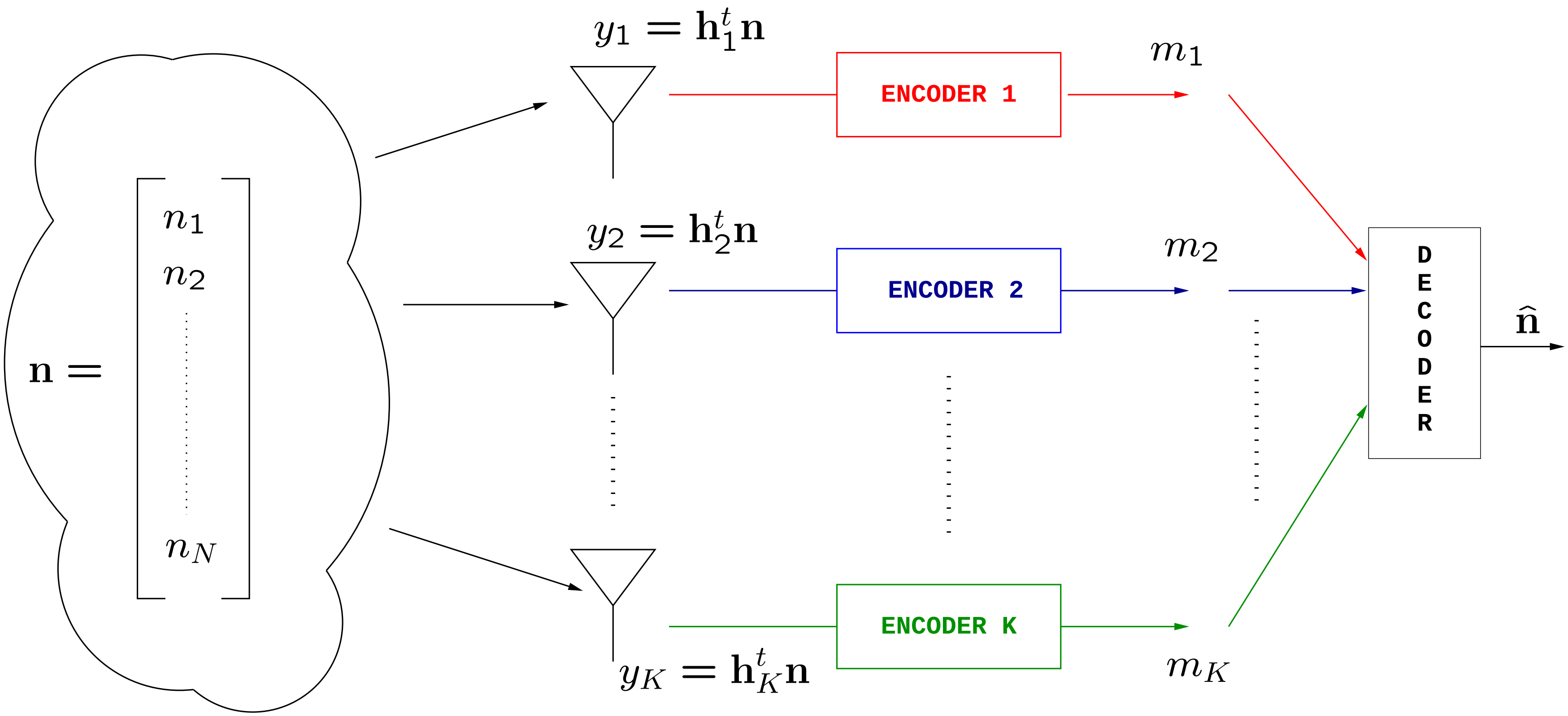
Sum Rate of Gaussian Multiterminal Source Coding

Pramod Viswanath
University of Illinois, Urbana-Champaign

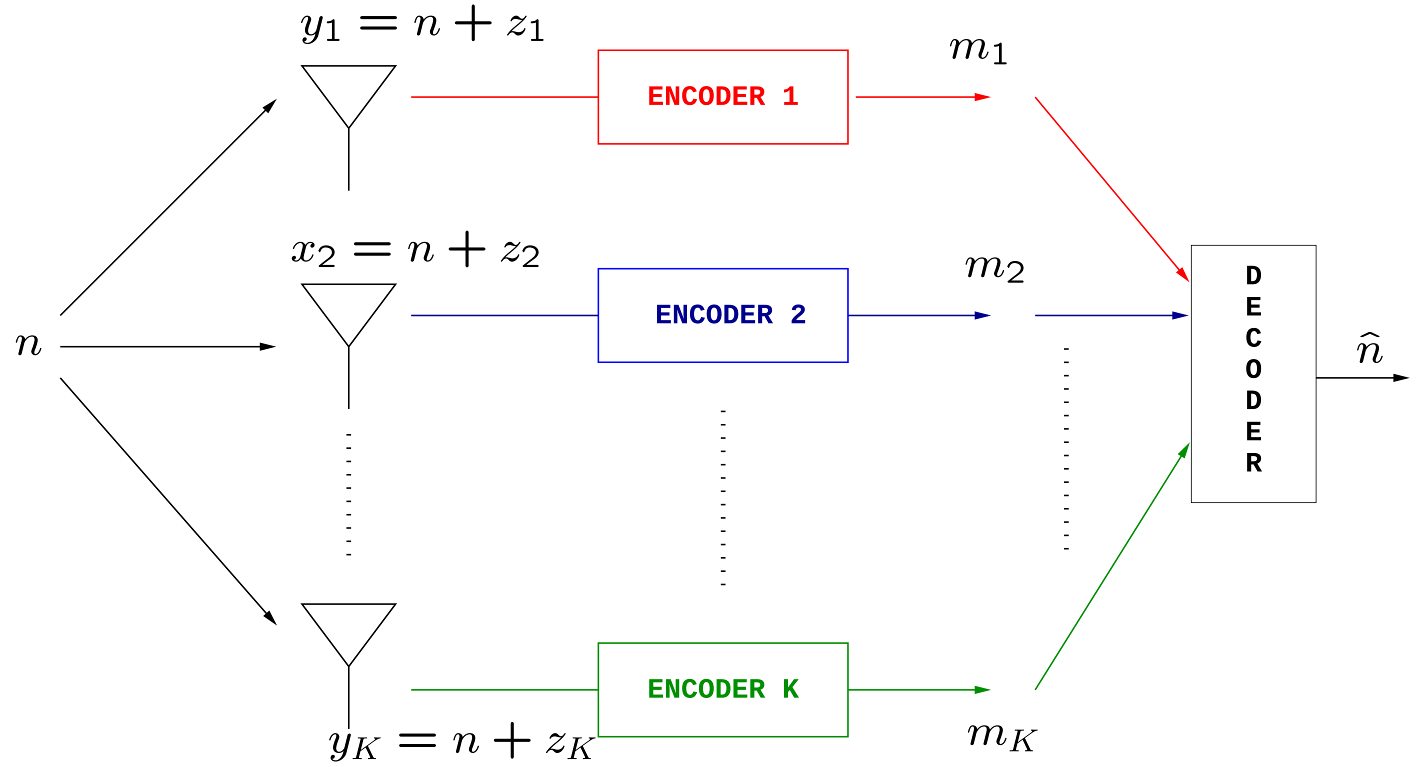
March 19, 2003

Gaussian Multiterminal Source Coding



Sum of rates of encoders R and distortion metric $d(\mathbf{n}, \hat{\mathbf{n}})$

Quadratic Gaussian CEO Problem



Sum of rates of encoders R and distortion metric $d\,(n, \widehat{n})$

Result: Quadratic Gaussian CEO

- quadratic distortion metric $d\,(n, \hat{n}) = (n - \hat{n})^2$

- For large number of encoders K,

$$R(D) = \frac{1}{2}\log^{+}\left(\frac{\sigma_n^2}{D}\right) + \frac{\sigma_z^2}{2\sigma_n^2}\left(\frac{\sigma_n^2}{D} - 1\right)^{+}$$

  – Second term is loss w.r.t. cooperating encoders

Outline

- Problem Formulation:

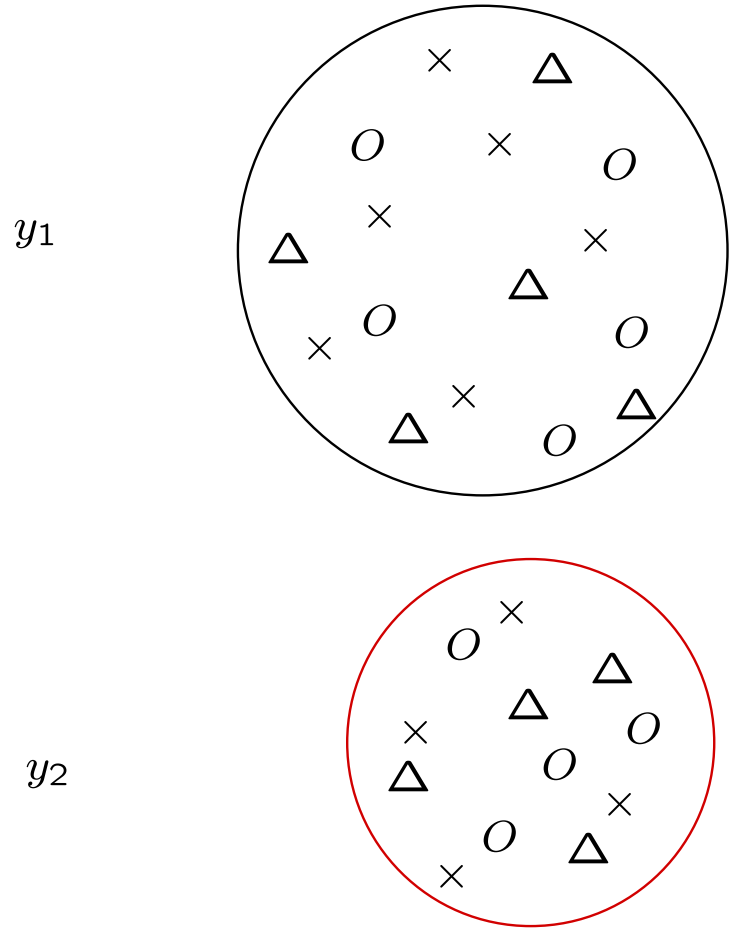
  Tradeoff between sum rate R and distortion (metric $d(\mathbf{n}, \hat{\mathbf{n}})$).

- Main Result:

  Characterize a class of distortion metrics for which no loss in sum rate compared with encoder cooperation

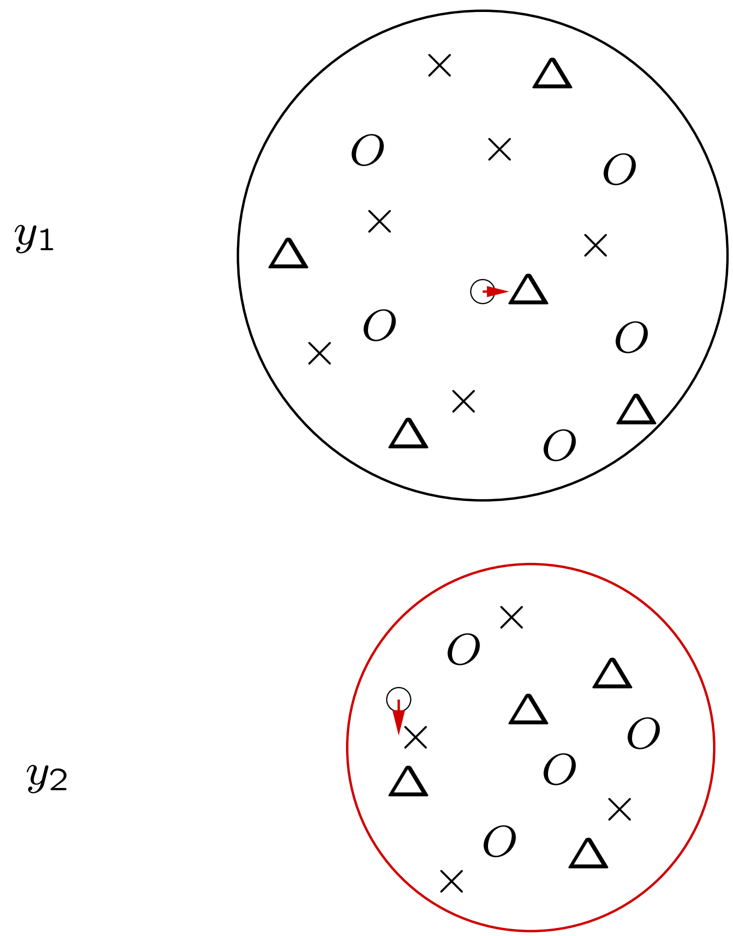
  – A multiple antenna test channel

Random Binning of Slepian-Wolf



- Rate is number of quantizers

Encoding in Slepian-Wolf

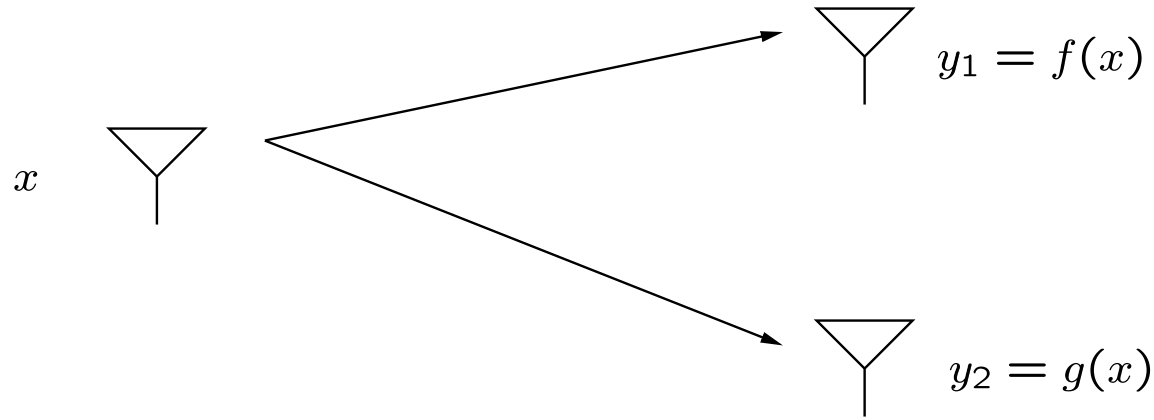


- Quantizer closest to realization

Decoding in Slepian-Wolf

- Decoder knows joint distribution of y_1, y_2
- It is given the two quantizer numbers from the encoders
- Picks the pair of points in the quantizers which best matches the joint distribution
  - For jointly Gaussian y_1, y_2 nearest neighbor type test
- $R_1 + R_2 = H(y_1, y_2)$ is sufficient for zero distortion

Deterministic Broadcast Channel



- Pick distribution on x such that y_1, y_2 have desired joint distribtion (Cover 98)

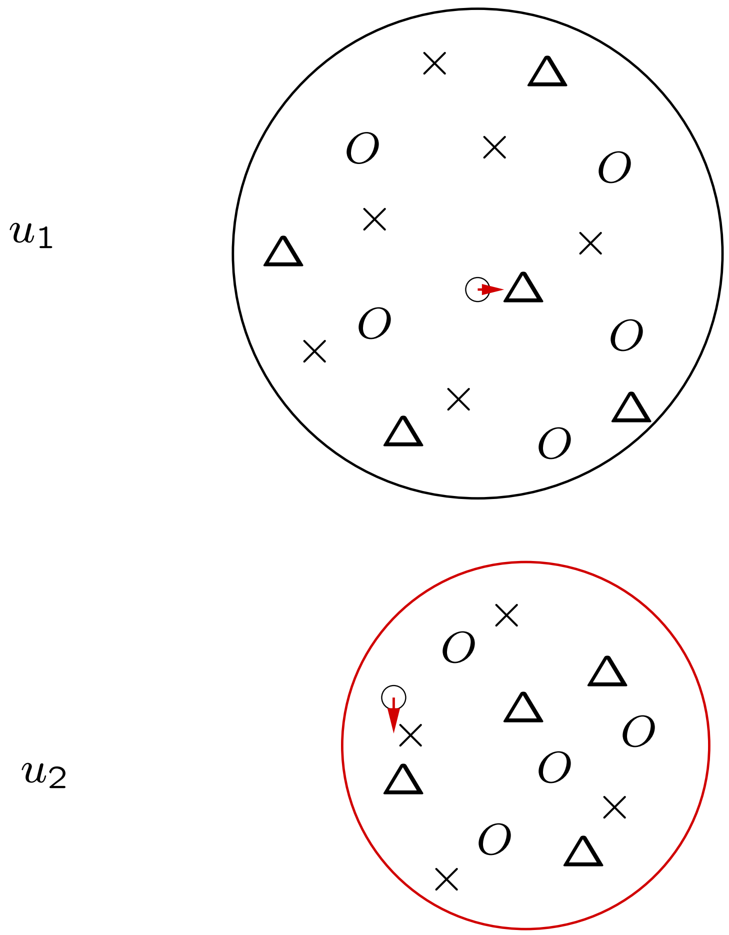
Slepian-Wolf code for Broadcast Channel

- Encoding: implement Slepian-Wolf decoder
  - given two messages, find the appropriate pair y_1, y_2 in the two quantizers
  - transmit x that generates this pair y_1, y_2.

Slepian-Wolf code for Broadcast Channel

- Encoding: implement Slepian-Wolf decoder
  - given two messages, find the appropriate pair y_1, y_2 in the two quantizers
  - transmit x that generates this pair y_1, y_2.
- Decoding: implement Slepian-Wolf encoder
  - quantize y_1, y_2 to nearest point
  - messages are the quantizer numbers

Lossy Slepian-Wolf Source Coding



- Approximate y_1, y_2 by u_1, u_2

Lossy Slepian-Wolf Source Coding

- Encoding: Find u_i that matches source y_i, separately for each i
  - For jointly Gaussian r.v. s, nearest neighbor calculation
  - Each encoder sends quantizer number containing the u picked

Lossy Slepian-Wolf Source Coding

- Encoding: Find u_i that matches source y_i, separately for each i
  - For jointly Gaussian r.v. s, nearest neighbor calculation
  - Each encoder sends quantizer number containing the u picked
- Decoding: Reconstruct the u's picked by the encoders
  - reconstruction based on joint distribution of u's
  - Previously $u_i = y_i$ were correlated
  - Here u's are independently picked

Lossy Slepian-Wolf

- We require

$$p\left[u_1,\ldots,u_K|y_1,\ldots,y_K\right]=\Pi_{i=1}^K p\left[u_i|y_i\right]$$

Lossy Slepian-Wolf

- We require

$$p\left[u_1,\ldots,u_K|y_1,\ldots,y_K\right]=\prod_{i=1}^{K}p\left[u_i|y_i\right]$$

- Generate $\widehat{n}_1,\ldots,\widehat{n}_K$ deterministically from reconstructed u's.

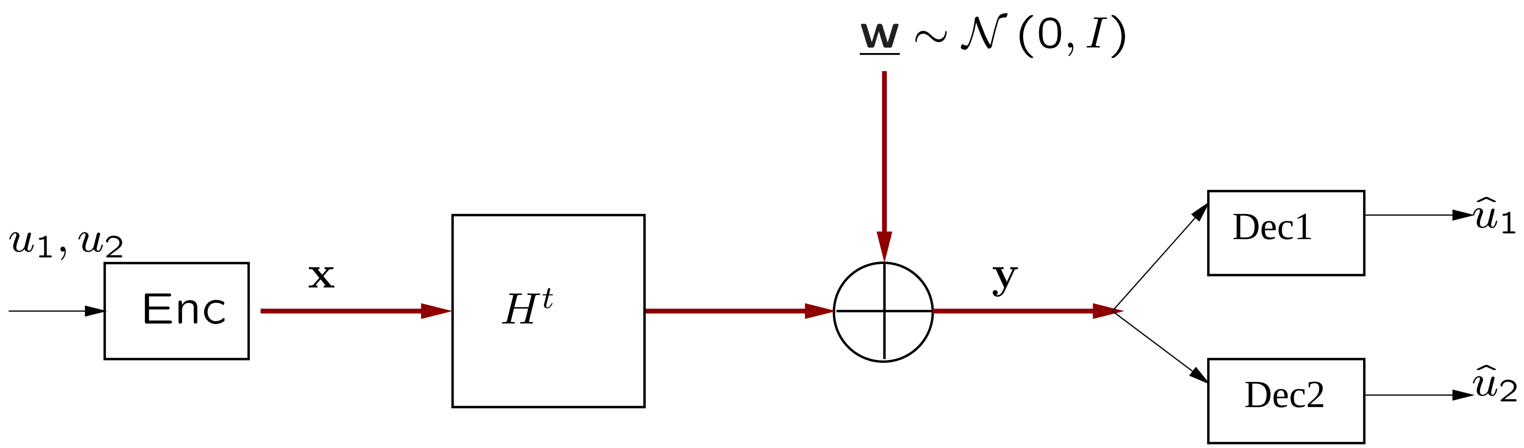
- Need sum rate

$$R_{\text{sum}}=I\left(u_1,\ldots,u_K;y_1,\ldots,y_K\right).$$

- Distortion equal to

$$E\left[d\left(\mathbf{n},\widehat{\mathbf{n}}\right)\right].$$

Marton Coding for Broadcast Channel



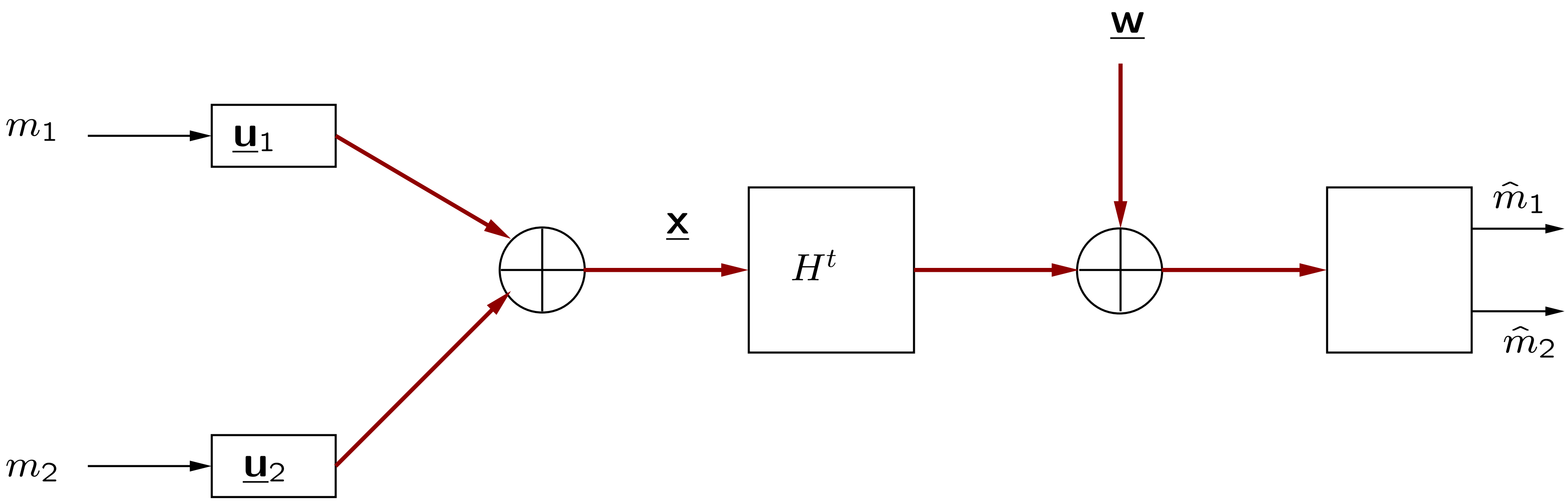
- Reversed encoding and decoding operations

- Sum rate $I(u_1, \ldots, u_K; y_1, \ldots, y_K)$.

- No use for the Markov property

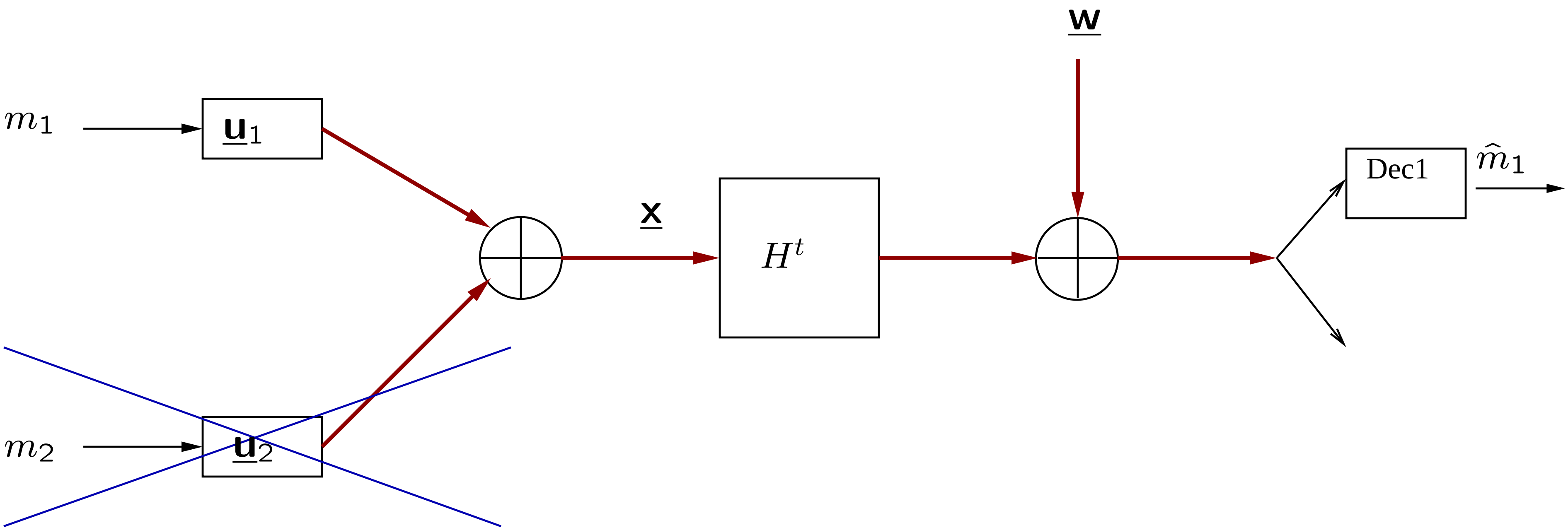
$$p[u_1, \ldots, u_K | y_1, \ldots, y_K] = \Pi_{i=1}^{K} p[u_i | y_i]$$

Achievable Rates: Costa Precoding



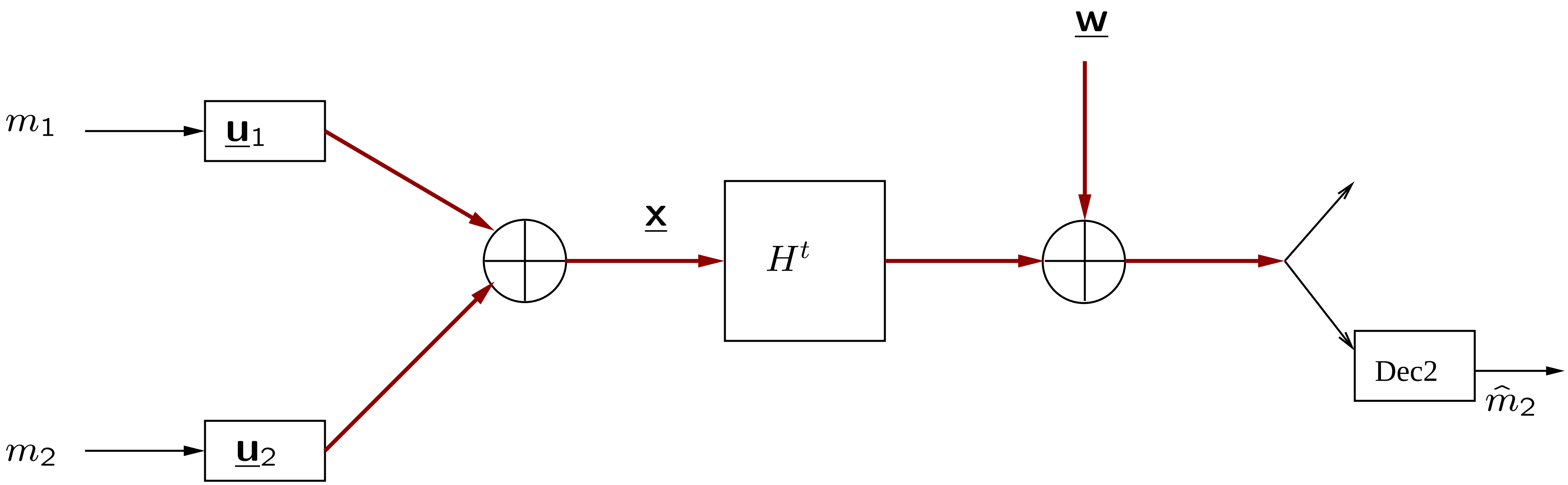
- Users' data modulated onto spatial signatures $\underline{u}_1, \underline{u}_2$

Stage 1: Costa Precoding



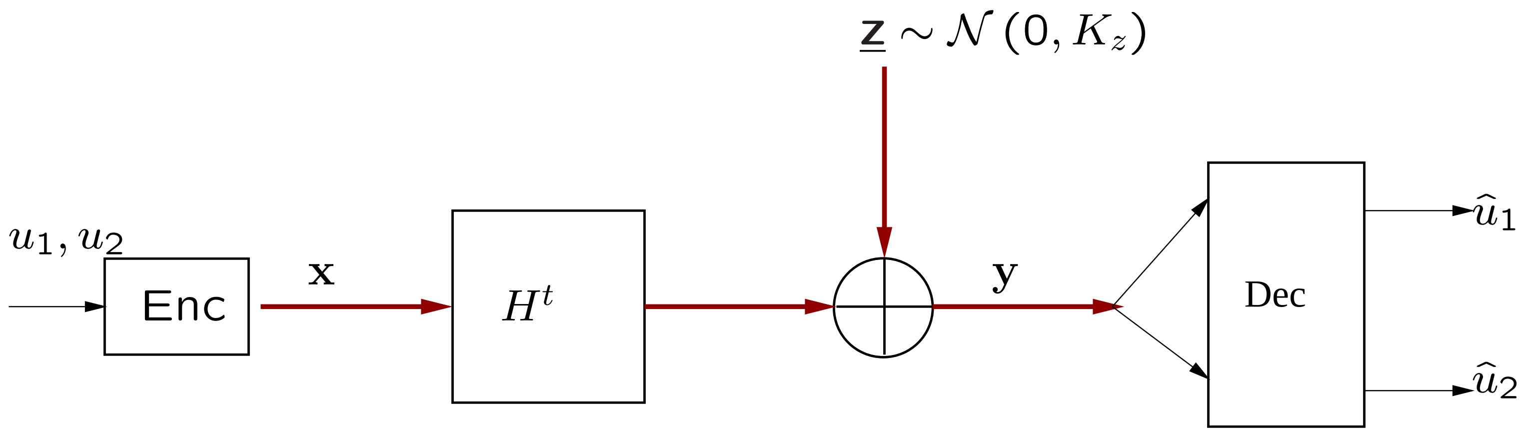
- Encoding for user 1 treating signal from user 2 as known interference at transmitter

Stage 2



- Encode user 2 treating signal for user 1 as noise

Adaptation to Lossy Slepian-Wolf



- Joint distribution of u's and y's depends on noise $\mathbf{z}$
- Performance independent of correlation in $\mathbf{z}$

Noise Coloring

- Fix particular Costa coding scheme - fixes u's and $\mathbf{x}$.

- Idea:

Choose $\mathbf{z}$ such that

$$p\left[u_1, \ldots, u_K | y_1, \ldots, y_K\right] = \prod_{i=1}^{K} p\left[u_i | y_i\right]$$

and $(K_z)_{ii} = 1$

- Then can adapt to Lossy Multiterminal Source Coding

Markov Condition and Broadcast Channel

- The Markov condition

$$p\left[u_1,\ldots,u_K|y_1,\ldots,y_K\right] = \Pi_{i=1}^{K} p\left[u_i|y_i\right]$$

  of independent interest in the broadcast channel

Markov Condition and Broadcast Channel

- The Markov condition

$$p\left[u_1,\ldots,u_K|y_1,\ldots,y_K\right]=\Pi_{i=1}^{K}p\left[u_i|y_i\right]$$

of independent interest in the broadcast channel

-

$$p\left[u_1,\ldots,u_K|y_1,\ldots,y_K\right]=\Pi_{i=1}^{K}p\left[u_i|y_1,\ldots,y_K,u_1,u_2,\ldots,u_{i-1}\right]$$

Markov Condition and Broadcast Channel

- The Markov condition

$$p\left[u_1, \ldots, u_K | y_1, \ldots, y_K\right] = \Pi_{i=1}^{K} p\left[u_i | y_i\right]$$

  of independent interest in the broadcast channel

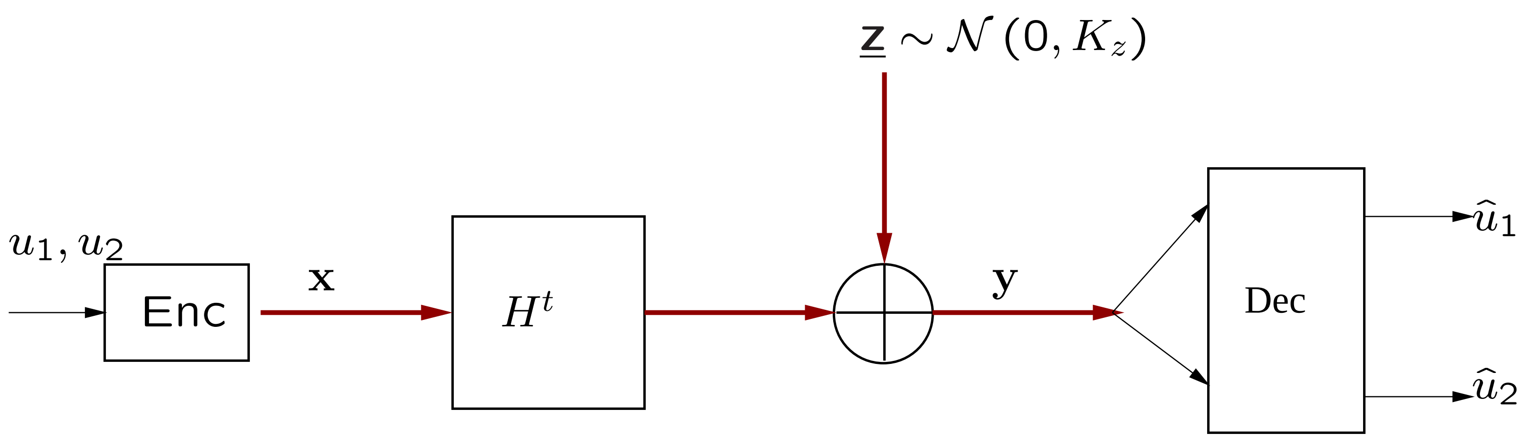
-

$$p\left[u_1, \ldots, u_K | y_1, \ldots, y_K\right] = \Pi_{i=1}^{K} p\left[u_i | y_1, \ldots, y_K, u_1, u_2, \ldots, u_{i-1}\right]$$

- Equivalent to: given $u_1, \ldots, u_{i-1}$

$$u_i \longrightarrow y_i \longrightarrow y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_K$$

Implication



- Need only y_1 to decode u_1

- Given u_1, need only y_2 to decode u_2

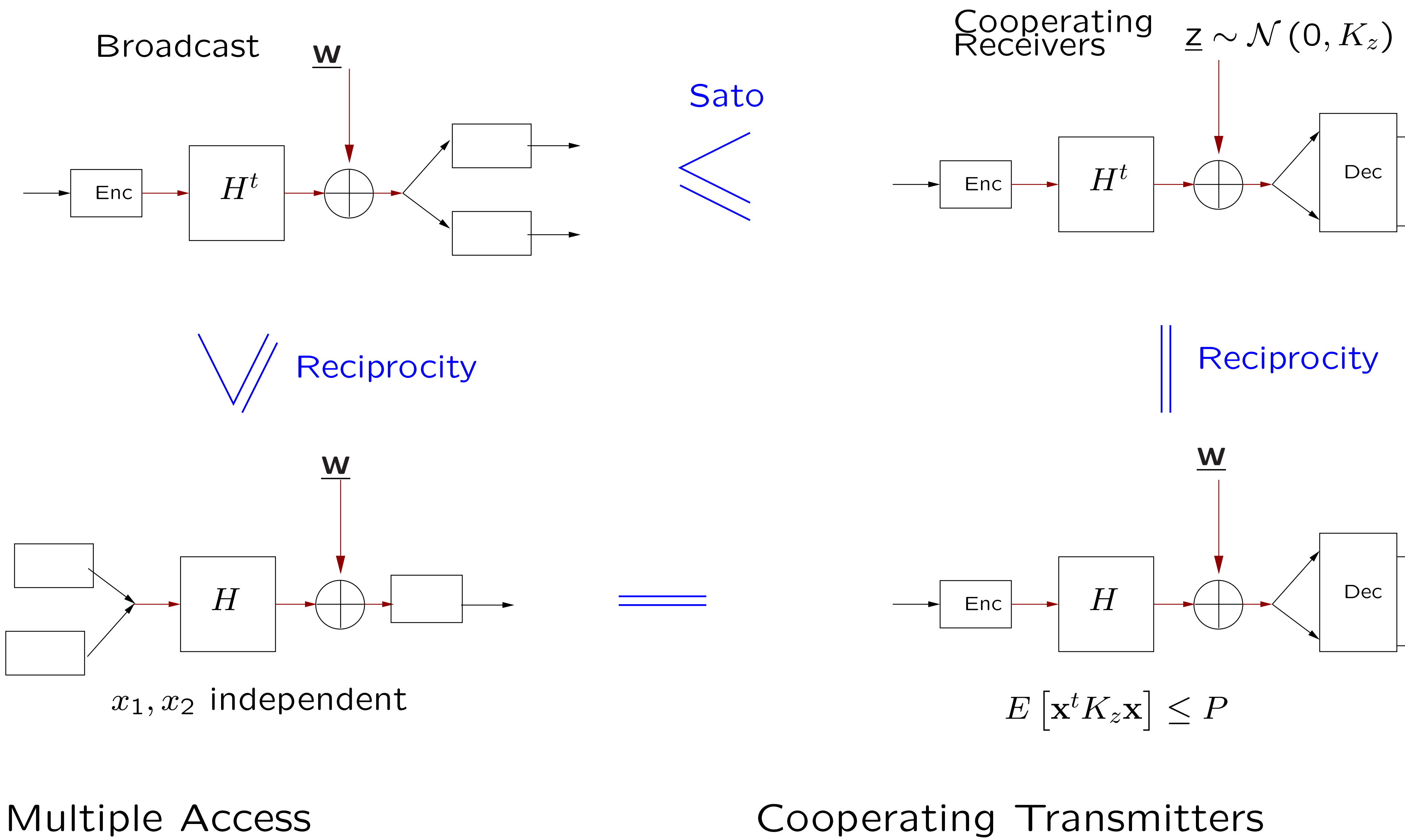
Performance of Costa scheme equals that when receivers cooperate

Markov Condition and Noise Covariance

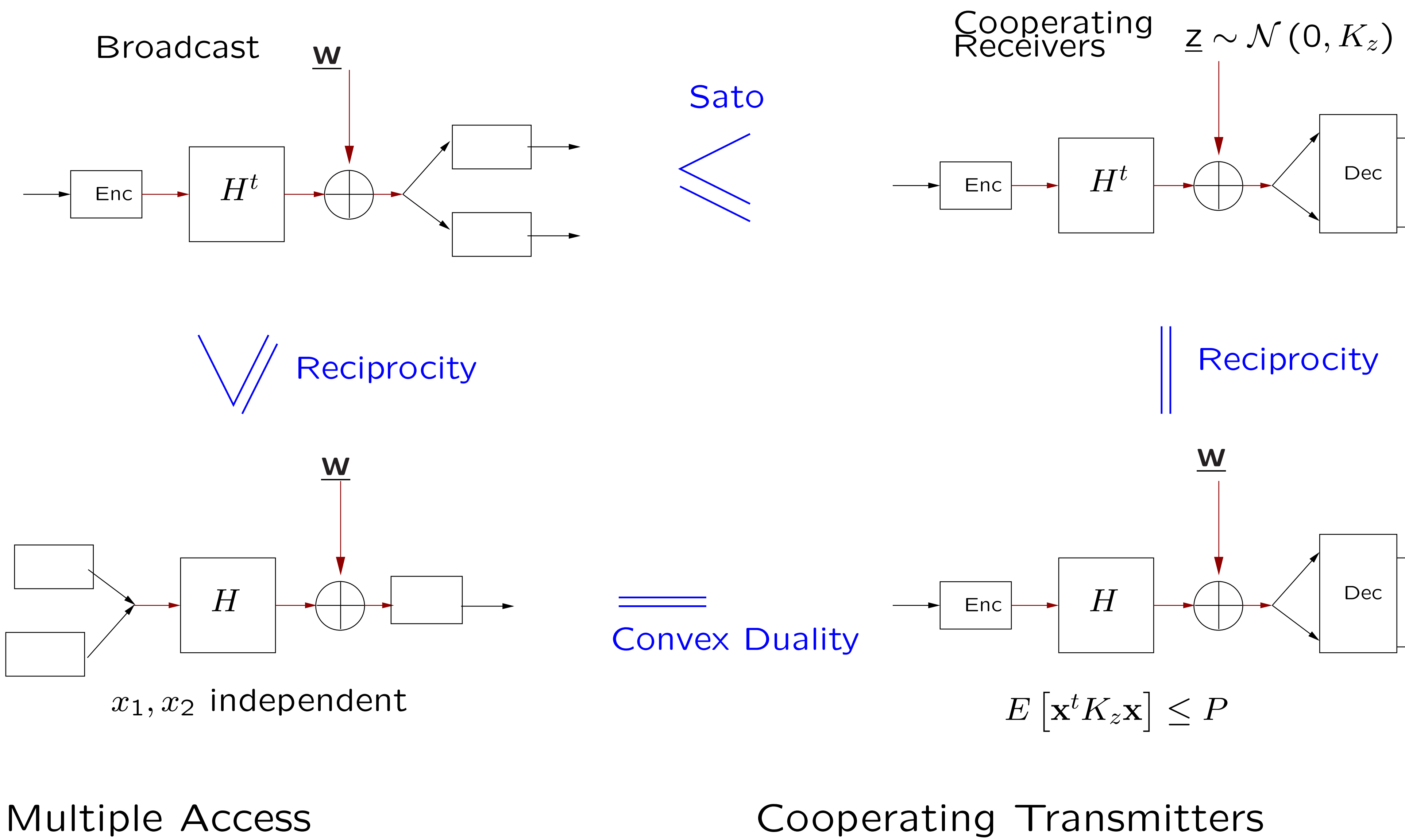
- The sum capacity is also achieved by such a scheme (CS 01, YC 01, VT 02, VJG 02)

- For every Costa scheme, there is a choice of K_z such that Markov condition holds (Yu and Cioffi, 01)

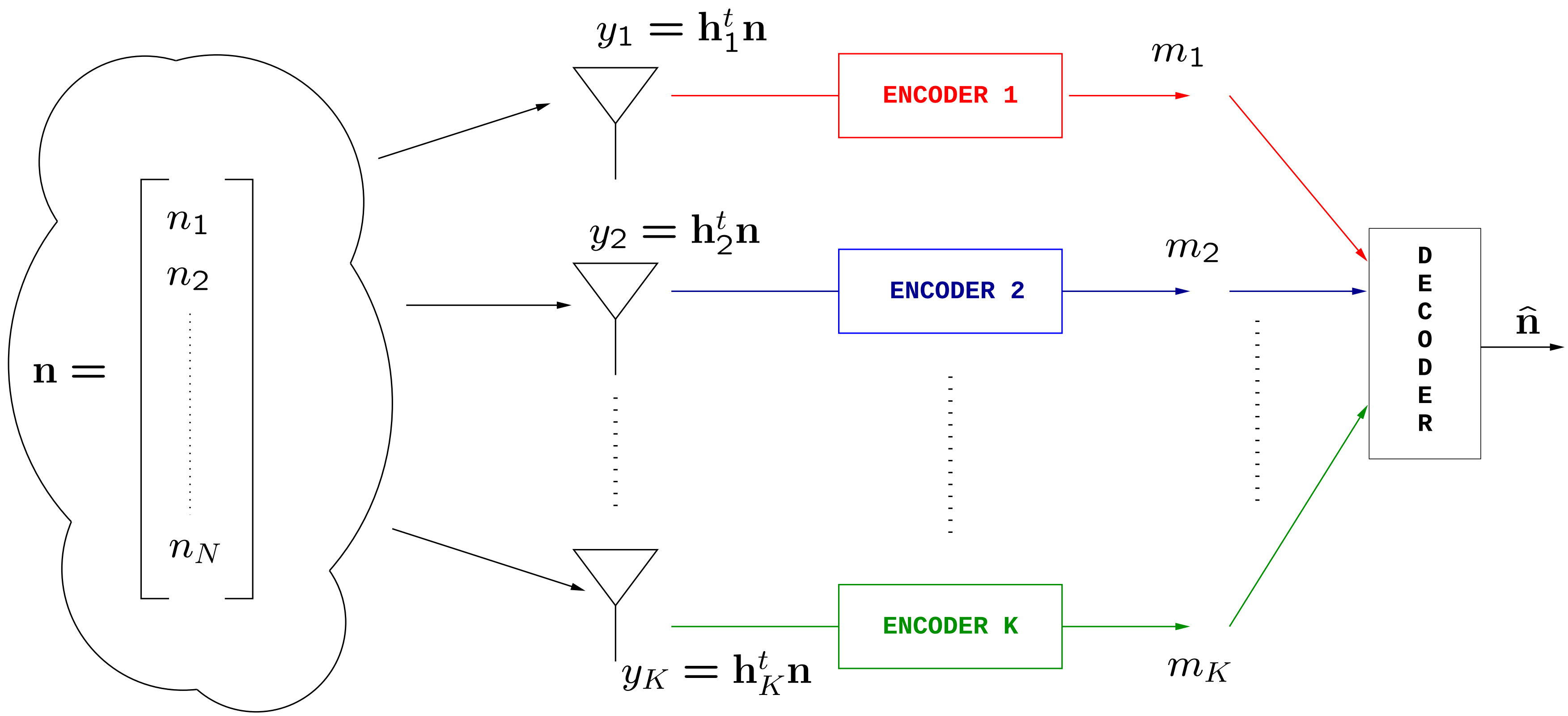
Sum Capacity



Sum Capacity



Gaussian Multiterminal Source Coding



$$H = [\mathbf{h}_1, \ldots, \mathbf{h}_K]$$

Sum of rates of encoders R and distortion metric $d(\mathbf{n}, \hat{\mathbf{n}})$

Main Result

- Distortion metric

$$d(\mathbf{n};\hat{\mathbf{n}}) = \frac{1}{N}(\mathbf{n}-\hat{\mathbf{n}})^t \left(I + H\text{diag}\{p_1,\ldots,p_K\} H^t\right)(\mathbf{n}-\hat{\mathbf{n}})$$

  – Here $p_1,\ldots,p_K$ - powers of users in reciprocal MAC

- Rate distortion function

$$\begin{aligned} R(D) &= \text{Sum rate of MAC} - N\log D \\ &= \text{Sum rate of Broadcast Channel} - N\log D \\ &= \log\det\left(I + HDH^t\right) - N\log D \end{aligned}$$

Bells and Whistles

- For quadratic distortion metric

$$d(\mathbf{n};\hat{\mathbf{n}}) = \frac{1}{N}(\mathbf{n}-\hat{\mathbf{n}})^t(\mathbf{n}-\hat{\mathbf{n}})$$

  set of $\mathbf{H}$ can be characterized

- Analogy with CEO problem:

For large number of encoders and random $\mathbf{H}$
characterization of $R(D)$ almost surely

Discussion

- A “connection” made between coding schemes for multiterminal source and channel coding

Discussion

- A "connection" made between coding schemes for multiterminal source and channel coding

- Connection somewhat superficial

  - relation between source coding and broadcast channel through a common random coding argument (PR 02, CC 02)

  - relation between source coding and multiple access channel through a change of variable (VT 02, JVG 01)

Discussion

- A "connection" made between coding schemes for multiterminal source and channel coding
- Connection somewhat superficial
  - relation between source coding and broadcast channel through a common random coding argument
  - relation between source coding and multiple access channel through a change of variable
- Connection is suggestive
  - a codebook level duality