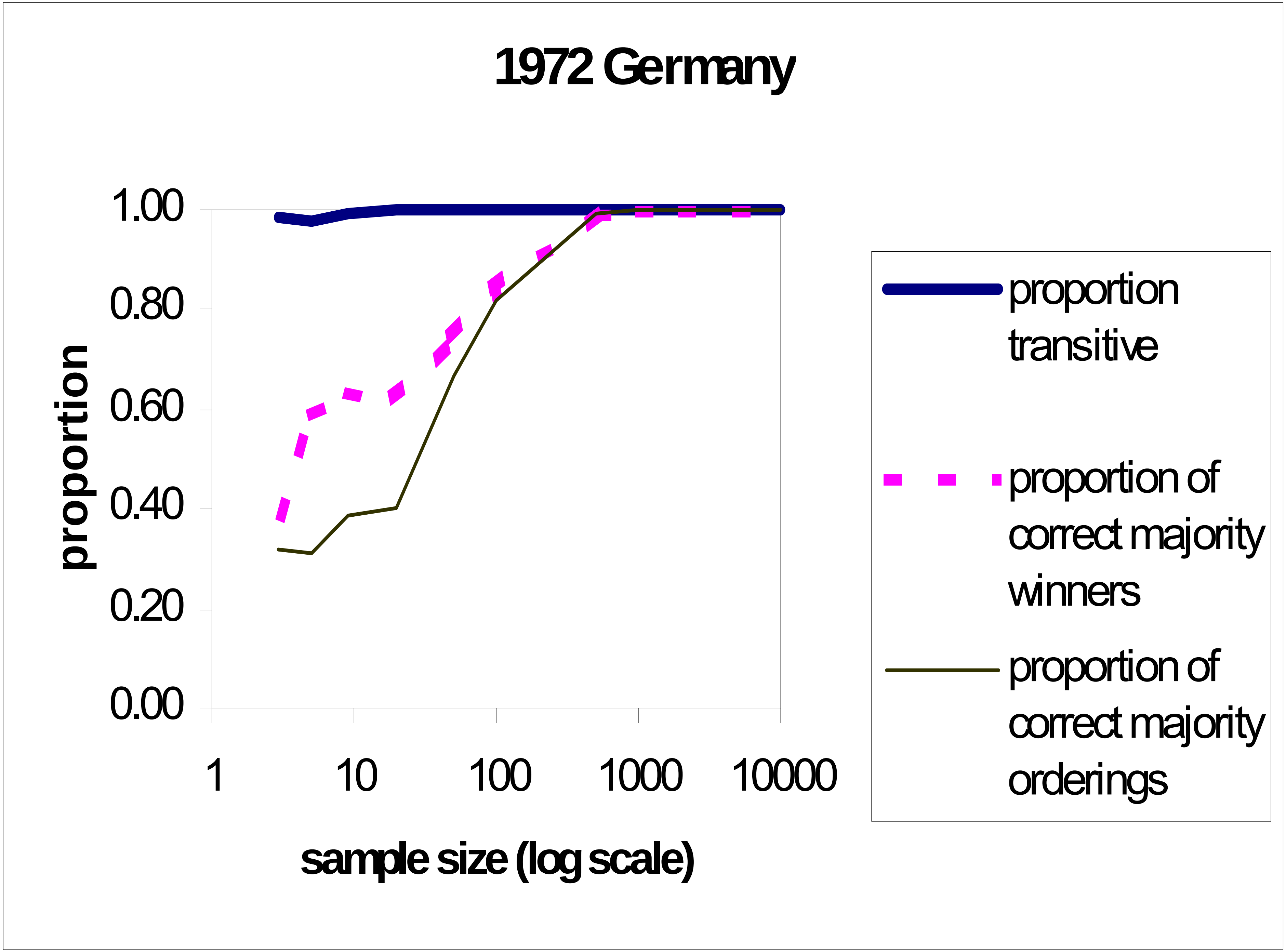
1992 ANES
proportion
1
0.8
0.6
0.4
0.2
0
1
10
100
1000
10000
sample size (log scale)
proportion transitive
proportion of correct majority winners
proportion of correct majority orderings

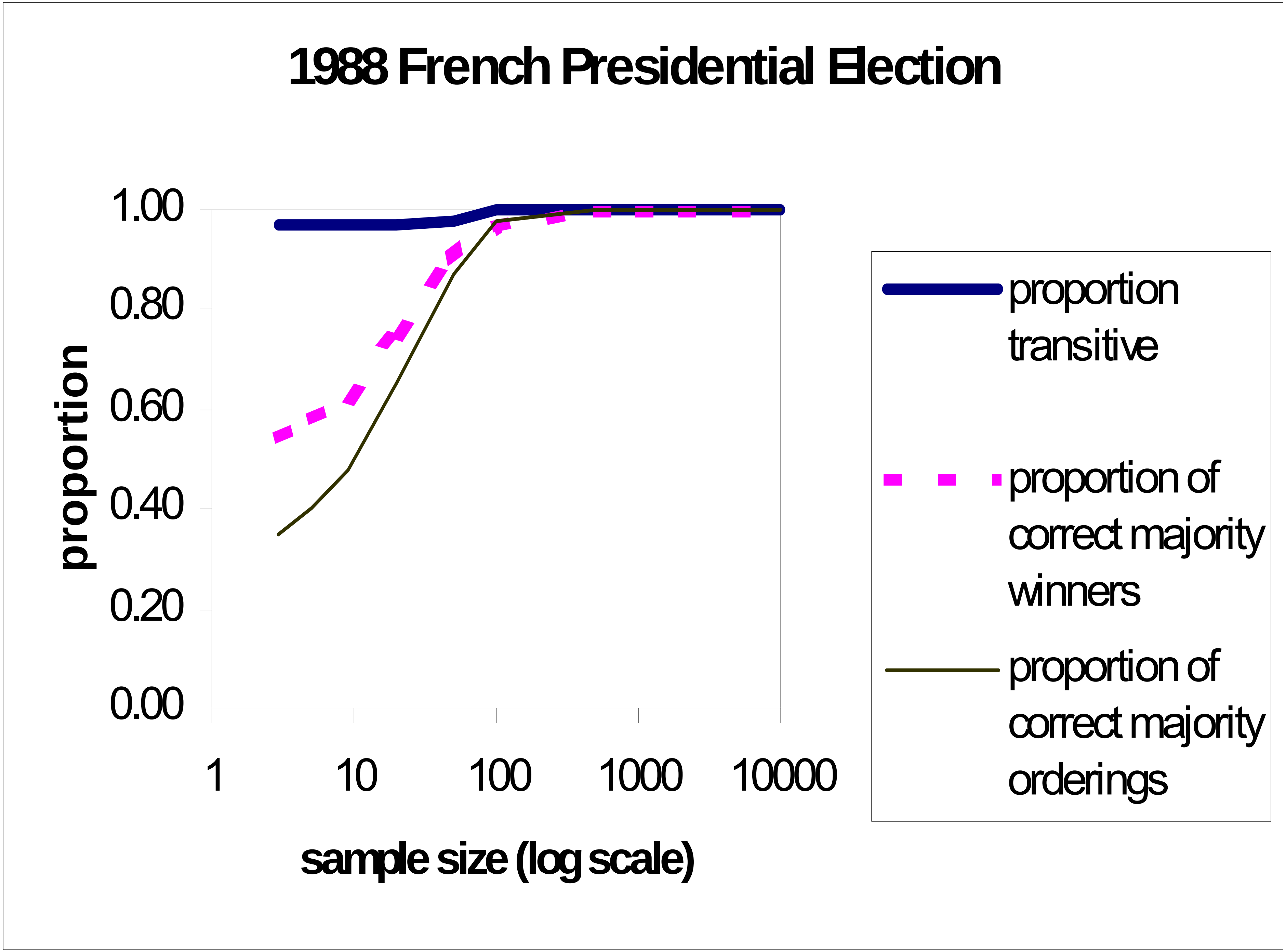

1961 Germany
1
0.8
0.6
0.4
0.2
0
proportion
1
10
100
1000
10000
sample size (on log scale)
proportion transitive
proportion of correct majority winners
proportion of correct majority orderings

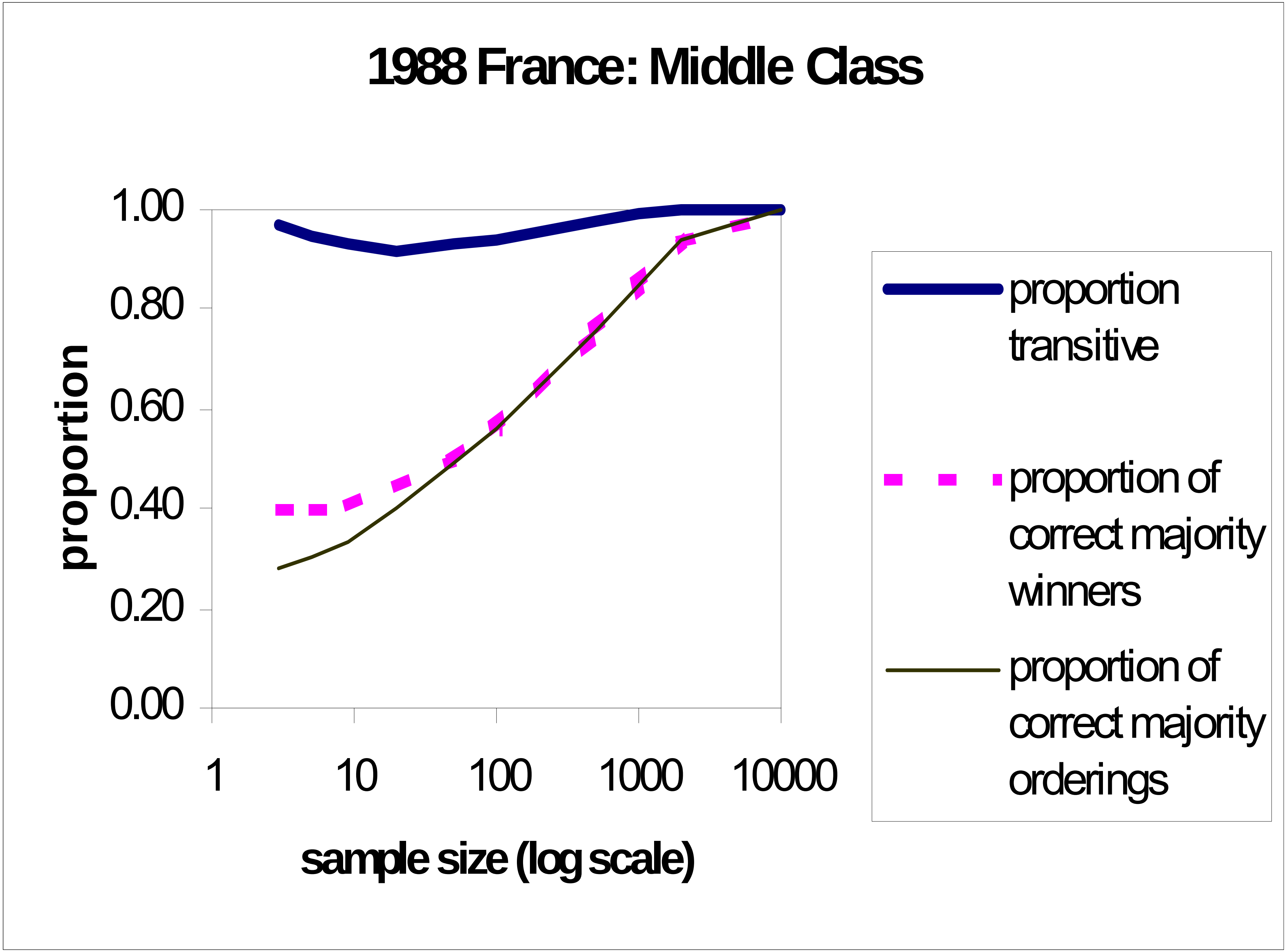


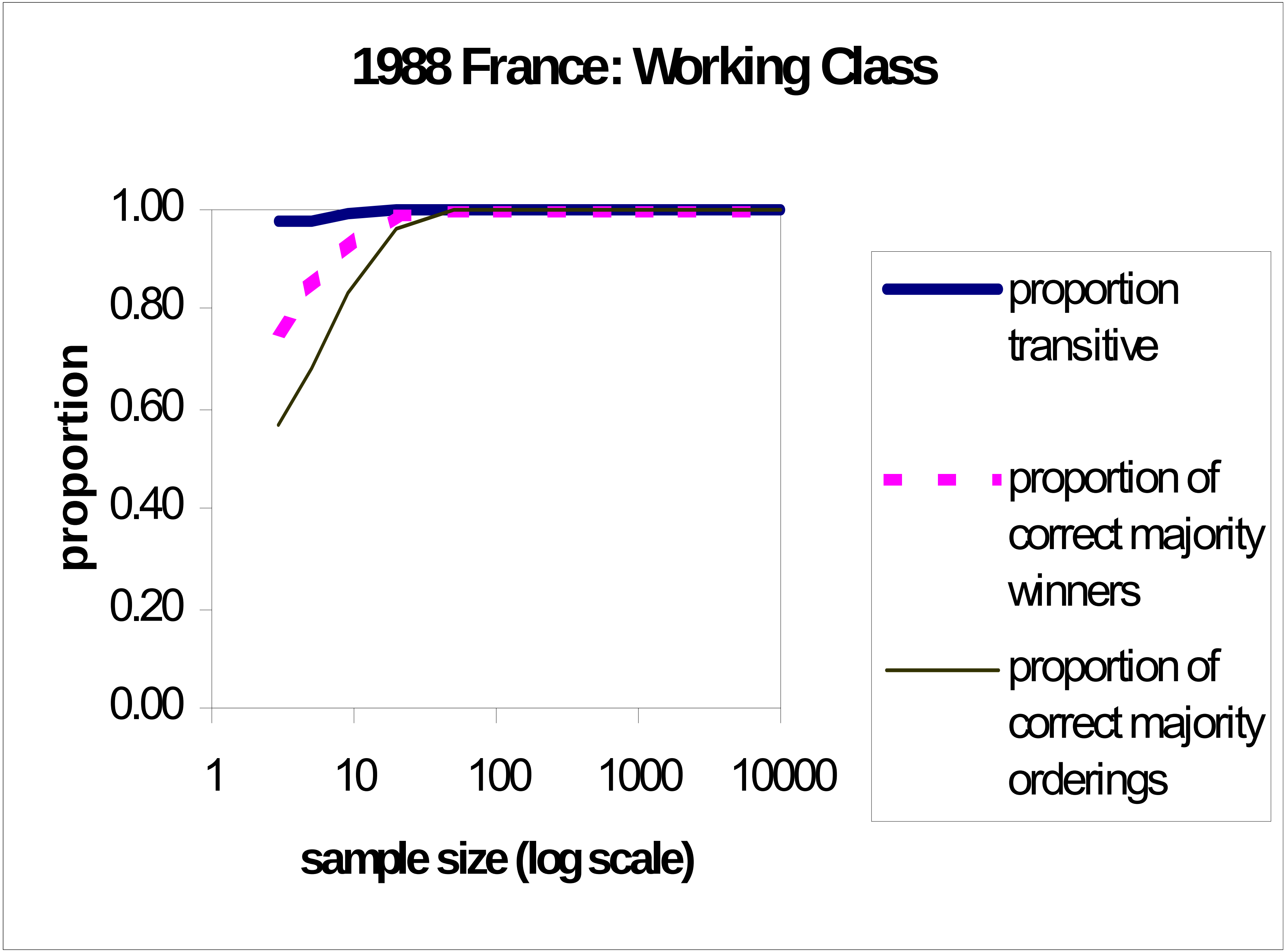
1965 Germany
proportion
1
0.8
0.6
0.4
0.2
0
1
10
100
1000
10000
sample size (log scale))
proportion transitive
proportion of correct majority winners
proportion of correct majority orderings


1969 Germany
proportion
1.00
0.80
0.60
0.40
0.20
0.00
1
10
100
1000
10000
sample size (log scale)
proportion transitive
proportion of correct majority winners
proportion of correct majority orderings

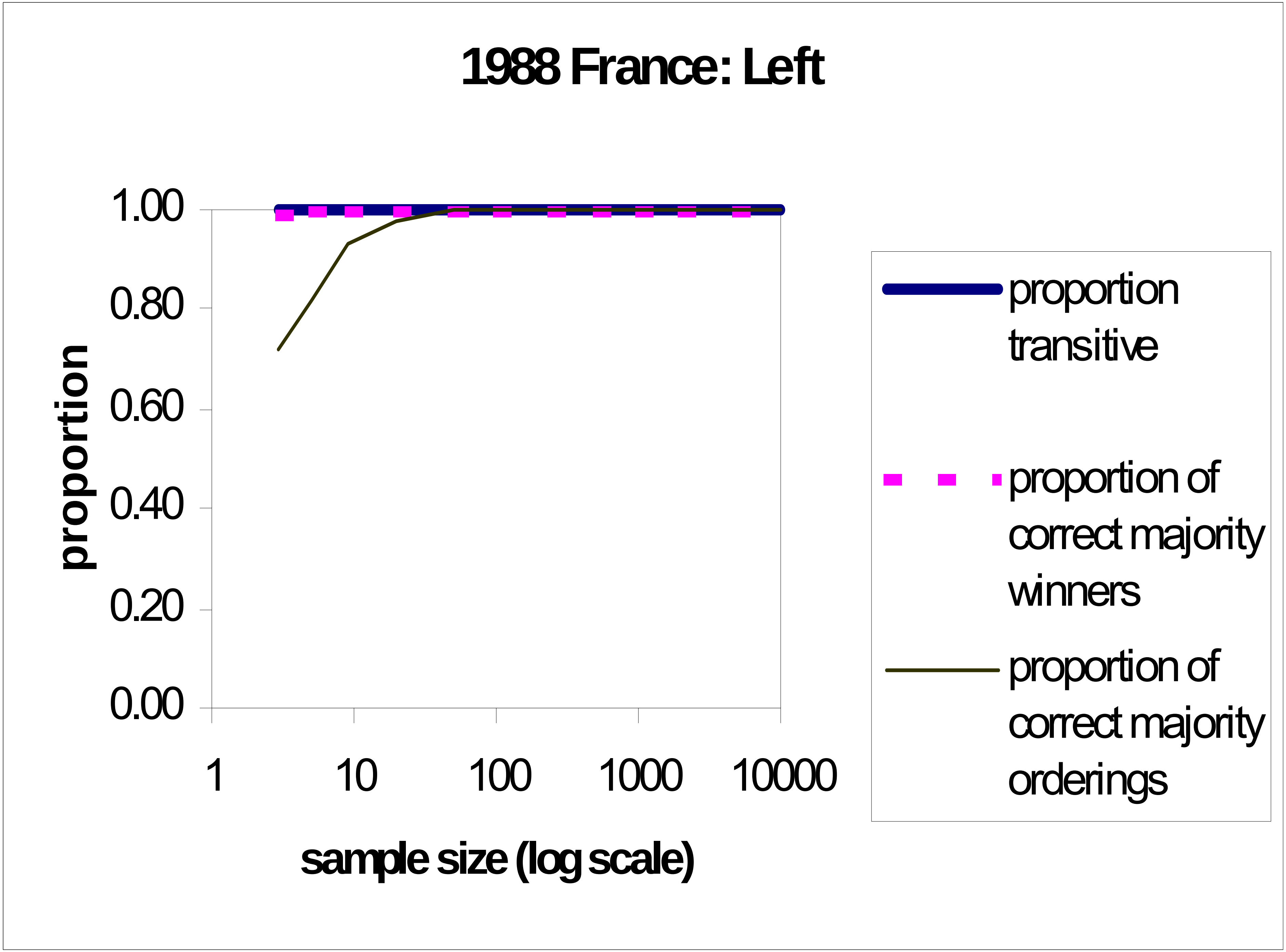

1972 Germany
1.00
0.80
0.60
0.40
0.20
0.00
proportion
1
10
100
1000
10000
sample size (log scale)
proportion transitive
proportion of correct majority winners
proportion of correct majority orderings


1988 French Presidential Election
proportion
1.00
0.80
0.60
0.40
0.20
0.00
1
10
100
1000
10000
sample size (log scale)
proportion transitive
proportion of correct majority winners
proportion of correct majority orderings

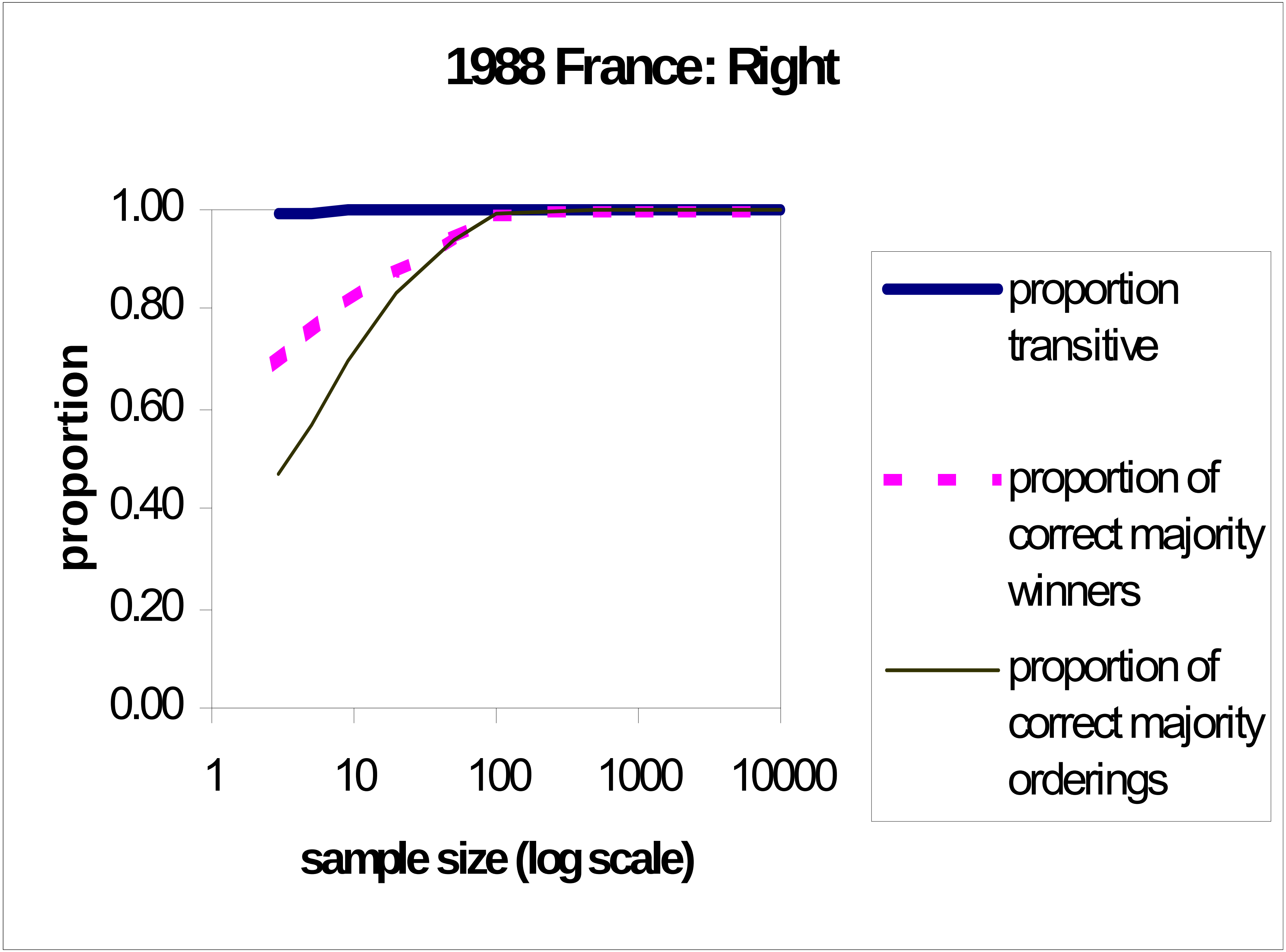

1988 France: Middle Class
proportion
1.00
0.80
0.60
0.40
0.20
0.00
1
10
100
1000
10000
sample size (log scale)
proportion transitive
proportion of correct majority winners
proportion of correct majority orderings

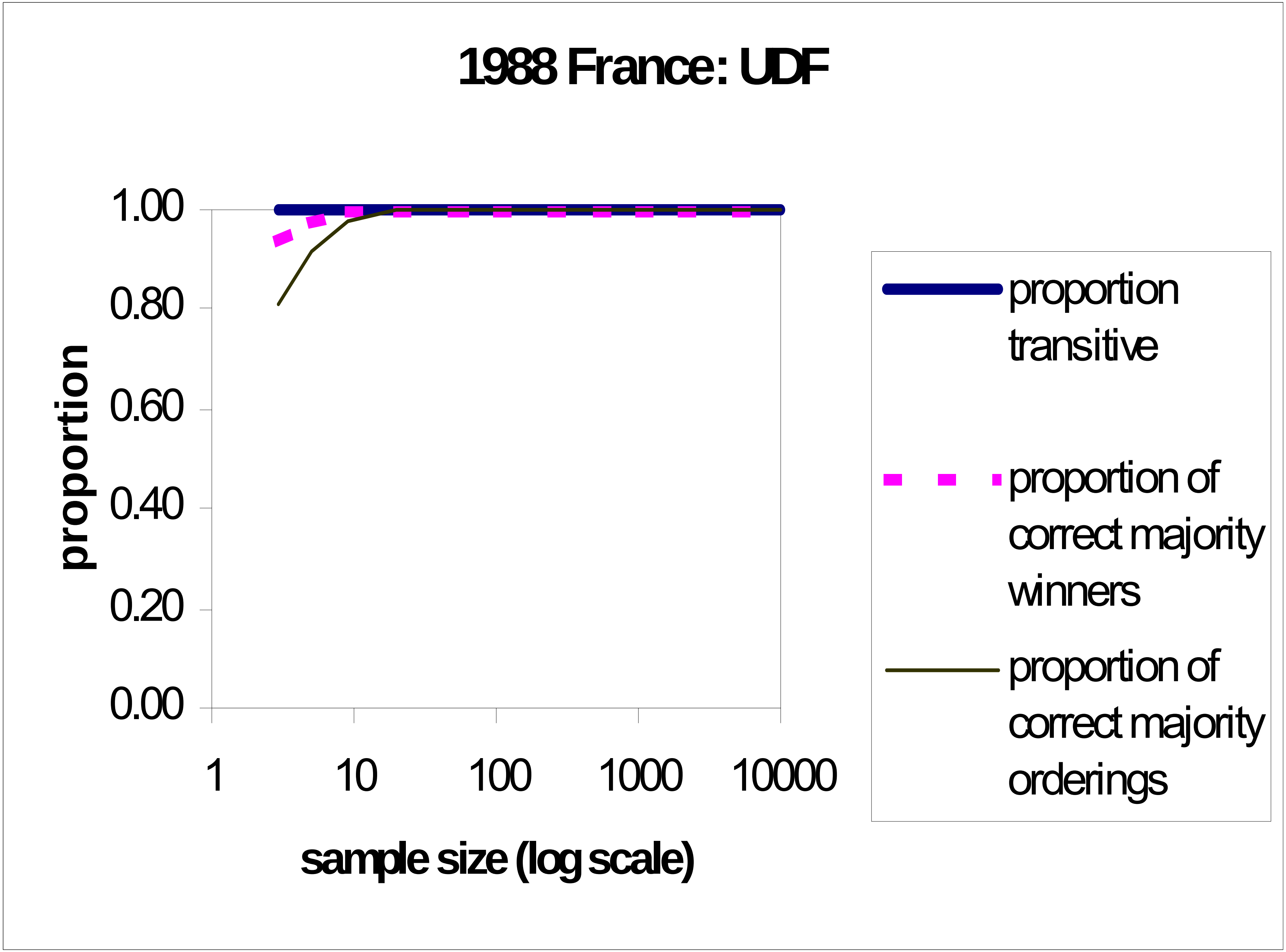


1988 France: Working Class
1.00
0.80
0.60
0.40
0.20
0.00
proportion
1
10
100
1000
10000
sample size (log scale)
proportion transitive
proportion of correct majority winners
proportion of correct majority orderings



1988 France: Left
proportion
1.00
0.80
0.60
0.40
0.20
0.00
1
10
100
1000
10000
sample size (log scale)
proportion transitive
proportion of correct majority winners
proportion of correct majority orderings


1988 France: Right
proportion
1.00
0.80
0.60
0.40
0.20
0.00
1
10
100
1000
10000
sample size (log scale)
proportion transitive
proportion of correct majority winners
proportion of correct majority orderings


1988 France: UDF
proportion
1.00
0.80
0.60
0.40
0.20
0.00
1
10
100
1000
10000
sample size (log scale)
proportion transitive
proportion of correct majority winners
proportion of correct majority orderings

Correct Majority Preference

<u>Sampling</u>

Population ⟶ Sample (Committee)

Majority Preference: *a* ♐ *b* ♐ *c*

Correct: *a* ♐ *b* ♐ *c*

Incorrect: ***any other***

<u>Inference</u>

Sample (Survey) ⟶ Population

Majority Preference: *a* ♐ *b* ♐ *c*

Correct: *a* ♐ *b* ♐ *c*

Incorrect: ***any other***

Look both at the probability of cycles and the probability of incorrect majority relations

Impartial Culture

Sampling

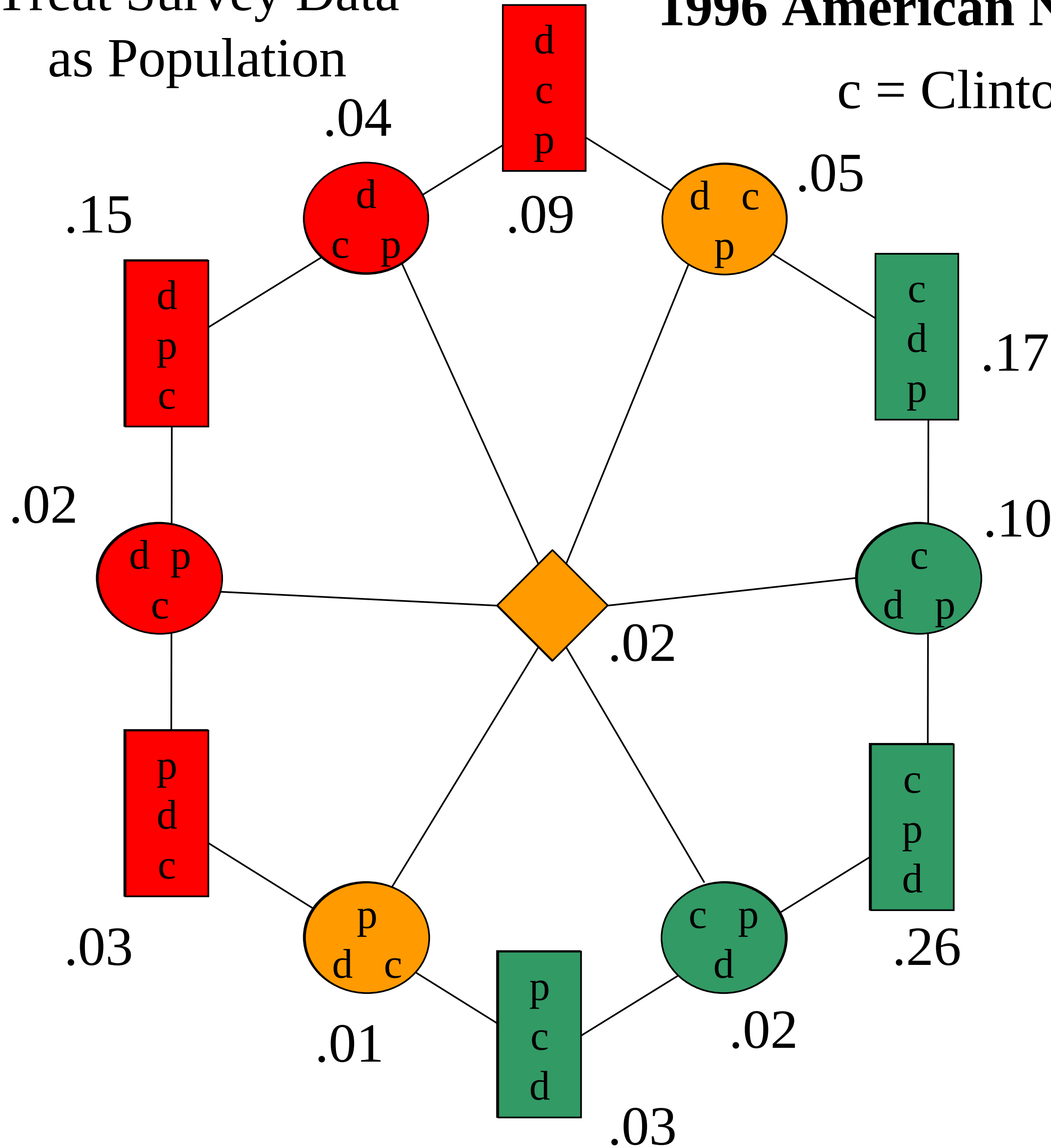
Population ⟶ Sample (Committee)

Majority Preference: $a \sim b \sim c$ Probability of cycles?

Treat Survey Data as Population

1996 American National Election Study

c = Clinton, d = Dole, p = Perot



Clinton - Dole

$p_{cBd} = 0.58 \qquad p_{dBc} = 0.33$

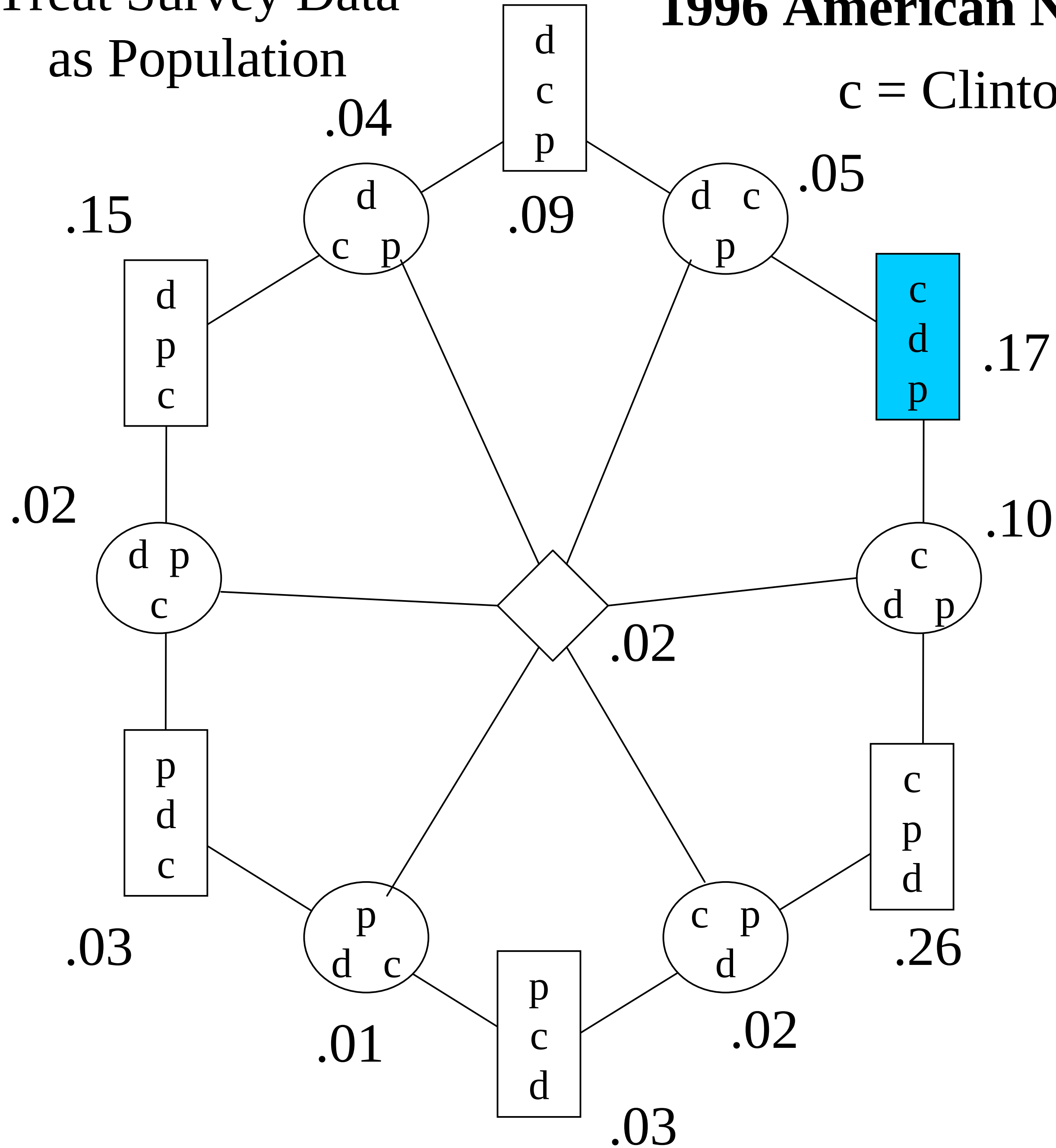
$p_{cEd} = 0.09$

$\delta_{cBd} = p_{cBd} - p_{dBc} = 0.25$

Treat Survey Data as Population

1996 American National Election Study

c = Clinton, d = Dole, p = Perot



Clinton - Dole

$p_{cBd} = 0.58 \quad p_{dBc} = 0.33$

$p_{cEd} = 0.09$

$\delta_{cBd} = 0.25$

Dole - Perot

$p_{dBp} = 0.50 \quad p_{pBd} = 0.36$

$p_{dEp} = 0.14$

$\delta_{dBp} = 0.09$

Clinton - Perot

$p_{cBp} = 0.68 \quad p_{pBc} = 0.24$

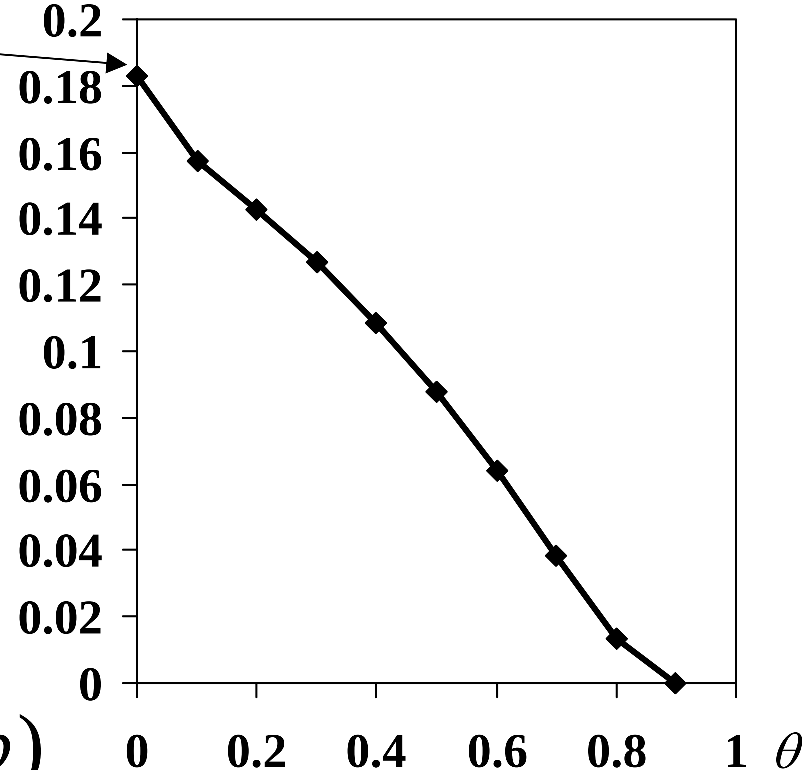
$p_{cEp} = 0.08$

$\delta_{cBp} = 0.43$

Pairwise comparison (sampling)

$$Err(N,\delta,\theta=0)=F_{Bin}\left(\left\lfloor\frac{N}{2}\right\rfloor, N, \frac{1+\delta}{2}\right)$$

$$Err(N=100,\delta=0.1,\theta)$$



$$a \nearrow_p b \Leftrightarrow p_{aBb} > p_{bBa}$$

$$\delta = p_{aBb} - p_{bBa} > 0$$

$$\theta = p_{aEb}$$

N - Sample Size

$$Err(N,\delta,\theta) = 1 - P(a \nearrow_s b)$$

Probability of incorrect majority relation between *a* and *b* in the Sample

So, for pairwise comparison (sampling):

Larger sample size (*N*)
Larger pairwise margin (δ) ← Strength of majority preferences

↓ (Properties of binomial distribution)

Smaller probability of Error (*Err(N, δ)*)
Higher confidence

Let us move from pairs of candidates to
the majority preference relation over all candidates

Upper and lower bounds on the joint event

$$P(A \cap B) \leq P(A)$$

	$\overline{A}$	A
B	$\overline{A} \cap B$	$A \cap B$
$\overline{B}$	$\overline{A} \cap \overline{B}$	$A \cap \overline{B}$

Upper and lower bounds on the joint event

$$P(A \cap B) \leq P(A)$$
$$P(A \cap B) \leq P(B)$$

$\Rightarrow$ $P(A \cap B) \leq \min\left(P(A), P(B)\right)$

	$\overline{A}$	A
B	$\overline{A} \cap B$	$A \cap B$
$\overline{B}$	$\overline{A} \cap \overline{B}$	$A \cap \overline{B}$

Upper and lower bounds on the joint event

$$P(A \cap B) \le P(A)$$
$$P(A \cap B) \le P(B)$$
$$\Rightarrow \quad P(A \cap B) \le \min\big(P(A), P(B)\big)$$

	$\overline{A}$	A
B	$\overline{A} \cap B$	$A \cap B$
$\overline{B}$	$\overline{A} \cap \overline{B}$	$A \cap \overline{B}$

$$1 - P(\overline{A}) - P(\overline{B}) \le P(A \cap B)$$

$$A = A_1 \cap A_2 \cap \ldots \cap A_K$$
$$P(\overline{A}_i) = Err_i$$
$$Err = \max_i \big(Err_i\big)$$
$$1 - K * Err \le P(A) \le 1 - Err$$

$\Downarrow$

Err is small - *P(A)* (confidence) is high,
Err is high - *P(A)* (confidence) is small

Application of bounds to the majority relations

In <u>Population</u> majority preference relation

is $a \succ b \succ c$

<u>In the Sample:</u>

1) Compute $Err(a,b); Err(b,c); Err(a,c)$

$$Err(a,b) = 1 - P(a \succ b)$$

2) Find $Err = \max(Err(a,b);\ Err(b,c);\ Err(a,c))$

3) Apply Bounds (in our case number of pairs K=3):

$$1\text{-}3Err \le P(a \succ b \succ c) \le 1\text{-}Err$$

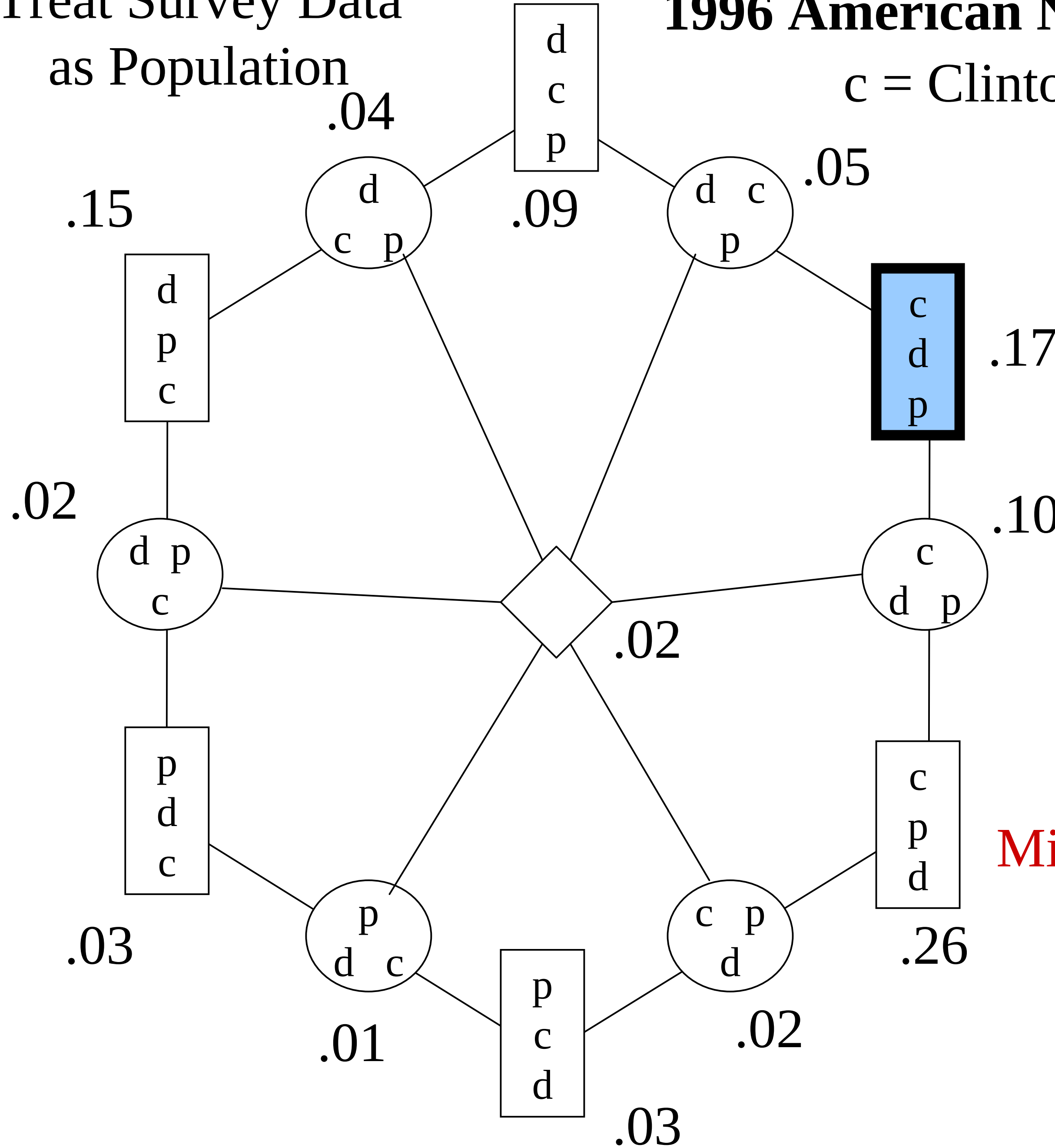
Lower Bound ↑ (left side) — Upper Bound ↑ (right side)

Let us compare bounds with the results of Monte-Carlo Simulations

Treat Survey Data as Population

1996 American National Election Study

c = Clinton, d = Dole, p = Perot



Clinton - Dole

$p_{cBd} = 0.58$ $p_{dBc} = 0.33$

$p_{cEd} = 0.09$

$\delta_{cBd} = 0.25$

Dole - Perot

$p_{dBp} = 0.50$ $p_{pBd} = 0.36$

$p_{dEp} = 0.14$

$\delta_{dBp} = 0.09$

Min

Clinton - Perot

$p_{cBp} = 0.68$ $p_{pBc} = 0.24$

$p_{cEp} = 0.08$

$\delta_{cBp} = 0.43$

Monte-Carlo simulations (Regenwetter et al. 2000) and bounds for ANES 1996 data.

		Majority Relation	$c \succ d \succ p$	$c \succ d \sim p$	$c \succ p \succ d$
			I	III	II
Sample Size	50	Monte-Carlo	0.80	0.03	0.13
	101	Monte-Carlo	0.93	0.01	0.06
	500	Monte-Carlo	1	0	0

Clinton definitely is unique majority winner; uncertainty Dole-Perot

Monte-Carlo simulations (Regenwetter et al. 2000) and bounds for ANES 1996 data.

		Majority Relation	$c \succ d \succ p$	$c \succ d \sim p$	$c \succ p \succ d$
	Formulae	Upper Bound	$P(d \succ p)$	$P(p \sim d)$	$P(p \succ d)$
	Formulae	Lower Bound	$P(d \succ p) - (1 - P(c \succ p)) - (1 - P(c \succ d))$	$P(d \sim p) - (1 - P(c \succ p)) - (1 - P(c \succ d))$	$P(p \succ d) - (1 - P(c \succ p)) - (1 - P(c \succ d))$
Sample Size	50	Monte-Carlo	0.80	0.03	0.13
Sample Size	101	Monte-Carlo	0.93	0.01	0.06
Sample Size	500	Monte-Carlo	1	0	0

Monte-Carlo simulations (Regenwetter et al. 2000) and bounds for ANES 1996 data.

		Majority Relation	$c \succ d \succ p$	$c \succ d \sim p$	$c \succ p \succ d$
	Formulae	Upper Bound	$P(d \succ p)$	$P(p \sim d)$	$P(p \succ d)$
	Formulae	Lower Bound	$P(d \succ p) - (1 - P(c \succ p)) - (1 - P(c \succ d))$	$P(d \sim p) - (1 - P(c \succ p)) - (1 - P(c \succ d))$	$P(p \succ d) - (1 - P(c \succ p)) - (1 - P(c \succ d))$
Sample Size	50	Upper Bound	0.841	0.034	0.125
		Monte-Carlo	0.80	0.03	0.13
		Lower Bound	0.807	0.000	0.091
	101	Upper Bound	0.930	0.013	0.057
		Monte-Carlo	0.93	0.01	0.06
		Lower Bound	0.926	0.009	0.053
	500	Upper Bound	1.000	6.08E-05	3.15E-04
		Monte-Carlo	1	0	0
		Lower Bound	1.000	6.08E-05	3.15E-04

Monte-Carlo simulations (Regenwetter et al. 2000) and bounds for ANES 1996 data.

		Majority Relation	$c \succ d \succ p$	$c \succ d \sim p$	$c \succ p \succ d$
	Formulae	Upper Bound	$P(d \succ p)$	$P(p \sim d)$	$P(p \succ d)$
		Lower Bound	$P(d \succ p) - (1 - P(c \succ p)) - (1 - P(c \succ d))$	$P(d \sim p) - (1 - P(c \succ p)) - (1 - P(c \succ d))$	$P(p \succ d) - (1 - P(c \succ p)) - (1 - P(c \succ d))$
Sample Size	50	Upper Bound	0.841	0.034	0.125
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		Lower Bound	0.926	0.009	0.053
	500	Upper Bound	1.000	6.08E-05	3.15E-04
		Monte-Carlo	1	0	0
		Lower Bound	1.000	6.08E-05	3.15E-04

Monte-Carlo simulations (Regenwetter et al. 2000) and bounds for ANES 1996 data.

		Majority Relation	$c \succ d \succ p$	$c \succ d \sim p$	$c \succ p \succ d$
	Formulae	Upper Bound	$P(d \succ p)$	$P(p \sim d)$	$P(p \succ d)$
	Formulae	Lower Bound	$P(d \succ p) - (1 - P(c \succ p)) - (1 - P(c \succ d))$	$P(d \sim p) - (1 - P(c \succ p)) - (1 - P(c \succ d))$	$P(p \succ d) - (1 - P(c \succ p)) - (1 - P(c \succ d))$
Sample Size	50	Upper Bound	0.841	0.034	0.125
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Sample Size	101	Lower Bound	0.926	0.009	0.053
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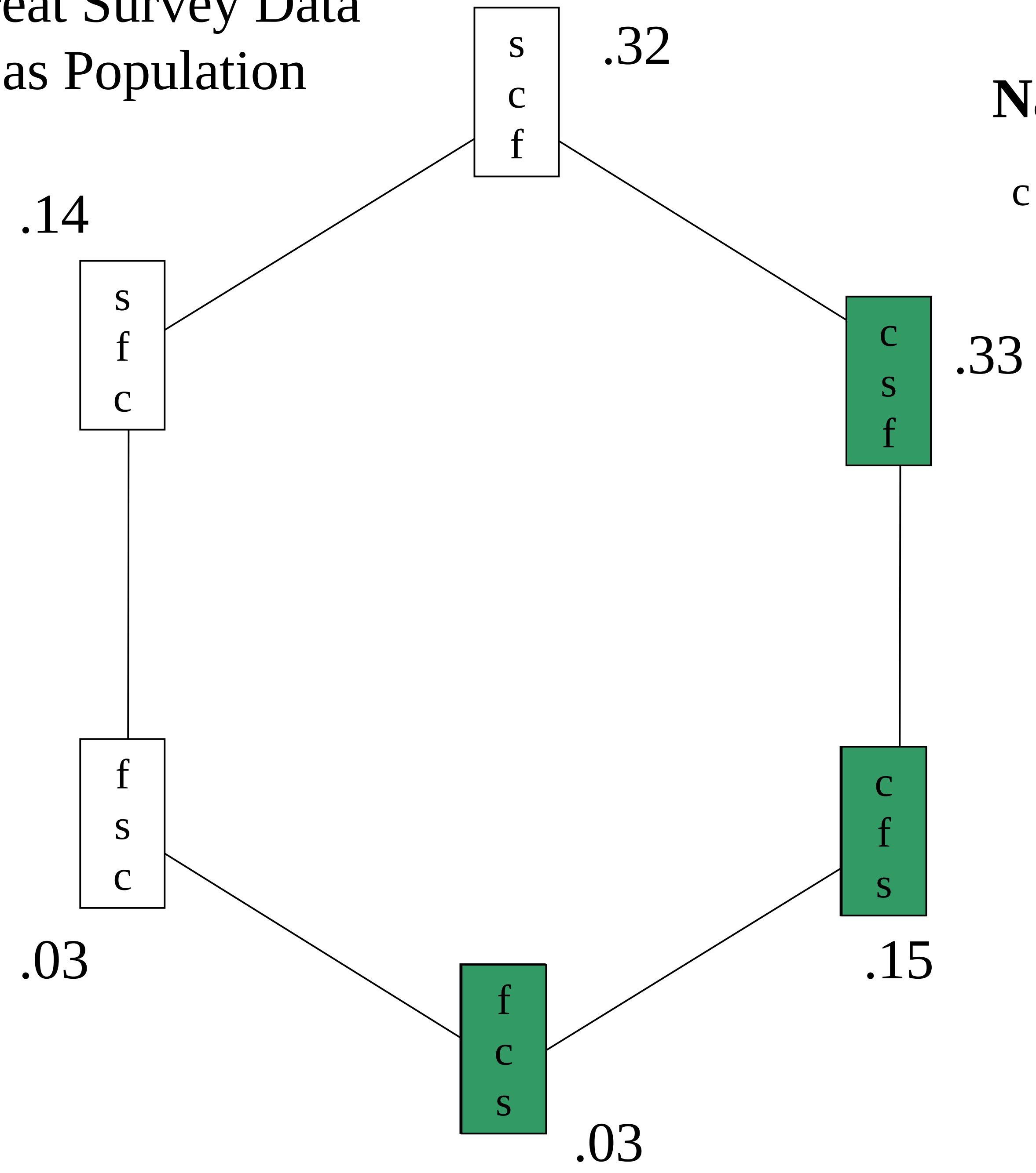
Sample Size	500	Monte-Carlo	1	0	0
Sample Size	500	Lower Bound	1.000	6.08E-05	3.15E-04

One more example, compare with Impartial Culture

Treat Survey Data as Population

1969 German National Election Study

c = CDU/CSU, s = SDP, f = FDP



CDU/CSU - SDP

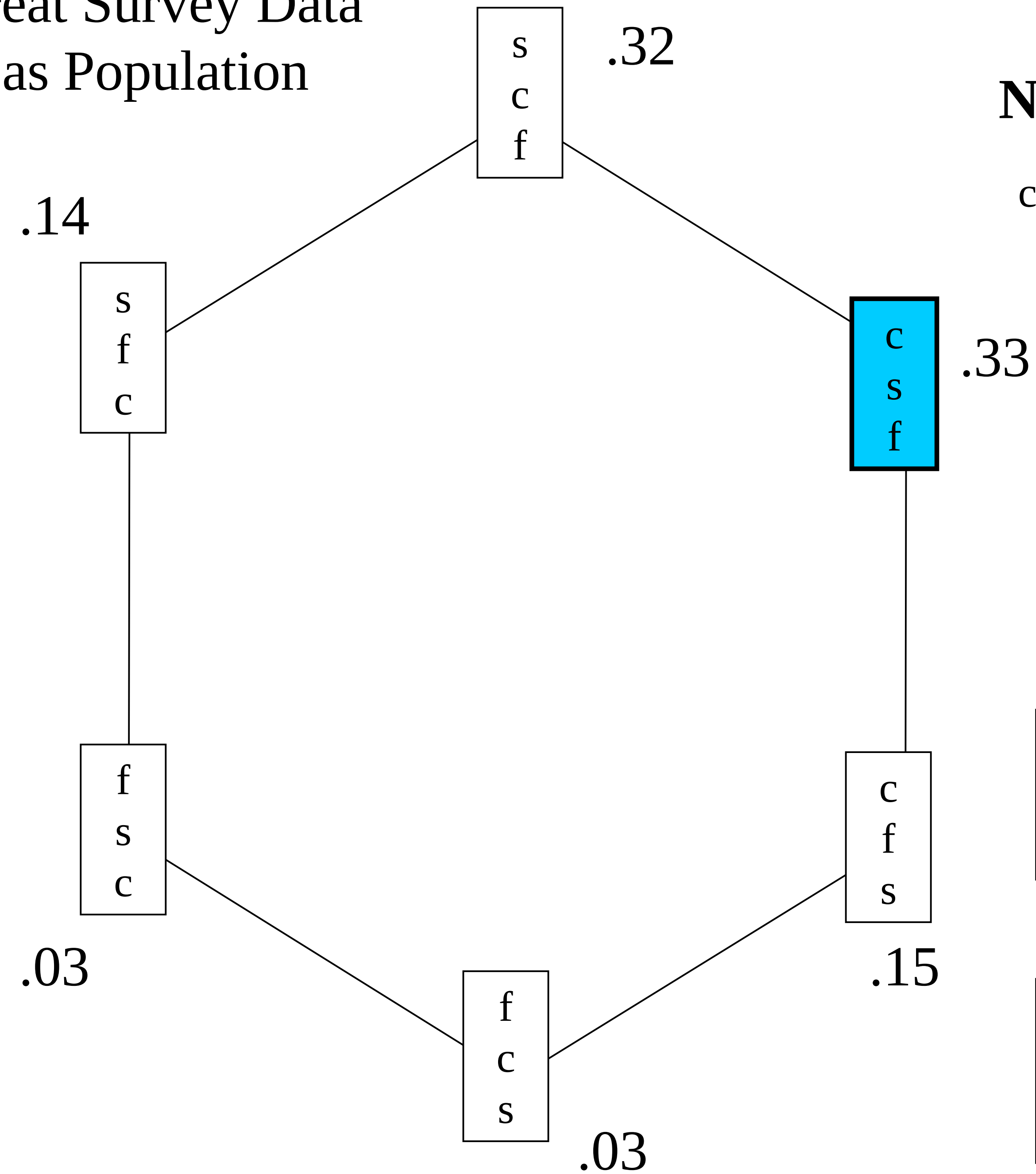
$$p_{cBs} = 0.51$$

$$\delta_{cBs} = 0.02$$

Treat Survey Data as Population

1969 German National Election Study

c = CDU/CSU, s = SDP, f = FDP



CDU/CSU - SDP

Min

$$p_{cBs} = 0.51$$
$$\delta_{cBs} = 0.02$$

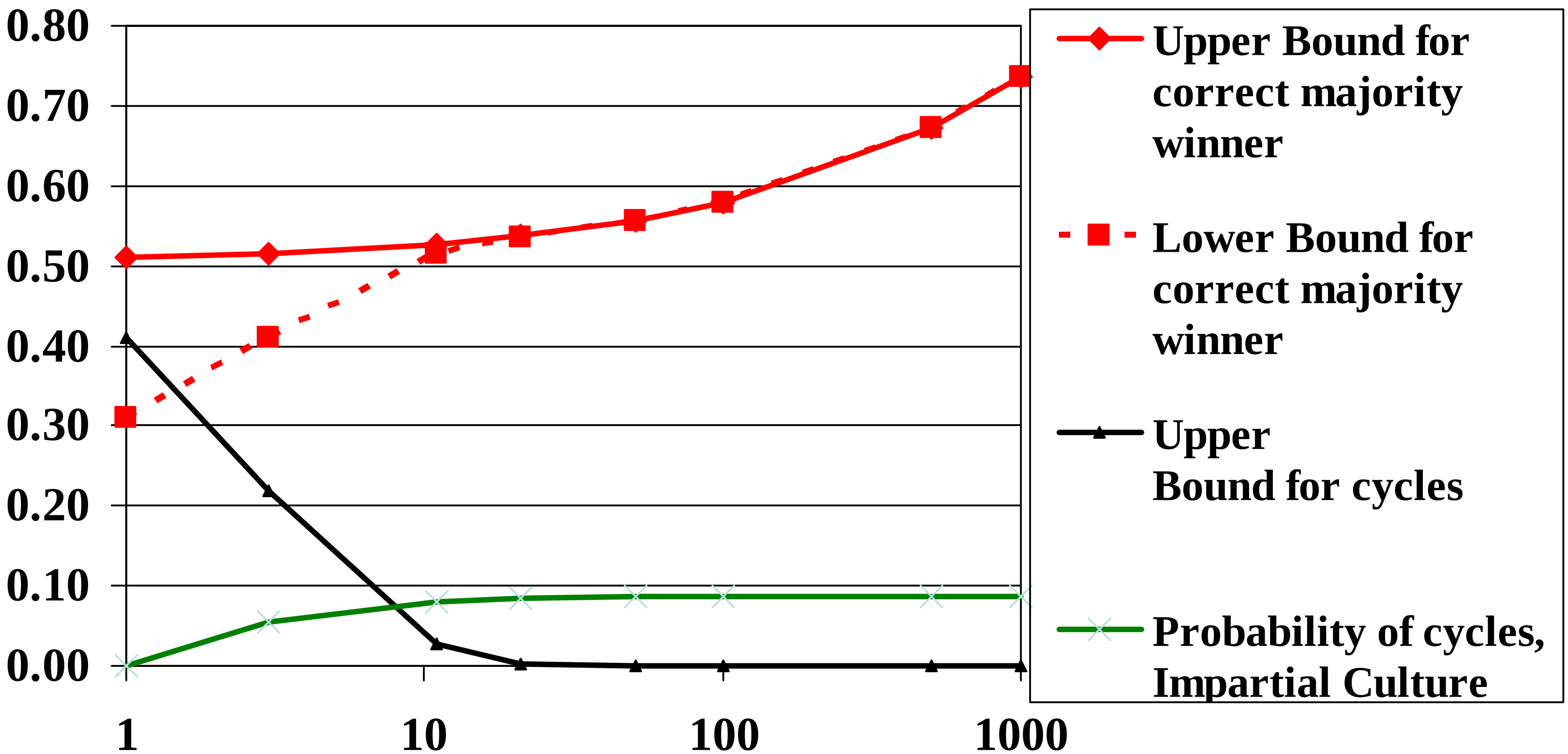
SDP - FDP

$$p_{sBf} = 0.79$$
$$\delta_{sBf} = 0.58$$

CDU/CSU - FDP

$$p_{cBf} = 0.80$$
$$\delta_{cBf} = 0.60$$

Probabilities of majority preference relations for GNES 1969 data and impartial culture (odd sample sizes)



Huge potential for incorrect majority relation

Conclusions from Sampling

- Whenever the population has an **asymmetric majority preference relation** (i.e. all pairwise margins are nonzero) we **recover it in the sample with probability close to 1** for large sample size

Population → Sample (Committee), $N \rightarrow \infty$

Linear Order → Linear Order

a ♐ *b* ♐ *c* → *a* ♐ *b* ♐ *c*

Cycle → Cycle

a ♐ *b*
b ♐ *c*
c ♐ *a*

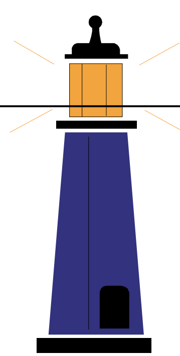
→

a ♐ *b*
b ♐ *c*
c ♐ *a*

Conclusions from Sampling

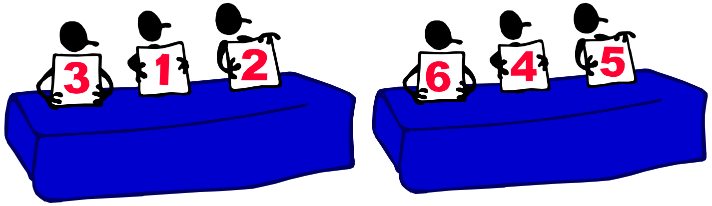
- Whenever the population has an **asymmetric majority preference relation** (i.e. all pairwise margins are nonzero) we **recover it in the sample with probability close to 1** for large sample size
- In particular, if **majority preference relation in the population is linear order**, **probability of cycles in the sample approaches zero** for large samples
- If property of **Moderate Stochastic Transitivity with Strict Inequalities** holds in the population, **the second most probable majority preference relation in the sample is a linear order** (of course, incorrect one).

Now let us move to the Inference Framework

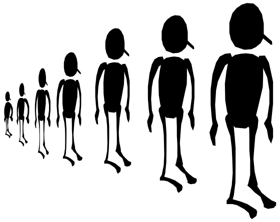


Inference of pairwise majority preference relation

<u>Sample (Survey/Committee)</u>



<u>Population</u>



$$\left.\begin{matrix} N_{aBb} \\ N_{bBa} \end{matrix}\right\} D$$

$$p_{aBb} \mid D? \qquad p_{bBa} \mid D?$$

$$N_{aBb} > N_{bBa} \Leftrightarrow a \succ_s b$$

$$\boxed{P\big((a \succ_p b) \mid D\big)?}$$

$$P\big((a \succ_p b) \mid D\big) = P\big((p_{aBb} > p_{bBa}) \mid D\big)$$

Apply Bayesian Inference

Bayesian Inference

<u>Sample (Survey/Committee)</u>

$$\left.\begin{matrix} N_{aBb} \\ N_{bBa} \end{matrix}\right\} D$$

<u>Population</u>

$$P\left((a \nearrow_p b) \mid D\right)?$$

<u>Beta-distribution:</u>

$$P\left((a \nearrow_p b) \mid D\right) = F_\beta\left(\frac{1}{2}, N_{bBa} + \alpha_{bBa}, N_{aBb} + \alpha_{aBb}\right)$$

$\alpha_{aBb}, \alpha_{bBa}$ - Prior parameters (prior Information)

No prior Information: $\alpha_{aBb} = 1, \alpha_{bBa} = 1.$

Paired Comparison + Method of Bounds
= Analysis of Survey Data

Treat survey data as a sample

1988 FNES, 961 respondents

m=Mitterand, b=Barre, c=Chirac, l=Lajoinie, p=Le Pen

x,y	$x=m, y=b$	$x=m, y=c$	$x=m, y=l$	$x=m$ $y=p$	$x=b$ $y=c$	$x=b, y=l$	$x=b$ $y=p$	$x=c$ $y=l$	$x=c$ $y=p$	$x=l$ $y=p$
N_{xBy}	538	546	786	734	442	648	764	577	720	483

538 respondents prefer Mitterand to Barre

Treat survey data as a sample

1988 FNES, 961 respondents

m=Mitterand, b=Barre, c=Chirac, l=Lajoinie, p=Le Pen

x,y	$x=m, y=b$	$x=m, y=c$	$x=m, y=l$	$x=m\ y=p$	$x=b\ y=c$	$x=b, y=l$	$x=b\ y=p$	$x=c\ y=l$	$x=c\ y=p$	$x=l\ y=p$
N_{xBy}	538	546	786	734	442	648	764	577	720	483
N_{yBx}	328	318	55	153	246	173	104	248	103	271

328 respondents prefer Barre to Mitterand

538>328, so Mitterand is preferred to Barre
by majority in the survey

Treat survey data as a sample

1988 FNES, 961 respondents

m=Mitterand, b=Barre, c=Chirac, l=Lajoinie, p=Le Pen

x,y	$x=m, y=b$	$x=m, y=c$	$x=m, y=l$	$x=m$ $y=p$	$x=b$ $y=c$	$x=b, y=l$	$x=b$ $y=p$	$x=c$ $y=l$	$x=c$ $y=p$	$x=l$ $y=p$
N_{xBy}	538	546	786	734	442	648	764	577	720	483
N_{yBx}	328	318	55	153	246	173	104	248	103	271
Probability of incorrect inference	3.8E -13	3.2E -15	3.7E -167	3.5E -92	2.8E -14	8.2E -66	2.1E -125	1.9E -31	3.0E -115	4.0E -15

Maximal probability of Error. Confidence is high.

1988 FNES

m=Mitterand, b=Barre, c=Chirac, l=Lajoinie, p=Le Pen

Most Probable Majority Preference Relation

Ranking	$m \succ b \succ c \succ l \succ p$	$b \succ m \succ c \succ l \succ p$	Any other
Upper Bound	1.0 - 3.8E-13	3.8E-13	2.8E-14
Lower Bound	1.0 - 4.2E-13	3.5E-13	

1988 FNES

m=Mitterand, b=Barre, c=Chirac, l=Lajoinie, p=Le Pen

Most Probable Majority Preference Relation → $m \succ b \succ c \succ l \succ p$

Second Most Probable Majority Preference Relation → $b \succ m \succ c \succ l \succ p$

Ranking	$m \succ b \succ c \succ l \succ p$	$b \succ m \succ c \succ l \succ p$	Any other
Upper Bound	1.0 - 3.8E-13	3.8E-13	2.8E-14
Lower Bound	1.0 - 4.2E-13	3.5E-13	

Bounds allow precise mapping of all majority relations in the sample

Key Questions:

* most probable majority relation	**Correct**
* probability of correct majority relation	**Close to 1**
* second most probable majority relation	**MSTwSI**
* probability of cycles	**Close to 0**

The only case when majority preference relations in the population and in the sample do not coincide with probability close to 1 for large samples is if some alternatives are majority tied.
(e.g. Impartial Culture)

- We have developed an approach for assessment of probabilities of possible majority preference relations both in sampling and inference frameworks.
- We have shown that the only case when majority preference relations in the population and in the sample do not coincide with probability close to one for large samples is if some alternatives are majority tied.
- We have demonstrated that cycles are second-order problem compared to the problem of finding correct majority preference relation.
- We have proven that if the property of Moderate Stochastic Transitivity with Strict Inequalities holds, then second most probable majority relation in the sample is transitive.

For Sampling…

Theorem (3 candidates)

Conjecture(> 3 candidates):

Impartial Culture maximizes the probability of majority cycles among Cultures of Indifference

$$(p_{aBb} = p_{bBc} = p_{aBc} = 1/2)$$

Sampling/Inference Framework

- Majority Rule
- All Positional Voting Methods (Scoring Rules), including Plurality and Borda
- Approval Voting

Inference: Social Welfare Orders

SSCW	{a, b}		{b, c}		{a, c}		{a, b, c}	
Voting Method	Preference	Confidence	Preference	Confidence	Preference	Confidence	Preference	Confidence
AV	a>b	98.53%	c>b	96.46%	a>c	62.24%	a>c>b	57.23%
Plur	a>b	99.55%	c>b	96.93%	a>c	77.43%	a>c>b	73.91%
AntiPlur	a>b	86.00%	c>b	98.56%	c>a	86.75%	c>a>b	71.31%
Maj	a>b	99.37%	c>b	95.08%	c>a	70.84%	c>a>b	65.29%
Borda	a>b	97.70%	c>b	99.29%	c>a	53.62%	c>a>b	50.61%

Inference: Social Welfare Orders from Approval Voting Data via SIM

TIMS C	{a, b}		{b, c}		{a, c}		{a, b, c}	
Voting Method	Pref	Conf	Pref	Conf	Pref	Conf	Pref	Conf
AV	b>a	100%	c>b	97.37%	c>a	100.00%	c>b>a	97.37%
Plurality	b>a	98.36%	c>b	79.21%	c>a	99.83%	c>b>a	77.40%
Anti-plurality	b>a	100%	c>b	98.43%	c>a	100.00%	c>b>a	98.43%
Borda	b>a	100%	c>b	98.02%	c>a	100.00%	c>b>a	98.02%

Inference: Social Welfare Orders from Approval Voting Data via SIM

SJDM	{a, b}		{b, c}		{a, c}		{a, b, c}	
Voting Method	Pref	Conf	Pref	Conf	Pref	Conf	Pref	Conf
AV	b>a	60.61%	b>c	98.62%	a>c	97.34%	b>a>c	56.58%
Plurality	b>a	61.35%	b>c	99.19%	a>c	98.50%	b>a>c	59.04%
Anti-plurality	a~b	50.00%	c>b	63.01%	c>a	63.01%	c>a~b	0.00%
Borda	b>a	55.37%	b>c	78.18%	a>c	73.89%	b>a>c	7.44%

Inference: Social Welfare Orders from Approval Voting Data via SIM

MAA	{a, b}		{b, c}		{a, c}		{a, b, c}	
Voting Method	Pref	Conf	Pref	Conf	Pref	Conf	Pref	Conf
AV	b>a	100%	b>c	100%	c>a	100%	b>c>a	100%
Plurality	b>a	100%	b>c	100%	c>a	100%	b>c>a	100%
Anti-plurality	b>a	100%	c>b	99.37%	c>a	100%	c>b>a	99.37%
Borda	b>a	100%	b>c	100%	c>a	100%	b>c>a	100%

Today:

- Statistical Sampling and Inference
- Why no Cycles? (General Value Restriction)
- Behavioral Social Choice Analysis of STV

General Concept of Majority Rule, Lack of Empirical Evidence for Cycles

Last Time: Defined Majority Rule for

- Random/Deterministic Utility Models
- Probability/Frequency Distributions over Binary Preference Relations

No Majority Cycles in

- 1969, 1972, 1976 GNES
- 1968, 1980, 1992, 1996 ANES
- 1988 FNES
- 7 Approval Voting elections (model based)

Model Dependence of Majority Rule Outcomes

A “preferred” to B
iff
Score A > Score B + Threshold

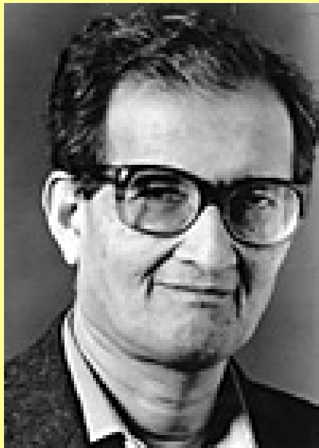
ANES	Threshold	SWO
1968	0, …, 96	Nixon Humphrey Wallace
ANES	**Threshold**	**SWO**
1992	0, …, 99	Clinton Bush Perot

ANES	Threshold	SWO
1980	0, …, 29	Carter Reagan Anderson
	30, …, 99	Reagan Carter Anderson
ANES	**Threshold**	**SWO**
1996	0, …, 49 85, …, 99	Clinton Dole Perot
	50, …,84	Dole Clinton Perot

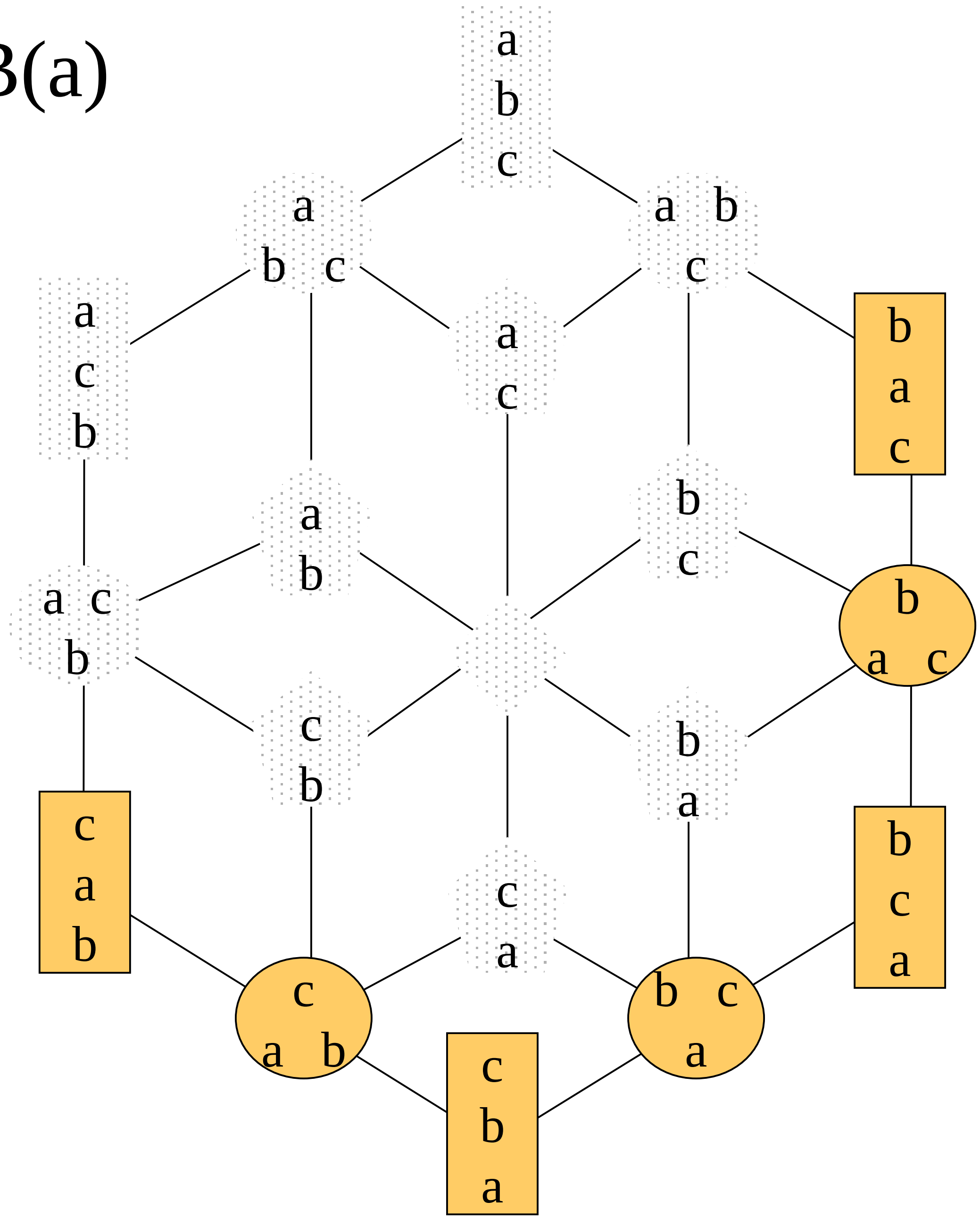
Let's forget about sampling… Instead…

A way out of Arrow's Impossibility:
Domain Restriction Conditions to eliminate Cycles

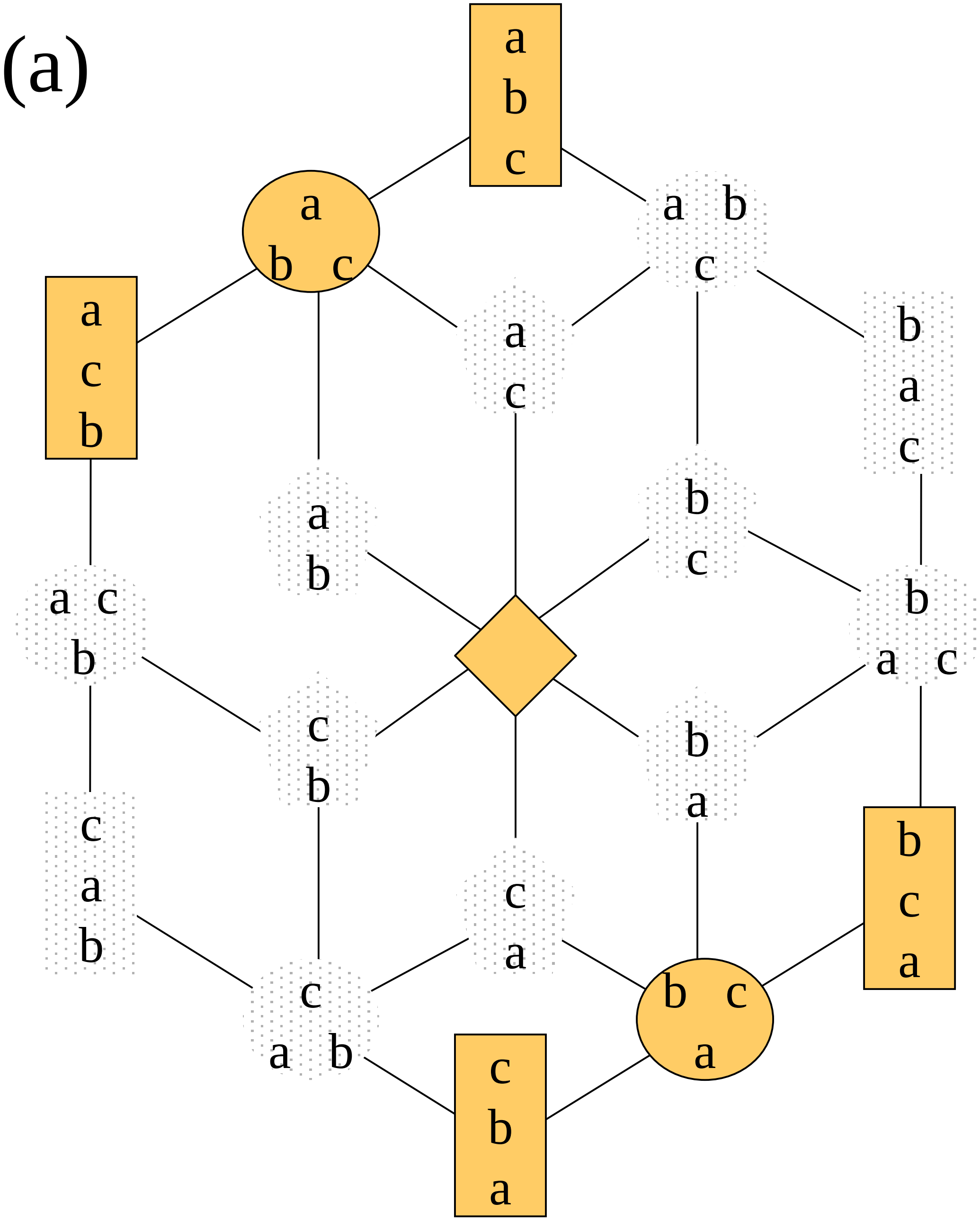
- Black's (1958) "single-peakedness"
- Sen's (1966, 1970) "value restriction"
  Never best, Never Middle, Never Worst



Sen's NB(a)

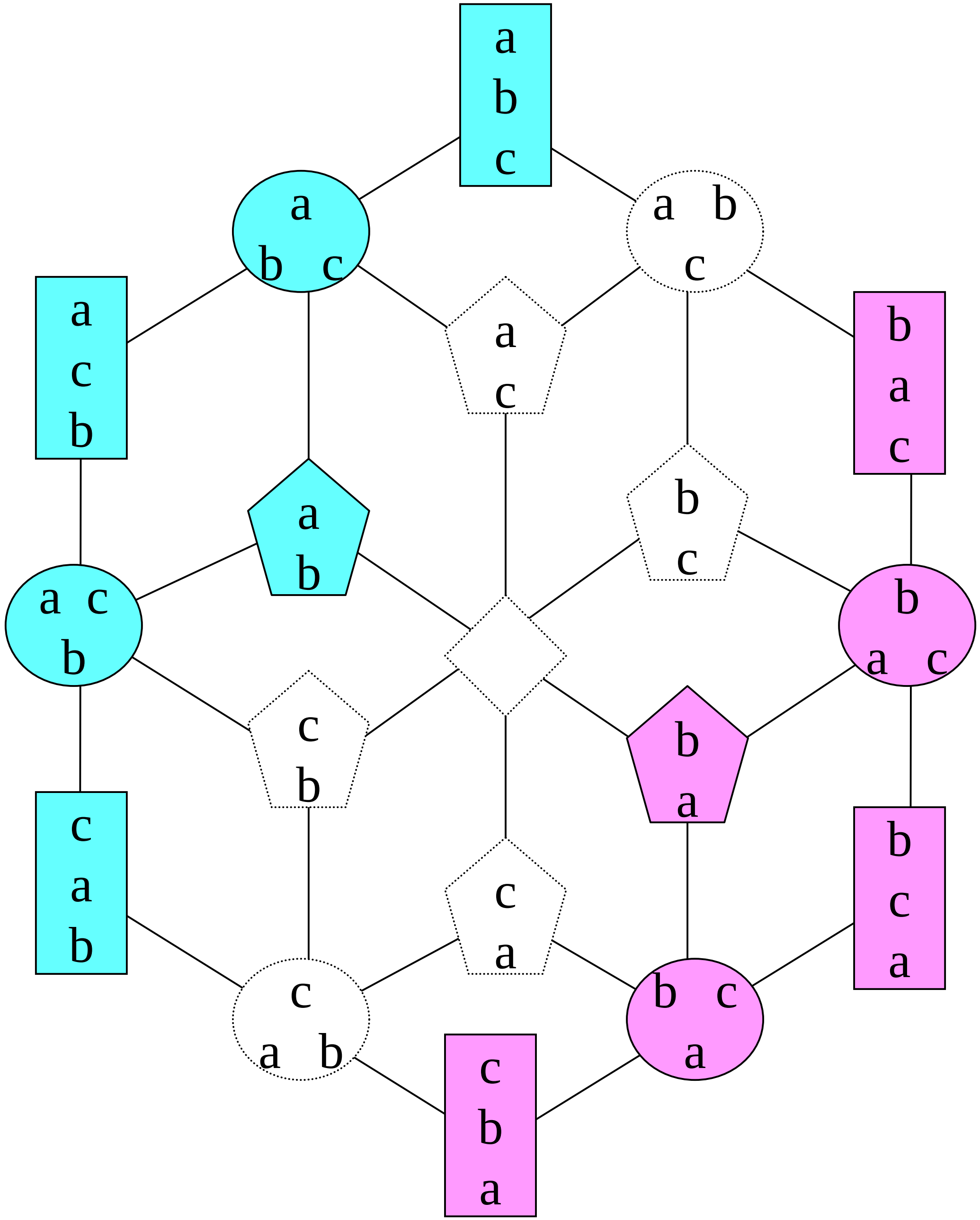


Sen's NM(a)



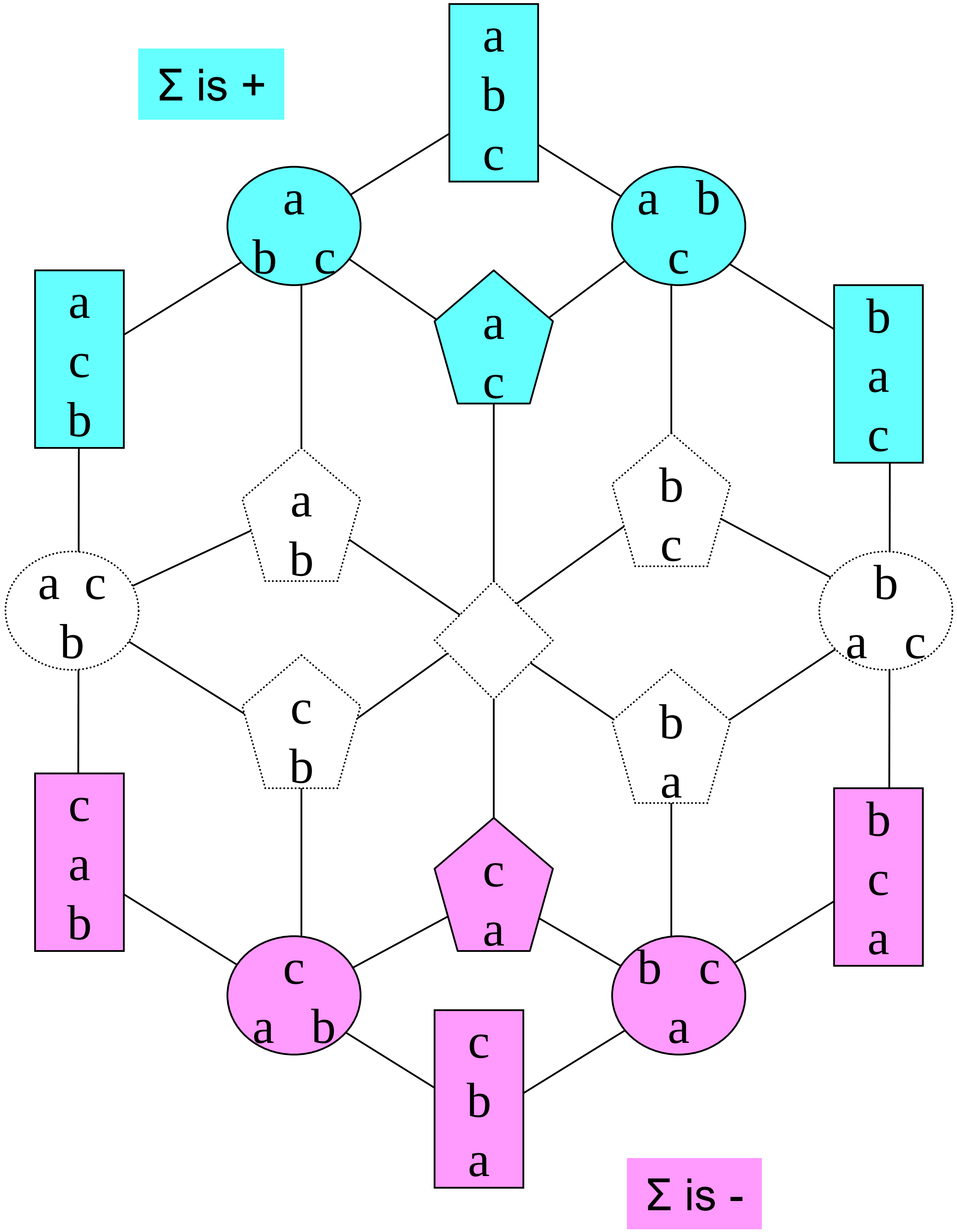
a versus b

Σ is +



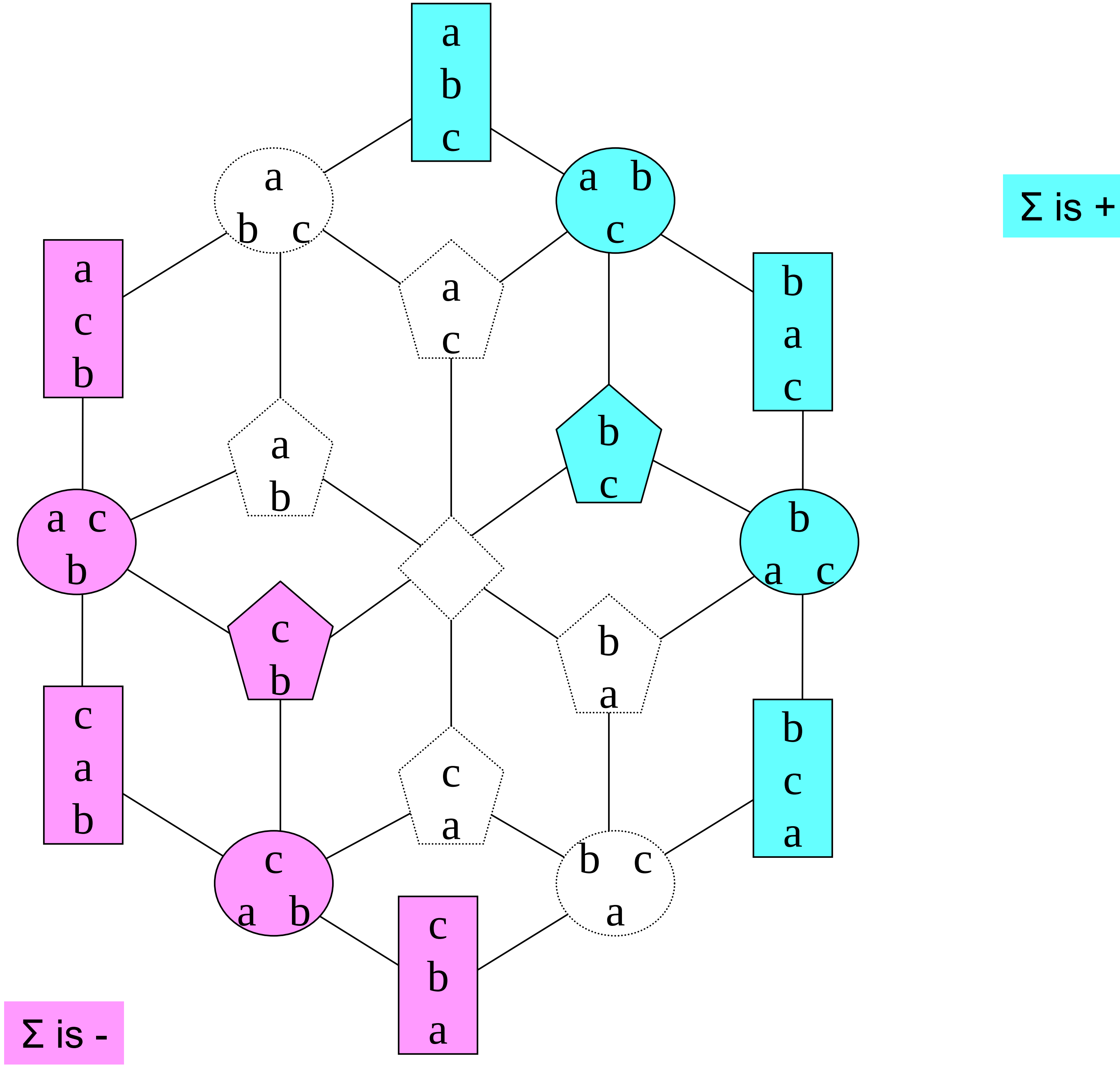
Σ is -

a versus c



Σ is +
a b c
a b c
a b c
a c b
a c
b a c
a b
b c
a c b
b a c
c b
b a
c a b
c a
b c a
c a b
b c a
c b a
Σ is -

b versus c



a b c
a b c
Σ is +
a b c
a c
a c b
b a c
a b
b c
a c b
b a c
c b
b a
c a b
c a
b c a
c a b
b c a
c b a
Σ is -

Definition 1.2.5 Consider a probability $\mathbb{P}$ on Π. We define a *weak majority preference relation* $\succsim$ and a *strict majority preference relation* $\succ$ through

$$c \succsim d \quad \Leftrightarrow \quad \mathbb{P}_{cd} \geq \mathbb{P}_{dc} \Leftrightarrow \mathbb{P}_{cd} \geq \frac{1}{2}, \tag{1.3}$$

$$c \succ d \quad \Leftrightarrow \quad \mathbb{P}_{cd} > \mathbb{P}_{dc} \Leftrightarrow \mathbb{P}_{cd} > \frac{1}{2}. \tag{1.4}$$

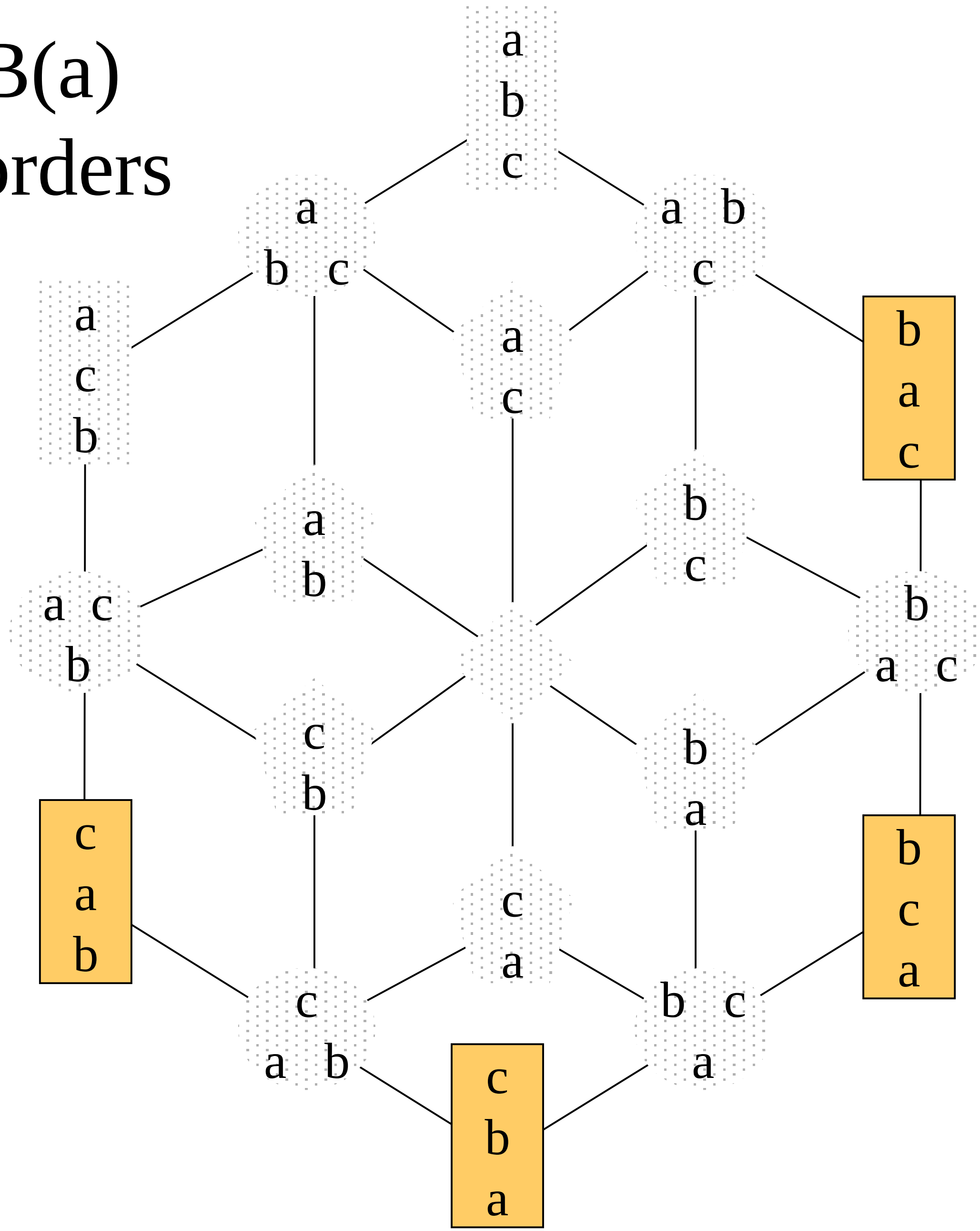
Definition 1.2.12 Given $N^{\mathbb{P}}$ on Π as before, for any triple $\{c, d, e\} \subseteq C$,

$$\begin{aligned}
N^{\mathbb{P}} \text{ satisfies } NW(c) \quad &\Leftrightarrow \quad N^{\mathbb{P}}_{cde} \leq 0 \;\&\; N^{\mathbb{P}}_{dec} \leq 0, \\
N^{\mathbb{P}} \text{ satisfies } NM(c) \quad &\Leftrightarrow \quad N^{\mathbb{P}}_{ecd} \leq 0 \;\&\; N^{\mathbb{P}}_{dce} \leq 0 \Leftrightarrow N^{\mathbb{P}}_{ecd} = 0, \\
N^{\mathbb{P}} \text{ satisfies } NB(c) \quad &\Leftrightarrow \quad N^{\mathbb{P}}_{cde} \leq 0 \;\&\; N^{\mathbb{P}}_{ced} \leq 0.
\end{aligned}$$

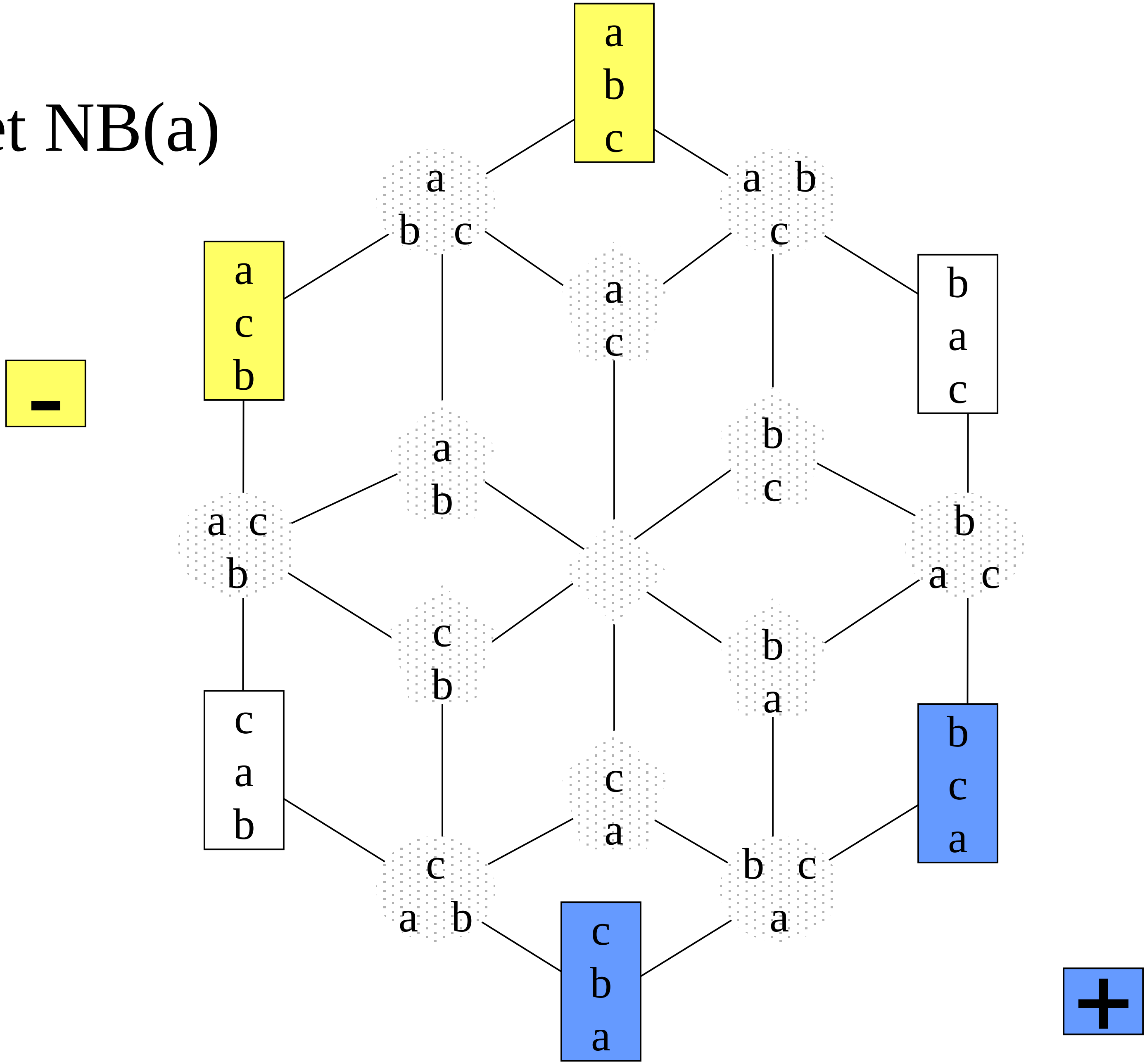
- $\mathbb{P}$ satisfies $NW(c) \Rightarrow N^{\mathbb{P}}$ satisfies $NW(c)$, but not conversely,
- $\mathbb{P}$ satisfies $NB(c) \Rightarrow N^{\mathbb{P}}$ satisfies $NB(c)$, but not conversely,
- $\mathbb{P}$ satisfies $NM(c) \Rightarrow N^{\mathbb{P}}$ satisfies $NM(c)$, but not conversely.

Clearly, domain restrictions imply distributional restrictions, but the converse does not generally hold.

Sen’s NB(a)
On linear orders



Net NB(a)



Definition 1.2.14 Given $\mathcal{NP}$ on Π as before, $\pi \in \Pi$ has a *net preference majority* if and only if

$$\mathcal{NP}(\pi) > \sum_{\substack{\pi' \in \Pi - \{\pi\}, \\ \mathcal{NP}(\pi') > 0}} \mathcal{NP}(\pi'). \tag{1.5}$$

Similarly, for any triple $\{c, d, e\} \subseteq \mathcal{C}$, cde has a *marginal net preference majority* if and only if

$$\mathcal{NP}_{cde} > \sum_{\substack{\pi' \in \{ced, dce, dec, ecd, edc\}, \\ \mathcal{NP}_{\pi'} > 0}} \mathcal{NP}_{\pi'}.$$

Theorem 1.2.15 *The weak majority preference relation $\succsim$ defined in Definition 1.2.5 is transitive if and only if for each triple $\{c, d, e\} \subseteq \mathcal{C}$ at least one of the following two conditions holds:*

1. *$\mathcal{NP}$ is marginally value restricted on $\{c, d, e\}$ and, in addition, if at least one net preference is nonzero then the following implication is true (with possible relabelings):*

$$\mathcal{NP}_{cde} = 0 \Rightarrow \mathcal{NP}_{dce} \neq \mathcal{NP}_{ced}.$$

2. *$\exists \pi_0 \in \{cde, ced, dce, dec, ecd, edc\}$ such that π_0 has a marginal net preference majority.*

Similarly, the strict majority preference relation $\succ$ is transitive if and only if for each triple $\{c, d, e\} \subseteq \mathcal{C}$ at least one of the following two conditions holds:

1. *$\mathcal{NP}$ is marginally value restricted on $\{c, d, e\}$.*

2. *$\exists \pi_0 \in \{cde, ced, dce, dec, ecd, edc\}$ such that π_0 has a marginal net preference majority.*

Net never best of a

$$
NP\begin{pmatrix} b \\ c \\ a \end{pmatrix} \leq 0,\ NP\begin{pmatrix} c \\ b \\ a \end{pmatrix} \leq 0,\ NP\begin{pmatrix} & b & \\ a & & c \end{pmatrix} \leq 0,\ NP\begin{pmatrix} b & & c \\ & a & \end{pmatrix} \leq 0,
$$

$$
NP\begin{pmatrix} & c & \\ a & & b \end{pmatrix} \leq 0,\ NP\begin{pmatrix} b \\ a \end{pmatrix} \leq 0,\ NP\begin{pmatrix} c \\ a \end{pmatrix} \leq 0,\ NP\begin{pmatrix} b \\ c \end{pmatrix} = 0,
$$

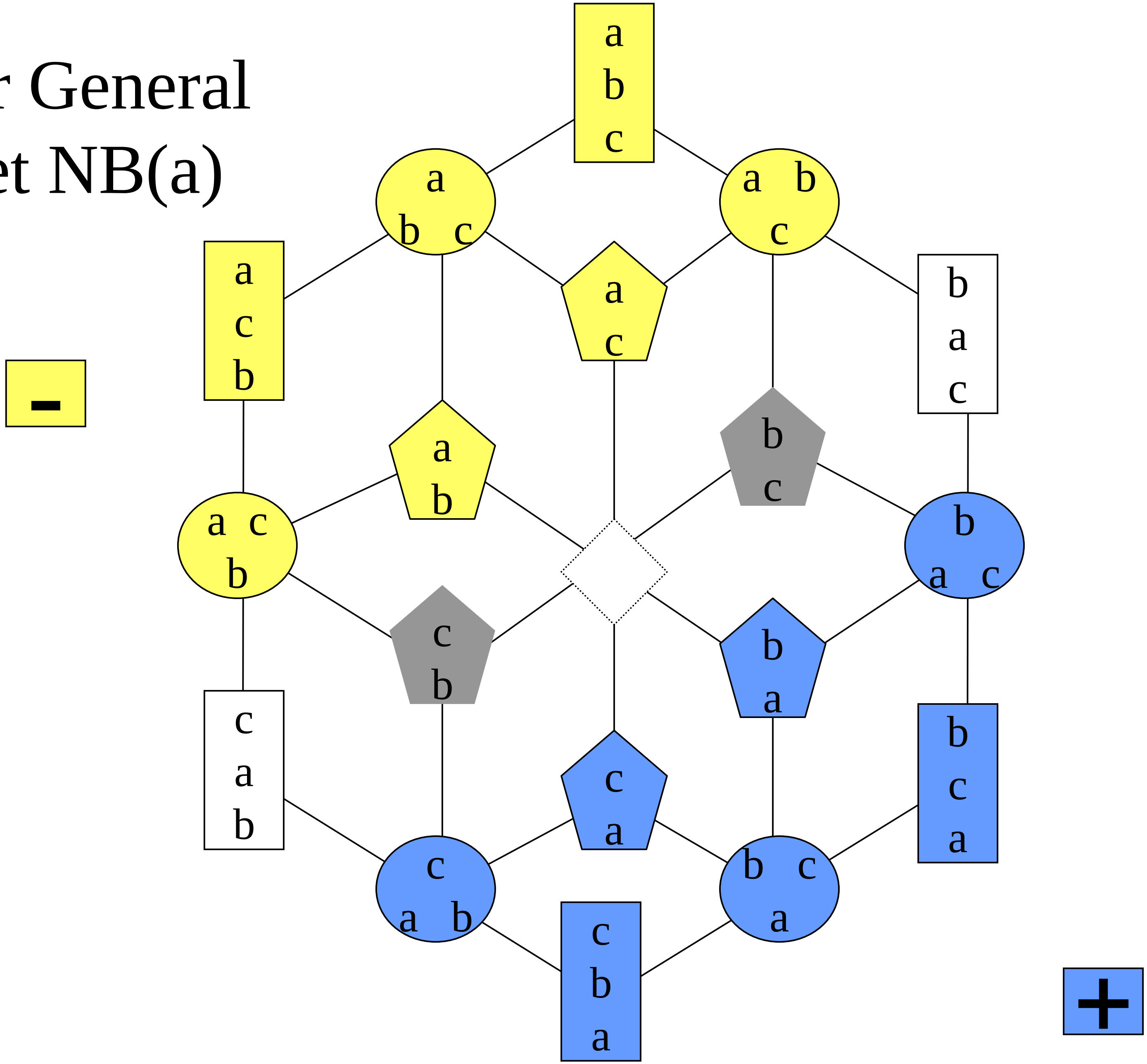
$$
NP\begin{pmatrix} a & \\ & > b \\ c & \end{pmatrix} = NP\begin{pmatrix} a & \\ & > c \\ b & \end{pmatrix} = NP\begin{pmatrix} b & \\ & > a \\ c & \end{pmatrix} = NP\begin{pmatrix} & a & \\ & \bigcirc & \\ c & & b \end{pmatrix} = 0.
$$

The marginal net preference probabilities derived on a triple $\{a, b, c\} \subseteq \mathcal{C}$ satisfy *net never middle of* a, denoted as $NM(a)$, if the following equalities hold:

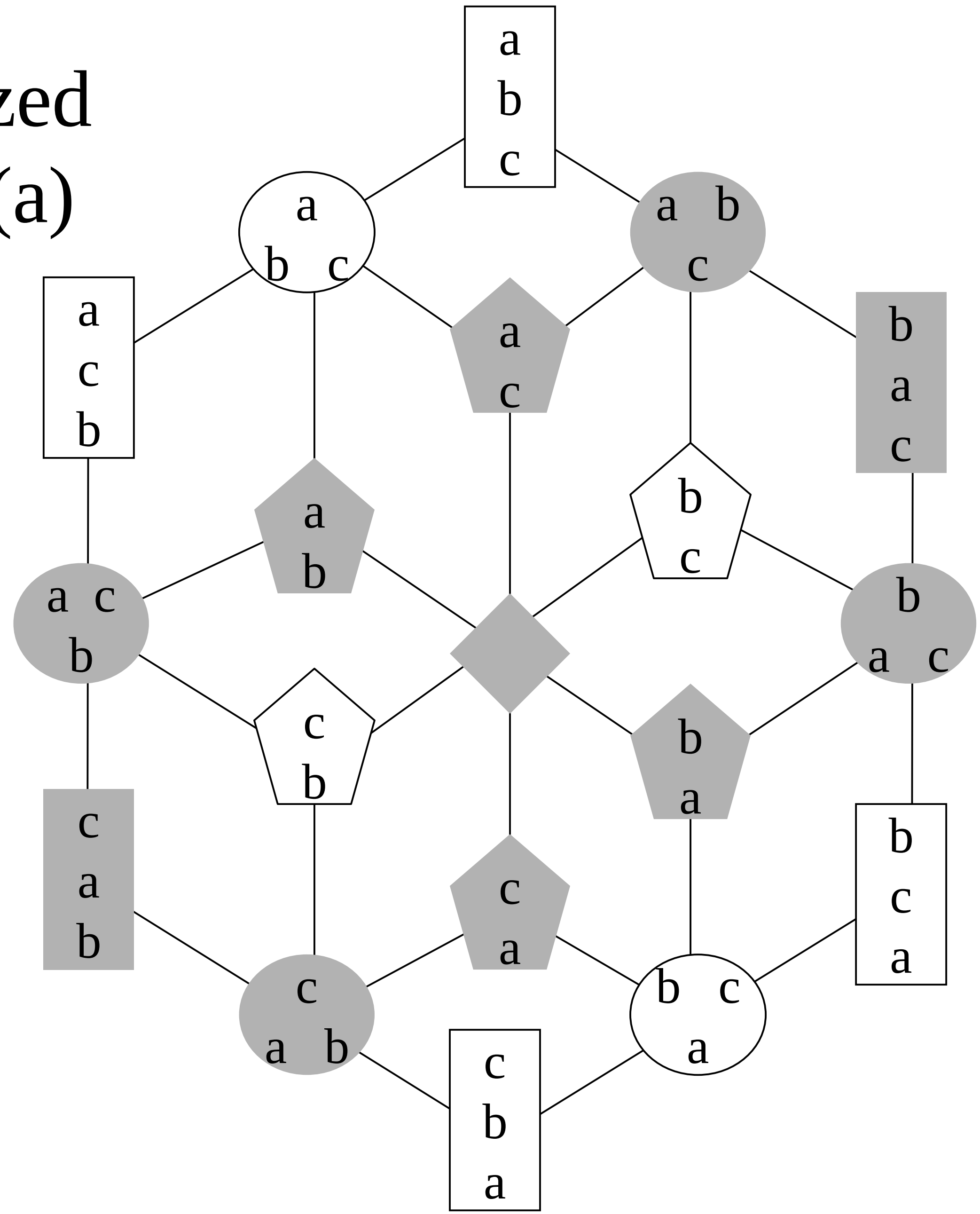
$$
NP\begin{pmatrix} b \\ a \\ c \end{pmatrix} = NP\begin{pmatrix} & b & \\ a & & c \end{pmatrix} = NP\begin{pmatrix} & c & \\ a & & b \end{pmatrix} = NP\begin{pmatrix} a \\ b \end{pmatrix} = NP\begin{pmatrix} a \\ c \end{pmatrix}
$$

$$
= NP\begin{pmatrix} a & \\ & > b \\ c & \end{pmatrix} = NP\begin{pmatrix} a & \\ & > c \\ b & \end{pmatrix} = NP\begin{pmatrix} b & \\ & > a \\ c & \end{pmatrix} = NP\begin{pmatrix} & a & \\ & \bigcirc & \\ c & & b \end{pmatrix} = 0.
$$

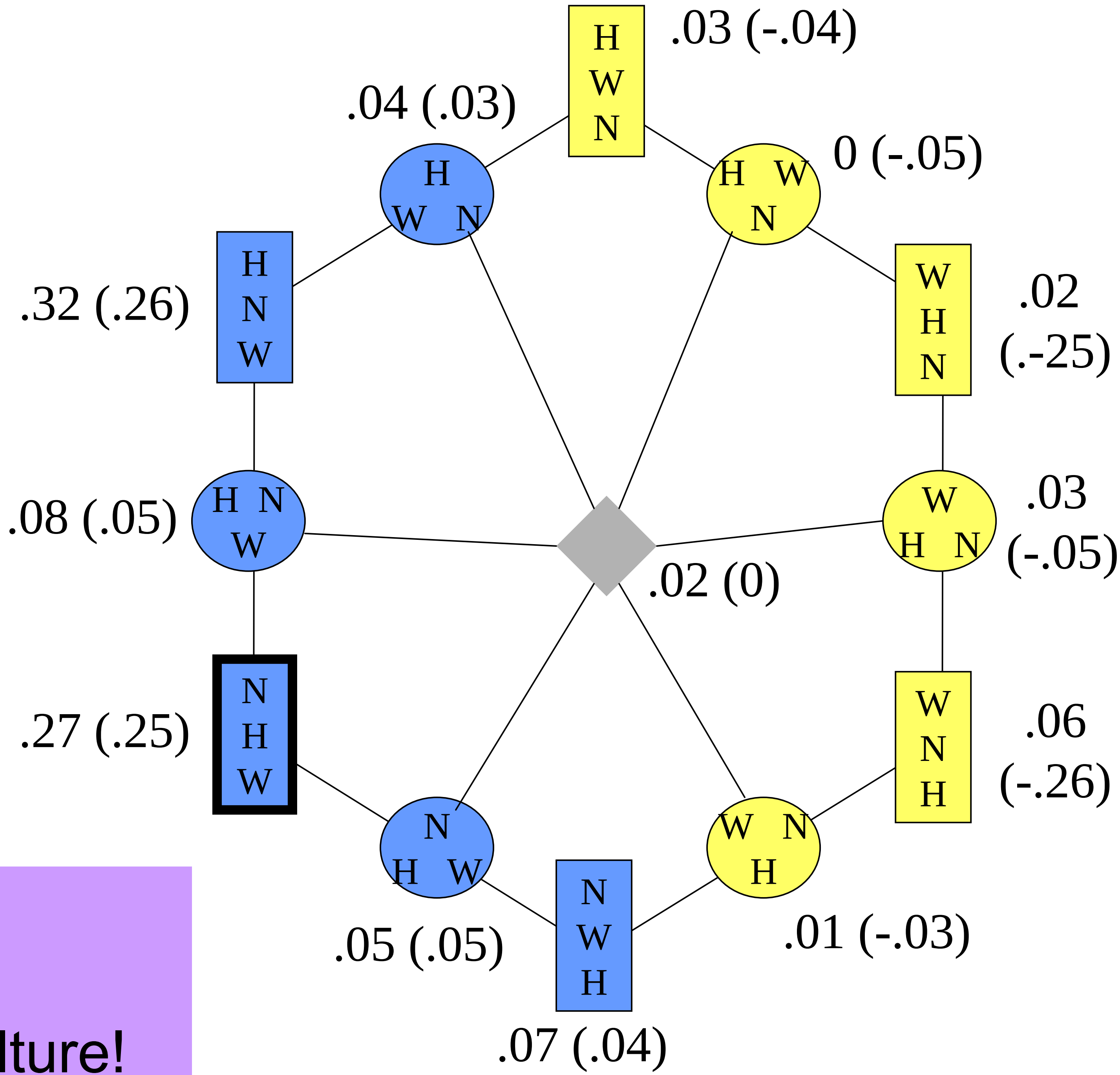
Our General Net NB(a)



Generalized Net NM(a)

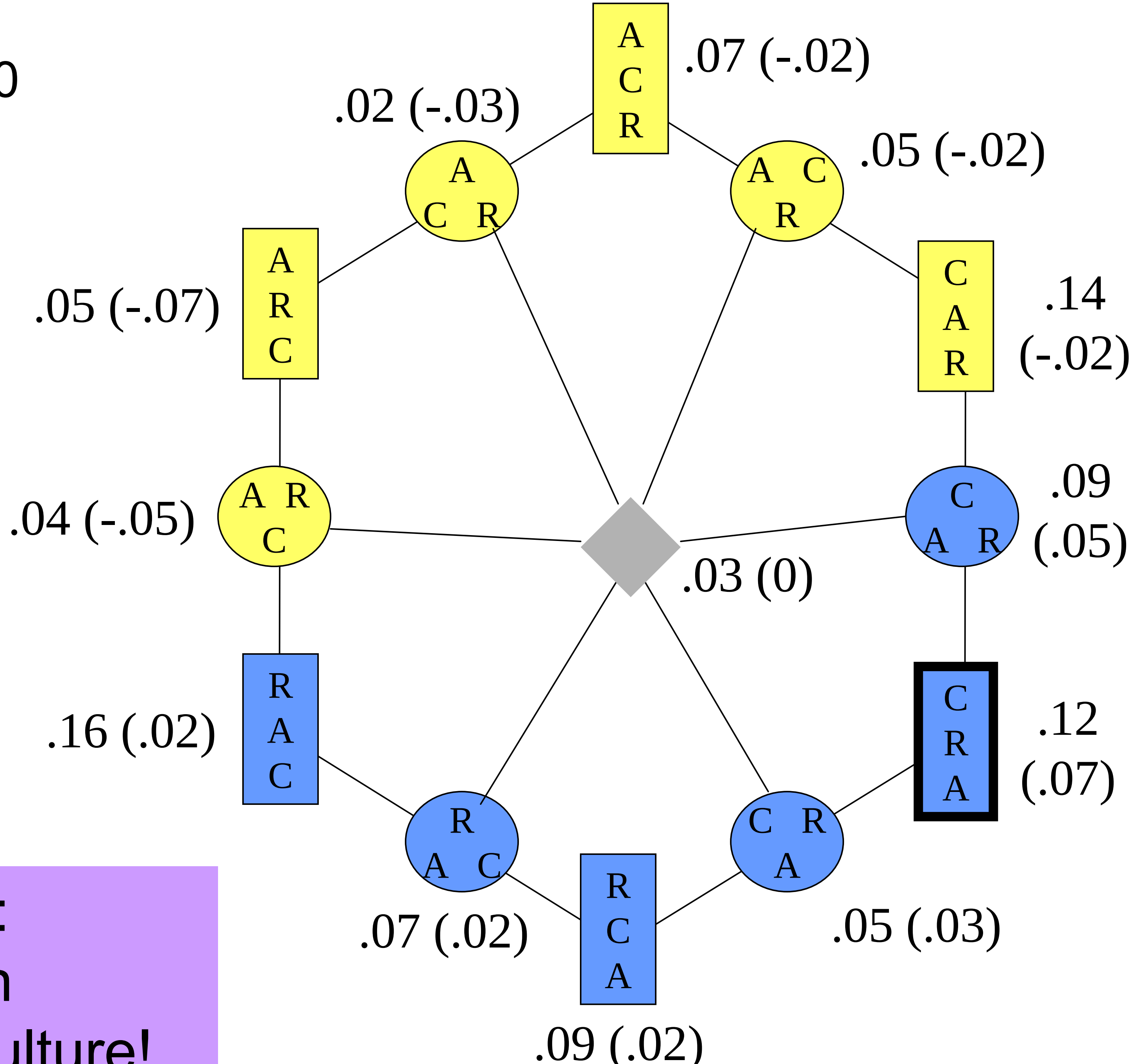


ANES 1968
Net MB(W)



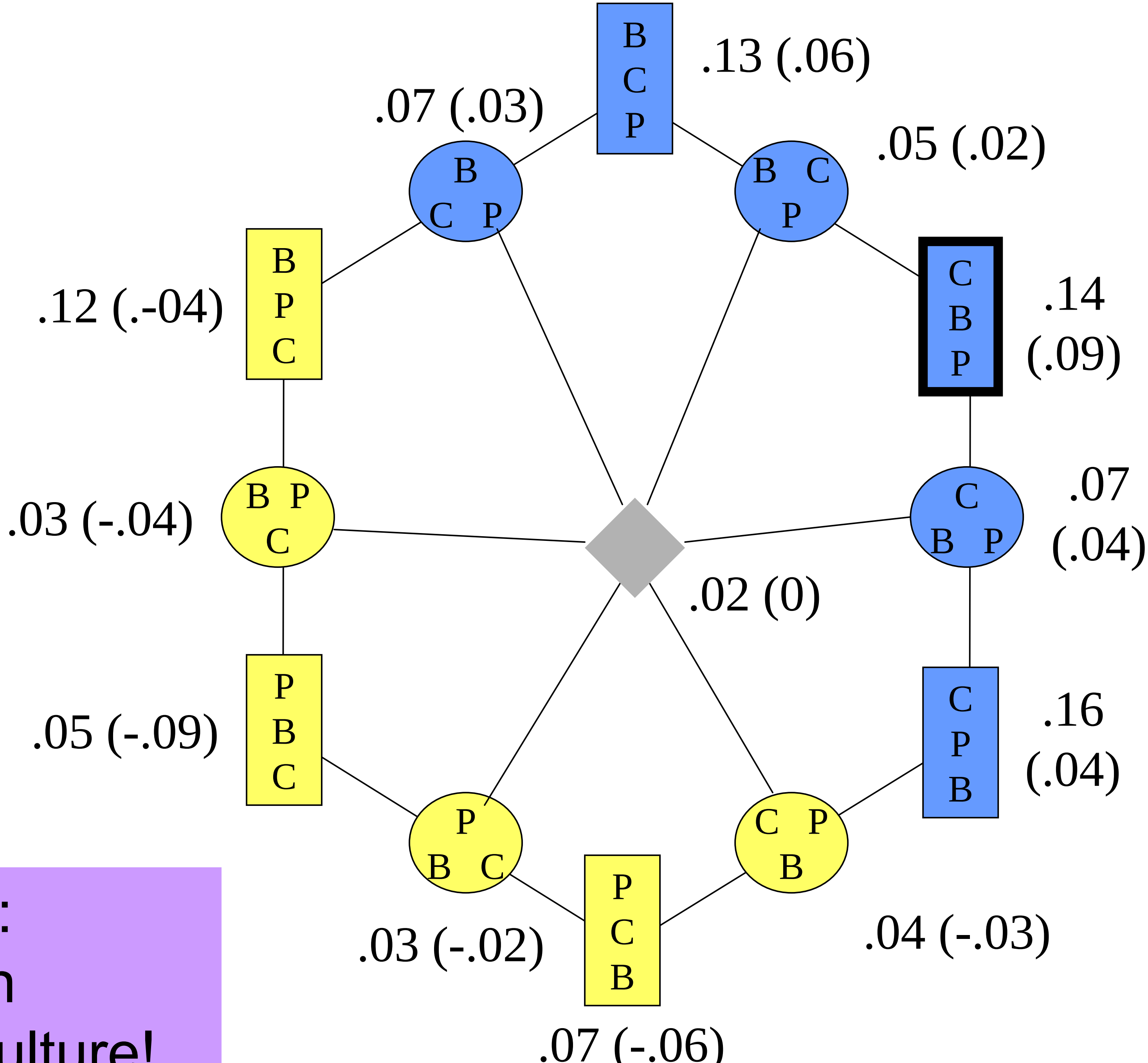
By the way:
Not from an
Impartial Culture!

ANES 1980
Net NB(A)



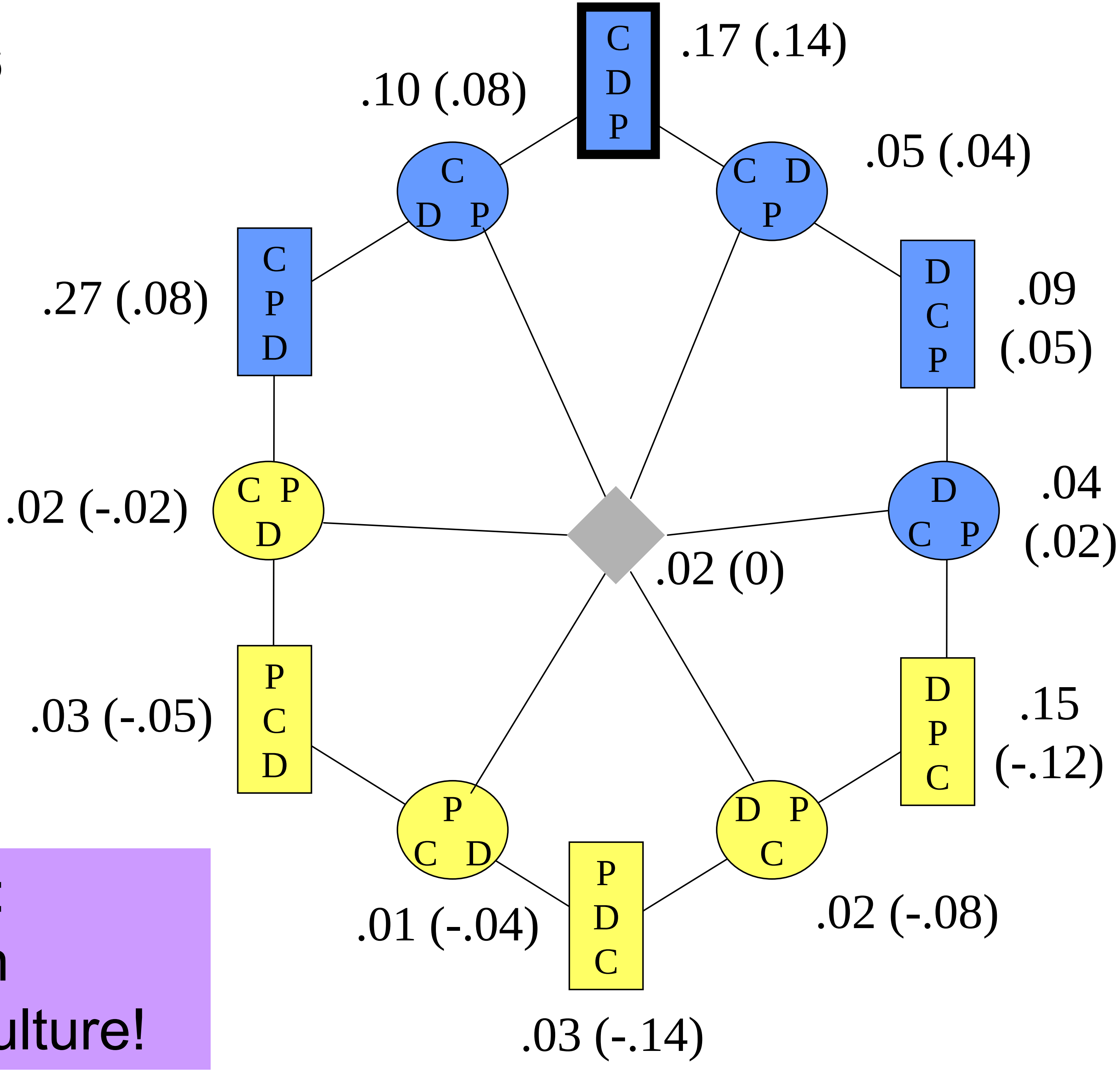
By the way:
Not from an
Impartial Culture!

ANES 1992
Net NB(P)



By the way:
Not from an
Impartial Culture!

ANES 1996
Net NB(P)



By the way:
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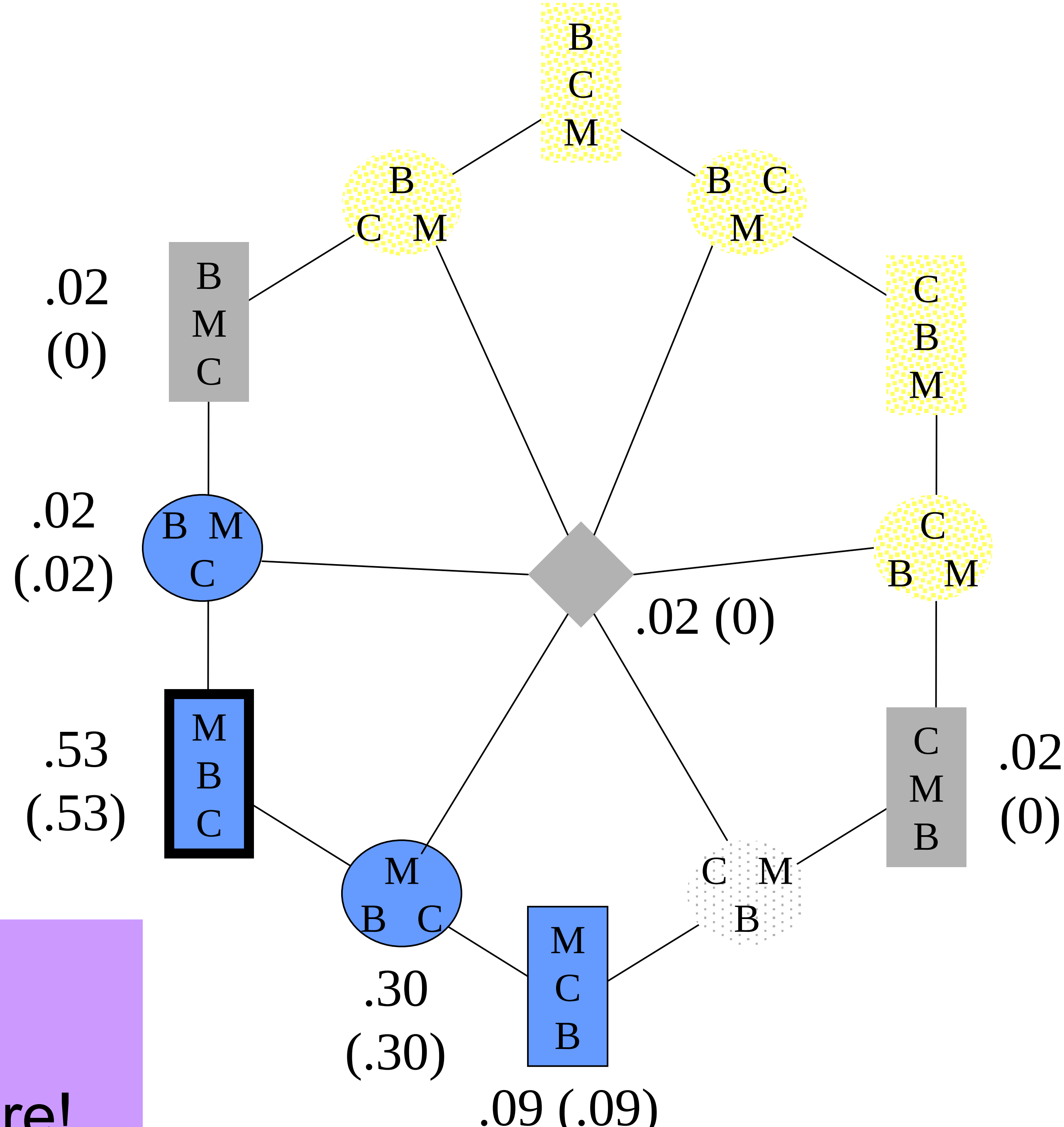
Impartial Culture!

1988 FNES:
Communists

Sen's NW(M)



By the way:
Not from an
Impartial Culture!

Definition 2.3.7 Given net preference probabilities $N\!P^{\mathcal{B}}$ as before, a binary (preference) relation B over $\{x, y, z\}$ has a *net preference majority* (among all members of a set $\mathcal{B}$ of binary relations) on $\{x, y, z\}$ if and only if

$$N\!P^{\mathcal{B}}(B) > \sum_{\substack{B' \in \mathcal{B} - \{B\} \\ N\!P^{\mathcal{B}}(B') > 0}} N\!P^{\mathcal{B}}(B'). \tag{2.30}$$

Theorem 2.3.8 *Given a net probability distribution $N\!P$ over all asymmetric binary relations over $\{a, b, c\}$, neither net value restriction of $N\!P$ nor net majority of a binary relation is necessary for $\succsim$ and/or $\succ$ to be transitive.*

Theorem 2.3.9 *Let $\mathcal{NP}$ be a net preference probability over asymmetric binary relations, as before.*

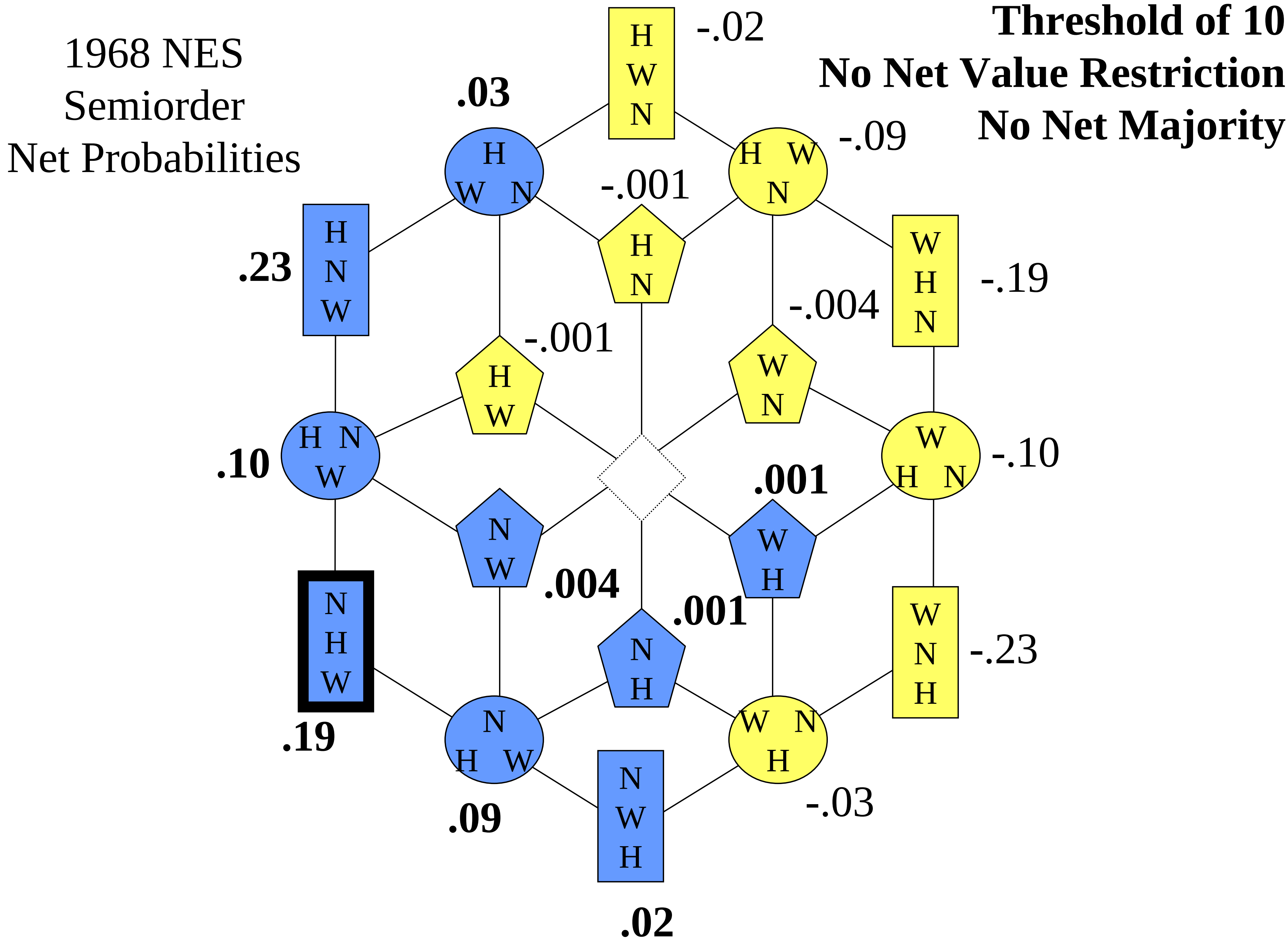
i) Sufficiency of net value restriction for transitive strict majority: *if net value restriction of $\mathcal{NP}$ holds then the strict majority preference relation $\succ$, as defined in Definition 2.1.3, is transitive. However,*

ii) Insufficiency of net value restriction for transitive weak majority: *if net value restriction of $\mathcal{NP}$ holds then the weak majority preference relation $\succsim$, as defined in Definition 2.1.3, need not be transitive.*

Theorem 2.3.10 *Let $\mathcal{NP}$ be a net preference probability over asymmetric binary relations on three elements.*

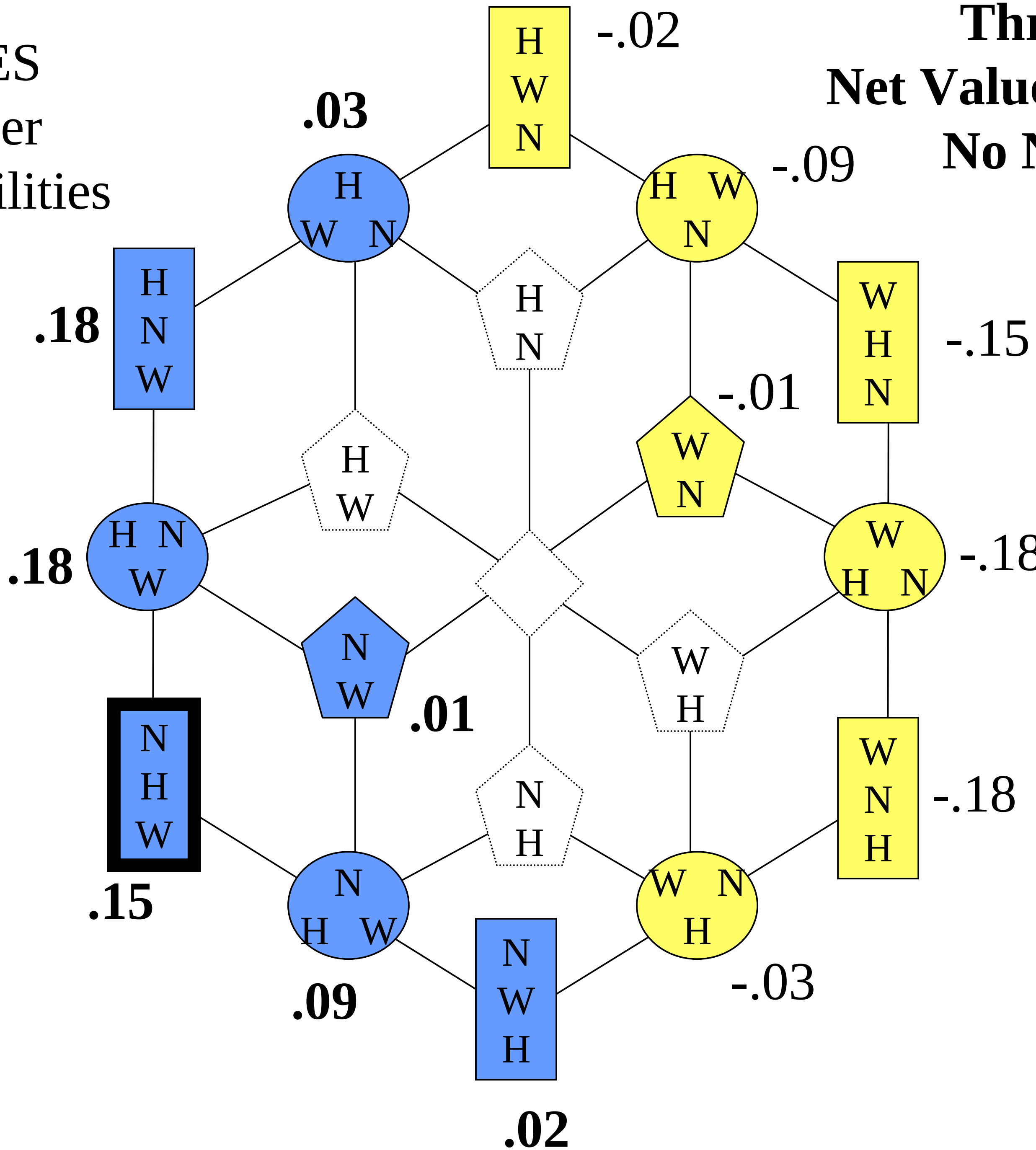
i) Sufficiency of net majority of a strict weak order: *if a strict weak order B has a net majority then $\succsim$ and $\succ$ are transitive. However,*

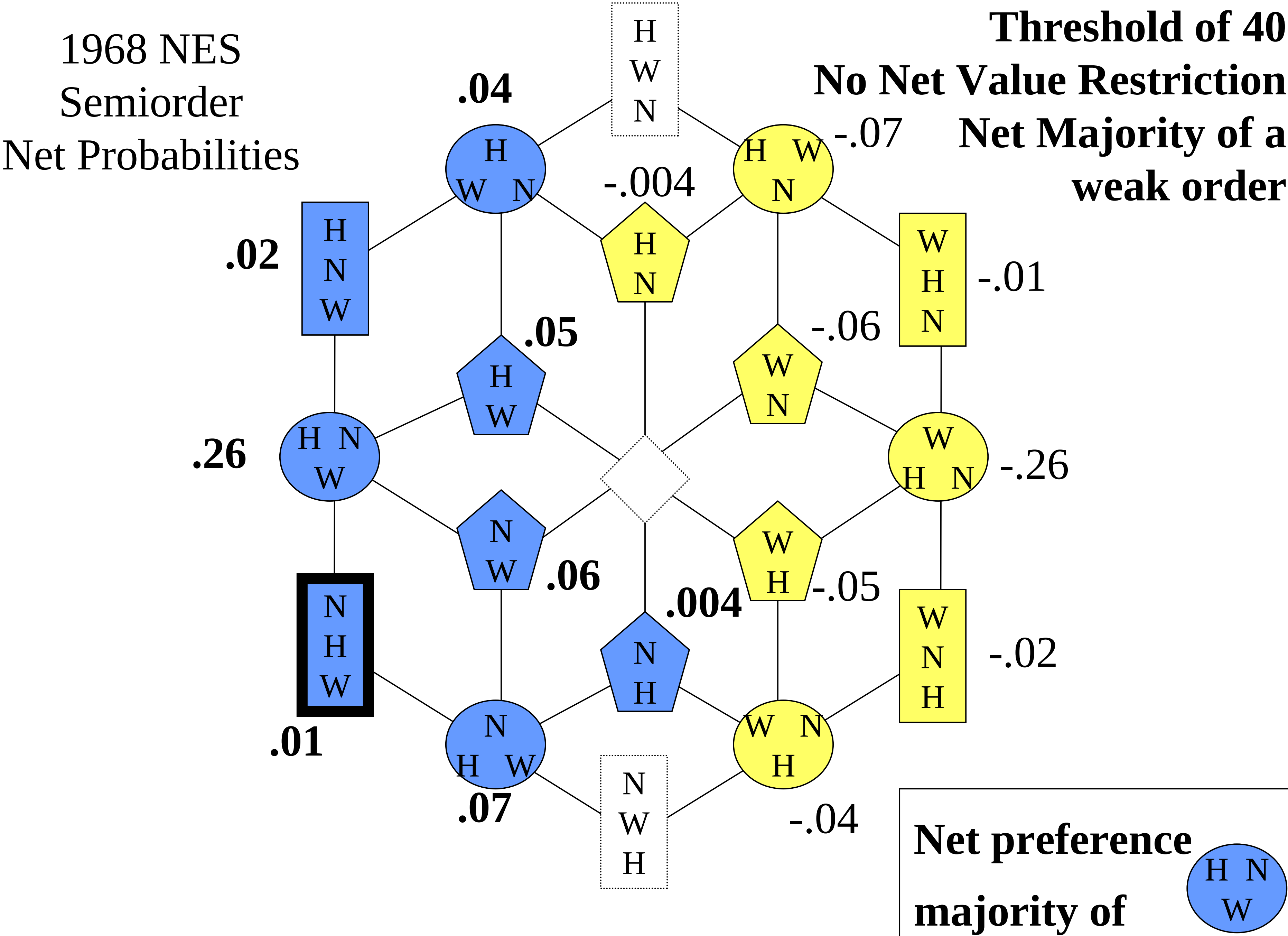
ii) Insufficiency of net majority of an asymmetric binary relation more general than a strict weak order: *if a semiorder, interval order, strict partial order, or more general asymmetric binary relation, B', has a net majority then neither $\succsim$ nor $\succ$ need be transitive.*


1968 NES
Semiorder
Net Probabilities
Threshold of 10
No Net Value Restriction
No Net Majority
H W N
-.02
.03
H W N
H W N
-.09
-.001
H N
.23
H N W
W H N
-.19
-.004
-.001
H W
W N
H N W
.10
W H N
-.10
.001
N W
W H
.004
N H W
.19
.001
N H
W N H
-.23
N H W
W N H
N W H
-.03
.09
.02

1968 NES
Semiorder
Net Probabilities

Threshold of 17
Net Value Restriction
No Net Majority

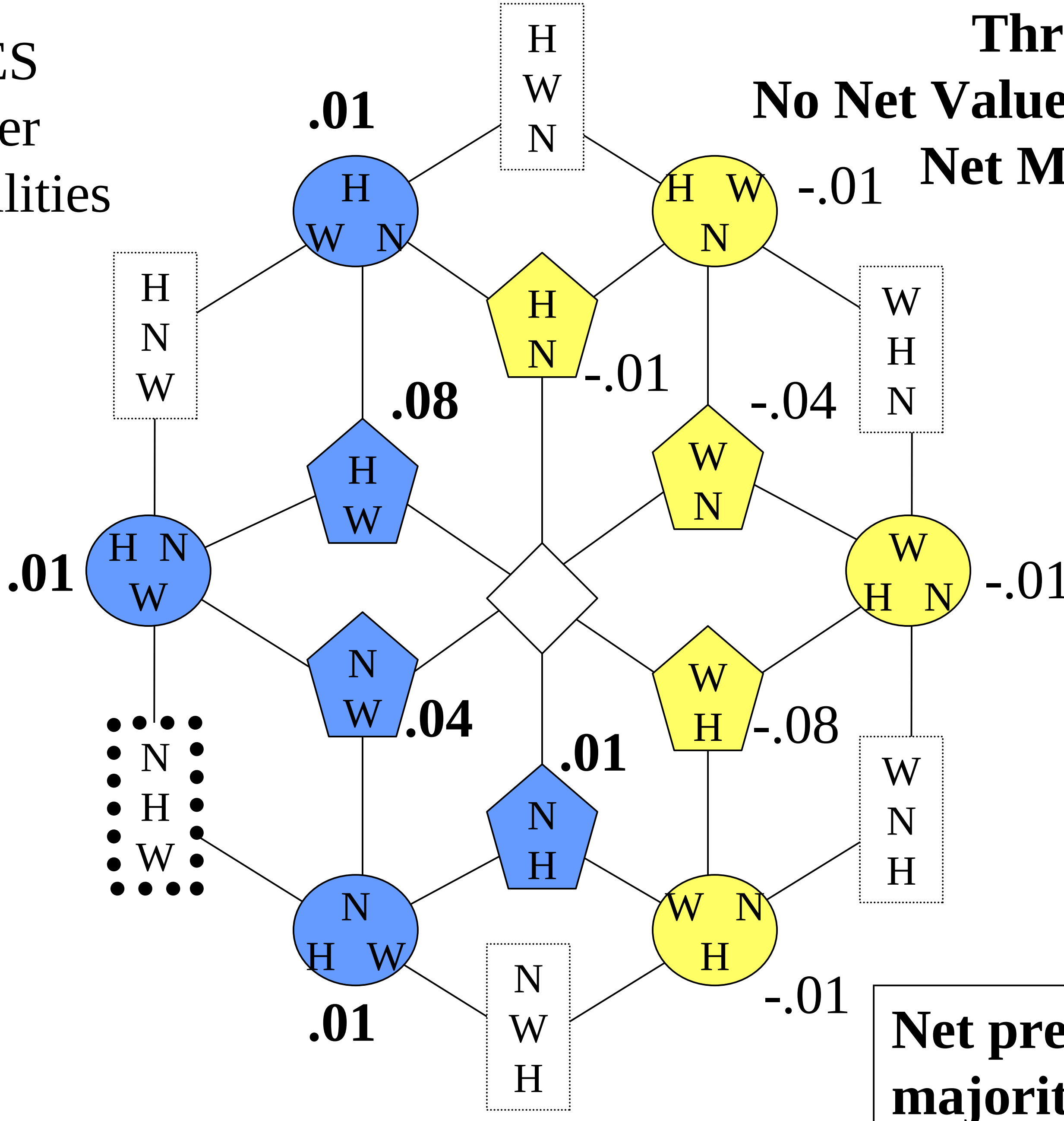




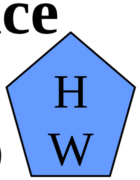
1968 NES
Semiorder
Net Probabilities
Threshold of 40
No Net Value Restriction
Net Majority of a
weak order
H W N
.04
H W N
-.07
H W N
-.004
H N
.02
H N W
W H N
-.01
.05
H W
-.06
W N
.26
H N W
W H N
-.26
N W
.06
W H
-.05
.004
N H
N H W
.01
W N H
-.02
N H W
.07
N W H
W N H
-.04
Net preference
majority of
H N W

1968 NES
Semiorder
Net Probabilities

Threshold of 90
No Net Value Restriction
Net Majority of a semiorder



Net preference majority of
(3 significant digits)



Theoretical primitives	Basic quantities	Conditions	Relationship to transitivity of $\succ$
Weak orders	tallies	NB, NM, NW of [Gär01, Sen66, Sen70]	sufficient but not necessary
Linear orders	net tallies	$NetNB, NetNM, NetNW$, net preference majority of [GH78, FG86a]	necessary and sufficient
Probabilities on linear orders	net probabilities	$NetNB, NetNM, NetNW$, net preference majority of Chapter 1	necessary and sufficient
Probabilities on partial orders	net probabilities	generalized $NetNB, NetNM, NetNW$, net majority (weak order)	sufficient but not necessary
		net majority (partial order) of Chapter 2	not sufficient

ANES	Threshold	SWO
1992	0, …, 99	Clinton Bush Perot

1992 ANES: The majority preference relation is $C \succ B \succ P$, for every value of ϵ, with $0 \leq \epsilon < 100$. Despite there being consistent transitivity of majority preferences across all threshold values, and despite the majority preference relation itself being robust as well, net value restriction holds only for thresholds of zero and 1. Furthermore, there is never any ordering with a net preference majority.

Near Net Value Restriction

Today:

- Statistical Sampling and Inference
- Why no Cycles? (General Value Restriction)
- Behavioral Social Choice Analysis of STV

American Psychological Association Presidential Elections

- Alternative Vote
- A.k.a. Instant Runoff Voting

Single Seat Special Case of

- Single Transferable Vote
- A.k.a. Hare System

THOMAS HARE (1806—91)

Charles Dodgson, a.k.a. Lewis Carroll

APA Elections: AV/STV

- Ballots: Partial/Full Rankings of 5 Candidates
- For m many seats, N many voters

  Droop Quota = N/(m+1) + 1

  Example: 1 seat, 100 voters, Droop Quota = 51
- Need Droop Quota of "First Rank" votes to win a seat
- Can't fill all seats by Droop Quota?

  (→ "Instant Runoff")

  Elimination by smallest # first rank votes

  Transfer to next on ballot

Example: 2001 APA

Seats: 1
Ballots Counted: 17911
Droop Quota: 8956

	1st Count	2nd Count	3rd Count	4th Count	5th Count
Candidate A	2599	400 2999	878 3877		
Candidate B	2412	422 2834			
Candidate C	4243	389 4632	730 5362	1558 6920	
Candidate D	1855				
✓ Candidate E	6802	458 7260	720 7980	1755 9735	
Exhausted Ballots		186 186	506 692	564 1256	
Totals	17911	17911	17911	17911	

✓ Elected

Example: 2001 APA

Seats: 1
Ballots Counted: 17911
Droop Quota: 8956

	1st Count (transfer)	1st Count	2nd Count (transfer)	2nd Count	3rd Count (transfer)	3rd Count	4th Count (transfer)	4th Count	5th Count (transfer)	5th Count
Candidate A		2599	400	2999	878	3877				
Candidate B		2412	422	2834						
Candidate C		4243	389	4632	730	5362	1558	6920		
Candidate D		1855								
✓ Candidate E		6802	458	7260	720	7980	1755	9735		
Exhausted Ballots	.		186	186	506	692	564	1256		
Totals		17911		17911		17911		17911		

✓ Elected

Example: 2001 APA

Seats: 1
Ballots Counted: 17911
Droop Quota: 8956

	1st Count	2nd Count	3rd Count	4th Count	5th Count
Candidate A	2599	400 2999	878 3877		
Candidate B	2412	422 2834			
Candidate C	4243	389 4632	730 5362	1558 6920	
✓ Candidate E	6802	458 7260	720 7980	1755 9735	
Exhausted Ballots		186 186	506 692	564 1256	
Totals	17911	17911	17911	17911	

✓ Elected

Example: 2001 APA

Seats: 1
Ballots Counted: 17911
Droop Quota: 8956

	1st Count	2nd Count	3rd Count	4th Count	5th Count
Candidate A	2599	400 2999	878 3877		
Candidate C	4243	389 4632	730 5362	1558 6920	
✓ Candidate E	6802	458 7260	720 7980	1755 9735	
Exhausted Ballots		186 186	506 692	564 1256	
Totals	17911	17911	17911	17911	

✓ Elected

Example: 2001 APA

Seats: 1
Ballots Counted: 17911
Droop Quota: 8956

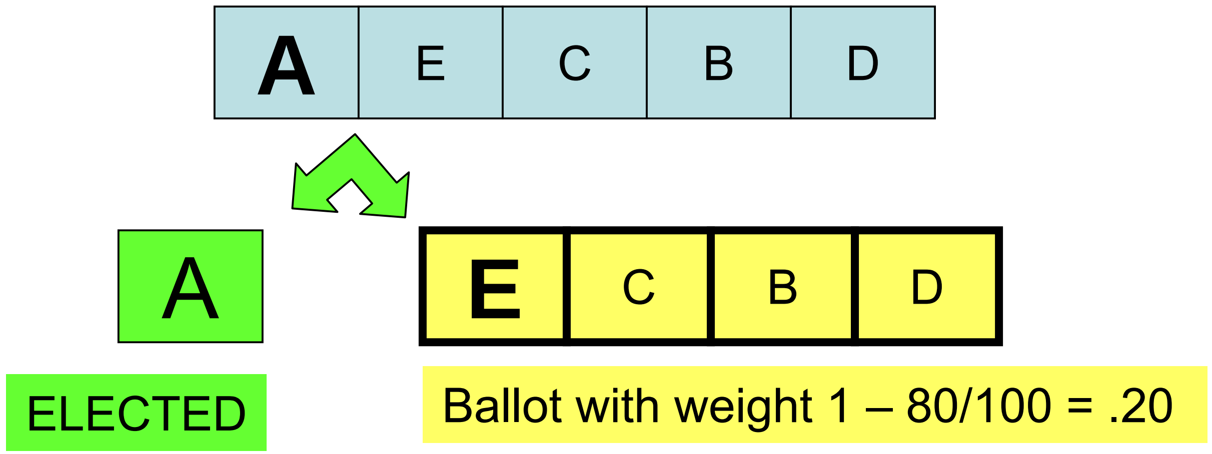
	1st Count		2nd Count		3rd Count		4th Count		5th Count	
Candidate C	4243		4632	389	5362	730	6920	1558		
✓ Candidate E	6802		7260	458	7980	720	9735	1755		
Exhausted Ballots		.	186	186	692	506	1256	564		
Totals	17911		17911		17911		17911			

✓ Elected

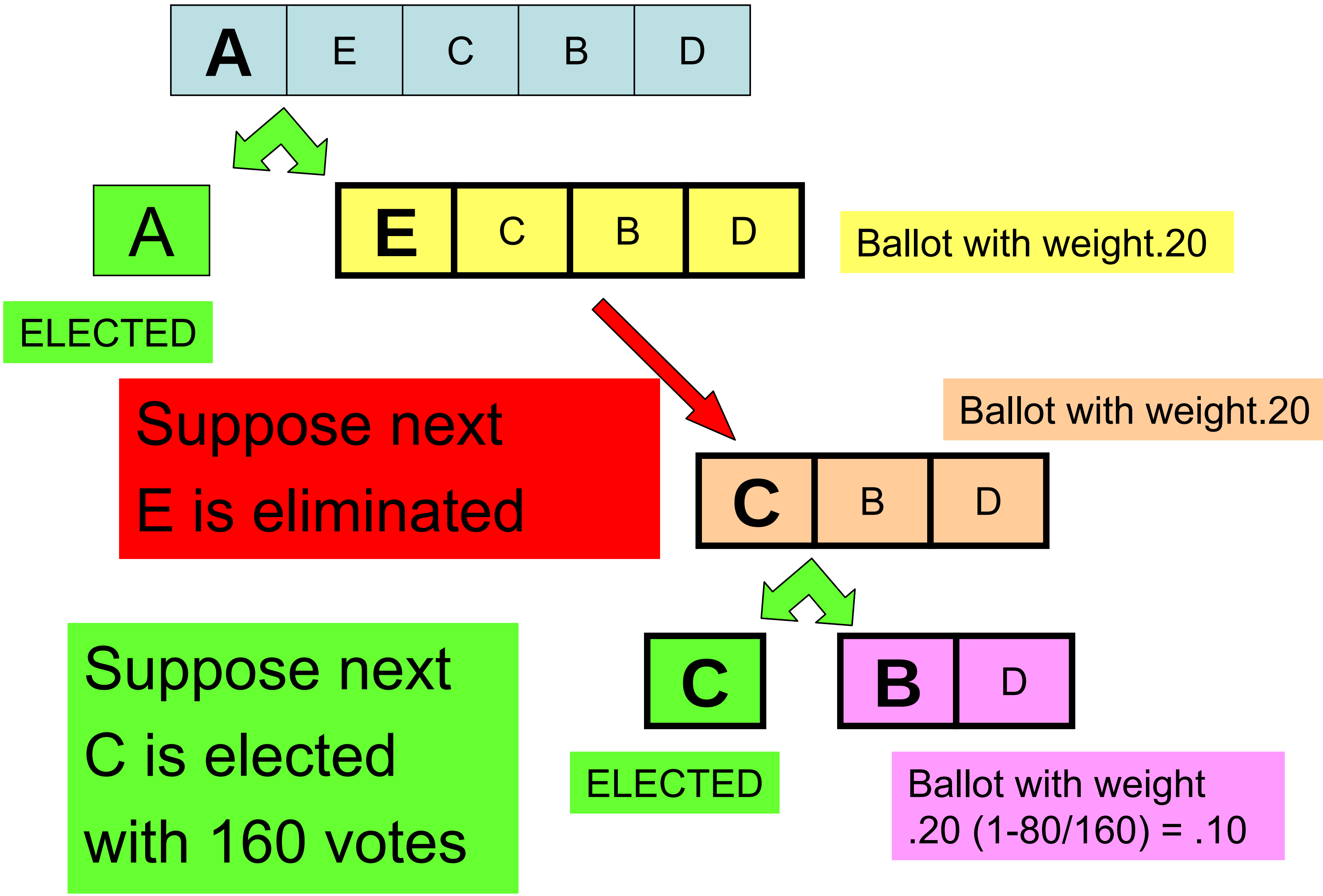
Multi-Seat Elections Transfer Procedure:

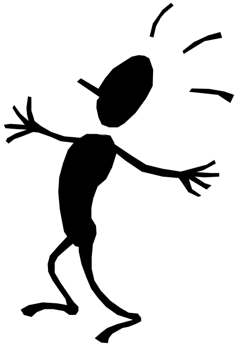
Suppose:

- Droop Quota = 80
- Candidate A received 100 first rank votes (including possible transfers from eliminated or already elected candidates)
- Find each ballot with A at first place and transfer:



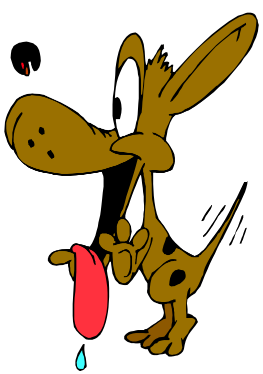
Multi-Seat Elections Transfer Procedure:

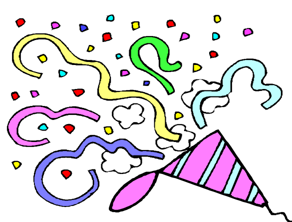




APA Data

- 1998-2001 Presidential Elections
- Partial Rankings on 5 Candidates
- N: 18,723; 18,398; 20,239; 17,911





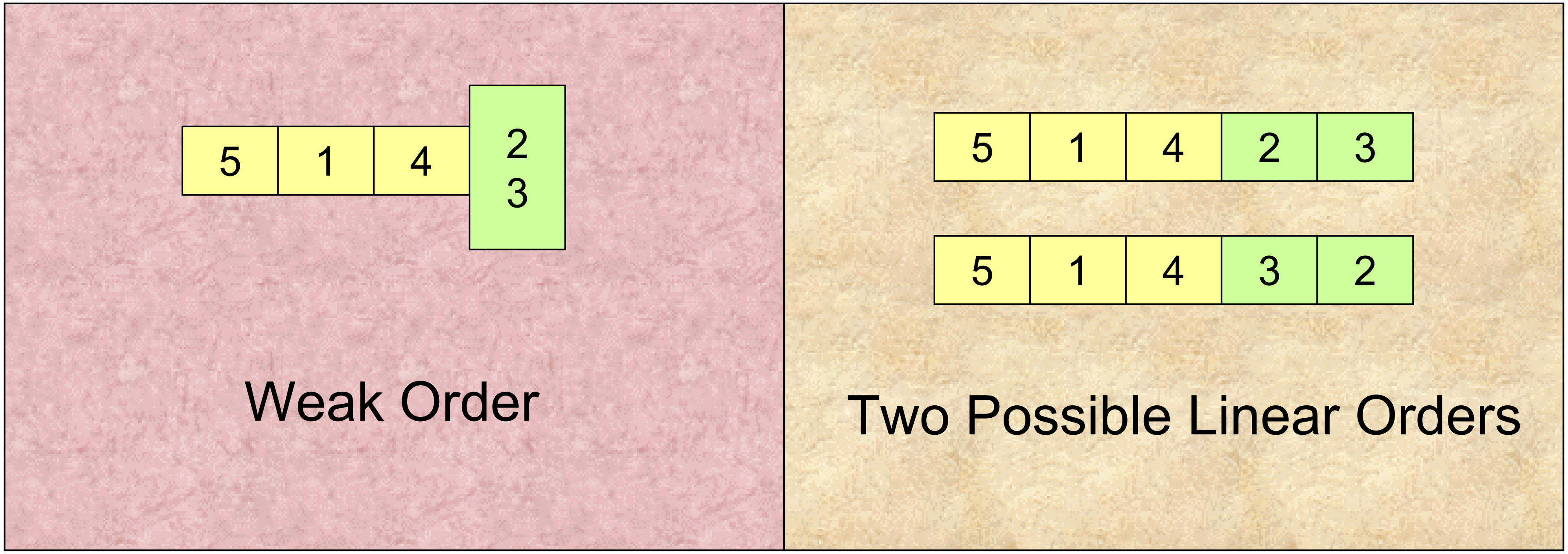
Two Methods of Analysis:

4	2	1	3	5

✓ Complete Ranking

5	1	4

❖ Partial Ranking



Two Methods of Analysis:

- Translate partial rankings into weak orders
- Compute social welfare functions: Majority, Borda, & plurality
- Bootstrap:

  Repeatedly (500 times) sample (w. replacement) of same sample size from original data & recompute social welfare functions

- Statistically infer model-based linear order probabilities from ballots
- Compute social welfare functions based on linear order probabilities
- Bootstrap:

  Repeatedly (500 times) sample (w.replacement) of same sample size from original data & reestimate model based predicted frequencies & social welfare functions

Two Methods of Analysis:

- Weak order based analysis
- Omitted candidates are treated as "tied at the bottom of the preference"
- Bootstrap confidence
- No statistical test

- Linear order based analysis
- All ballots are assumed to originate from linear order
- Size-Independent Model of partial ranking data
- Bootstrap confidence
- Statistical test

Condorcet and Arrow Revisited

Weak Order Analysis Majority Preference:	Linear Order Analysis Majority Preference:
1998: 32145	1998: **32415**
1999: 43215	1999: **43215**
2000: **52134**	2000: **52134**
2001: 53124	2001: **51324**

Bootstrapped Confidence
bold > 95%

NO CYCLES
Majority preferences are linear orders
in all 4 data sets by both methods of analysis

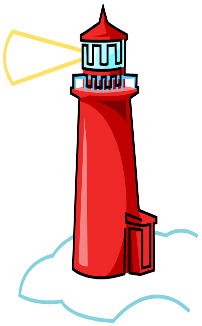
Condorcet versus Borda

Majority / Borda:	
1998:	32145 / **32145**
1999:	43215 / 43125
2000:	**52134** / **52134**
2001:	53124 / **53124**

Majority / Borda:	
1998:	**32415** / **32415**
1999:	**43215** / **43215**
2000:	**52134** / **52134**
2001:	**51324** / 51324

Bootstrapped Confidence
bold > 95%

(almost) NO DISAGREEMENT!
Majority orders and Borda orders
are virtually identical by both methods of analysis

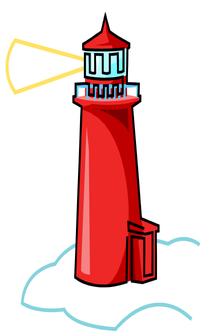


Plurality Scoring rule:

- 1^{st} ranked candidate gets 1 point,
- other candidates get 0 points.

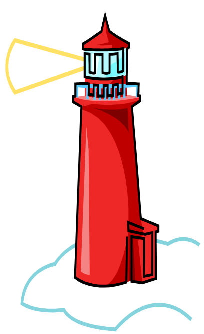
STV versus Majority, Borda, Plurality: Weak Order Based Analysis

	STV	Majority	Borda	Plurality
1998	3 31 315 3512	32145	**32145**	35124
1999	4 43 431 4312	43215	43125	43152
2000	5 52 523 5321	**52134**	**52134**	53214
2001	5 53 531 5312	53124	**53124**	**53124**



STV versus Majority, Borda, Plurality: Linear Order Based Analysis

	STV	Majority	Borda	Plurality
1998	3 31 315 3512	**32415**	**32415**	35124
1999	4 34 431 4315	**43215**	**43215**	43152
2000	5 52 523 5321	**52134**	**52134**	53214
2001	5 53 531 5312	**51324**	51324	**53124**



Model Fit: Size-Independent Model

♐: Partial ranking

$\#_{♐}$: Number of objects that are ranked in ♐

$\Pi_{♐}$: Set of complete rankings that start with ♐

♐

5	1	4

$\#_{♐} = 3$

$\Pi_{♐}$

5	1	4	2	3
5	1	4	3	2

$$P(♐) = P(S = \#_{♐})P(R \in \Pi_{♐})$$

Model Fit: Size-Independent Model

$$P(\vec{x}) = P(S = \#_{\vec{x}})P(R \in \Pi_{\vec{x}})$$

Data:

$$5! + 5! + (5)(4)(3) + (5)(4) + 5 - 1$$

$$= 324 \text{ degrees of freedom}$$

Model:

$$5 - 1 + 5! - 1$$

$$= 123 \text{ free parameters}$$

-2 Log Likelihood Ratio (G^2)

SIM against Multinomial:

$$324 - 123 = 301 \text{ degrees of freedom}$$

Model Fit: Size-Independent Model

	N	Multi LnLik	Model LnLik	G-Square
1998	18723	-702	-1108	811
1999	18298	-720	-1163	885
2000	20239	-722	-1593	1743
2001	17911	-723	-1292	1138

Model Fit: Size-Independent Model

	N	Multi LnLik	Model LnLik	G-Square	Agresti D
1998	18723	-702	-1108	811	.07
1999	18298	-720	-1163	885	.08
2000	20239	-722	-1593	1743	.10
2001	17911	-723	-1292	1138	.09

Model Fit: Size-Independent Model

	N	Multi LnLik	Model LnLik	G-Square	Agresti D	R-Sqre
1998	18723	-702	-1108	811	.07	96%
1999	18298	-720	-1163	885	.08	93%
2000	20239	-722	-1593	1743	.10	92%
2001	17911	-723	-1292	1138	.09	93%

Model Fit: Size-Independent Model (for Size > 1 only)

	N	Multi LnLik	Model LnLik	G-Square	Agresti D	R-Sqre
1998	18723	-702	-950	494	.07	99%
1999	18298	-720	-999	558	.07	98%
2000	20239	-722	-1400	1356	.09	97%
2001	17911	-723	-993	541	.07	99%

Hybrid Model Based Analysis:

- Fit size-independent model to partial rankings with # > 1
- Use estimated parameters to predict partial rankings for all #
- Choose P(S=1) as big as possible without over predicting any # = 1 partial rankings
- Treat all remaining # = 1 partial rankings as weak orders
- Compute social welfare outcomes

Model Dependence Check: 1998

	STV	Majority	Borda	Plurality
All partial rankings	**3** **31** **315** *3512*	32145	**32145**	35124
Partial rankings #4 or #5	3 31 312 3125	32415	32415	31524
Complete rankings	3 31 312 3125	32415	32415	31524
Size-independent model	**3** 32 312 *3512*	**32415**	**32415**	35124
Hybrid model	**3** 32 312 *3512*	**32415**	32145	35124

Model Dependence Check: 1999

	STV	Majority	Borda	Plurality
All partial rankings	**4** **43** **431** *4315*	43215	**43125**	43152
Partial rankings #4 or #5	4 43 431 4312	43215	43215	43152
Complete rankings	4 43 431 4312	43215	43215	43152
Size-independent model	**4** *34* **431** *4315*	**43215**	**43215**	43152
Hybrid model	**4** *34* **431** *4315*	**43215**	**43215**	43152

Model Dependence Check : 2000

	STV	Majority	Borda	Plurality
All partial rankings	**5** **52** **523** 5321	**52134**	**52134**	5<u>321</u>4
Partial rankings #4 or #5	5 52 523 5231	52134	52134	5<u>231</u>4
Complete rankings	5 52 523 5231	52134	52134	5<u>321</u>4
Size-independent model	**5** **52** **523** 5321	**52134**	**52134**	5<u>321</u>4
Hybrid model	**5** **52** **523** *5321*	**52134**	52134	5<u>321</u>4

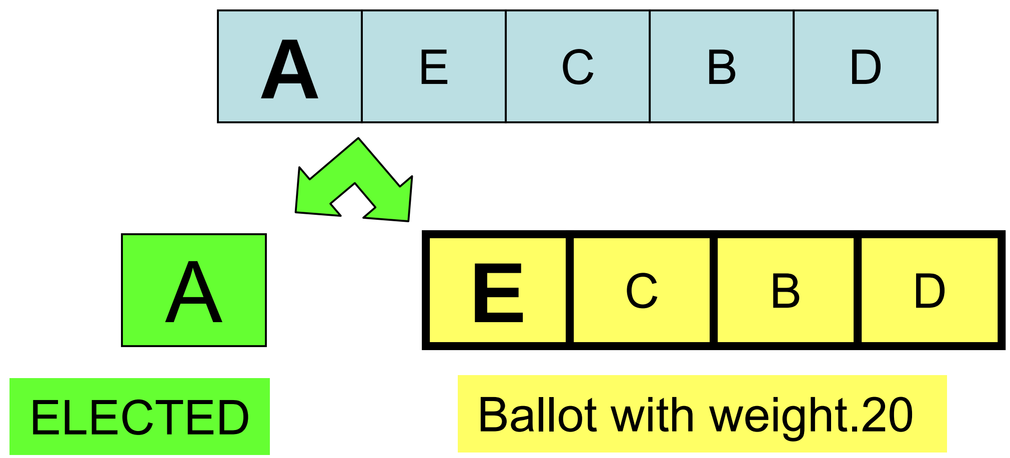
Model Dependence Check : 2001

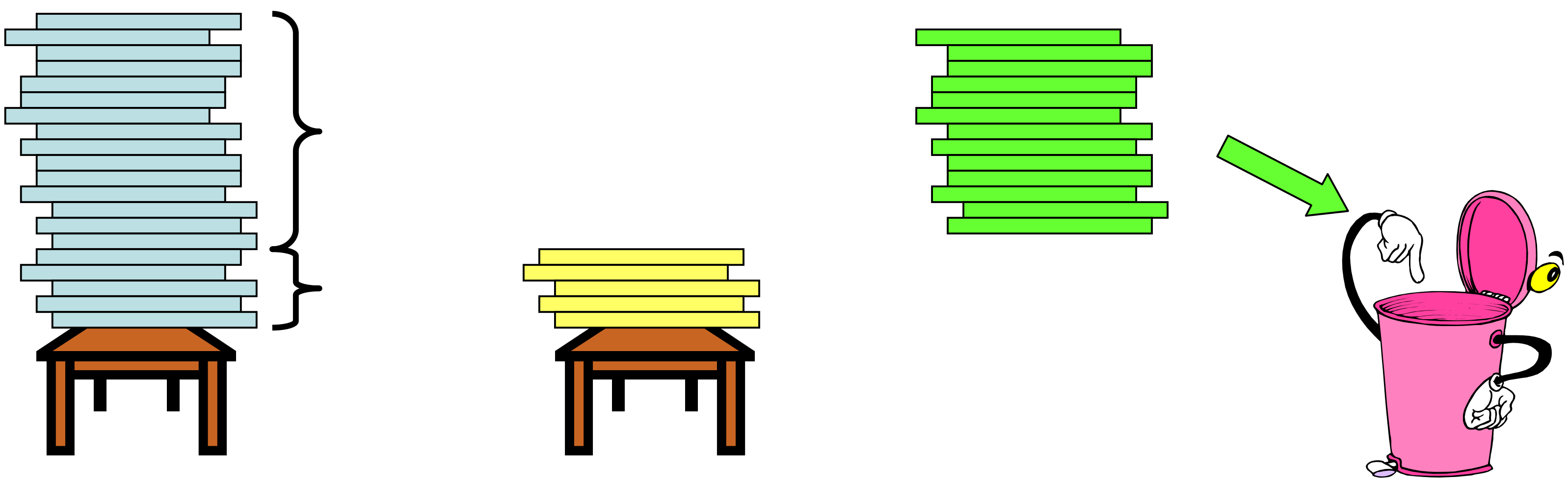
	STV	Majority	Borda	Plurality
All partial rankings	**5 53 531 5312**	53124	**53124**	**53124**
Partial rankings #4 or #5	5 53 531 5312	51324	51324	53124
Complete rankings	5 53 531 5312	51324	51324	53124
Size-independent model	**5 53 531 5312**	**51324**	51324	**53124**
Hybrid model	**5** 51 **531 5312**	51324	**53124**	**53124**

Hand Tallies (& some Computer Tallies):

Suppose:

- Droop Quota = 80
- A is elected with 100 votes
- Transfer 20% of each ballot won by A to next on the ballot





Monte Carlo Simulation of Probabilistic Tallies (100,000 repetitions)

- Can only affect multi-seat case
- 1998: very slight chance of “incorrect” outcomes for 4 seats
- 1999: matches deterministic tally throughout
- 2001: matches deterministic tally throughout
- 2000: matches deterministic tally for full set of partial ranking ballots

Monte Carlo Simulation of Probabilistic Tallies

If voters are required to rank at least 4 of the 5 candidates, 2000 election, 3-seat case:

{5,2,1} 2.8%

versus

{5,2,3} 97.2%

If voters are required to rank all 5 candidates, 2000 election, 3-seat case:

{5,2,1} 44.4%

versus

{5,2,3} 55.5%

Behavioral Social Choice

- Practical and Theoretical Challenge of Impartial Culture
- Limited Relevance of Majority Cycles:
  - Model Dependence vs. Cycles
  - Erroneous Assessment outweighs Cycles (sampling)
  - Generalized Domain Restrictions (Distributional Restrictions)
- Empirical Congruence among

  Condorcet & Borda (& Plurality winner)
- Sampling/Inference Framework
  - (Condorcet's) Majority
  - Borda, Plurality and other Scoring Rules
  - Approval Voting
- Testable models to reconstruct preferences from incomplete data