The Exact Round Complexity of Secure Computation

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joint work with
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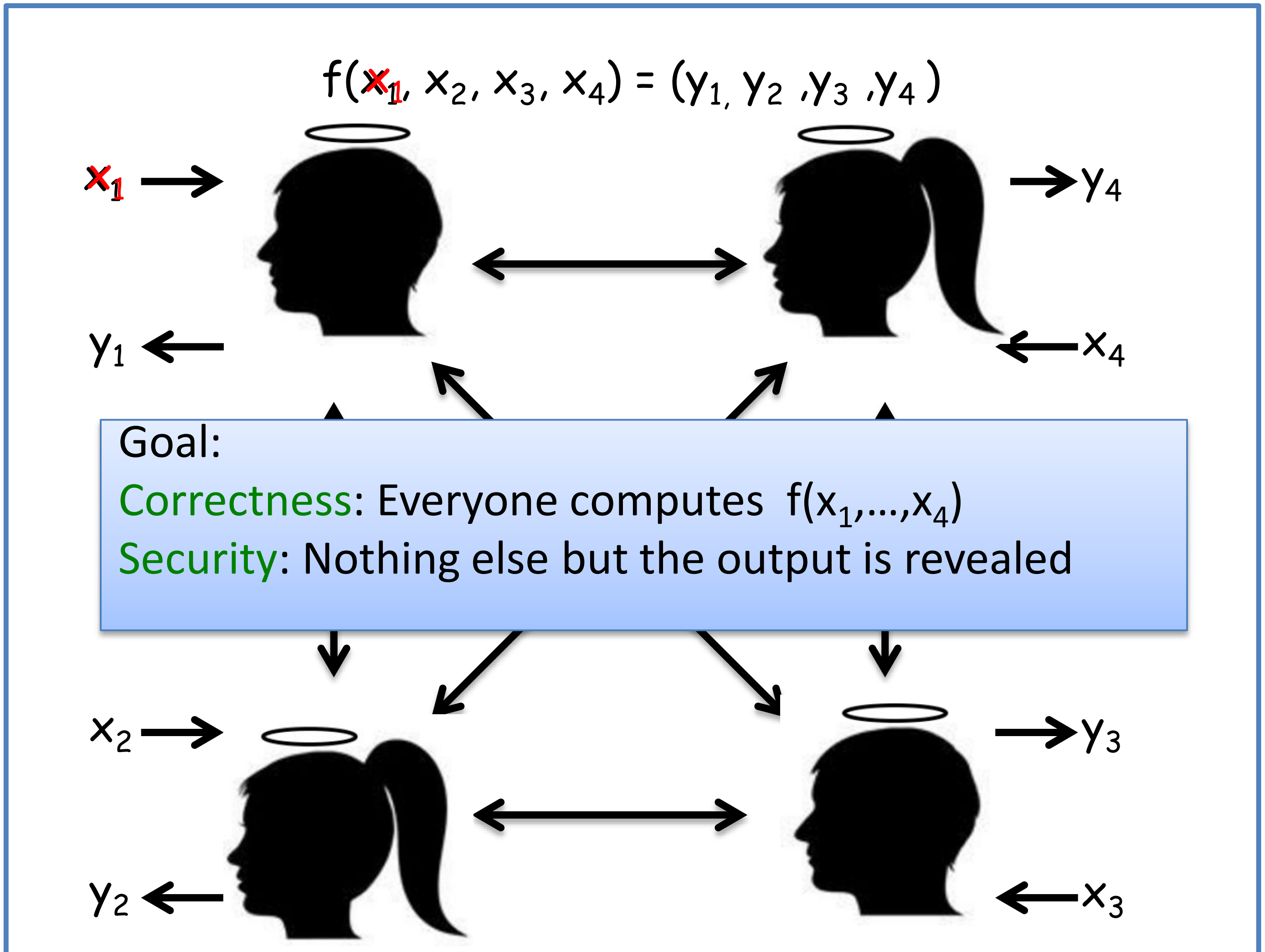
Background: Secure Multi-Party Computation



Goal:

Correctness: Everyone computes $f(x_1,\ldots,x_4)$

Security: Nothing else but the output is revealed

Adversary:

PPT

Malicious

Static

Motivating Questions

Lower bounds on the round complexity of secure protocols.

Construct **optimal** round secure protocols.

State of the Art: Information-Theoretic Setting

Communication Complexity	Round Complexity
$O(n\|C\|)$	$O(depth_C)$

State of the Art: Information-Theoretic Setting

Communication Complexity	Round Complexity
$\Omega(n\|C\|)$ [DN**P**R16]	$\Omega(\text{depth}_C)$ [DN**P**R16]

Novel approach must be found to construct **O(1)** round protocols
(that beat the complexities of BGW, CCD, GMW, SPDZ etc.)

State of the Art: Computational Setting

Communication Complexity	Round Complexity	
<<|C|	2PC	MPC

FHE

State of the Art: Computational Setting

Round Complexity	
2PC	MPC
5 rounds [KO04,ORS15]	O(1)*

No CRS
No Preprocessing

*[BMR90,KOS03,Pas04,DI05,DI06,PPV08,IPS08,Wee10,Goy11,LP11,GLOV12]

State of the Art: Computational Setting

Round Complexity	
2PC	MPC
5 rounds [KO04,ORS15]	O(1)

What is the exact round complexity of secure MPC?

Standard Communication Model for MPC

Simultaneous Message Exchange Channel

Standard Communication Model for MPC

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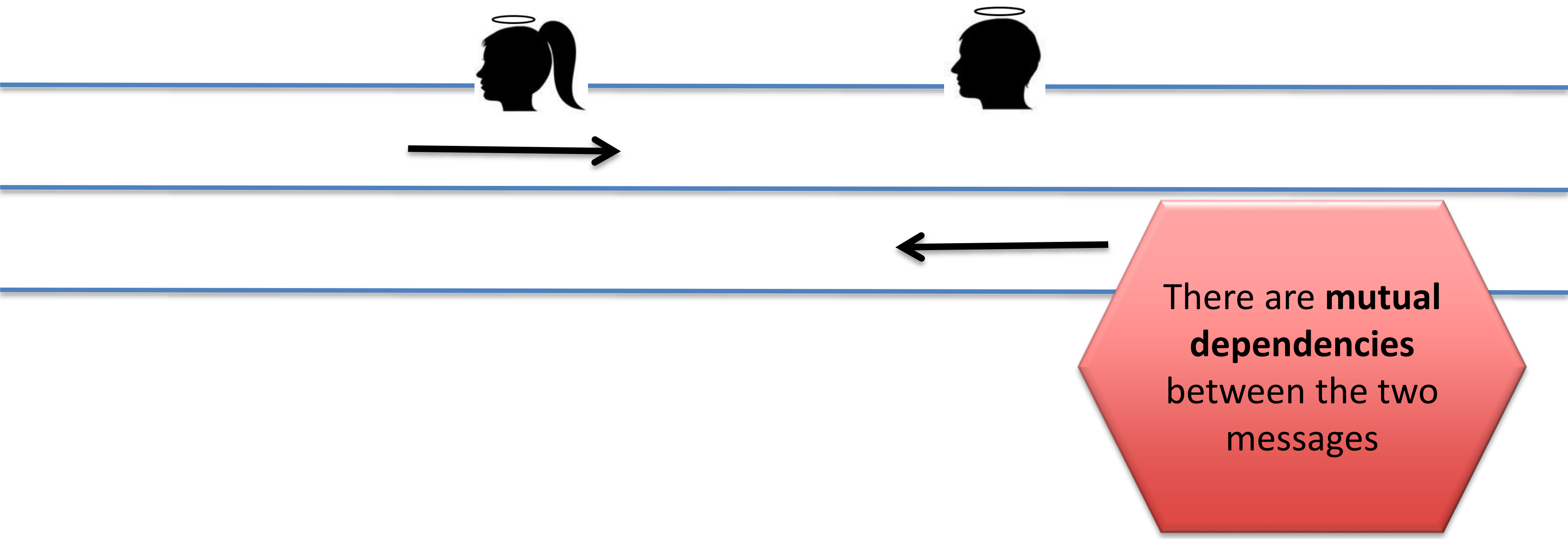
Simultaneous Message Exchange Channel

Standard Communication Model for MPC

Simultaneous Message Exchange Channel

Communication Model for 2PC

Non-Simultaneous Message Exchange Channel



State of the Art: Computational Setting

Round Complexity	
2PC	MPC
5 rounds [KO04]	O(1)

What is the exact round complexity of secure **MPC**?

How many simultaneous message exchange rounds are necessary for **2PC**?

Our Results

Round Complexity	
2PC	MPC
5 rounds [KO04]	O(1)

- **(3-round Impossibility)**:

There does **not** exist a **3-round protocol** for the **two-party coin-flipping** functionality.

Our Results

Round Complexity	
2PC	MPC
max(4,k+1)[1]	O(1)

[1] k-round NMCOM

Suppose that there exists a k-round NMCOM scheme; then

- **(2PC)**: there exists a **max(4, k + 1)-round** protocol for securely realizing every two-party functionality.

The use of NMCOM is not a coincidence [LPV09,Goy11,LP11,LPTV10,GLOV12]

Our Results

Round Complexity	
2PC	MCF*
max(4,k+1)[1]	max(4,k+1)

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Suppose that there exists a k-round NMCOM scheme; then

- **(2PC)**: there exists a **max(4, k + 1)-round** protocol for securely realizing every two-party functionality;
- **(MPC)**: there exists a **max(4, k + 1)-round** protocol for securely realizing the multi-party coin-flipping functionality.

Our Results

Round Complexity	
2PC	MCF*
max(4,k+1)[1]	max(4,k+1)

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Suppose that there exists a k-round NMCOM scheme; then

- **(2PC)**: there exists a **max(4, k + 1)-round** protoc -
  party functionality;
- **(MPC)**: there exists a **max(4, k + 1)-round** proto -
  party coin-flipping functionality.

Four rounds are both **necessary and sufficient** for both the results based on 3-round NMCOMs [PPV08,GPR16,COSV16].

Outline

1. Lower bound on the two-party coin-flipping.
2. 4-round 2PC protocol.

Our Results

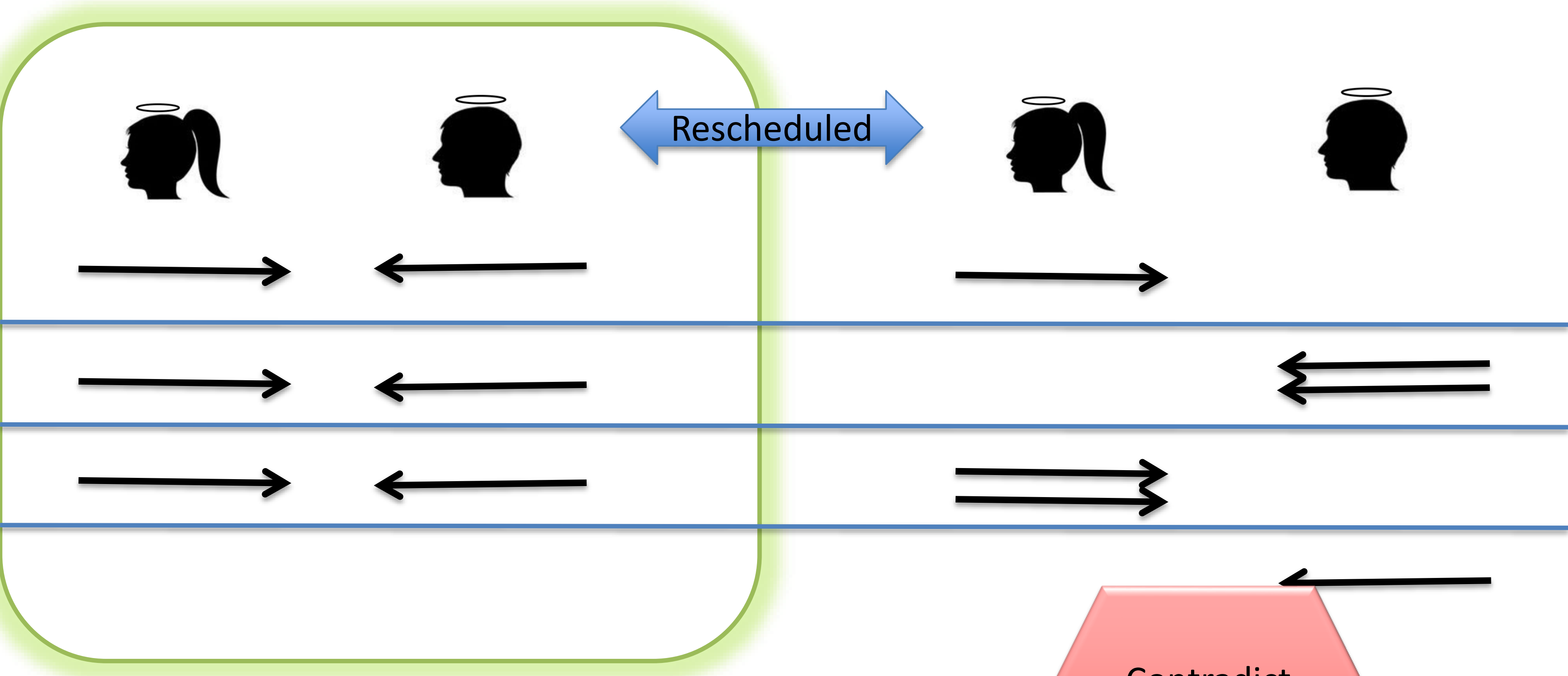
Theorem 1. There does **not** exist a **3-round protocol** for the **two-party coin-flipping** functionality

- for tossing $\omega(\log \lambda)$ coins,
- with a black-box simulation,
- in the simultaneous message exchange model.

where λ is the security parameter

Proof (sketch)

Suppose that there exists a protocol which realizes simulatable coin-flipping in 3 rounds.



Contradict the result of [KO04]

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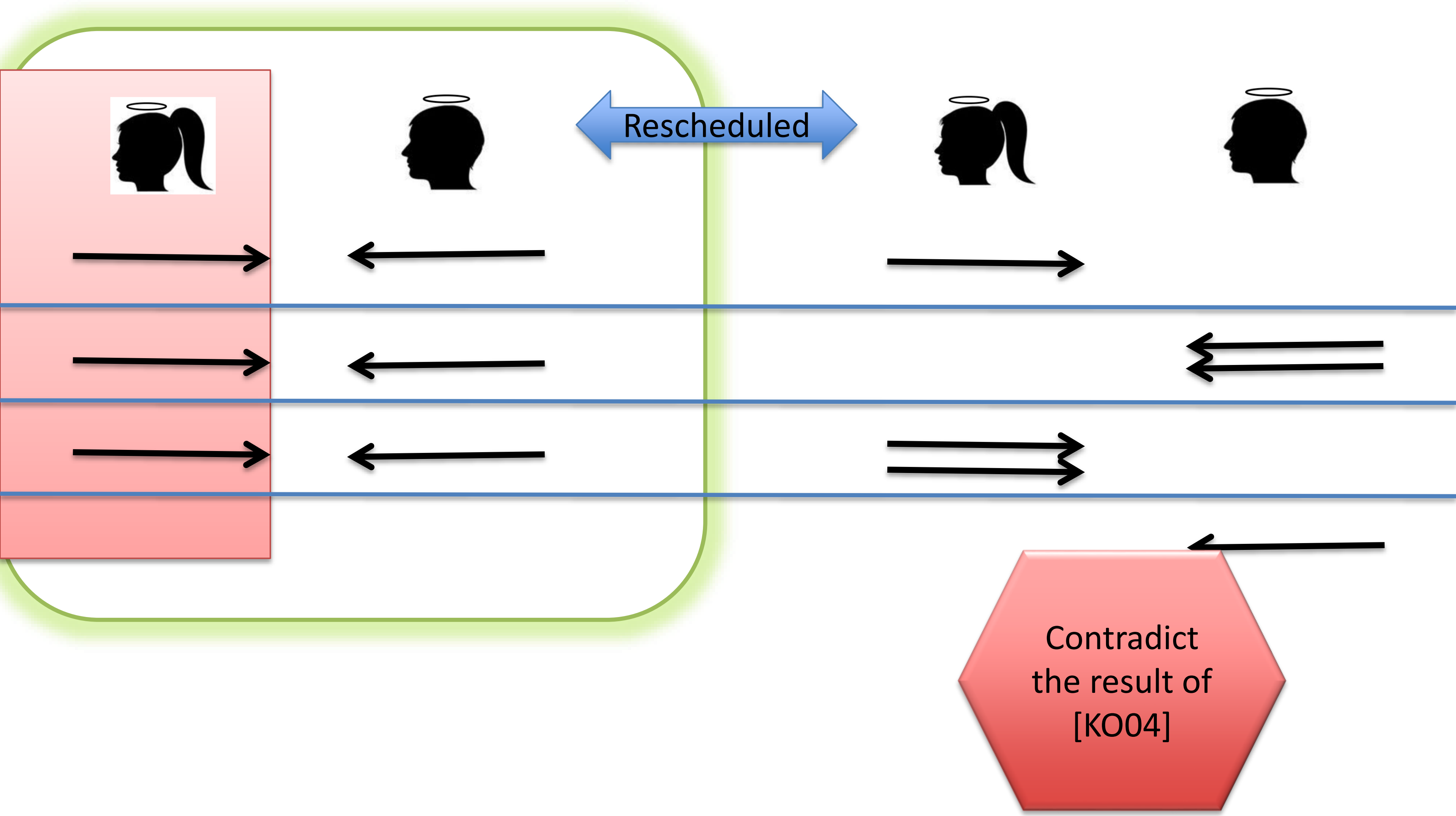

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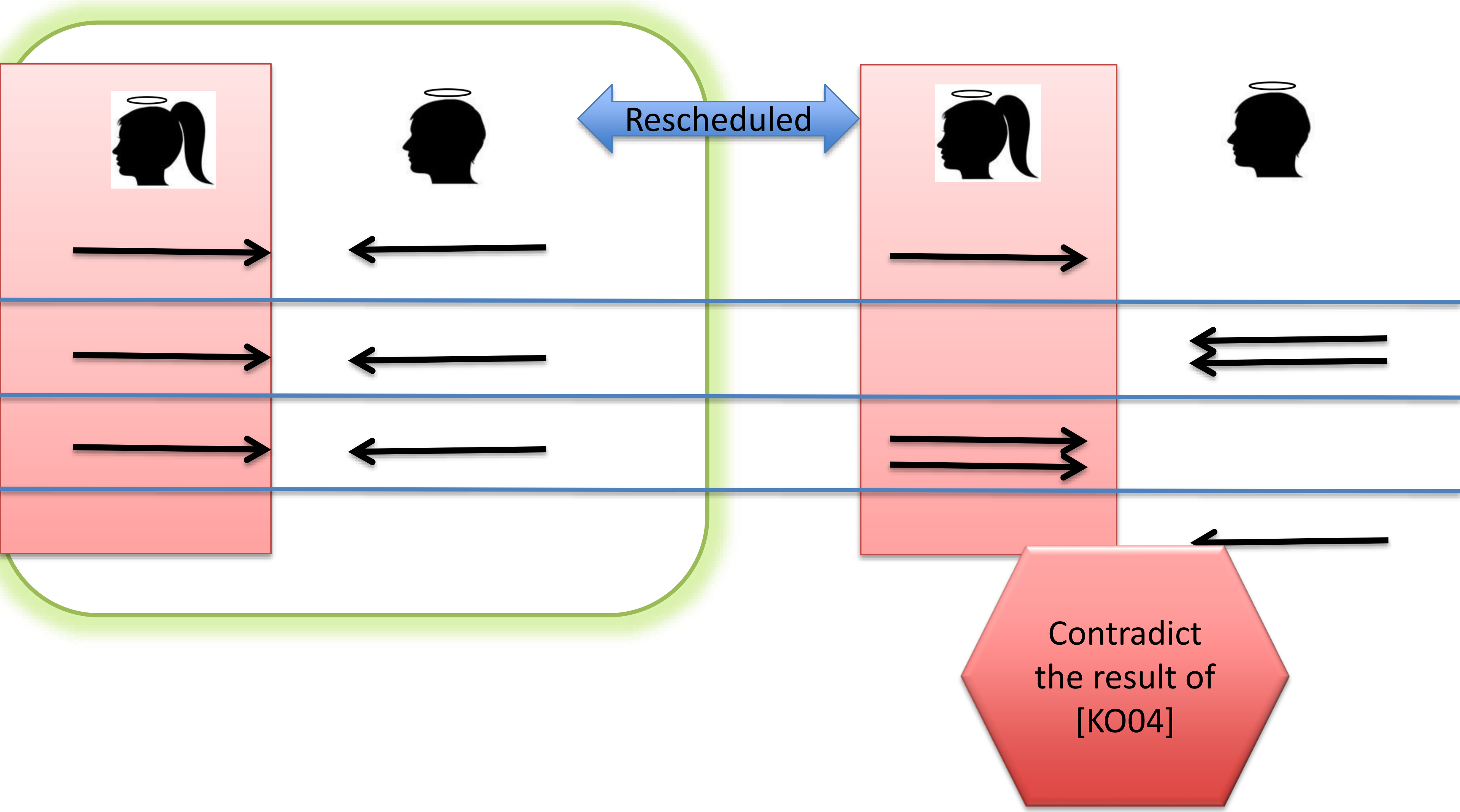

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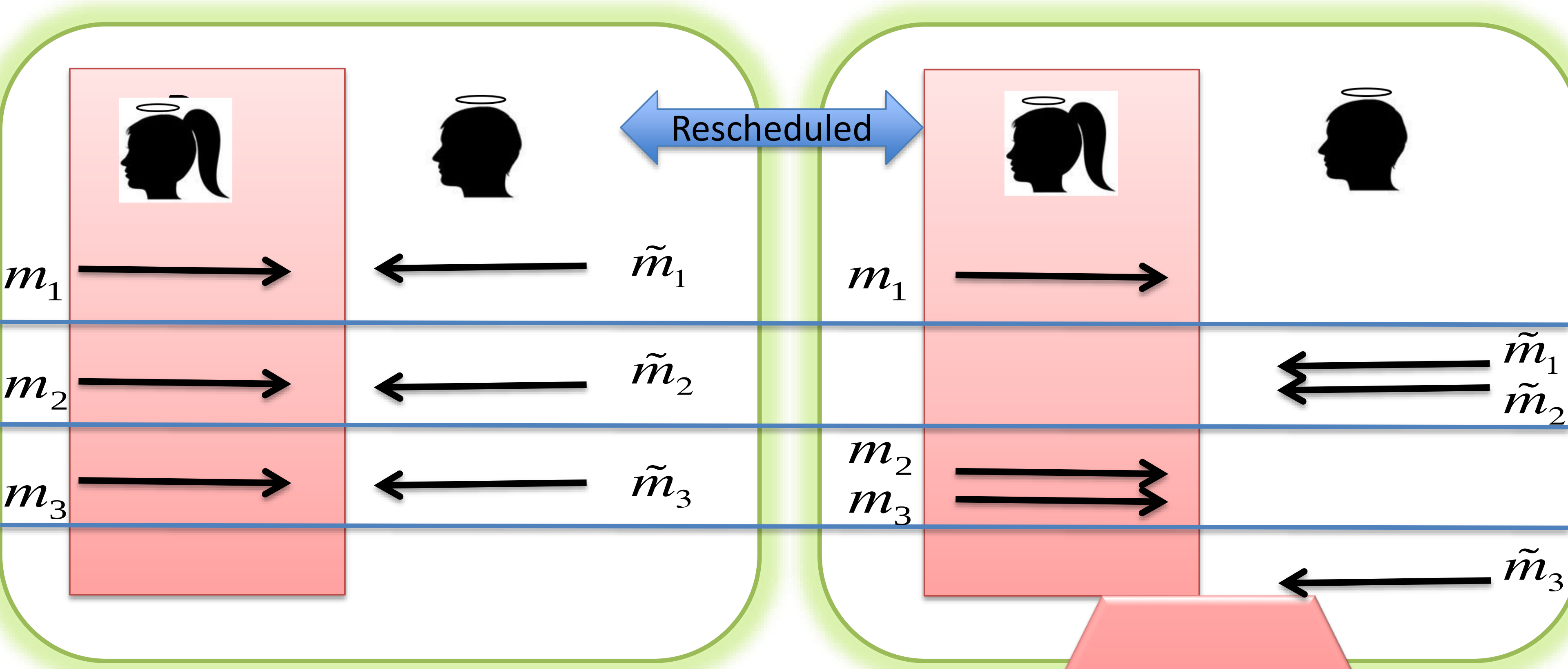
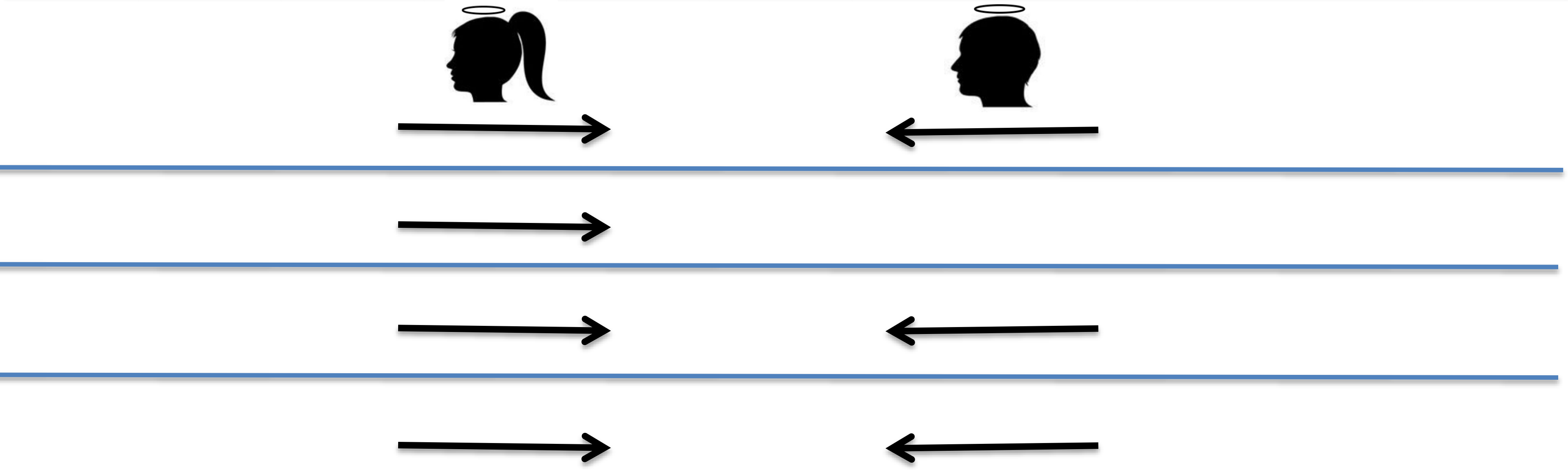


Contradict the result of [KO04]

Our Results

Theorem 2. There does **not** exist a **4-round protocol** for the **two-party coin-flipping** functionality

- for tossing ω(log λ) coins,
- with a black-box simulation,
- in the simultaneous message exchange model,
- with **at least one unidirectional round.**



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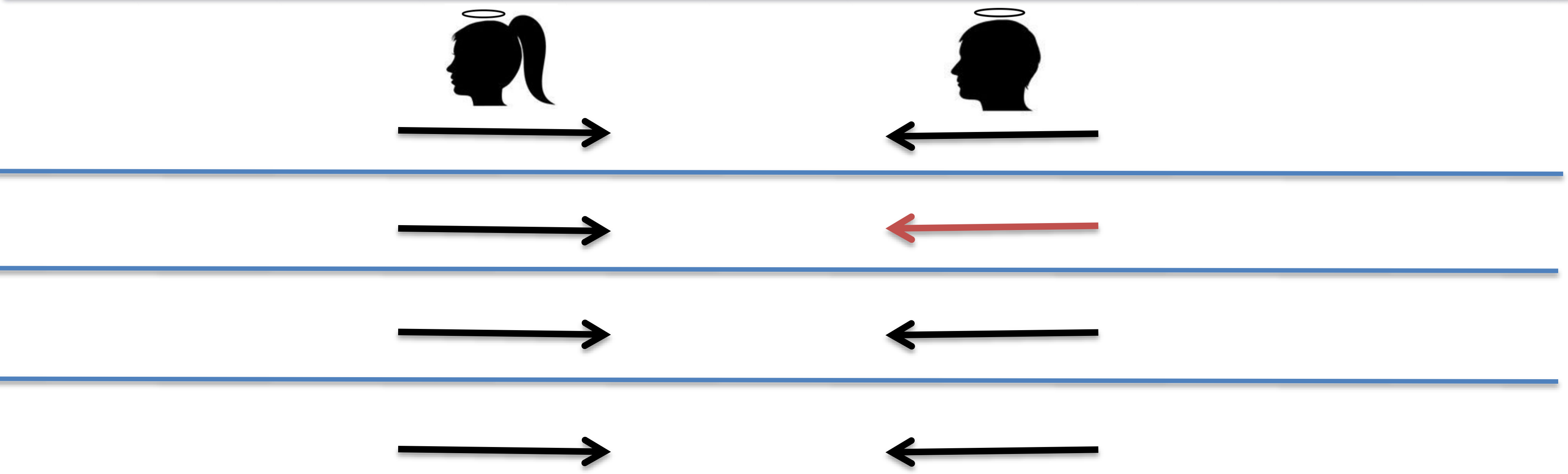
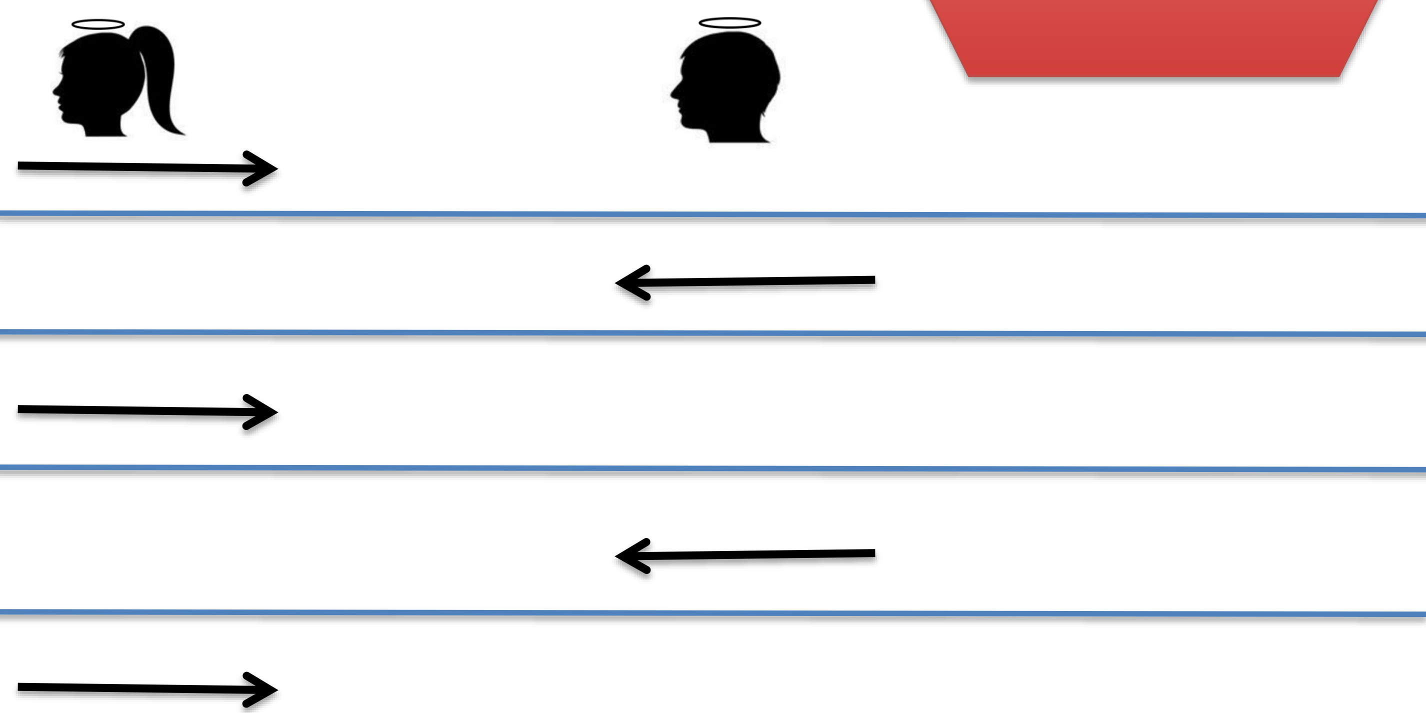


Our Approach for 2PC

Starting point: Katz-Ostrovsky (KO) protocol [KO04] which is a 4-round protocol for **only** one-sided functionalities and 5-rou… …d functionalities.

Is it still **5** rounds with simultaneous transmission?

5-round [KO04]:



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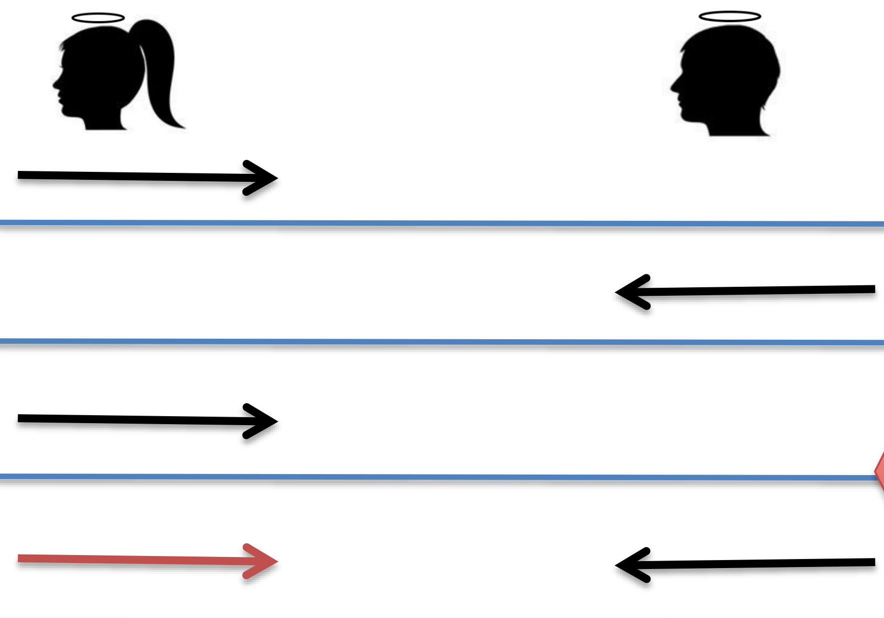
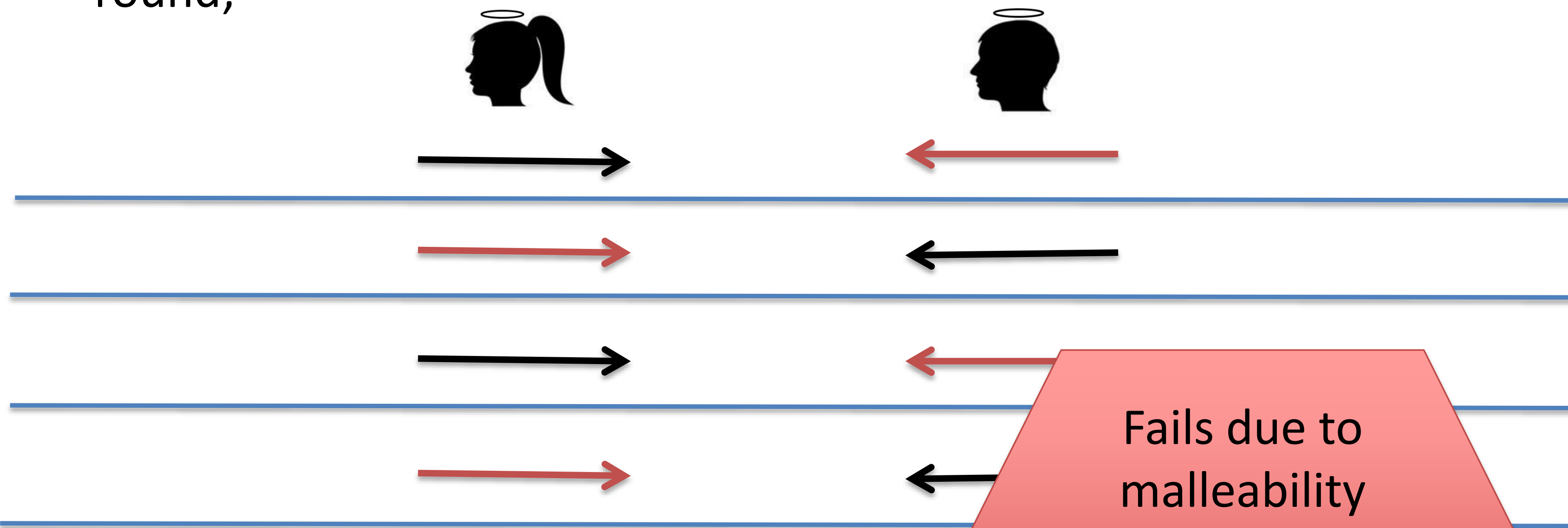
4-round attempt:



Such a 4-round protocol fails due to Theorem 2.

Our Approach for 2PC

Must use the simultaneous message exchange channel in each round;



Fails due to malleability and input consistency issues.

Run two executions of a 4-round protocol (party learns the output) in “opposite” directi

Our Approach for 2PC

Simultaneous Executions

+

3-round NMCOM

+

...

↓

4-round 2PC

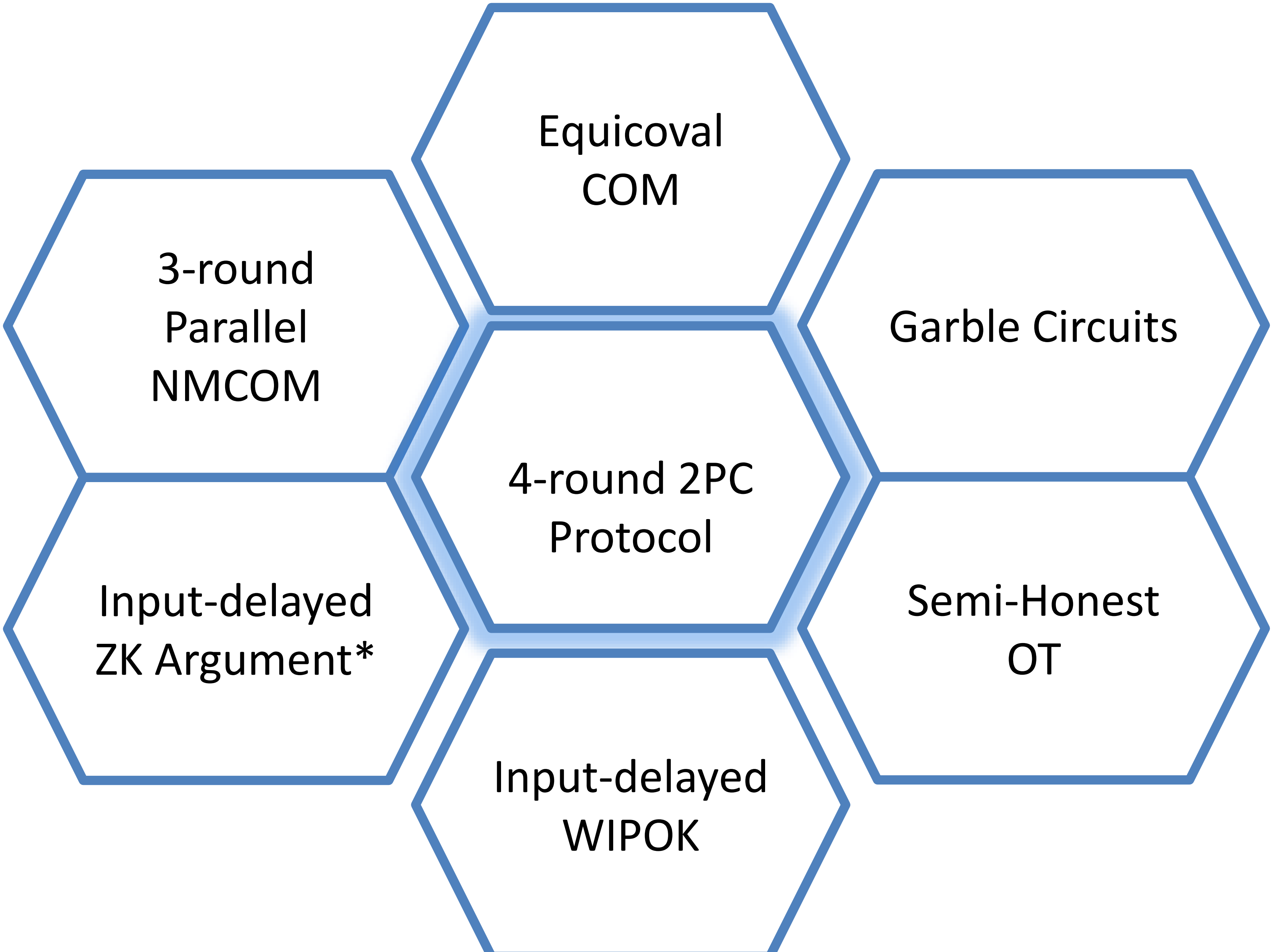
max(4, k + 1)-round 2PC protocols

Theorem 3.

TDP + k-round (parallel) **NMCOM → max(4, k + 1)**-round **2PC** protocol

- with black-box simulation,
- in the presence of a malicious adversary,
- in the simultaneous message exchange model.

Tools for our 2PC Protocol



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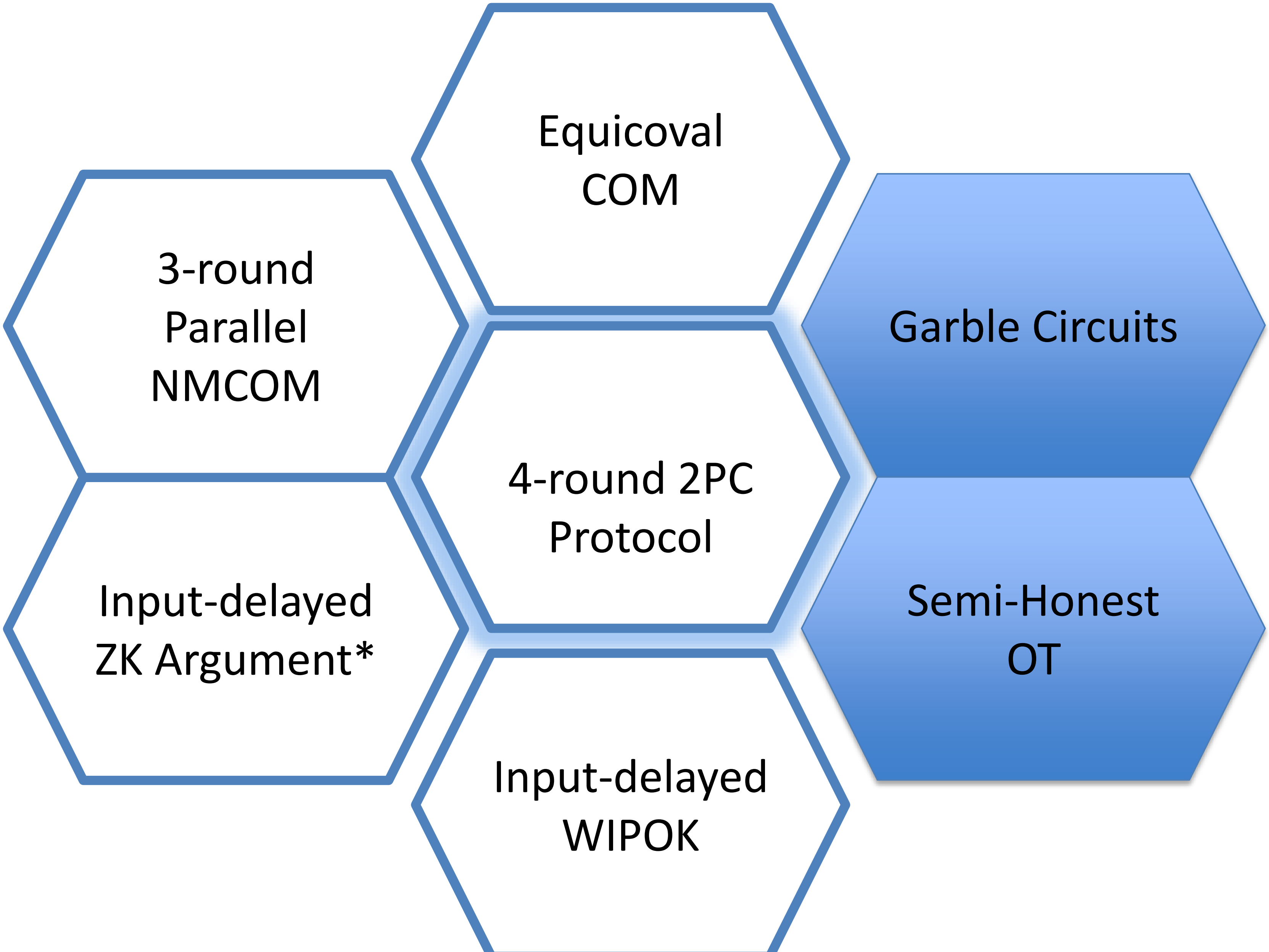
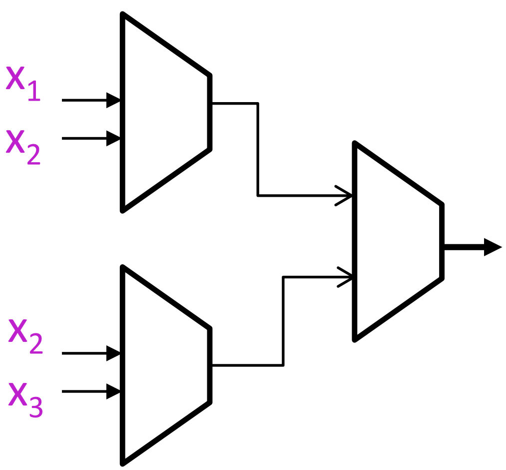
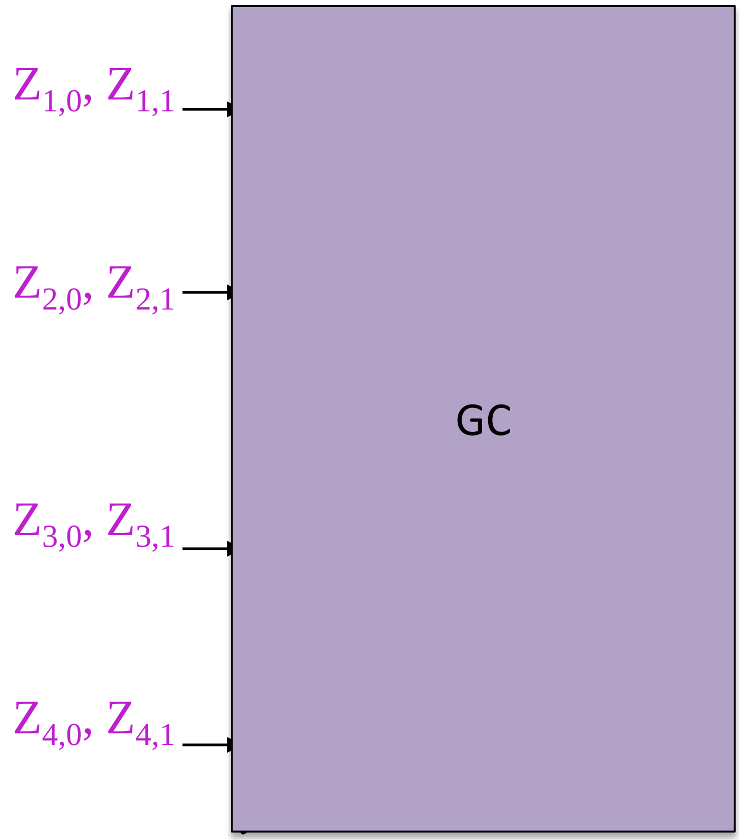


Garble Circuit Construction [Yao80]

Boolean Circuit C



Garbled Circuit GC



Pairs of λ-bit keys

Garble Circuit Construction [Yao80]

Boolean Circuit C

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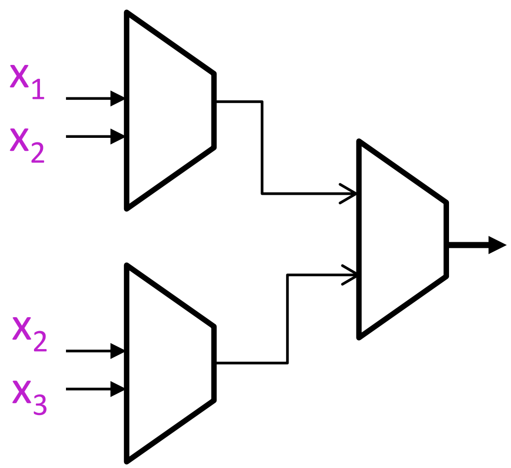

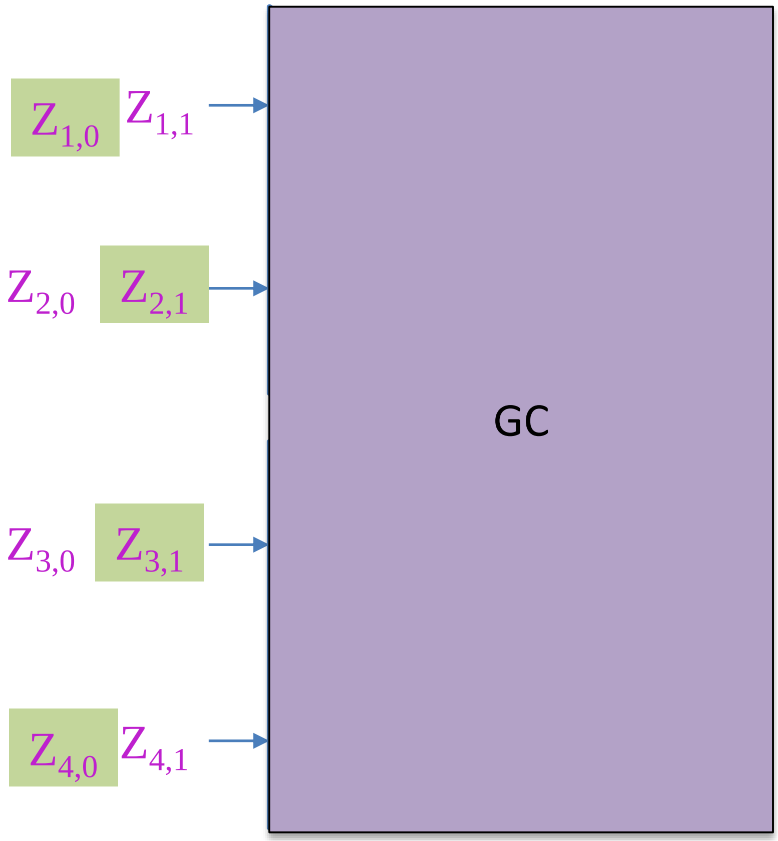

Pairs of λ-bit keys

$$C(x) \xleftarrow{\text{Decoder}} GC, Z_x$$

Semi-Honest Secure 2PC

 (x)

$C(x, y)$

(y) 

$\xleftarrow{(GC_y, f_{i,b})}$

$Z_{i,b}, GC_y, f_{i,b}, f_{i,b}^{-1}$

$z'_{i,b} \leftarrow \{0,1\}^\lambda, z_{i,b} = f_{i,b}(z'_{i,b}),$
$z_{i,1-b} = z'_{i,1-b}$

$\xrightarrow{z_{i,b}}$

$\xleftarrow{W_{i,b}}$

$W_{i,b} = Z_{i,b} \oplus H(f_{i,b}^{-1}(z_{i,b}))$

$Z_{i,x_i} = W_{i,x_i} \oplus H(z_{i,x_i})$

$v = EvalGC(GC_y, Z_{i,x_i})$
$where\ v = C(x, y)$

$i \in \{0,1\}^\lambda, b \in \{0,1\}$

Semi-Honest Secure 2PC

 (x) $C(x, y)$ (y) 

$s_1 = (GC_y, f_{i,b})$ $\longleftarrow$

$Z_{i,b}, GC_y, f_{i,b}, f_{i,b}^{-1}$

$z'_{i,b} \leftarrow \{0,1\}^\lambda, z_{i,b} = f_{i,b}(z'_{i,b}),$
$z_{i,1-b} = z'_{i,1-b}$

$s_2 = (z_{i,b})$ $\longrightarrow$

$s_3 = (W_{i,b})$ $\longleftarrow$

$W_{i,b} = Z_{i,b} \oplus H(f_{i,b}^{-1}(z_{i,b}))$

$Z_{i,x_i} = W_{i,x_i} \oplus H(z_{i,x_i})$

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3-round SH 2PC: (S_1 , S_2 , S_3)

 (x)

$C(x, y)$

(y) 

S_1 $\longleftarrow$

$Z_{i,b}, GC_y, f_{i,b}, f^{-1}_{i,b}$

$$z'_{i,b} \leftarrow \{0,1\}^\lambda, z_{i,b} = f_{i,b}(z'_{i,b}), \quad z_{i,1-b} = z'_{i,1-b}$$

S_2 $\longrightarrow$

S_3 $\longleftarrow$

$$W_{i,b} = Z_{i,b} \oplus H(f^{-1}_{i,b}(z_{i,b}))$$

$$Z_{i,x_i} = W_{i,x_i} \oplus H(z_{i,x_i})$$

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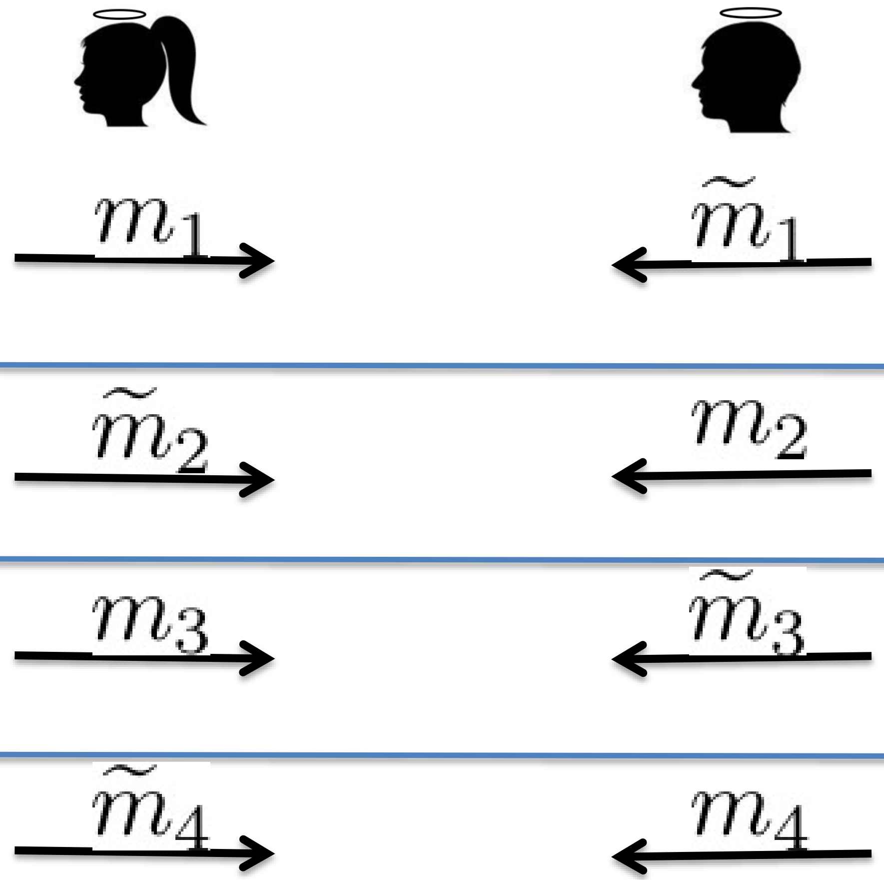
Our 2PC Protocol



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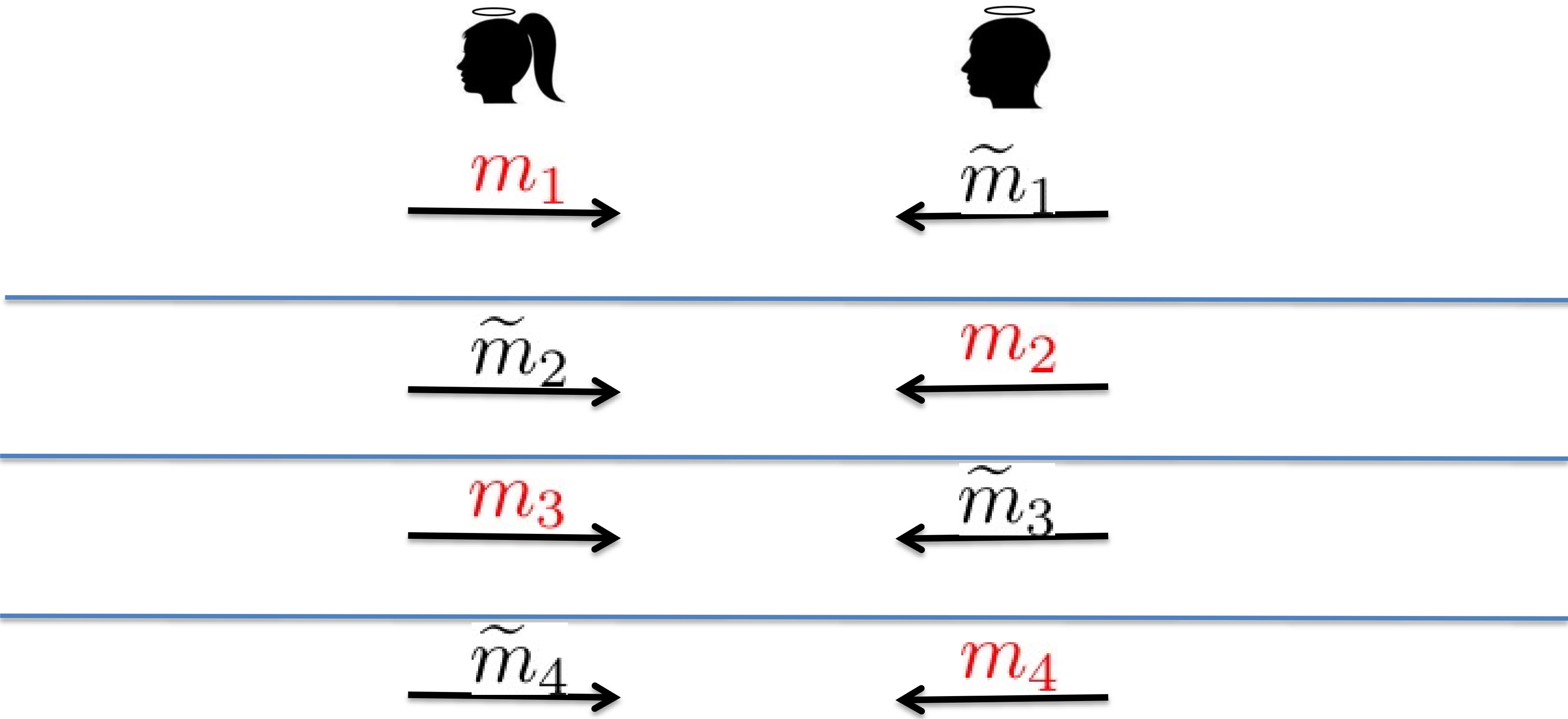


Our 2PC Protocol

3-round SH 2PC: (s_1 , s_2 , s_3)

3-round NMCOM: (nm_1 , nm_2 , nm_3)

3-round Π_{WIPOK}: (p_1 , p_2 , p_3)

4-round Π_{FS}: (fs_1 , fs_2 , fs_3 , fs_4)

Our 2PC Protocol

 (x)

(y) 

fs_1 p_1 nm_1

$$r_{i,b} \leftarrow \{0,1\}^{\lambda}, nmcom(id_1, r_{i,b})$$

fs_2 p_2 nm_2 S_1 C_{GC} C_Z **r’**

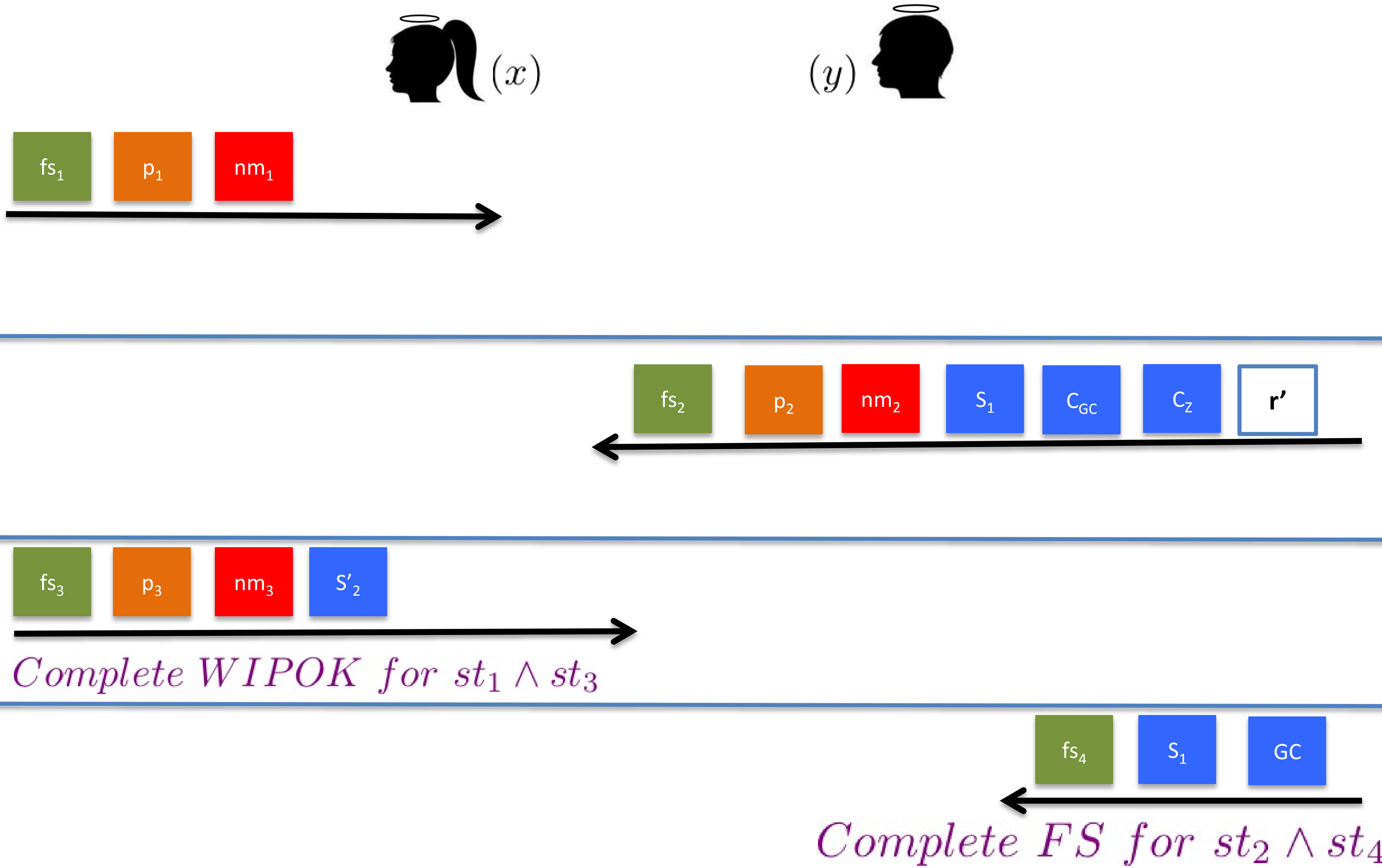
$$r'_{i,b} \leftarrow \{0,1\}^{\lambda}$$

fs_3 p_3 nm_3 S'_2

$$If\ x_i = 1,\ z'_{i,1} \leftarrow \{0,1\}^{\lambda}, z_{i,1} = f^{\lambda}_{i,1}(z'_{i,1}),$$
$$z_{i,0} = r_{i,0} \oplus r'_{i,0}.$$
$$If\ x_i = 0,\ z'_{i,0} \leftarrow \{0,1\}^{\lambda}, z_{i,0} = f^{\lambda}_{i,0}(z'_{i,0}),$$
$$z_{i,1} = r_{i,1} \oplus r'_{i,1}.$$

fs_4 S_1 GC

Our 2PC Protocol



Proof Systems

- 3-round Π_{WIPOK} public-coin, witness-indistinguishable proof-of-knowledge [FLS99] for NP $(st_1 \wedge st_2)$

- 4-round Π_{FS} zero-knowledge argument-of knowledge protocol [FS90] for NP (*thm*) based on NMCOM and Π_{WIPOK}.

1st Π_{WIPOK}: V sets $t_1=f(w_1)$, $t_2=f(w_2)$

and proves knowledge of a w for $t_1 \vee t_2$

2nd Π_{WIPOK}: P proves knowledge of a witness to

$thm \vee (t_1 \vee t_2)$

Proof Systems

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Crucial Change

1st Π_{WIPOK}: V sets $t_1=nm^{\sigma 1}$, $t_2=nm^{\sigma 2}$

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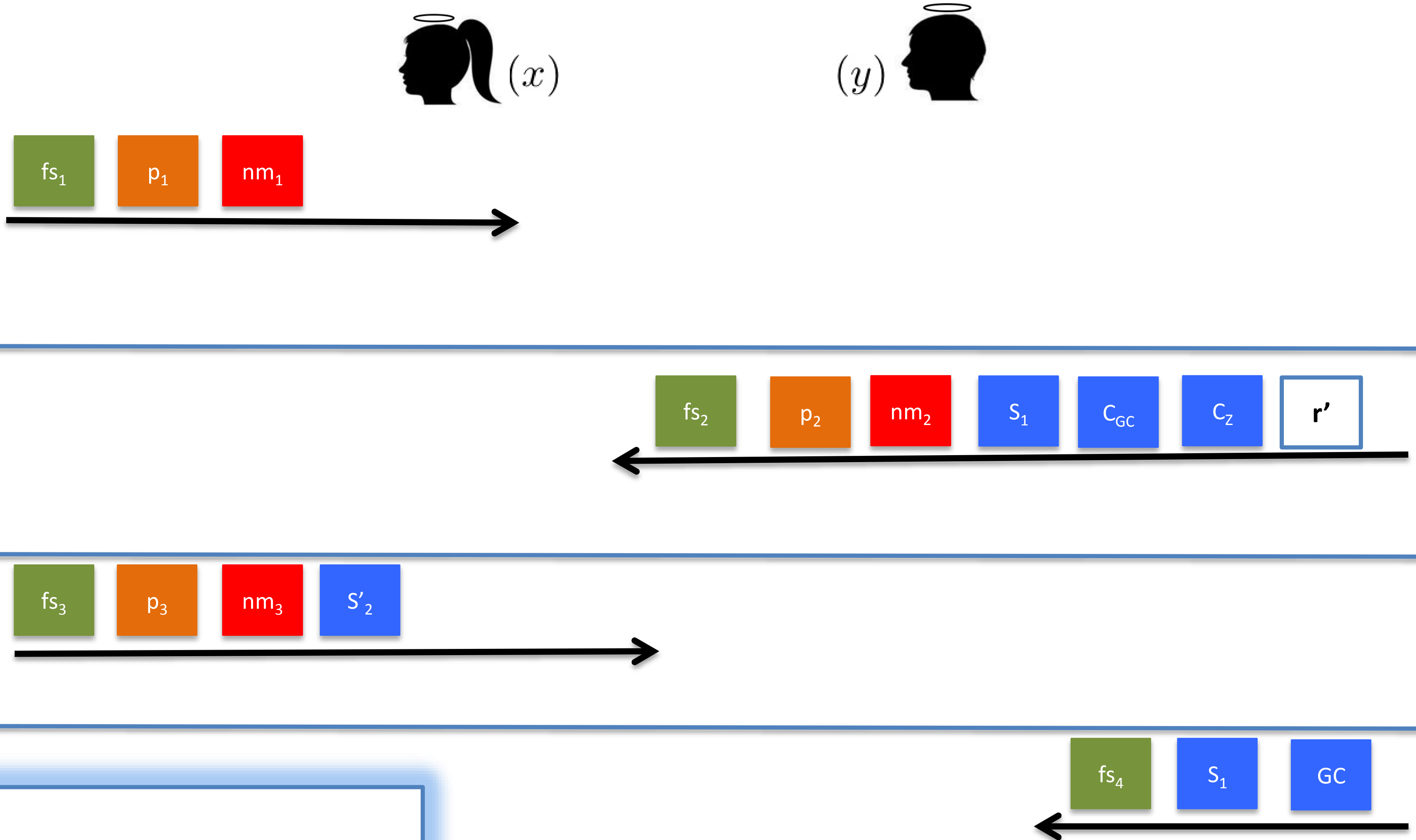
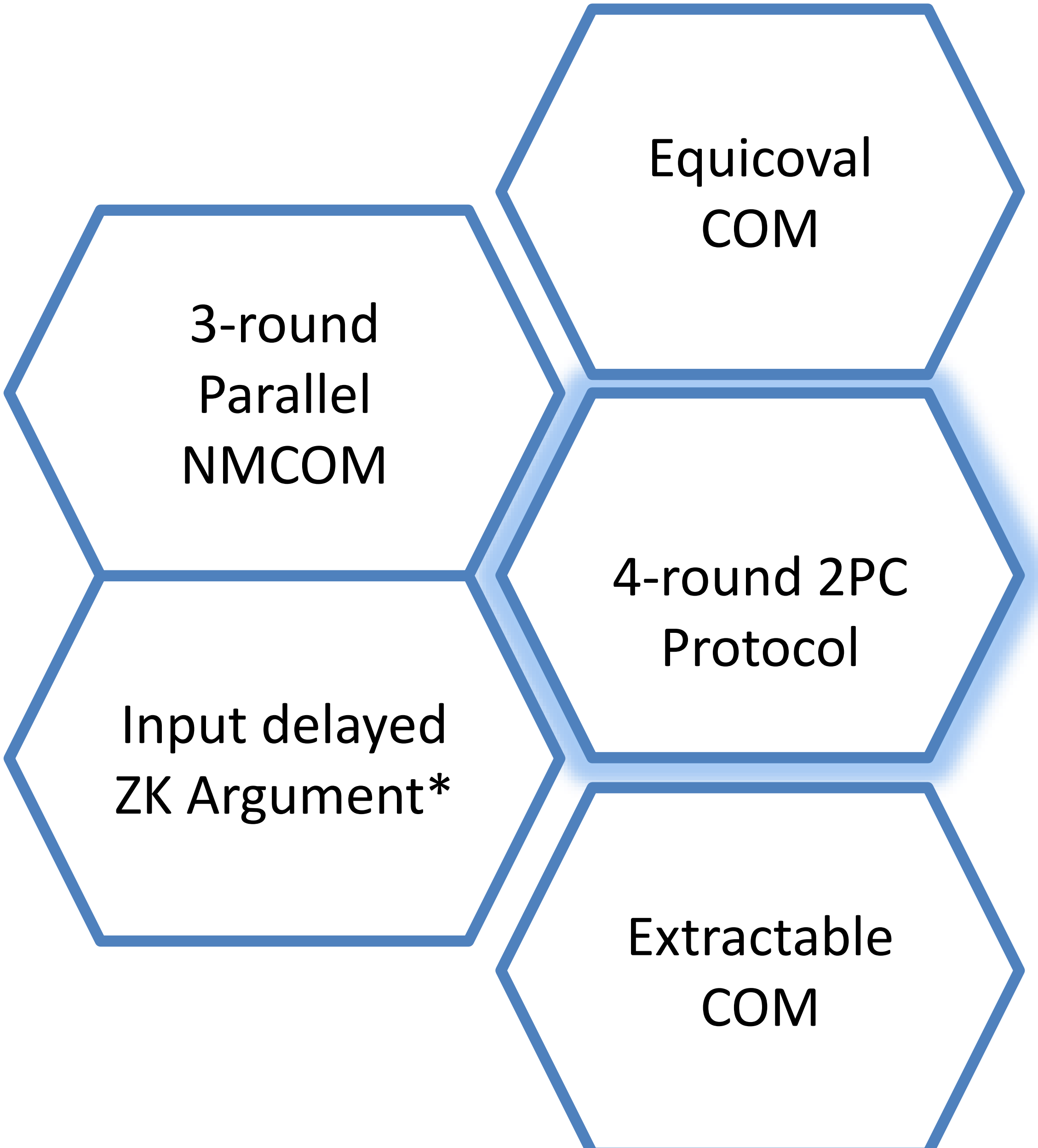
Input-Delayed Proof Systems

Our 2PC Protocol



Simulation Soundness

Tools for our Coin-Flipping Protocol



Conclusion

Round Complexity	
2PC	MPC
5 rounds [KO04]	O(1)

- **(3-round Impossibility)**:

There does **not** exist a **3-round protocol** for the **two-party coin-flipping** functionality.

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max(4,k+1)[1]	max(4,k+1)

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Suppose that there exists a k-round NMCOM scheme; then

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- **(MPC)**: there exists a **max(4, k + 1)-round** proto -
  party coin-flipping functionality.

Four rounds are both **necessary and sufficient** for both the results based on the 3-round NMCOM of [GPR16].

4-round 2PC protocols

Theorem [GMP**P**16]

TDP + k-round (parallel) **NMCOM → max(4, k + 1)**-round **2PC** protocol

[GMP**P**16]: **TDP +** **3-round NMCOM** [PPV08] → **4**-round **2PC** protocol

[H**P**V16]: **TDP +** **OWF** → **4**-round **2PC** protocol

4-round MPC protocols

[GMP**P**16]:	**TDP +**	LWE	➔ **6**-round **MPC** protocol
[GMP**P**16]:	**TDP +**	iO	➔ **5**-round **MPC** protocol
[H**P**V16]:	**TDP +**	iO	➔ **4**-round **MPC** protocol*

Open Problems

Crypto Assumption	Plain Model	CRS Model
MPC protocols		
Semi-Honest OT	O(1) rounds [BMR90…]	4 rounds [GMW87+AIK05]
LWE	6 rounds [this work]	2 rounds [MW15]
iO	4 rounds [H**P**V16]	2 rounds [GGHR14]

Can we get optimal-round static MPC protocols from different/weaker assumptions?

Thank you!