Logarithmic Time Prediction

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DIMACS Workshop on Big Data through the Lens of Sublinear Algorithms

The Multiclass Prediction Problem

Repeatedly

1. See x
2. Predict $\hat{y} \in \{1, ..., K\}$
3. See y

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Goal: Find $h(x)$ minimizing error rate:

$$\Pr_{(x,y)\sim D}(h(x) \neq y)$$

with $h(x)$ fast.

Why?

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dimacs.rutgers.edu/**Workshops**/Parallel**Algorithms**

DIMACS Workshop on Big Data through the Lens of Sublinear Algorithms August 27 - 28, 2015 **DIMACS** Center, CoRE Building, Rutgers University Organizers:

my slice of pizza: **Big Data**, **Sublinear Algorithms** ...

mysliceofpizza.blogspot.com/2015/07/**big-data-sublinear-algorithms**...

Jul 24, 2015 · Grigory Yaroslavtsev, **Alexandr Andoni** and I are organizing the **DIMACS workshop on Big Data through the Lens of Sublinear Algorithms**, Aug 27--28, at …

Big Data Through the Lens of Sublinear Algorithms

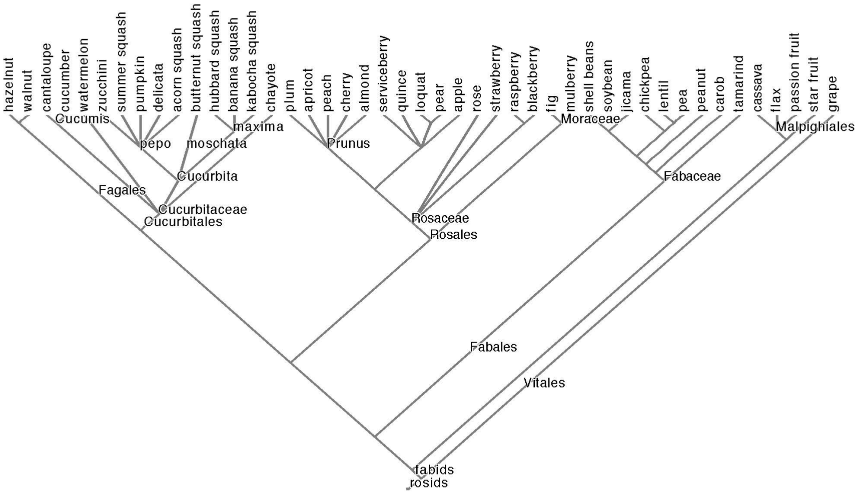
grigory.us/mpc-**workshop-dimacs**.html

Why?

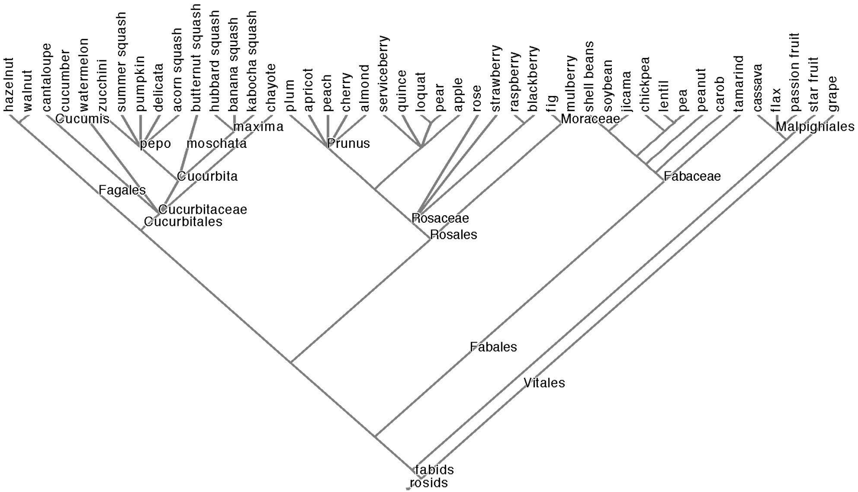
Trick #1

K is small

Trick #2: A hierarchy exists

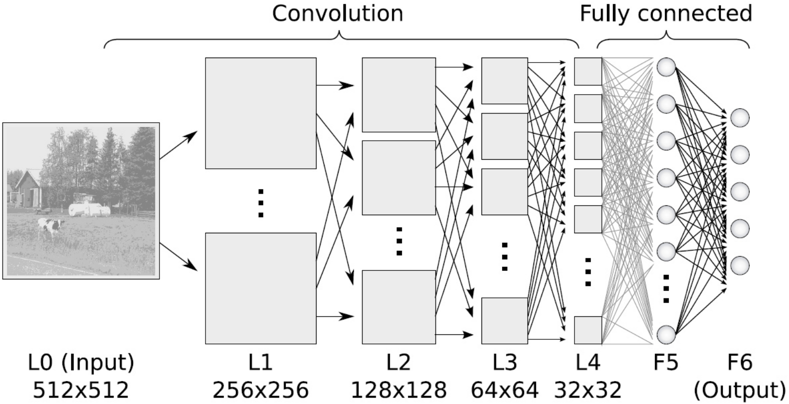


Trick #2: A hierarchy exists

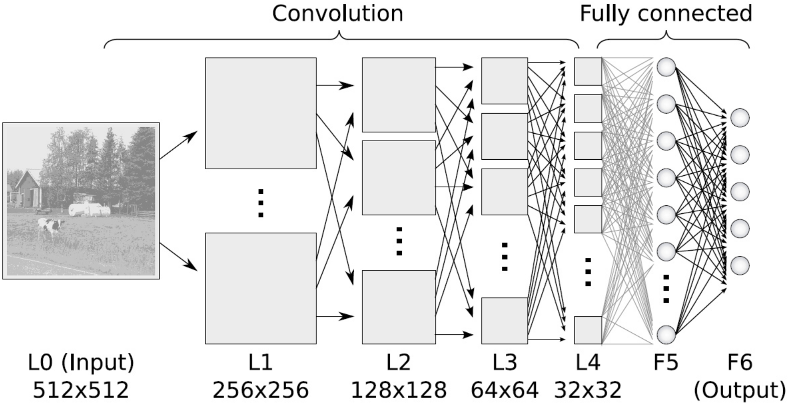


So use Trick #1 repeatedly.

Trick #3: Shared representation

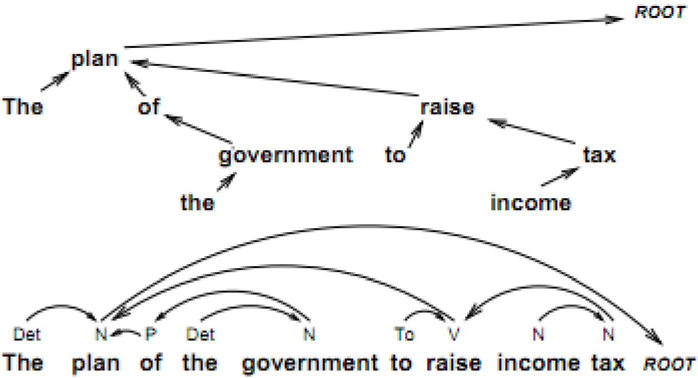


Trick #3: Shared representation

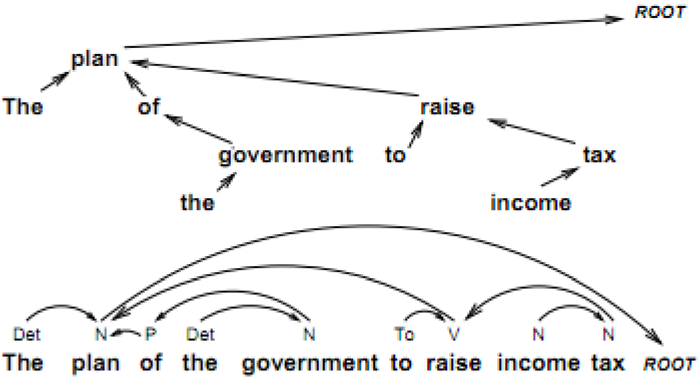


Very helpful... but computation in the last layer can still blow up.

Trick #4: "Structured Prediction"



Trick #4: "Structured Prediction"



But what if the structure is unclear?

Trick #5: GPU

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4 Teraflops is great… yet still burns energy.

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Theorem: There exists multiclass classification problems where achieving 0 error rate requires $\Omega(\log K)$ time to train or test per example.

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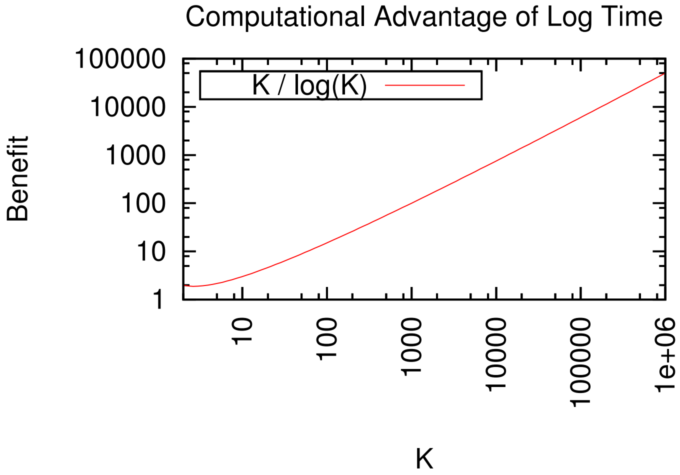
Any prediction algorithm outputting less than $\log_2 K$ bits loses with constant probability.

Any training algorithm reading an example requires $\Omega(\log_2 K)$ time.

Can we predict in time $O(\log_2 K)$?



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Prediction is $\Omega(K)$

Not it #2: Hierarchy Construction

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Training is $\Omega(K)$ or worse.

Not it #3: Unnormalized learning

Train K regressors by
For each example (x, y)

1. Train regressor y with $(x, 1)$.

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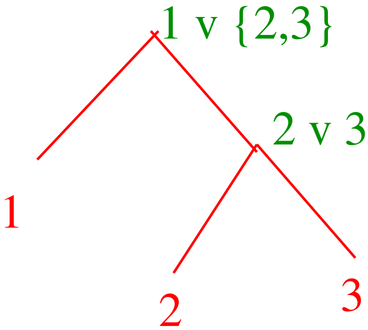
Prediction is still $\Omega(K)$.

Can we predict in time $O(\log_2 K)$?

Is logarithmic time even possible?

P(y=1) = .4
P(y=2) = .3
P(y=3) = .3



$P(\{2,3\}) > P(1) \Rightarrow$ lose for divide and conquer

Filter Trees [BLR09]

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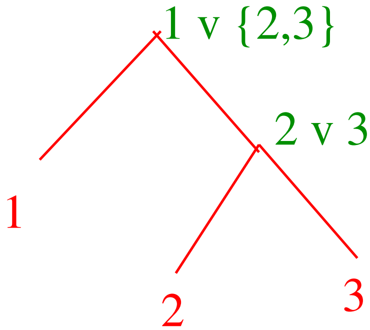
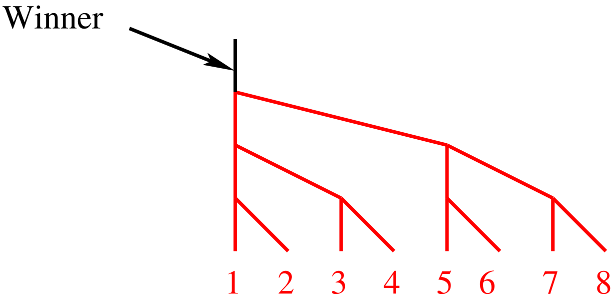
$P(y=2) = .3$
$P(y=3) = .3$



1. Learn $2v3$ first
2. Throw away all error examples
3. Learn 1 v Survivors

Theorem: For all multiclass problems, for all binary classifiers,
Multiclass Regret $\leq$ Average Binary Regret * $\log(K)$

Can you make it robust?



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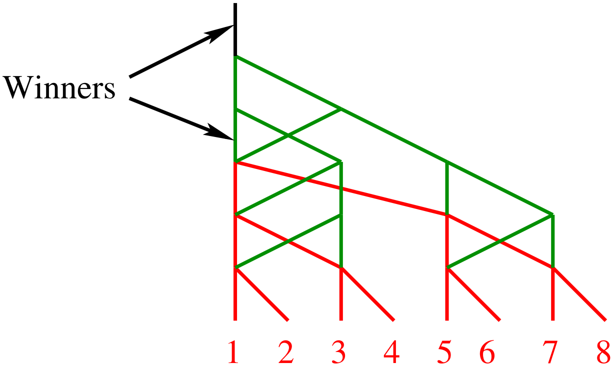

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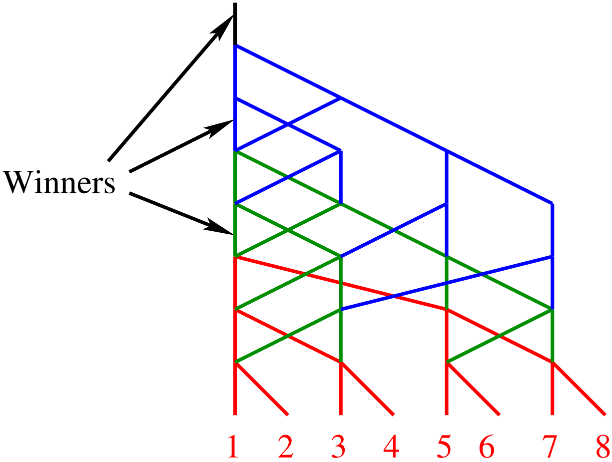

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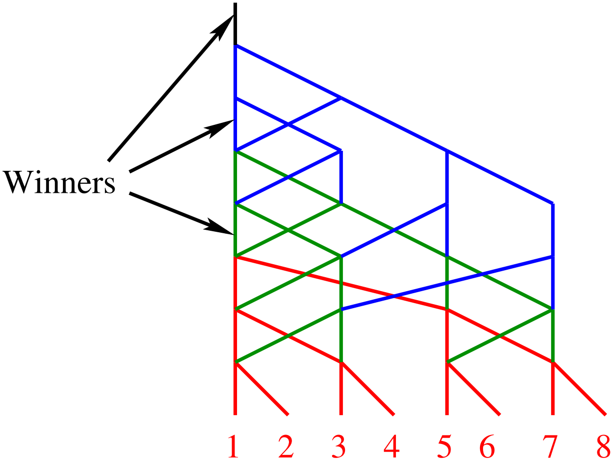

Theorem: [BLR09] For all multiclass problems, for all binary classifiers, a log(K)-correcting tournament satisfies:

Multiclass Regret $\leq$ Average Binary Regret * 5.5

Determined best paper prize for ICML2012 (area chair decisions).

How do you learn structure?

Not all partitions are equally difficult.
Compare $\{1,7\}v\{3,8\}$ to $\{1,8\}v\{3,7\}$
What is better?

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[BWG10]: Better to confuse near leaves than near root.
Intuition: the root predictor tends to be overconstrained while the leafwards predictors are less constrained.

The Partitioning Problem [CL14]

Given a set of n examples each with one of K labels, find a partitioner h that maximizes:

$$E_{x,y} |\Pr(h(x) = 1, y) - \Pr(h(x) = 1)\Pr(y)|$$

The Partitioning Problem [CL14]

Given a set of n examples each with one of K labels, find a partitioner h that maximizes:

$$E_x \sum_y \Pr(y) |\Pr(h(x) = 1 | x \in X_y) - \Pr(h(x) = 1)|$$

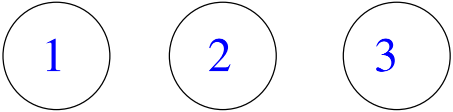
where X_y is the set of x associated with y.

The Partitioning Problem [CL14]

Given a set of n examples each with one of K labels, find a partitioner h that maximizes:

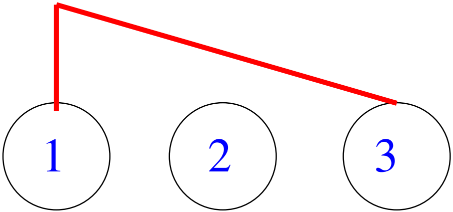
Nonconvex for any symmetric hypothesis class (ouch)

Bottom Up doesn't work



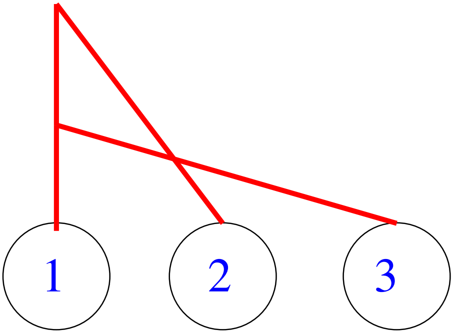
Suppose you use linear representations.

Bottom Up doesn't work



Suppose you use linear representations.
Suppose you first build a 1v3 predictor.

Bottom Up doesn't work



Suppose you use linear representations.
Suppose you first build a 1v3 predictor.
Suppose you then build a 2v{1v3} predictor.
You lose.

Does partitioning recurse well?

Theorem: If at every node n,

$$E_{x,y}\left|\Pr(h(x)=1,y)-\Pr(h(x)=1)\Pr(y)\right|>\gamma$$

then after

$$\left(\frac{1}{\epsilon}\right)^{\frac{4(1-\gamma)^2\ln k}{\gamma^2}}$$

splits, the multiclass error is less than ϵ.

Online Partitioning

Relax the optimization criteria:
$E_{x,y}\left|E_{x|y}\left[\hat{y}(x)\right]-E_{x}\left[\hat{y}(x)\right]\right|$
... and approximate with running average

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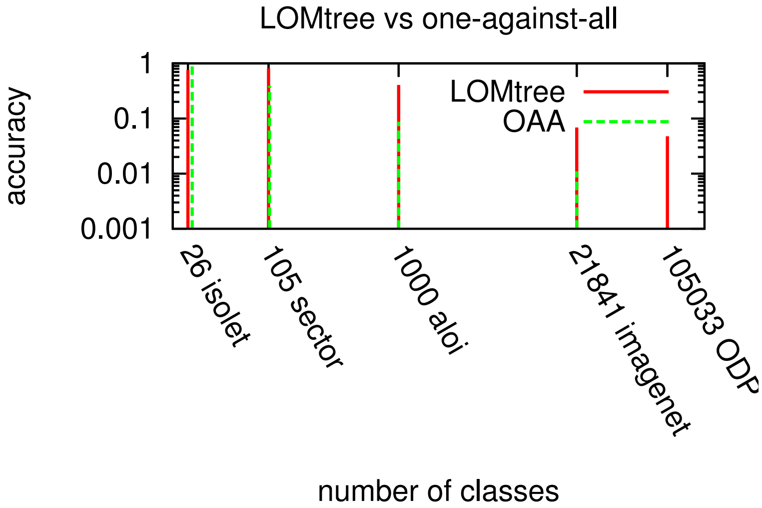
... and approximate with running average

Let $e = 0$ and for all y, $e_y = 0, n_y = 0$
For each example (x, y)

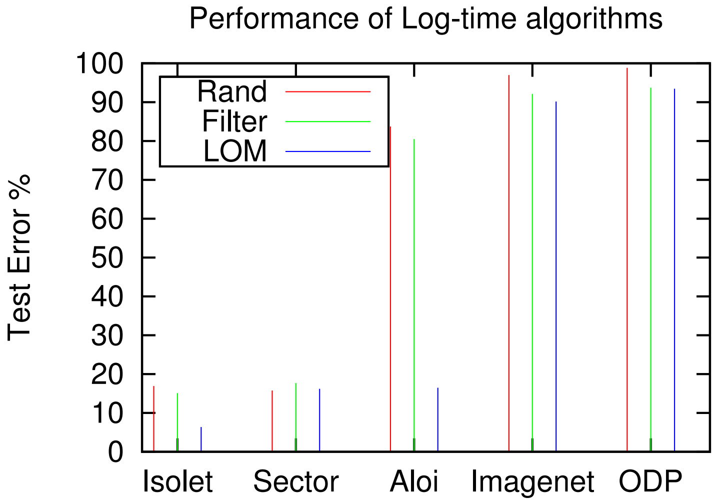
1. if $e_y < e$ then $b = -1$ else $b = 1$
2. Update w using (x, b)
3. $n_y \leftarrow n_y + 1$
4. $e_y \leftarrow \frac{(n_y - 1)e_y}{n_y} + \frac{\hat{y}(x)}{n_y}$
5. $e \leftarrow \frac{(t-1)e}{t} + \frac{\hat{y}(x)}{t}$

Apply recursively to construct a tree structure.

Accuracy for a fixed training time



Test Error %, optimized, no train-time constraint



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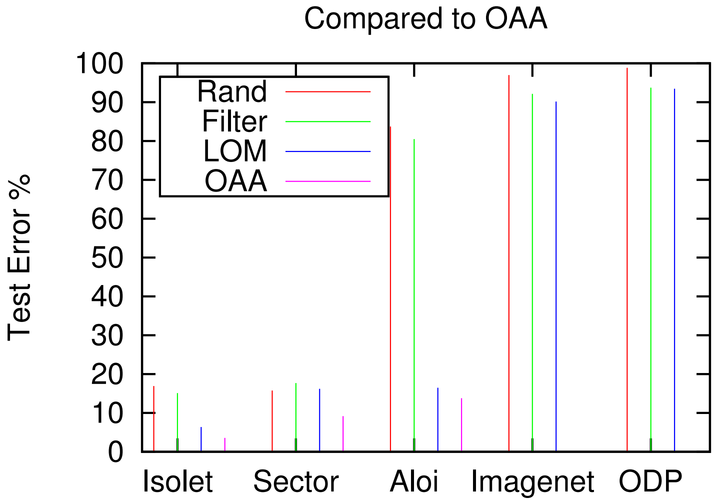
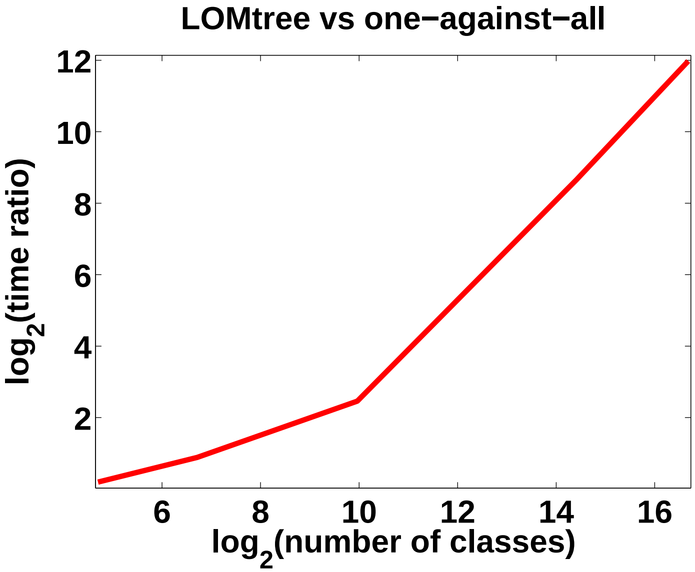


Classes vs Test time ratio



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What is the right way to achieve consistency and dynamic partition?

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What is the right way to achieve <u>consistency</u> and dynamic partition?

How can you balance representation complexity and sample complexity?

Bibliography

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