Power grid vulnerability analysis

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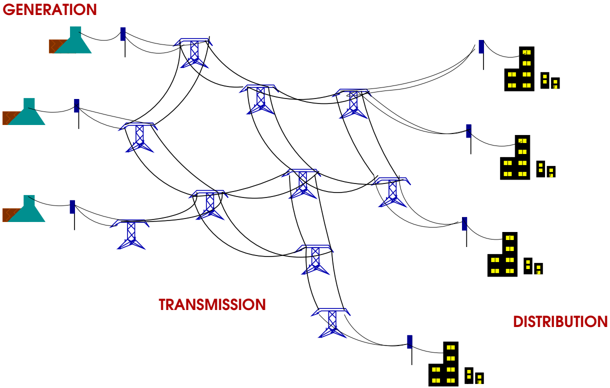
Dimacs 2010

Background: a power grid is three systems



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- Power grids are subject to “noise” which is difficult to model accurately.
- Power grids can exhibit non-monotone behavior as a result of control or adversarial actions.
- Power grids can cascade.

AC power flows – polar coordinates

→ Voltage at a node ("bus") k is of the form $U_k\, e^{j\theta_k}$, where $j = \sqrt{-1}$

→ Power flowing on edge ("line") $\{k, m\}$ equals $p_{km} + jq_{km}$, where

$$p_{km} = U_k^2\, g_{km} - U_k\, U_m\, g_{km} \cos\theta_{km} - U_k\, U_m\, b_{km} \sin\theta_{km}$$

$$q_{km} = -U_k^2\, (b_{km} + b_{km}^{sh}) + U_k\, U_m\, b_{km} \cos\theta_{km} - U_k\, U_m\, g_{km} \sin\theta_{km}$$

Here, $\theta_{km} \doteq \theta_k - \theta_m$

g_{km}, b_{km}, b_{km}^{sh} are known *parameters* (series conductance, series reactance, shunt susceptance)

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and, ideally, meet thermal constraints (flow limits) on the power lines

Research challenges

$\longrightarrow$ Do we have **fast** and **reliable** algorithm for the power flow problem?

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What about the cases where multiple solutions exist?

- After a contingency has take place, or a control has been applied: which solution should be instantiated?
- What if all solutions are “bad”?

Solution methodologies

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- **New result:** Low et al (2010). Some (many?) optimal power flow problems can be solved using semidefinite programming.

Linearized ("DC") model: $sin(\theta_i - \theta_j) \approx (\theta_i - \theta_j)$ for $\theta_i \approx \theta_j$

A **power flow** is a solution f, θ to:

- $\sum_{ij} f_{ij} - \sum_{ij} f_{ji} = b_i$, for all i, where

  $b_i > 0$ for each generator i,

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Lemma: Given a choice for b with $\sum_i b_i = 0$ (a requirement), the system has a **unique** (in f) solution.

A quote from:

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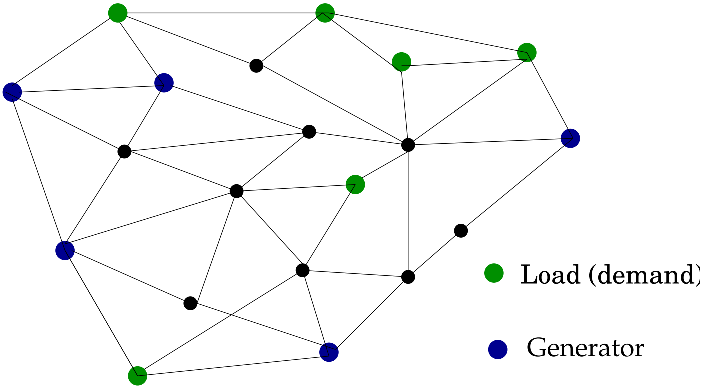
Cause 1 was "inadequate system understanding" – stated 20 times

Cause 2 was "inadequate situational awareness" – stated 14 times

Cause 3 was "inadequate tree trimming" – stated 4 times

Cause 4 was "inadequate RC diagnostic support" – stated 5 times

Cascades



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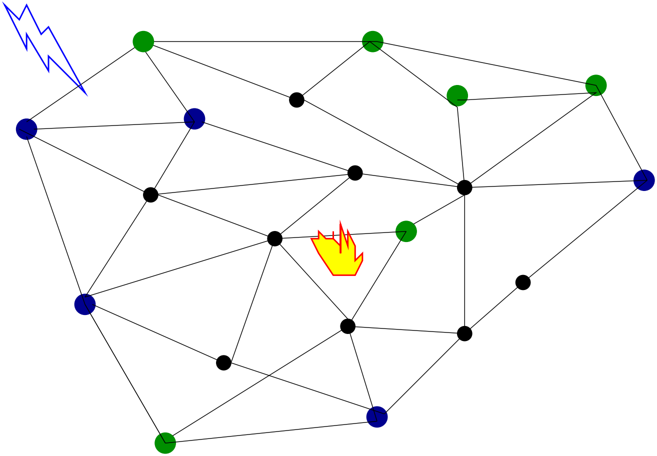

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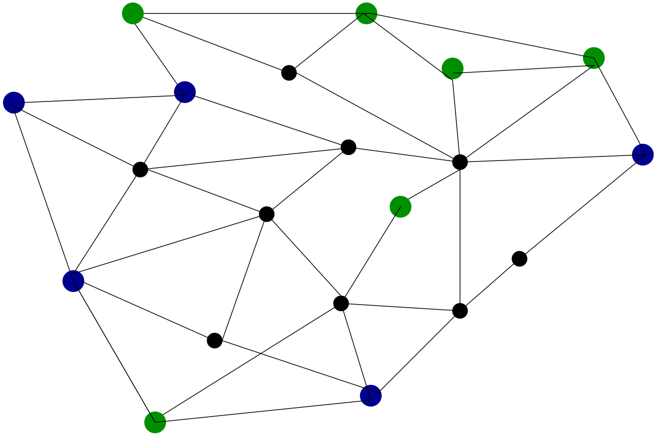

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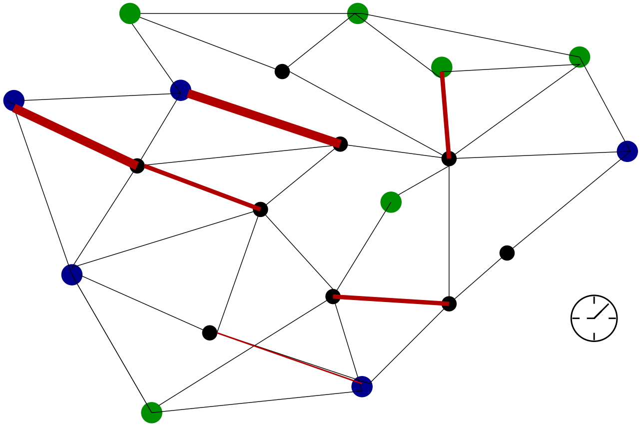

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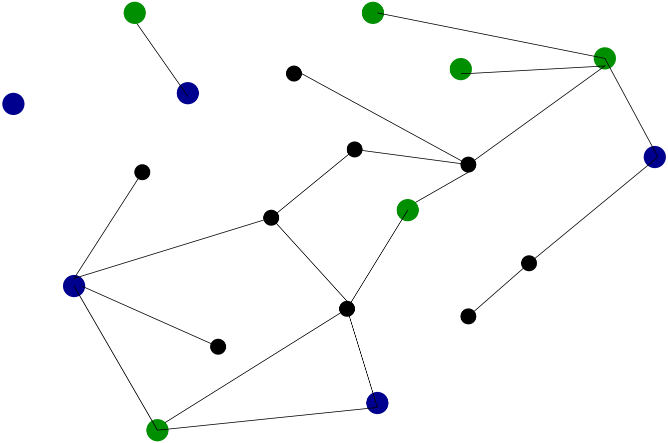

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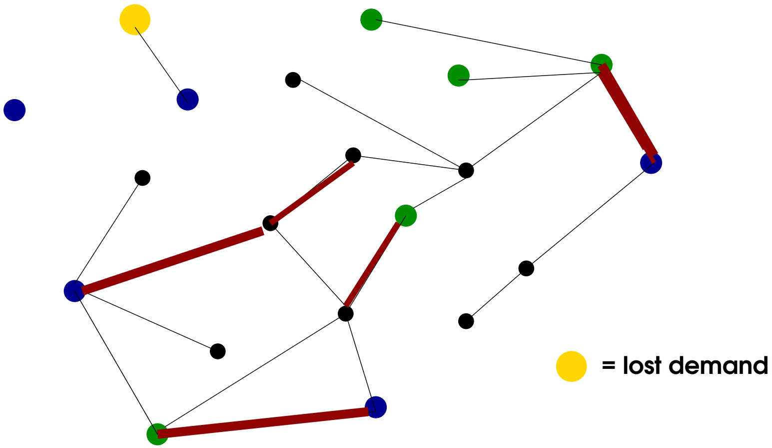

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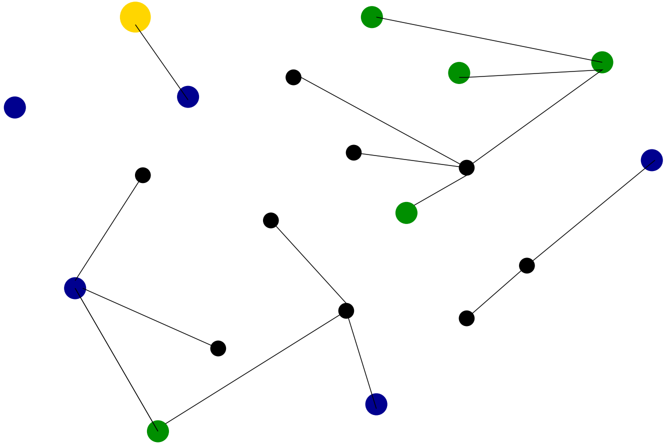

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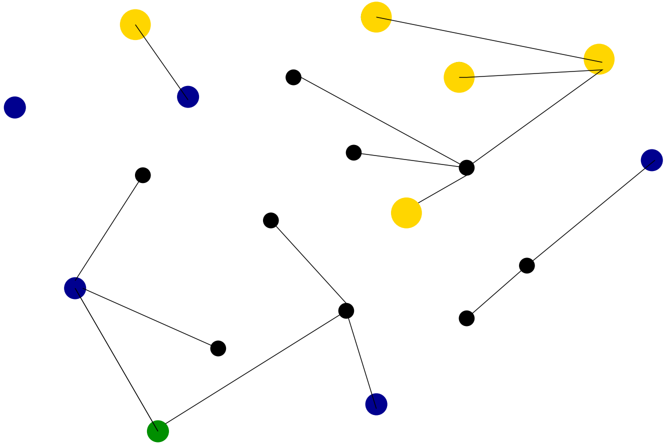

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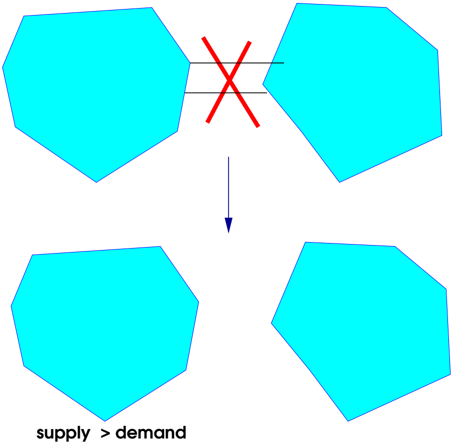

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if $(1-\gamma)u_e \leq \tilde{f}^r_e \leq u_e$ then e fails **with probability 1/2**

Formal cascade model (Dobson et al)

$\longrightarrow$ Initial outage event takes place (an “act of God”).

For $r = 1, 2, \ldots,$

1. Reconfigure demands and generator output levels.
2. New power flows are instantiated.
3. The next set of outages takes place.
   (Stochastic or history-dependent criterion)

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→ If no more faults occur or too much demand has been lost, STOP

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At round R, reduce demands so as to remove any line overloads.

Deterministic, no history model

"Optimal" control via integer programming formulation

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“Optimal” control via integer programming formulation ?

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"Optimal" control via integer programming formulation ?

- f_j^r = flow on arc j at round r
- y_j^r = $\mathbf{1}$, if arc j fails in round r, $\mathbf{0}$ otherwise
- d_j^r = demand at node i in round r
- and many other variables

$$\max \sum_{i \in \mathcal{D}} d_i^R$$

Subject to:

$$\sum_{j \in \delta^+(i)} f_j^r - \sum_{j \in \delta^-(i)} f_j^r = \begin{cases} s_i^r & i \in \mathcal{G} \\ -d_i^r & i \in \mathcal{D} \\ 0 & \text{otherwise} \end{cases} \quad \forall\, 1 \le r \le R \tag{1}$$

$$f_j^r = \pi_j^r - \nu_j^r \quad \forall\, j \in \mathcal{A} \text{ and } 1 \le r \le R \tag{2}$$

$$\pi_j^r \le \tilde{D} p_j^r, \quad \nu_j^r \le \tilde{D} n_j^r, \quad \forall\, j \in \mathcal{A} \text{ and } 1 \le r \le R \tag{3}$$

$$p_j^r + n_j^r = 1 - \sum_{h=1}^{r-1} y_j^h, \quad \forall\, j \in \mathcal{A} \text{ and } 1 \le r \le R \tag{4}$$

$$\pi_j^r + \nu_j^r - u_j \le \tilde{D} y_j^r \quad \forall\, j \in \mathcal{A} \text{ and } 1 \le r \le R \tag{5}$$

$$\pi_j^r + \nu_j^r \ge u_j y_j^r \quad \forall\, j \in \mathcal{A} \text{ and } 1 \le r \le R-1 \tag{6}$$

$$\pi_j^R + \nu_j^R \le u_j \quad \forall\, j \in \mathcal{A} \tag{7}$$

$$|\phi_i^r - \phi_j^r - x_j f_j^r| \le M_j \sum_{h=1}^{r-1} y_j^h \quad \forall j \in \mathcal{A} \tag{8}$$

$$0 \le s_i^r \le \tilde{s}_i \quad \forall\, i \in \mathcal{G}, \quad 0 \le d_i^r \le \tilde{d}_i \quad \forall\, i \in \mathcal{D}, \tag{9}$$

$$p_j^r,\ n_{ij}^r,\ y_j^r = 0 \text{ or } 1, \quad \forall\, j \in \mathcal{A} \text{ and } 1 \le r \le R \tag{10}$$

$$0 \le \pi_j^r,\ 0 \le \nu_j^r, \quad \forall\, j \in \mathcal{A} \text{ and } 1 \le r \le R. \tag{11}$$

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At round $\boldsymbol{r}$,

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- If $\kappa > c_v^r$, demand at v reduced (scaled) by a factor

$$\max\left\{1,\ s_v^r\,(c_v^r - \kappa)\ +\ b_v^r\right\}.$$

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$$\max\left\{1,\ s_v^r\,(c_v^r - \kappa)\ +\ b_v^r\right\}.$$

The goal: pick control to maximize demand being served at the end of round R.

For each demand v, and round r, control $c_v^r,\ b_v^r,\ s_v^r$

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Then, equivalent problem:

- In round r, let $\alpha^r(K) \leq 1$ be chosen for each *component* of the network in round r
- If node $v \in$ component K, then its demand is scaled by $\alpha^r(K)$

Notation:

- $\hat{\beta}$ = supply/demand vector at time 0
- $\hat{f}$ = corresponding power flows at time 0
- $\Theta^R(t, \beta) : \mathcal{R}_+ \rightarrow \mathcal{R}_+$ = total demand, at the end of round R, using optimal control, if the supply/demand vector is $t\ \beta$

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- Robust/stochastic version?

General case: simulation-based optimization

Given a control vector $\tilde{u} = (c_v^r, b_v^r, s_v^r)$ (over all v and r),

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$\Theta(\tilde{u})$ is obtained through a simulation

Derivative-free optimization

Conn, Scheinberg, Vicente, others

Rough description:

- *Sample* a number of control vectors $\tilde{\boldsymbol{u}}$
- Use the sample points to construct a convex approximation to $\tilde{\boldsymbol{\Theta}}$
- Optimize this approximation; this yields a new sample point

Scalability to large dimensionality?

"First order" method

Given a control vector $\tilde{u}$

1. Estimate the "gradient" $g = \nabla\tilde{\Theta}(\tilde{u})$ through finite differences.

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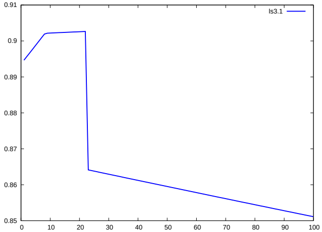
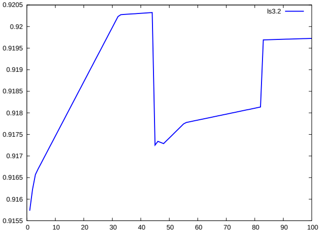
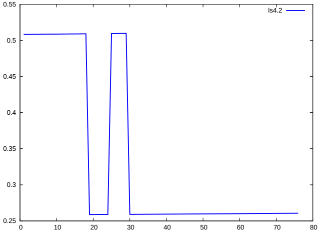
$\longrightarrow$ Easily parallelizable

Line searches







Current parallel implementation: boss-nerd

- Boss carries out search algorithm
- Nerds simulate cascades with given control
- Communication using Unix sockets

Scaling

Example: 10000 nodes, 19309 lines

5 gradient steps

8-core i7 CPUs (3 machines total)

cores	wall-clock sec
2	94379
4	47592
8	28136
16	14618
24	9918

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  1. Generate an interdiction of the grid (“initial event”)
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  3. At least three rounds of cascade after initial event

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(3) Perform full gradient search starting from the output in (2).

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- **K** random lines taken out
- highly loaded lines more likely to be taken out; connectivity preserved

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K	yield, (%) no control	yield, control	wallclock (sec)
1	90.04	95.03	134
2	12.54	50.13	87
5	32.94	81.05	107
10	2.02	36.97	97
20	1.64	27.84	159
50	0.83	16.96	209

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- It is best to stop the cascade in the first round
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(Answer: both wrong)

Details: cascade with 50 (highly loaded) random lines taken out

- No control $\Rightarrow$ yield = **0%**
- Optimal round 1 only constant control $\Rightarrow$ yield = **38%**
- Optimal scaling control $\Rightarrow$ yield = **45%**
- Plus segmented gradient seach $\Rightarrow$ yield = **50%**

Load distribution at time zero (load of arc $j = \frac{|f_j|}{u_j}$)

load	no. of arcs
1505	1
58	1
48	2
32	1
22	2
19	1
11	1
7	2
6	2
5	4
4	6
3	18
2	181

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Optimal round 1 scale = **0.51**, so **44** faults

Out-of-sample testing: use stochastic faults

at round r,

e **fails** if $u_e < \tilde{f}_e^r$,

e **does not fail** if $(1 - \gamma) u_e > \tilde{f}_e^r$, ($\gamma$ = tolerance)

if $(1 - \gamma) u_e \leq \tilde{f}_e^r \leq u_e$ then e fails **with probability 1/2**

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What is the impact of γ?

$\gamma = 0.03, 0.10, 0.20,$

10000 runs