Probabilistic Polynomial-Time Process Calculus for Security Protocol Analysis

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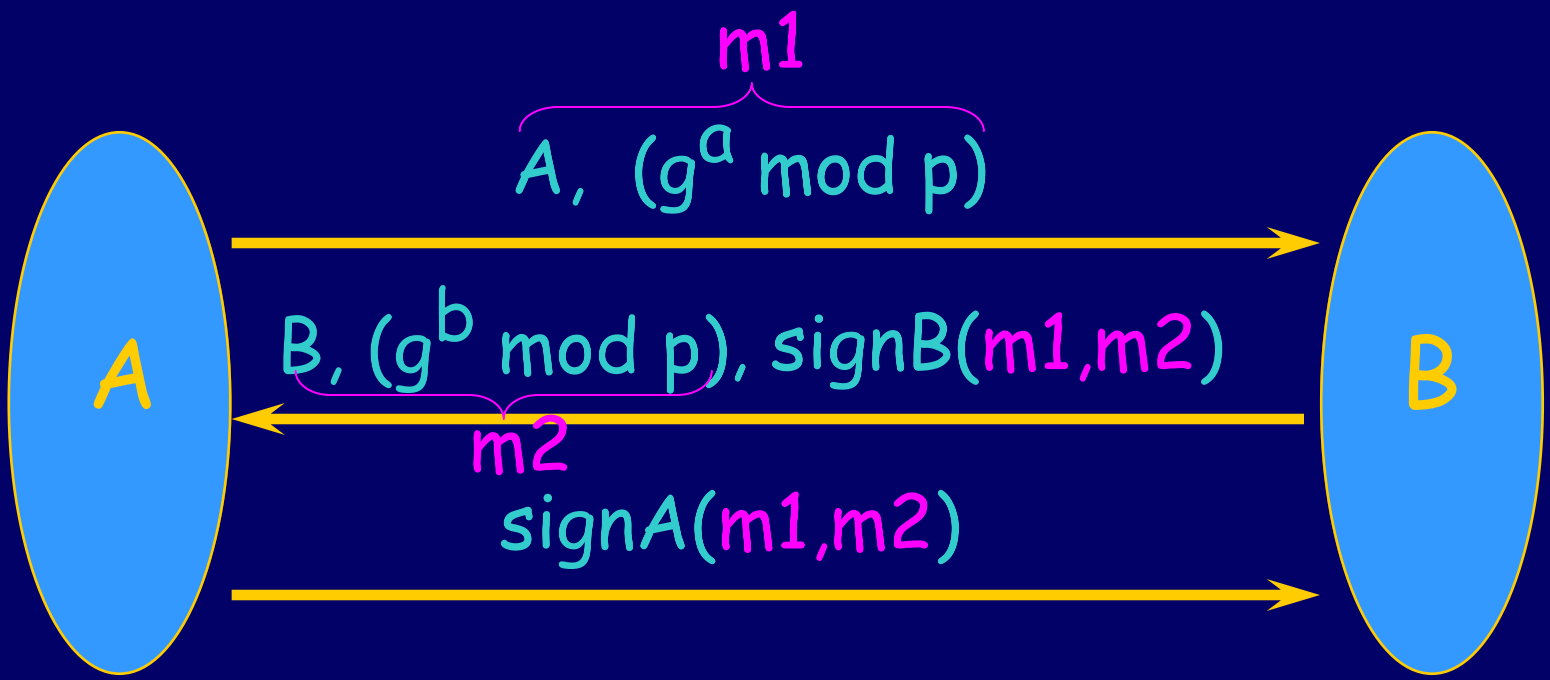
Outline

- Security protocols
- Research goals
- Specific process calculus
  - Probabilistic semantics & complexity
  - Asymptotic equivalence & bisimulation
  - Equational proof system
  - Examples
    - Computational indistinguishability
    - Decision Diffie-Hellman & ElGamal encryption

Protocol security

- Cryptographic Protocol
  - Program distributed over network
  - Use cryptography to achieve goal
- Attacker
  - Intercept, replace, remember messages
  - Guess random numbers, some computation
- Correctness
  - Attacker cannot learn protected secret or cause incorrect conclusion

IKE subprotocol from IPSEC



Result: A and B share secret g^{ab} mod p

Analysis involves probability, modular exponentiation, complexity, digital signatures, communication networks

Compositionality

- Confidentiality
  - $A \rightarrow B$: $\text{encrypt}_{KB}(\text{msg})$
- Authentication
  - $A \rightarrow B$: $\text{sign}_{KA}(\text{msg})$
- Composition
  - $A \rightarrow B$: $\text{encrypt}_{KB}(\text{msg}), \text{sign}_{KA}(\text{msg})$
  - Broken! $\text{sign}_{KA}(\text{msg})$ can leak info abt. msg
  - Right way: $\text{encrypt}_{KB}(\text{msg}), \text{sign}_{KA}(\text{cipher})$

Standard analysis methods

- Model-checking (finite state analysis)
- Automated theorem provers
  - Symbolic search of protocol runs
  - Correctness proofs in formal logic (Dolev-Yao)

- Computational model
  - Consider probability and complexity
    - More realistic intruder model
    - Interaction between protocol and cryptography

Easier

Harder

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One General Starting Point

- Express security properties in terms of *comparison to an ideal protocol*
- Protocol is secure if no adversary can distinguish it from *some idealized version* of the protocol
  - Beaver '91, Goldwasser-Levin '90, Micali-Rogaway '91
- Security properties should be compositional

Language approach

Roscoe '95, Schneider '96, Abadi-Gordon'97

- Write protocol in process calculus
  - Dolev-Yao model
- Express security using observational equivalence
  - Standard relation from programming language theory

    $P \approx Q$ iff for all contexts $C[\ ]$, same observations about $C[P]$ and $C[Q]$
  - Inherently compositional
  - Context (environment) represents adversary
- Use proof rules for $\approx$ to prove security
  - Protocol is secure if no adversary can distinguish it from some idealized version of the protocol

Probabilistic poly-time process calculus

- Probabilistic polynomial-time execution model
- Specify security via equivalence to "ideal" protocol
- Also state cryptographic assumptions via equivalences
- Leads to new proof system
  - Equational reasoning
  - Based on probabilistic bisimulation, asymptotic equivalence
- Connections with modern crypto
  - Characterize computational indistinguishability
  - Formal derivation of semantic security from computational assumption DDH (both stated as equations) and vice versa (indistinguishability of encryptions)

Neighbors

- Canetti;   B. Pfitzmann, Waidner, Backes
  - Interactive Turing machines
  - General framework for crypto properties
  - Protocol ***realizes*** an ideal setting
  - Universally composable security
- Abadi, Rogaway, Jürjens;
  Micciancio, Warinschi; Corin, Laud;
  Horwitz, Gligor; Herzog
  - Toward transfer principles between formal Dolev-Yao model and computational model
- Impagliazzo, Kapron
  - Logic of the computational model

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Syntax

Expressions have size poly in |n|

- Bounded CCS with integer terms

$P ::= 0$

$\mid$	$out(c_{q(\lvert n\rvert)}, T).\ P$	send up to q(|n|) bits
$\mid$	$in(c_{q(\lvert n\rvert)}, x).\ P$	receive
$\mid$	$\upsilon c_{q(\lvert n\rvert)}.(P)$	private channel
$\mid$	$[T=T]\ P$	test
$\mid$	$P \mid P$	parallel composition
$\mid$	$!_{q(\lvert n\rvert)}.\ P$	bounded replication

Terms may contain symbol n; channel width and replication bounded by poly in |n|

Evaluation

- Reduction
  - Evaluate unguarded terms and matches
  - Local computation embodied in terms
- Scheduling
  - Probabilistically pick a type of action
- Communication
  - Pick a particular action of the chosen type uniformly at random
  - During an actual run only pick input/output actions.

Nondeterminism vs probabilism

- Alice encrypts msg and sends to Bob

  $A \rightarrow B: \{ msg \}_K$

- Adversary uses nondeterminism

  Process E_0 out(c,0) | … | out(c,0)

  Process E_1 out(c,1) | … | out(c,1)

  Process E

  $in(c, b_1) \ldots in(c, b_n).out(d, b_1 b_2 \ldots b_n, msg)$

In reality, at most 2^{-n} chance to guess n-bit key

Complexity results

- Polynomial time
  - For each closed process expression P, there is a polynomial $q(x)$ such that
    - For all n
    - For all probabilistic polynomial-time schedulers

    eval of P halts in time $q(|n|)$

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How to define process equivalence?

- **Intuition**
  - $| \mathrm{Prob}\{ C[P] \rightarrow o \} - \mathrm{Prob}\{ C[Q] \rightarrow o \} | < \varepsilon$
- **Difficulty**
  - How do we choose ε?
    - Less than 1/2, 1/4, … ? (not equiv relation)
    - Vanishingly small ? As a function of what?
- **Solution**
  - Use security parameter
    - Protocol is family $\{ P_n \}_{n>0}$ indexed by key length
  - Asymptotic form of process equivalence

$P \approx Q$ if for all polynomials p, observables $\varepsilon < 1/p(n)$

One way to get equivalences

- Labeled transition system
  - Allow process to send any output, read any input
  - Label with numbers "resembling probabilities"
- Probabilistic bisimulation relation
  - Relation $\sim$ on processes
  - If $P \sim Q$ and $P \xrightarrow{r} P'$, then exists Q' with $Q \xrightarrow{r} Q'$ and $P' \sim Q'$, and vice versa
  - Reactive form of bisimulation (scheduling)
  - van Glabbeek, Smolka, Steffen '95

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Provable equivalences

- Assume scheduler is stable under bisimulation

- $P \sim Q \quad \Rightarrow \quad C[P] \sim C[Q]$
- $P \sim Q \quad \Rightarrow \quad P \approx Q$
- $P \mid (Q \mid R) \approx (P \mid Q) \mid R$
- $P \mid Q \approx Q \mid P$
- $P \mid 0 \approx P$

Provable equivalences

- $P \approx \nu\, c.\ (\ out(c,T)\ |\ in(c,x).P)$ $\quad x \notin FV(P)$
- $P\{a/x\} \approx \nu\, c.\ (\ out(c,a)\ |\ in(c,x).P)$

  bandwidth of c large enough
- $P \approx 0$ if no public channels in P
- $P \approx Q \quad \Rightarrow \quad P\{d/c\} \approx Q\{d/c\}$

  c , d same bandwidth, d fresh
- $out(c,T) \approx out(c,T')$

  $Prob[T \rightarrow a] = Prob[T' \rightarrow a]$ all a

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Computational indistinguishability

- $T(i,n)$, $T'(i,n)$ terms in the calculus
  - T, T' represent uniform prob. poly-time function ensembles f_i , g_i : { } $\rightarrow \{0,1\}^{q(|n|)}$

- $out(c,T) \approx out(c,T')$ says exactly that the function ensembles f_i , g_i are *indistinguishable by prob. poly-time statistical tests*

- Yao '82: fundamental notion in crypto

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Connections with modern crypto

- Ciphersystem consists of three parts
  - Key generation
  - Encryption (often probabilistic)
  - Decryption
  - Formal derivation of semantic security of ElGamal from DDH and *vice versa*
    - Well known fact in crypto [Tsiounis & Yung '98]

ElGamal cryptosystem

- n security parameter (*e.g.*, key length)
  G_n cyclic group of prime order p,
  length of p roughly n, g generator of G_n
- Keys
  - public $\langle g, y \rangle$, private $\langle g, x \rangle$ s.t. $y = g^x$
- Encryption of $m \in G_n$
  - for random $k \in \{0, \ldots, p\text{-}1\}$ outputs $\langle g^k, m\, y^k \rangle$
- Decryption of $\langle v, w \rangle$ is $w\,(v^x)^{-1}$
  - For $v = g^k$, $w = m\, y^k$ get
    $w\,(v^x)^{-1} = m\, y^k / g^{kx} = m\, g^{xk} / g^{kx} = m$

Semantic security

- Known equivalent:
  indistinguishability of encryptions
  - adversary can't tell from the traffic which of the two chosen messages has been encrypted
  - ElGamal:

    $$\langle 1^n , g^k , m\, y^k \rangle \approx \langle 1^n , g^{k'} , m'\, y^{k'} \rangle$$
- In case of ElGamal known to be equivalent to DDH [Tsiounis-Yung]
- *Formally derivable using the proof rules*

Decision Diffie-Hellman (DDH)

- Standard crypto assumption
- n security parameter (*e.g.*, key length)
  G_n cyclic group of prime order p,
  length of p roughly n,
  g generator of G_n
- For random $a, b, c \in \{0, \ldots, p-1\}$
  $\langle g^a, g^b, g^{ab} \rangle \approx \langle g^a, g^b, g^c \rangle$

DDH implies sem. sec. of ElGamal

- Start with $\langle g^a, g^b, g^{ab} \rangle \approx \langle g^a, g^b, g^c \rangle$ (random a,b,c)
- Build up statement of sem. sec. from this.
  - $in(c,\langle x,y \rangle).out(c, \langle g^r, x.g^{rx} \rangle) \approx$
    $in(c,\langle x,y \rangle).out(c, \langle g^r, y.g^{rx} \rangle)$
- The proof consists of
  - Structural transformations
    - E.g., $out(c,T(r); r \text{ random}) \approx out(c,U(r))$ (any r) implies
      $in(c,x).out(c,T(x)) \approx in(c,x).out(c,U(x))$
  - Domain-specific axioms
    - E.g., $out(c, \langle g^a, g^b, g^{ab} \rangle) \approx out(c, \langle g^a, g^b, g^c \rangle)$ implies
      $out(c, \langle g^a, g^b, Mg^{ab} \rangle) \approx out(c, \langle g^a, g^b, Mg^c \rangle)$ (any M)

Sem. sec. of ElGamal implies DDH

- Harder direction. Compositionality of $\approx$ makes 'building up' easier than breaking down.
- Want to go from

  $\mathrm{in}(c,\langle x,y\rangle).\mathrm{out}(c,\langle g^r, x.g^{rx}\rangle) \approx \mathrm{in}(c,\langle x,y\rangle).\mathrm{out}(c,\langle g^r, y.g^{rx}\rangle)$

  to

  $\langle g^x, g^r, g^{rx}\rangle \approx \langle g^x, g^r, g^c\rangle$
- Proof idea: if x = 1, then we essentially have DDH.
- The proof 'constructs' a DDH tuple by
  - Hiding all public channels except the output challenge
  - Setting a message to 1
- Need structural rule equating a process with the term simulating the process
  - We use special case where process only has one public output

Current State of Project

- Compositional framework for protocol analysis
  - Precise language for studying security protocols
  - Replace nondeterminism with probability
  - Equivalence based on ptime statistical tests
- Probabilistic ptime language
- Methods for establishing equivalence
  - Probabilistic bisimulation technique
- Notion of compositionality
- Examples
  - Decision Diffie-Hellman, semantic security, ElGamal encryption, computational indistinguishability

Conclusion

- Future work
  - Simplify semantics
  - Weaken bisimulation technique to generate asymptotic equivalences
  - Apply to more complex protocols
    - Bellare-Rogaway, Oblivious Transfer, Computational Zero Knowledge, …
  - Studying various models of compositionality for security protocols (WITS '04)
    - Canetti (ITMs), Pfitzmann-Waidner (IOAs)

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