Non-malleable codes in the split-state model

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- Consider a tamperable communication channel.

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We want

- Correctness: $\forall m,\ \text{Dec}(\text{Enc}(m)) = m$.
- Simulation: $\forall\, f \in \mathcal{F},\quad \exists\, g \in \mathcal{G},\quad$ where
  - $\mathcal{F}$ is large and realistic against attacks/channels.
  - $\mathcal{G}$ small and "easy to handle".

Example: Error-correcting codes

$$m \xrightarrow{\text{Enc}} c \xrightarrow{f} c^* \xrightarrow{\text{Dec}} m^* \qquad \textit{(Real)}$$

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$$\boxed{\quad \mathcal{F} \quad} \longrightarrow \boxed{\mathcal{G} \quad \mathrm{Id}(m) = m}$$

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$$\boxed{\begin{array}{r} \mathcal{F} \\ \Delta(c, c^*) <= \rho \end{array}} \longrightarrow \boxed{\begin{array}{r} \mathcal{G} \\ \text{Id}(m) = m \end{array}}$$

- $\mathcal{G} = \{Id\}$ is “easy to handle".
- $\mathcal{F}$ realistic/useful.
- Constructions: Hadamard, Reed-Solomon, Reed-Muller, etc..

Example: Error-detecting codes

$$m \xrightarrow{\text{Enc}} c \xrightarrow{f} c^* \xrightarrow{\text{Dec}} m^* \qquad \textit{(Real)}$$

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$$\boxed{\mathcal{F}: \quad \Delta(c, c^*) <= 2\rho} \longrightarrow \boxed{\mathcal{G}: \quad \begin{array}{l} \text{Id}(m) = m \\ \perp(m) = \perp \end{array}}$$

Same constructions as those for ECC.

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$$\boxed{\mathcal{F}: \quad f_\delta(c) = c + \delta} \longrightarrow \boxed{\mathcal{G}: \quad \begin{array}{l} \mathrm{Id}(m) = m \\ \perp(m) = \perp \end{array}}$$

AMD Codes: Application in robust fuzzy extractors and secret sharing [C**D**FPW12], NM-codes [DPW10], etc.

Error-correction/detection impossible

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$$\boxed{\begin{array}{l} \hfill \mathcal{F} \\ \text{Constant} \\ \text{functions} \\ f_{c^*}(c) = c^* \end{array}} \xrightarrow{??} \boxed{\begin{array}{l} \hfill \mathcal{G} \\ \mathrm{Id}(m) = m \\ \perp(m) = \perp \end{array}}$$

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$\mathcal{F}$: Constant functions

$f_{c^*}(c) = c^*$

$\not\longrightarrow$

$\mathcal{G}$:

$\text{Id}(m) = m$

$\perp(m) = \perp$

Let $c^* = \text{Enc}(m')$ for some fixed m'.

Thus, $\text{Dec}(c^*) = m' \notin \{m, \perp\}$.

Non-malleable codes

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Is NM "realistic/easy-to-handle"? When is it useful?

Application of Non-malleable codes

- Consider $\mathrm{Sign}_{sk}(\mathrm{userID}, m)$.
- Task: How to protect sk against tampering attack.
- Encode sk using non-malleable code.
- Thus, $sk^* = \mathrm{Dec}(f(\mathrm{Enc}(sk)))$ is either equal to sk or unrelated.
- Thus, cannot use $\mathrm{Sign}_{sk^*}(\mathrm{userID}, \cdot)$ to forge $\mathrm{Sign}_{sk}(\mathrm{userID}', \cdot)$.

Non-malleable codes: Formal Definition

Let (Enc, Dec) be a coding scheme with Enc **randomized**, and Dec deterministic, s.t. $\forall m \; \mathrm{Dec}(\mathrm{Enc}(m)) = m$,

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Note: T is independent of m.
Thus, intuitively, either $m^* = m$ or they are unrelated.

Which realistic families $\mathcal{F}$ can we tolerate?

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$$\boxed{\mathcal{F}_{\text{all}}} \quad \not\longrightarrow \quad \boxed{\begin{array}{l} \text{NM} \\ \text{Id}(m) = m \\ g_{m^*}(m) = m^* \end{array}}$$

Impossible [DPW10].

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Impossible [DPW10].

$\forall \; g \in \mathcal{F}_{\text{all}}, \;$ let $\; f(c) = \text{Enc}(g(\text{Dec}(c)))$.

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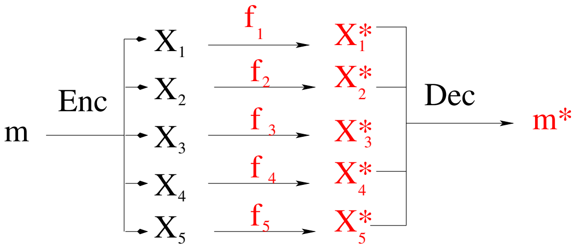
  YES (this talk). We show several constructions, including $t = 2$ and constant rate (i.e. code length is $\Theta(k)$).

NM-codes in the t-split state model



The coding scheme is **non-malleable** w.r.t. family $\mathcal{F}_{\text{t-split}}$, if

$\forall\ f_1, \ldots, f_t, \quad \exists T$ which is a **probabilistic combination** of:

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Common outline for our results: Non-malleable reductions [A**D**KO15]

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An NM-code for $\mathcal{F}$ can be viewed as $\mathcal{F} \Rightarrow$ NM, where NM is the function family comprising of

- constant functions
- identity function

Non-malleable Reduction: Composability

Theorem

For all $\mathcal{F}$, $\mathcal{G}$, $\mathcal{H}$, we have that

$$\mathcal{F} \Rightarrow \mathcal{G}, \text{ and } \mathcal{G} \Rightarrow \mathcal{H}, \text{ implies } \mathcal{F} \Rightarrow \mathcal{H}\ .$$

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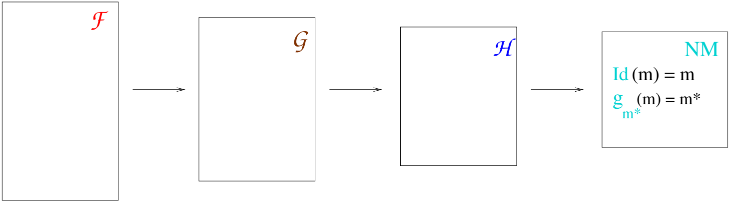
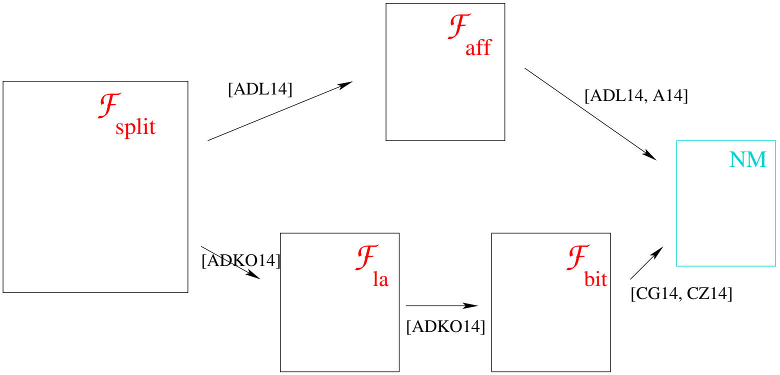


Make families simpler, until non-malleable.

Our results



ADL14 gives a scheme for encoding k-bit messages to $\Theta(k^7)$-bit codewords.

ADKO15 gives a scheme for encoding k-bit messages to $\Theta(k)$-bit codewords.

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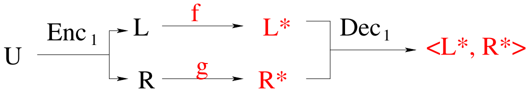
- Argue non-malleability only for a uniformly random message M.

$\mathcal{F}_{\text{split}} \Rightarrow \mathcal{F}_{\text{affine}}$

$U = U_{\mathbb{F}_p}$, $p = \text{poly}(k)$ is a prime

$\text{Enc}_1(U) = L, R \in \mathbb{F}_p^n$ s.t. $\langle L, R \rangle = U$, $n = \text{poly}(\log k)$.



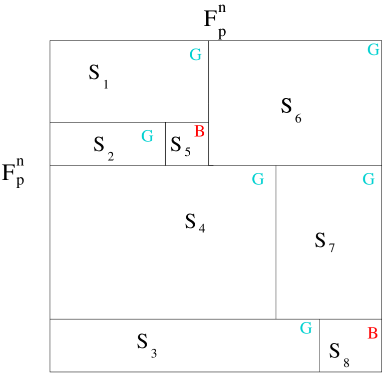
We show:

$$\forall \ f, g, \quad (\langle L, R \rangle, \ \langle f(L), g(R) \rangle) \ \approx \ (U, \ A_{f,g} U + B_{f,g}) \ .$$

Proof Step 1: Partitioning Lemma

Fix f, g. Let $\phi(L, R) := (\langle L, R\rangle,\ \langle f(L), g(R)\rangle)$

$$\mathcal{D} := \{D : D \text{ is a conv. comb. of } (U, aU + b),\ a, b \in \mathbb{F}\}$$



It is enough to partition $\mathbb{F}_p^n \times \mathbb{F}_p^n$ into "good" and "bad" rectangles such that

- If S is a good set, then $\phi(L, R)|_{(L,R)\in S}$ is close to some distribution in $\mathcal{D}$.
- The union of all bad sets has size much smaller than p^{2n}.

Our partitioning

We partition $\mathbb{F}_p^n \times \mathbb{F}_p^n$ into four type of rectangles.

- **Type 1**: $g(R) = a$ for some $a \in \mathbb{F}_p^n$. Then $\phi = (\langle L, R\rangle \ , \ \langle f(L), g(R)\rangle)$ is close to $(U_{\mathbb{F}_p}, \ \langle f(L), a\rangle)$ which belongs to $\mathcal{D}$.

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- **Type 3**: $f(L) = AL$ for some $A \in \mathbb{F}_p^{n\times n}$, and $A^T g(R) = cR + d$, for $c \in \mathbb{F}_p$, and $d \in \mathbb{F}_p^n$, which implies

$$\phi = (\langle L, R\rangle\ ,\ \ c\langle L, R\rangle + \langle L, d\rangle)\ ,$$

which is in $\mathcal{D}$ if the partition S is large enough.

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We show that the set $\mathbb{F}_p^n \times \mathbb{F}_p^n$ can be partitioned into sets of the above four types such that the total size of "bad" sets is much smaller than p^{2n}.

Main tools used for the proof

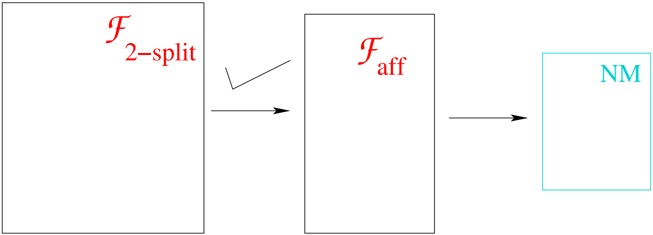
- Linearity test [BSG94, Sam07, San12] : For $f : \mathbb{F}_p^n \mapsto \mathbb{F}_p^n$

  $$\Pr(f(L) - f(L') = f(L - L')) \geq \varepsilon \;\; \Rightarrow \;\; \exists A \;\; \Pr(f(L) = AL) \geq p^{-\log^6(1/\varepsilon)} .$$

  - We need a generalized version, for which we show that essentially the same proof works.

- Hadamard Extractor: $\langle \cdot, \cdot \rangle$ is a strong 2-source extractor.

- (Generalized) Vazirani's XOR Lemma:
  (X_1, X_2) is close to uniform in $\mathbb{F}_p \times \mathbb{F}_p$ if and only if $aX_1 + bX_2$ is close to uniform in $\mathbb{F}_p$ for all $a, b \in \mathbb{F}_p$, not both zero.


$\mathcal{F}_{2\text{-split}}$
$\mathcal{F}_{\mathrm{aff}}$
NM

Step two: $\mathcal{F}_{\text{affine}} \Rightarrow$ NM

$$m \xrightarrow{\text{Enc}_2} c \xrightarrow{h_{A,B}} Ac + B \xrightarrow{\text{Dec}_2} m^*$$

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Define an *affine-evasive set* $\mathcal{C}$ of $\mathbb{F}_p$ as a set s.t. for C chosen uniformly at random from $\mathcal{C}$,

$\forall\ a,\ b \in \mathbb{F}_p \times \mathbb{F}_p$ s.t. $a \neq 0$ and $(a, b) \neq (1, 0)$

Step two: $\mathcal{F}_{\text{affine}} \Rightarrow$ NM

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Partition $\mathcal{C}$ into equal parts $\mathcal{C}_1, \ldots, \mathcal{C}_{|\mathcal{M}|}$ and define

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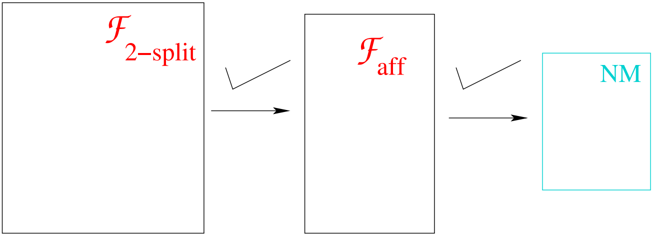
An affine-evasive set construction modulo p [A14]:

$$S := \left\{ \frac{1}{q} \pmod{p} \;\middle|\; q \text{ is prime},\ \ q < \frac{p^{1/4}}{2} \right\}\ .$$

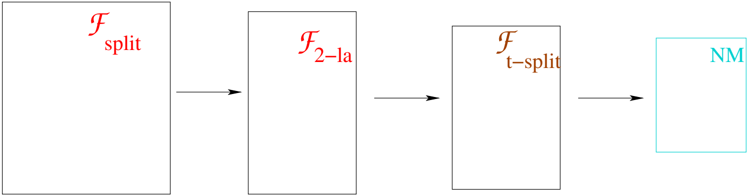


$\mathcal{F}_{2\text{-split}}$
$\mathcal{F}_{\text{aff}}$
NM

Our second result [ADKO15]

NM-reduction from 2-split to t-split for large constant t

k-bit messages $\Longrightarrow \Theta(k)$-bit codewords.



Some natural tampering families

- $\mathcal{S}_n^t$ denotes the tampering family in the *t-split-state model* with each part having length n.

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- $\mathcal{S}_n^t$ denotes the tampering family in the *t-split-state model* with each part having length n.
- $\mathcal{L}_n^{\leftarrow t}$ denotes the class of *lookahead manipulation functions* l that can be rewritten as $l = (l_1, \ldots, l_t)$, for $l_i : \{0,1\}^{in} \to \{0,1\}^n$, where

$$l(x) = l_1(x_1)||l_2(x_1, x_2)|| \ldots ||l_i(x_1, \ldots, x_i)|| \ldots ||l_t(x_1, \ldots, x_t)$$

.

$\mathcal{S}^2_{3tn}$ $(\Rightarrow)$ $\mathcal{L}_n^{\leftarrow t}$

Quentin: Q, S_1		Wendy W
S_1	$\xrightarrow{S_1}$	
	$\xleftarrow{R_1}$	$R_1 = \mathrm{Ext}(W; S_1)$
$S_2 = \mathrm{Ext}(Q; R_1)$	$\xrightarrow{S_2}$	
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$S_t = \mathrm{Ext}(Q; R_{t-1})$	$\xrightarrow{S_t}$	
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Figure: Alternating Extraction

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- $\mathrm{Dec}((Q, S_1), W) = S_1, \ldots, S_t$.
- Alternating Extraction Theorem [DP07] shows:

$$S_{i+1}, \ldots, S_t \approx U, \text{ given } S_1, \ldots, S_i, S'_1, \ldots, S'_i \; .$$

- Intuitively, this implies

$$\forall i, \quad S'_i \text{ is independent of } S_{i+1}, \ldots, S_t \; .$$

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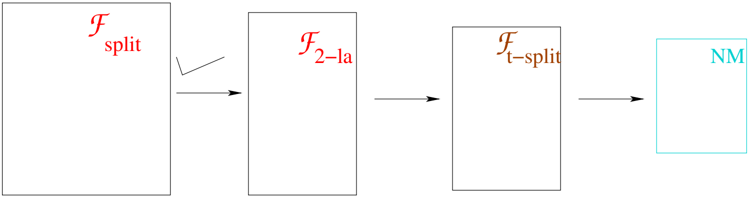
$\mathcal{F}_{\text{split}}$
$\mathcal{F}_{2\text{-la}}$
$\mathcal{F}_{\text{t-split}}$
NM

$\mathcal{L}_{2t\ell}^{\leftarrow t} \times \mathcal{L}_{2t\ell}^{\leftarrow t} \Rightarrow \mathcal{S}_{\ell}^{t}$

Define the reduction by the following:

$$\mathrm{Dec}(L, R) := (\langle L_t, R_1\rangle, \langle L_{t-1}, R_2\rangle, \ldots \langle L_1, R_t\rangle) \ ,$$

where $\langle\cdot,\cdot\rangle$ is the ℓ-bit inner product (interpreting L_i, R_i as elements of $\mathbb{F}_{2^n}^{2t}$.

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Intuitively, the result follows from the observation (using the Hadamard two-source extractor property) that $b_i = \langle L_{t-i+1}, R_i\rangle$ is close to uniform given $b'_j = \langle L'_{t-j+1}, R'_j\rangle$ for $j \neq i$.

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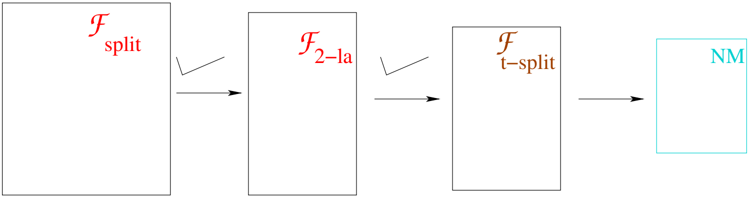
Formal proof: More subtle due to joint distributions. See paper.



$\mathcal{F}_{\text{split}}$
$\mathcal{F}_{2\text{-la}}$
$\mathcal{F}_{\text{t-split}}$
NM

Summarizing and Composing the two reductions

We showed:

- $\mathcal{S}^2_{3tn}$ $(\Rightarrow)$ $\mathcal{L}^{\leftarrow t}_{n}$

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This implies:

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Our work combined with an independent work [CZ14] gives constant rate 2-split NM-Codes.

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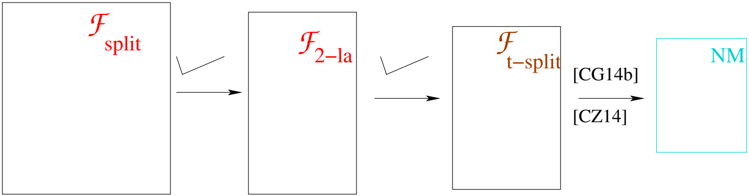
[CZ14] showed: $\mathcal{S}^{10}_{\Theta(\ell)} \Rightarrow \mathsf{NM}_\ell$.

This combined with our reduction gives:

$$\mathcal{S}^{2}_{\Theta(\ell)} \Rightarrow \mathsf{NM}_\ell \; .$$



Future work

The following are major open questions in this area.

- Optimizing the rate of the NM-code construction in split-state model, either by improving our proof techniques, or using some other construction.
- Proposing other useful tampering models.
- Other applications of NM-codes. There has been some recent work in this direction by [CMTV14] and [AGMPP14].

Thank You