HYBRID ARQ IN WIRELESS NETWORKS

Emina Soljanin
Mathematical Sciences Research Center, Bell Labs

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ACKNOWLEDGEMENTS

Alexei Ashikhmin
Jaehyiong Kim
Sudhir Ramakrishna
Adriaan van Wijngaarden

Automatic Repeat Request

- The receiving end detects frame errors and requests retransmissions.
- P_e is the frame error rate, the average number of transmissions is

$$1 \cdot (1 - P_e) + \cdots + n \cdot P_e^{n-1}(1 - P_e) + \cdots = \frac{1}{1 - P_e}$$

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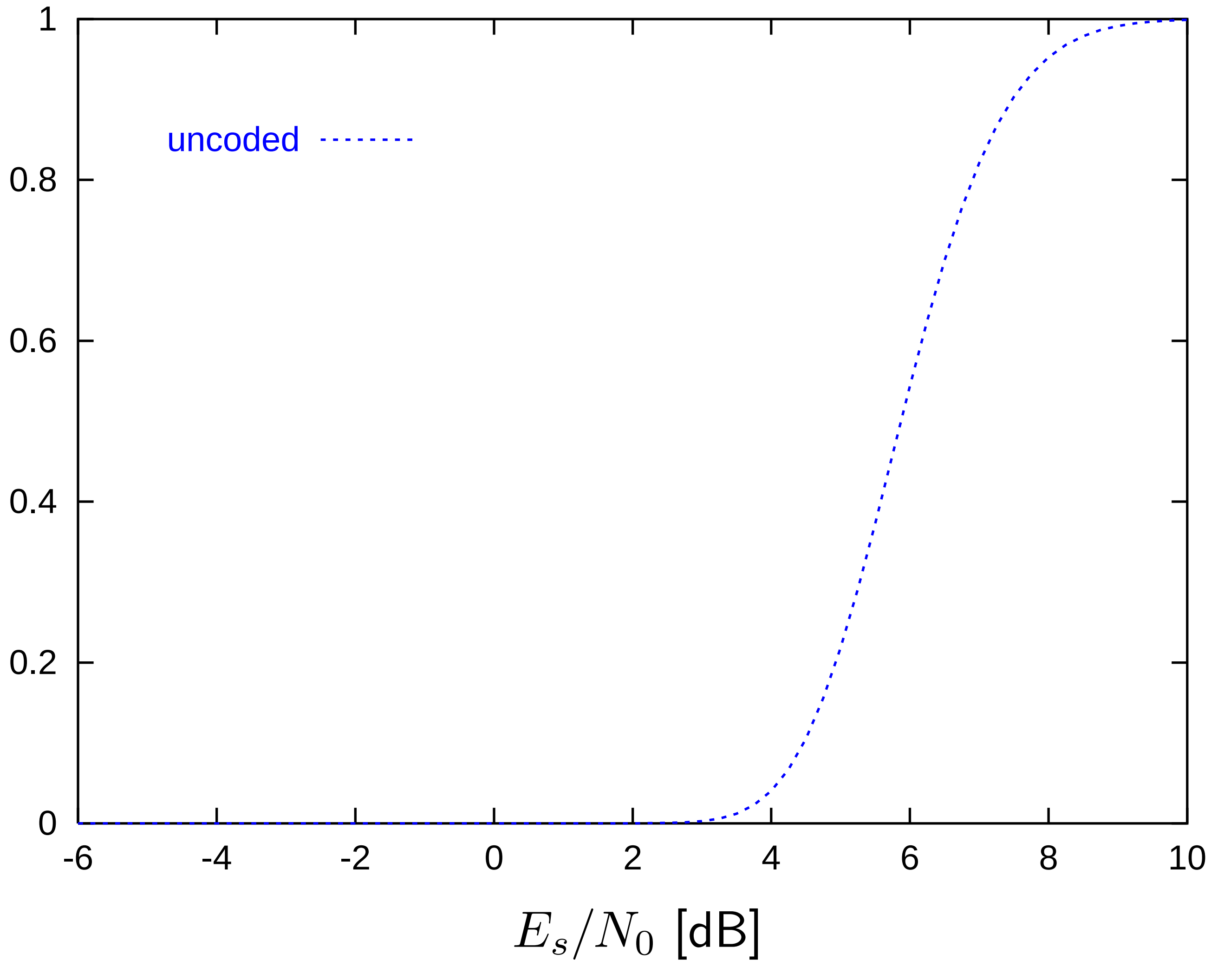
- Hybrid ARQ uses a code that can correct some frame errors.
- In HARQ schemes
  - the average number of transmissions is reduced, but
  - each transmission carries redundant information.

THROUGHPUT IN HYBRID ARQ
BPSK, AWGN, BCH Coded



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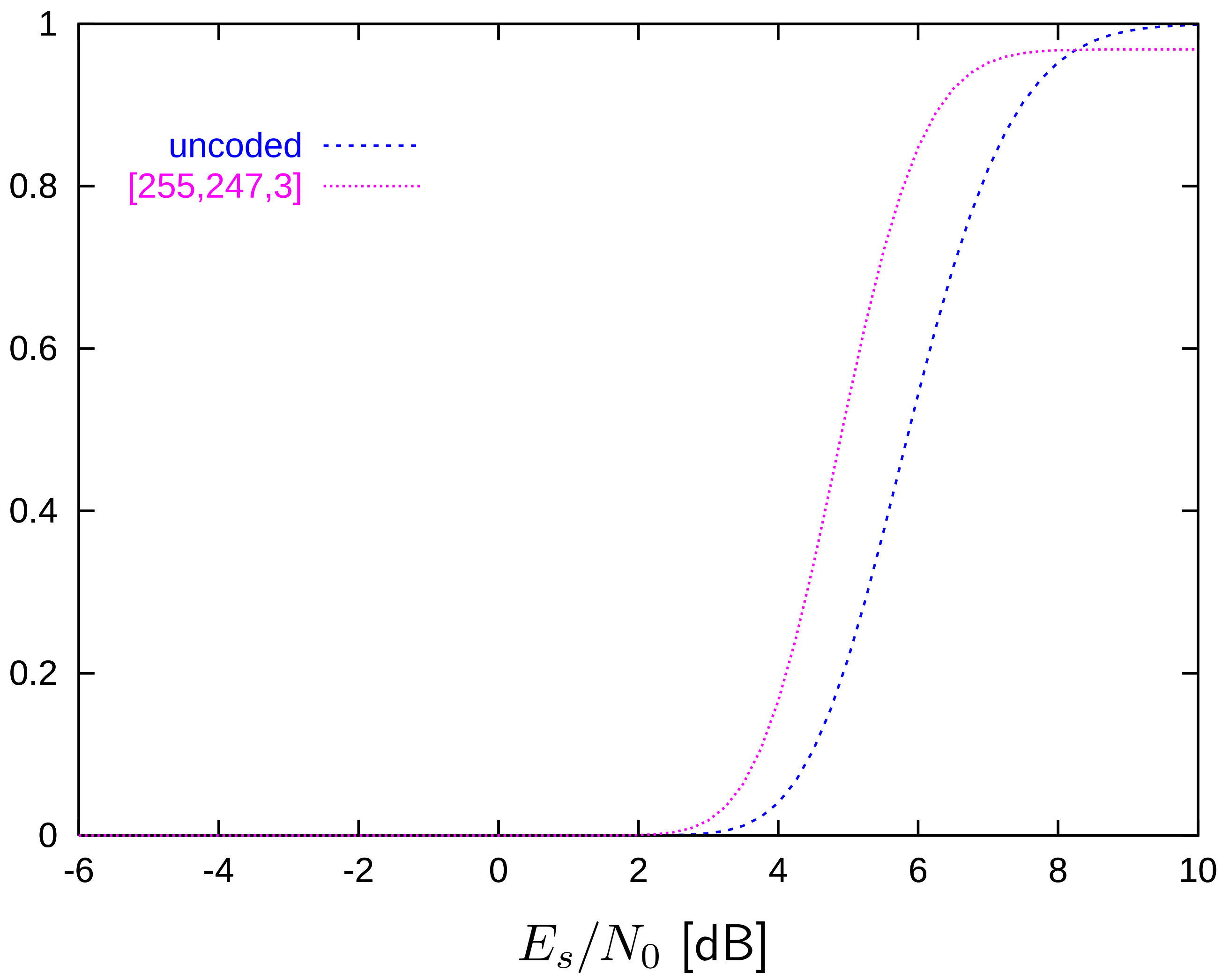

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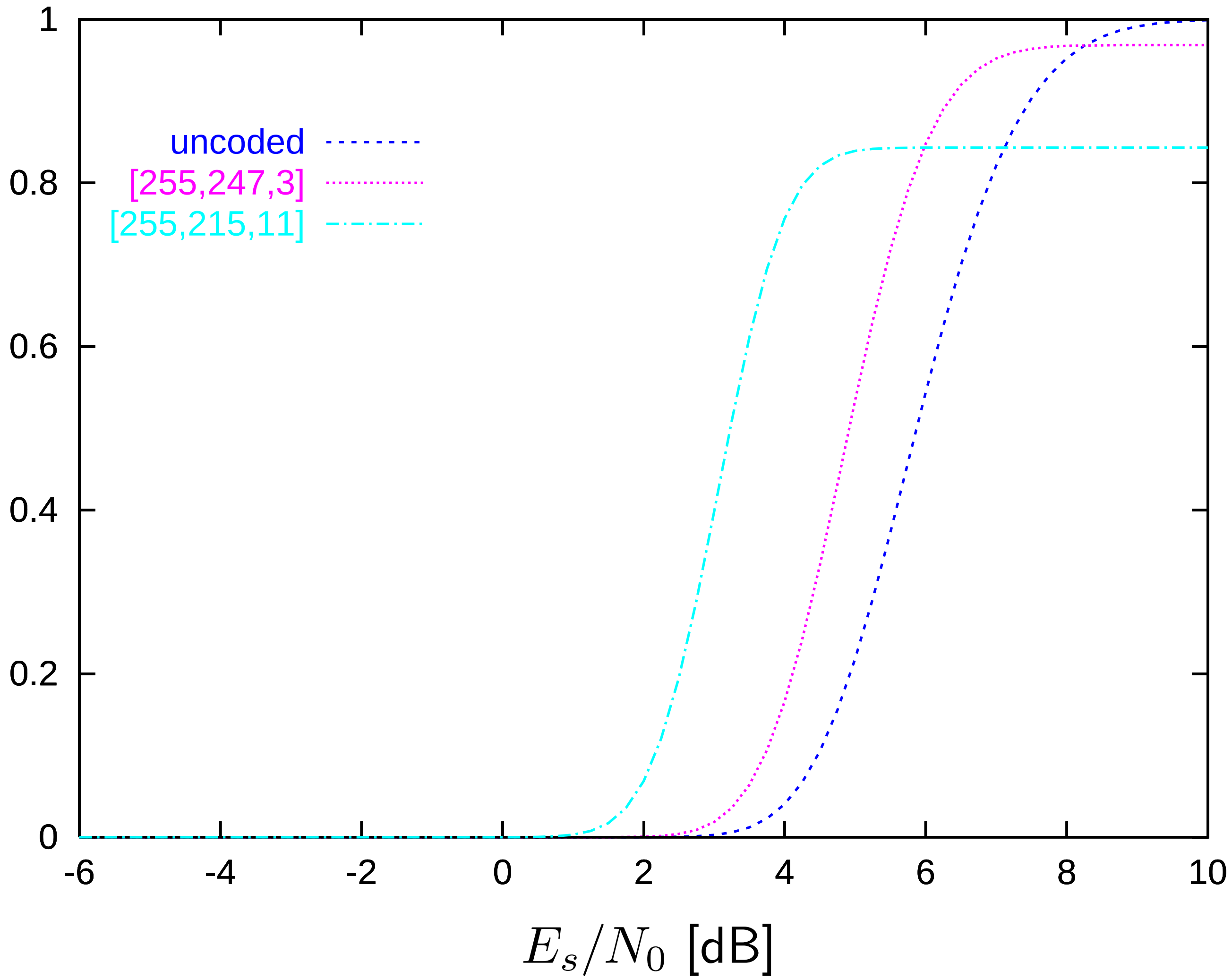

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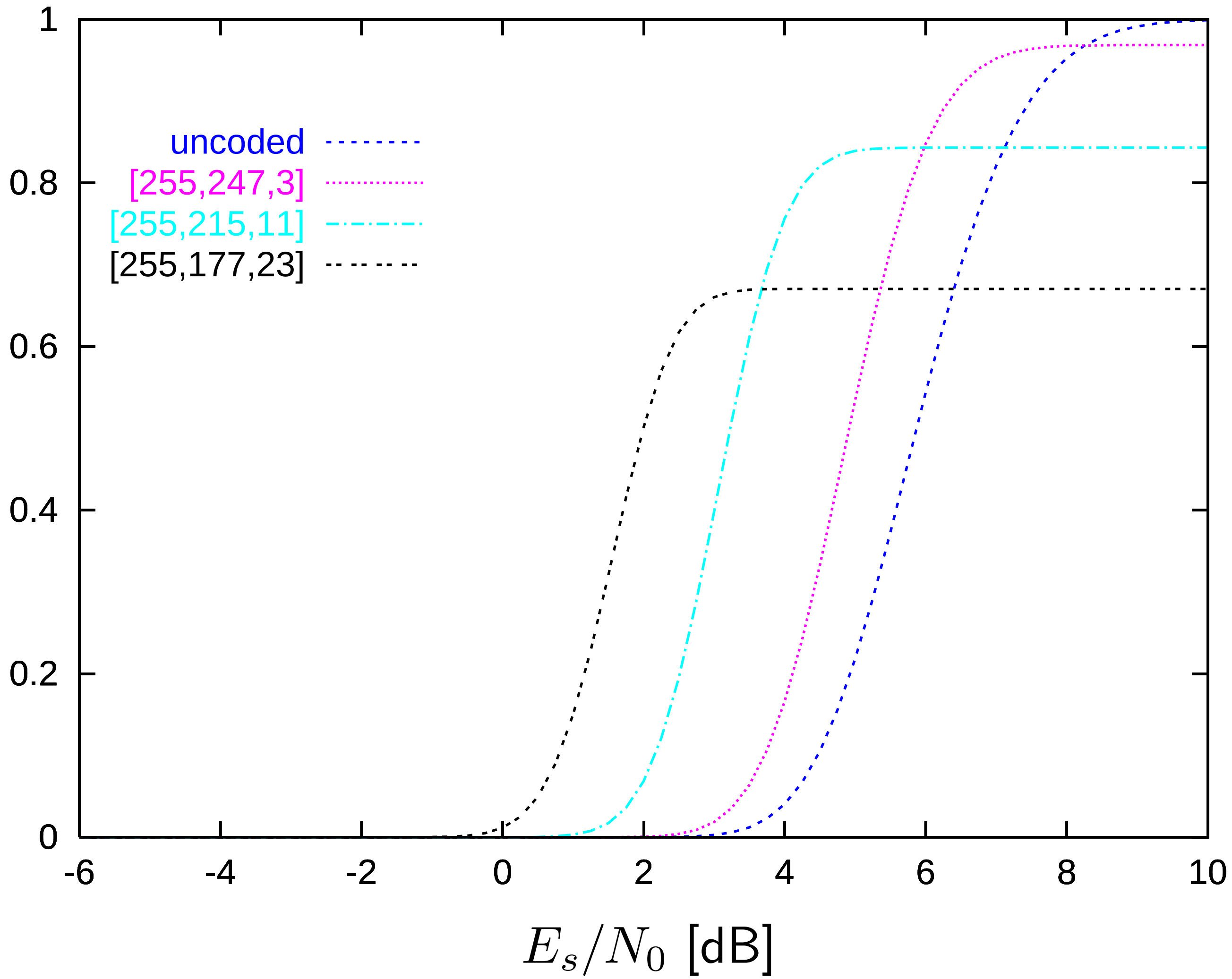

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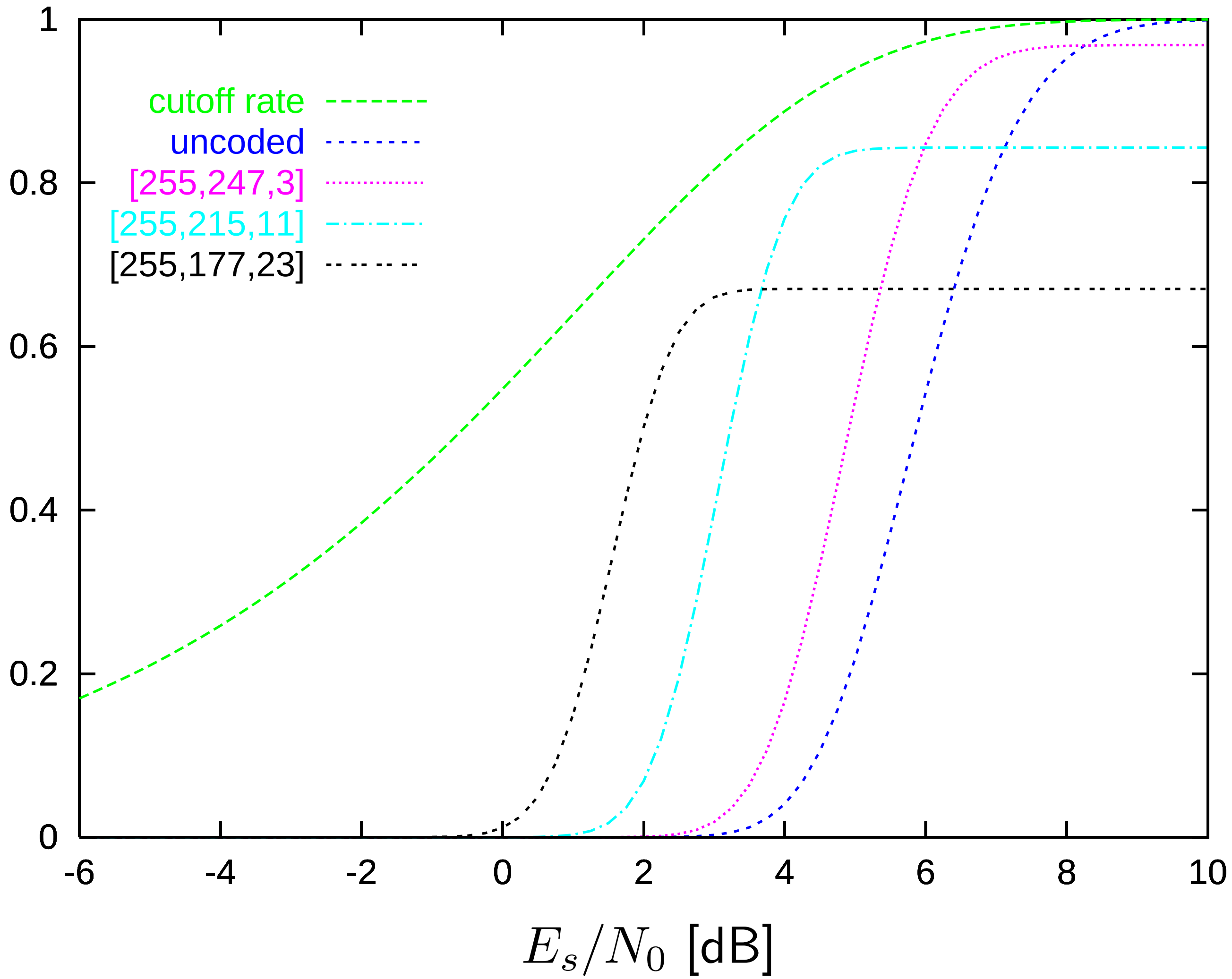

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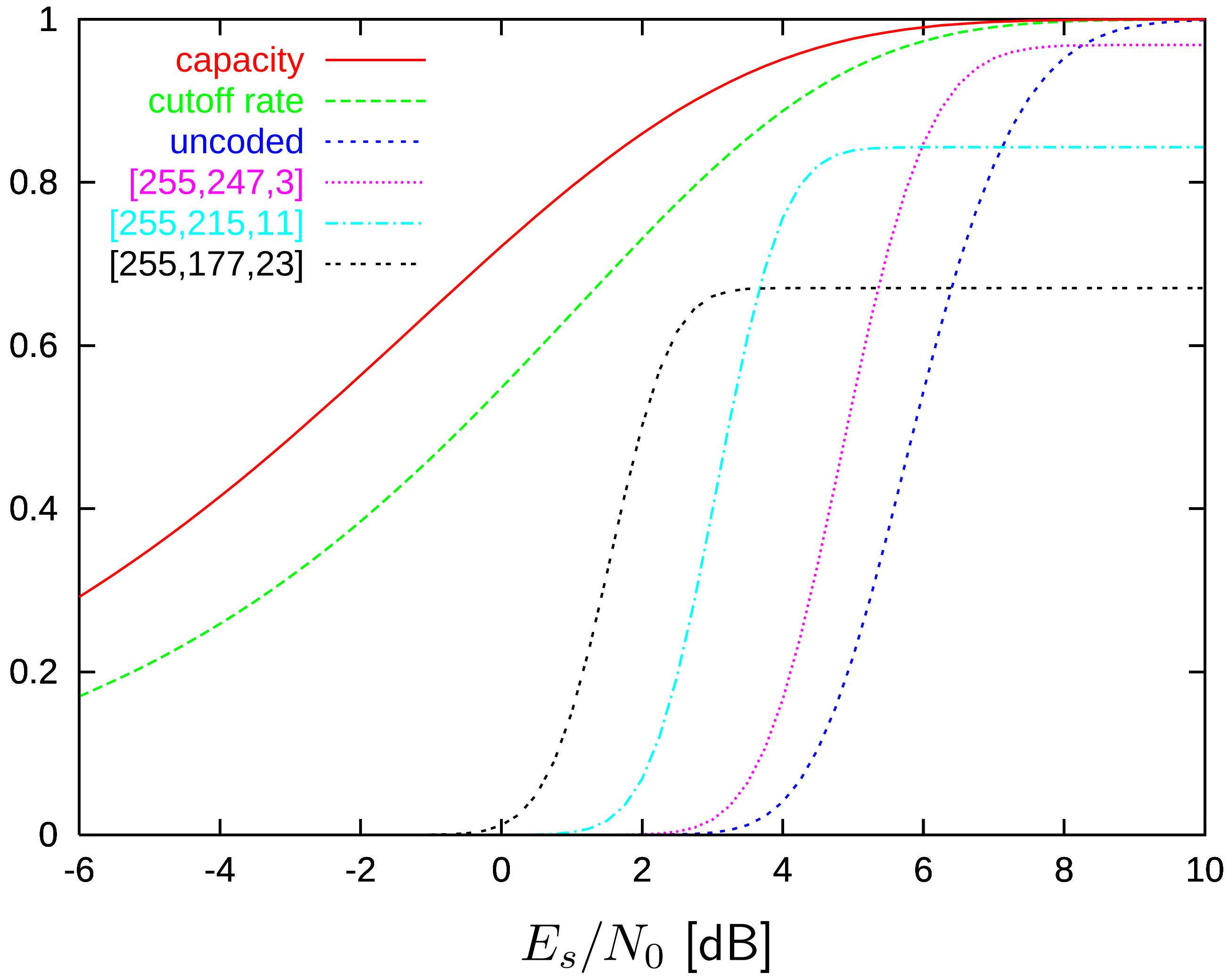

TYPE II HYBRID ARQ
Incremental Redundancy

- Information bits are encoded by a (low rate) mother code.
- Information and a selected number of parity bits are transmitted.

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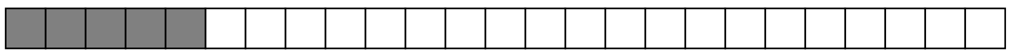
- Information and a selected number of parity bits are transmitted.
- If a retransmission is not successful:
  - transmitter sends additional selected parity bits
  - receiver puts together the new bits and those previously received.
- Each retransmission produces a codeword of a stronger code.
- Family of codes obtained by puncturing of the mother code.

INCREMENTAL REDUNDANCY

A Rate $1/5$ Mother Code



at the transmitter

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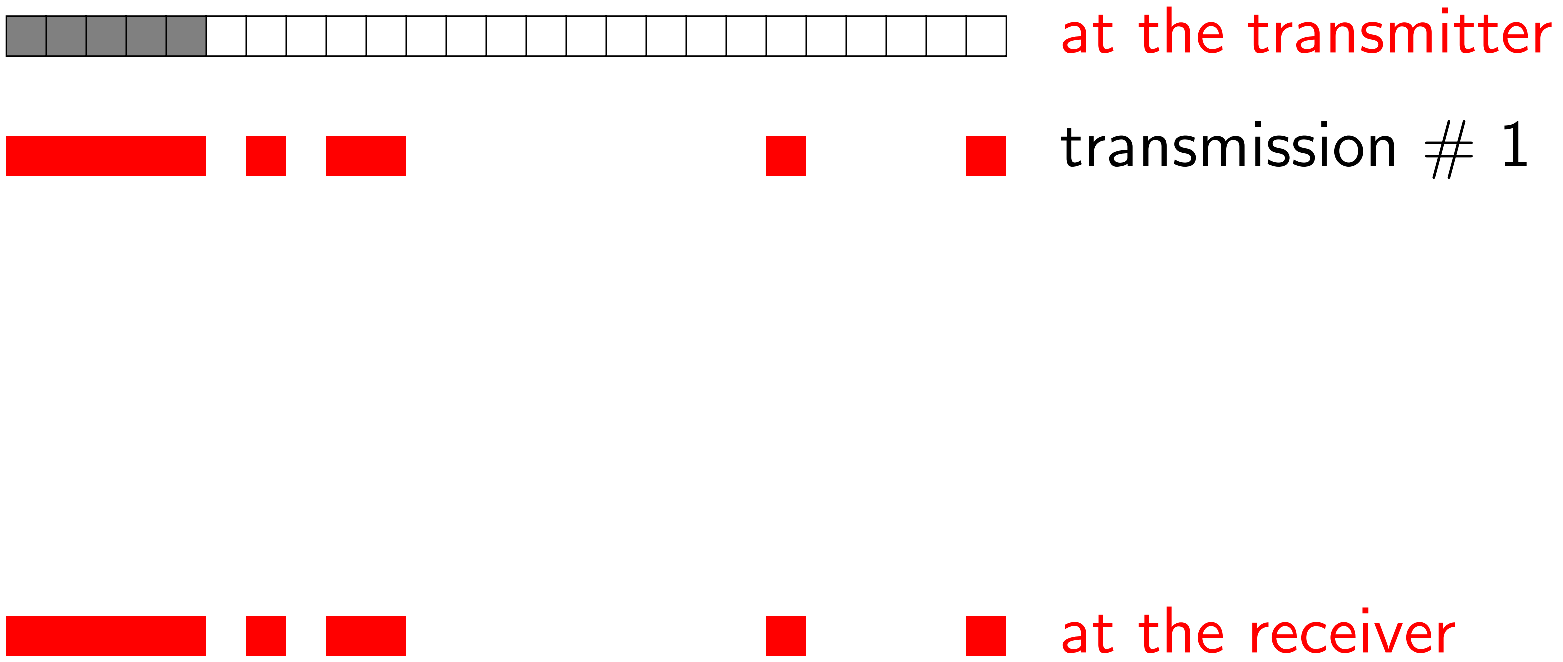

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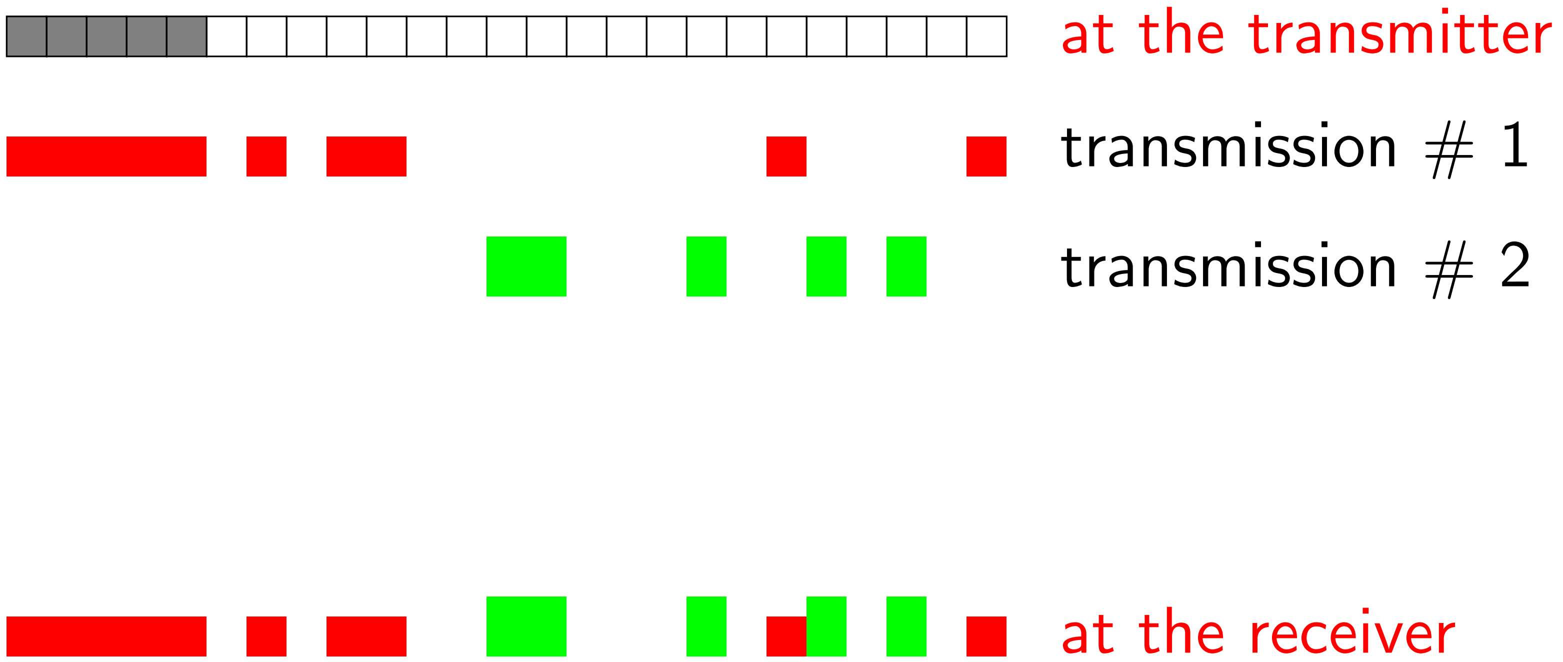

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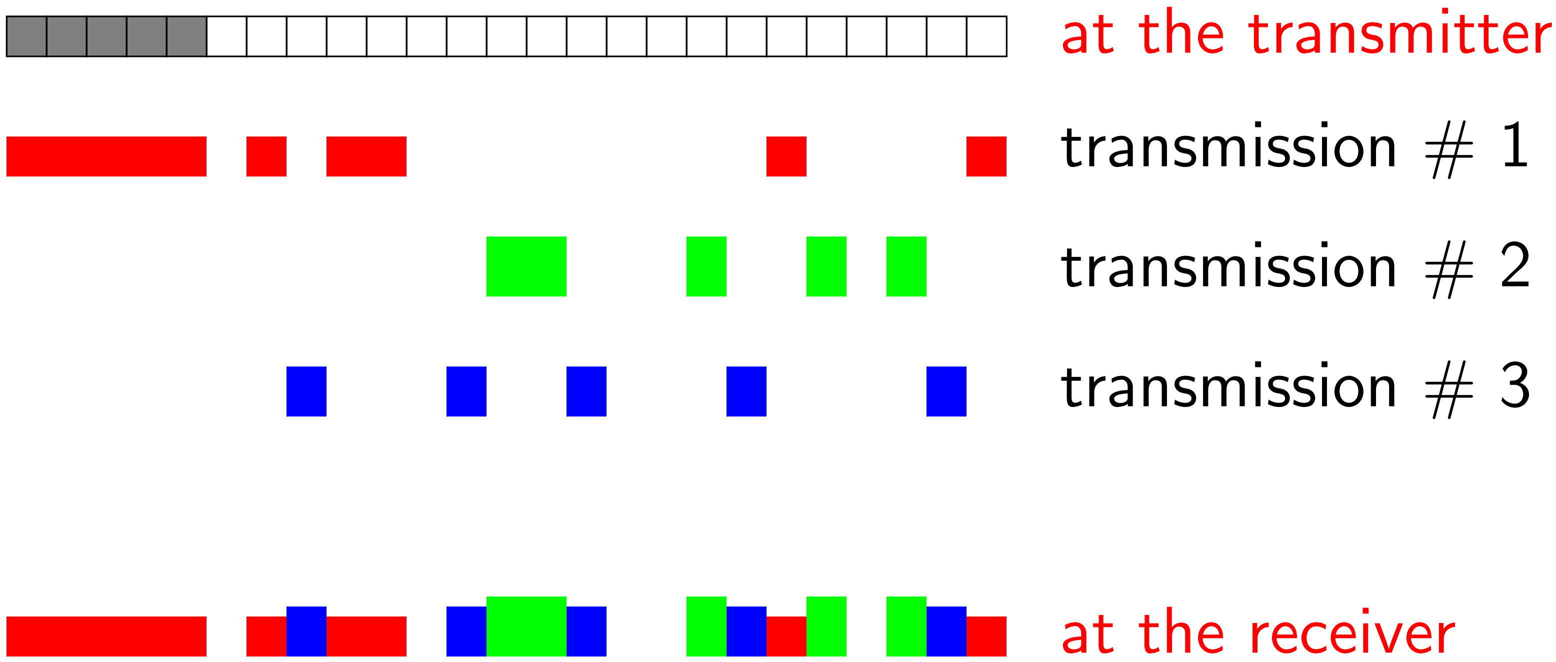

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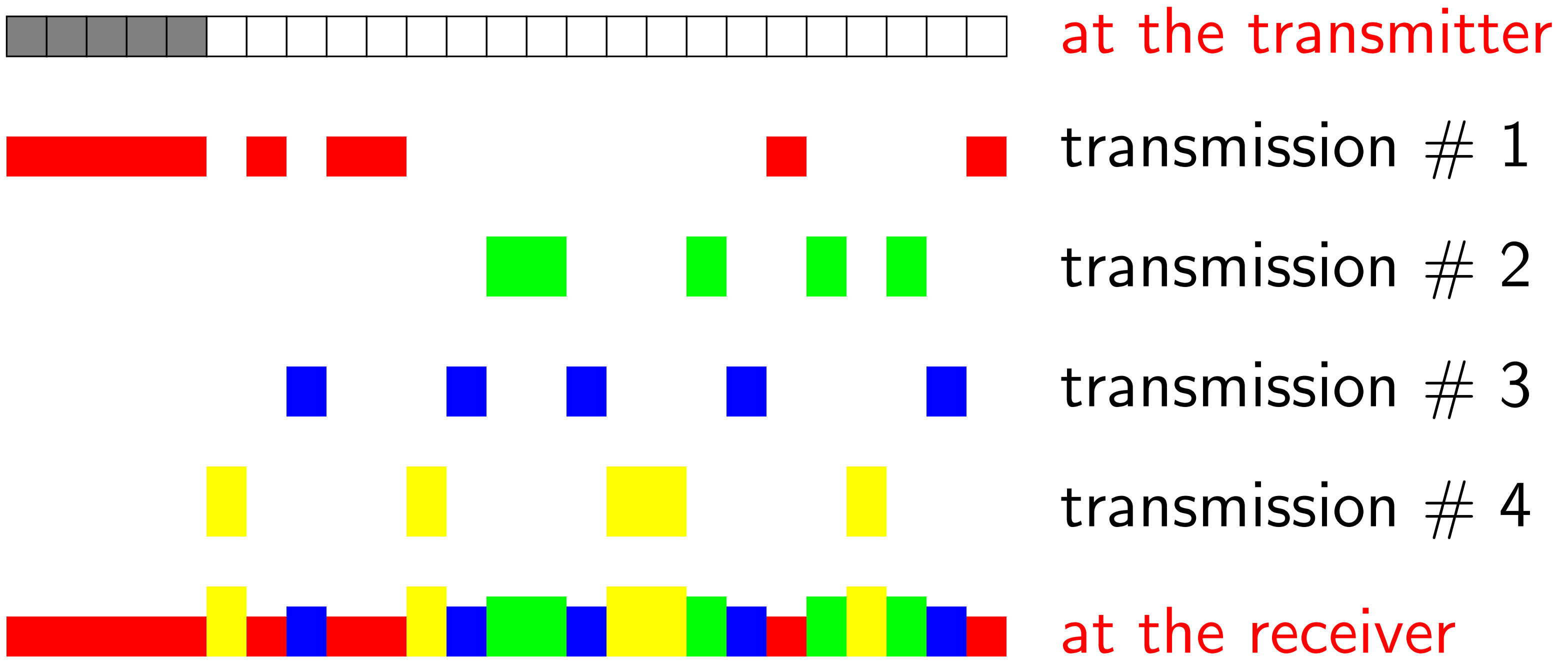

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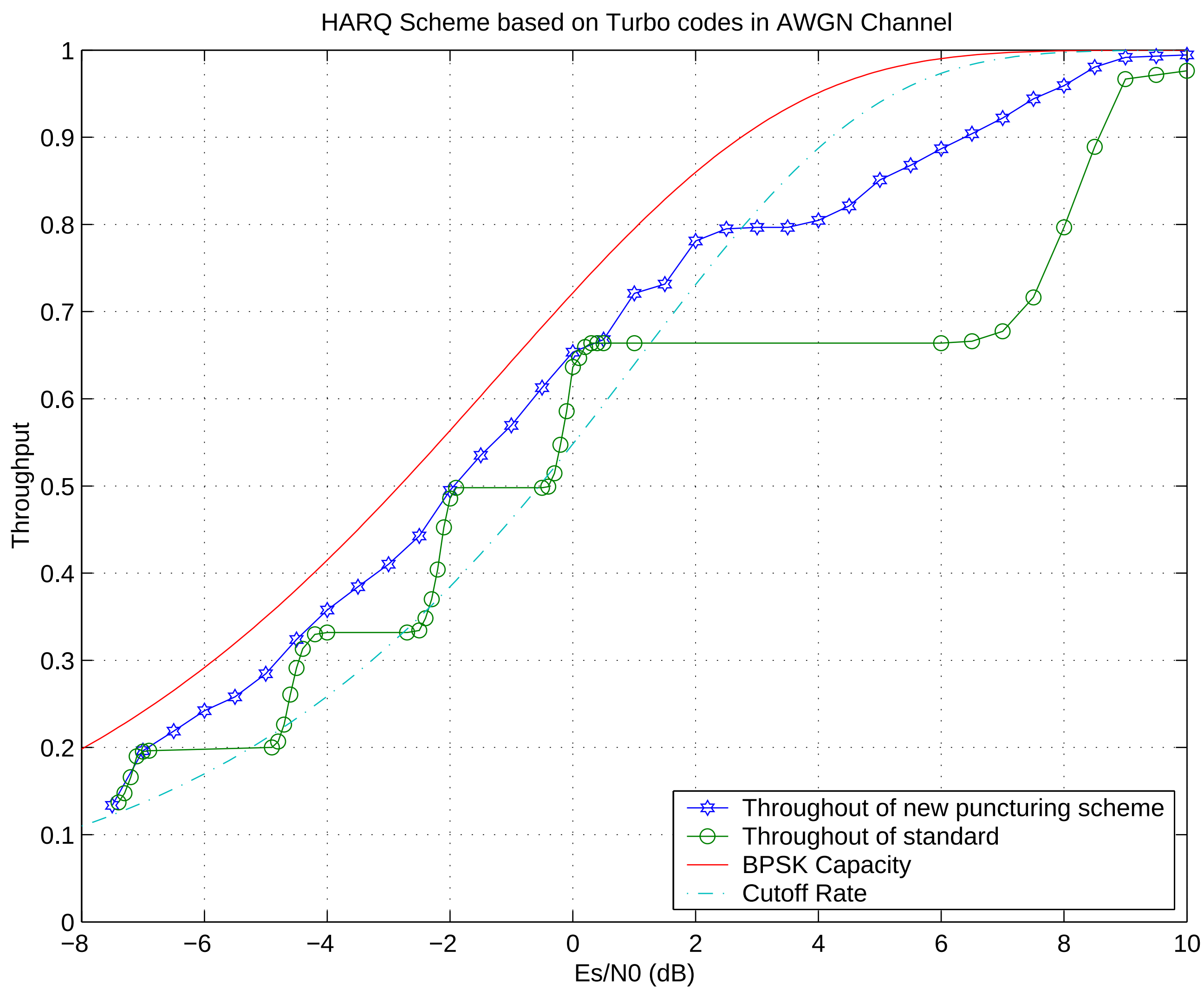

RANDOMLY PUNCTURED CODES

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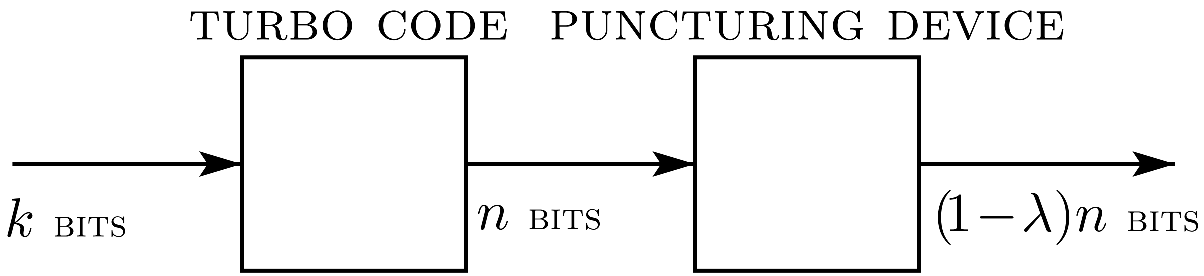
- Each bit is punctured independently with probability λ.
- The expected rate of the punctured code is $R/(1-\lambda)$.
- For large n we have



A FAMILY OF RANDOMLY PUNCTURED CODES

Rate Compatible Puncturing

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- λ_j for $j = 1, 2, \ldots, m$ are puncturing rates, $\lambda_j > \lambda_k$ for $j < k$.
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- If the i-th bit is punctured in the k-th code and $j < k$, then it was punctured in the j-th code.
- θ_i for $i = 1, 2, \ldots, n$ are uniformly distributed over $[0, 1]$.
- If $\theta_i < \lambda_l$, then the i-th bit is punctured in the l-th code.

MEMORYLESS CHANNEL MODEL

- Binary input alphabet $\{0, 1\}$ and output alphabet $\mathcal{Y}$.
- Constant in time with transition probabilities $W(b|0)$ and $W(b|1)$, $b \in \mathcal{Y}$.

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- Constant in time with
  transition probabilities $W(b|0)$ and $W(b|1)$, $b \in \mathcal{Y}$.

- Time varying with
  transition probabilities at time i $W_i(b|0)$ and $W_i(b|1)$, $b \in \mathcal{Y}$.

- $W_i(\cdot|0)$ and $W_i(\cdot|1)$ are known at the receiver.

PERFORMANCE MEASURE

Time Invariant Channel

- Sequence $\boldsymbol{x} \in \mathcal{C} \subseteq \{0,1\}^n$ is transmitted, and $\boldsymbol{x'}$ decoded.

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$$P_e(\boldsymbol{x}, \boldsymbol{x'}) \leq \gamma^d = \exp\{-d\alpha\},$$

where γ is the Bhattacharyya noise parameter:

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and $\alpha = -\log \gamma$ is the Bhattacharyya distance.

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- Weight distribution A_d for a turbo code?

- Consider a set of codes $[\mathcal{C}]$ corresponding to all interleavers.

- Use the average $\overline{A}_d^{[\mathcal{C}](n)}$ instead of A_d for large n.

TURBO CODE ENSEMBLES

A Coding Theorem by Jin and McEliece

- There is an ensemble distance parameter $c_0^{[\mathcal{C}]}$ s.t. for large n

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- For a channel whose Bhattacharyya distance $\alpha > c_0^{[\mathcal{C}]}$, we have

$$\overline{P}_W^{[\mathcal{C}](n)} = O(n^{-\beta}).$$

- $c_0^{[\mathcal{C}]}$ is the ensemble noise threshold.

PUNCTUREDTURBO CODE ENSEMBLES
ITW, April 2003

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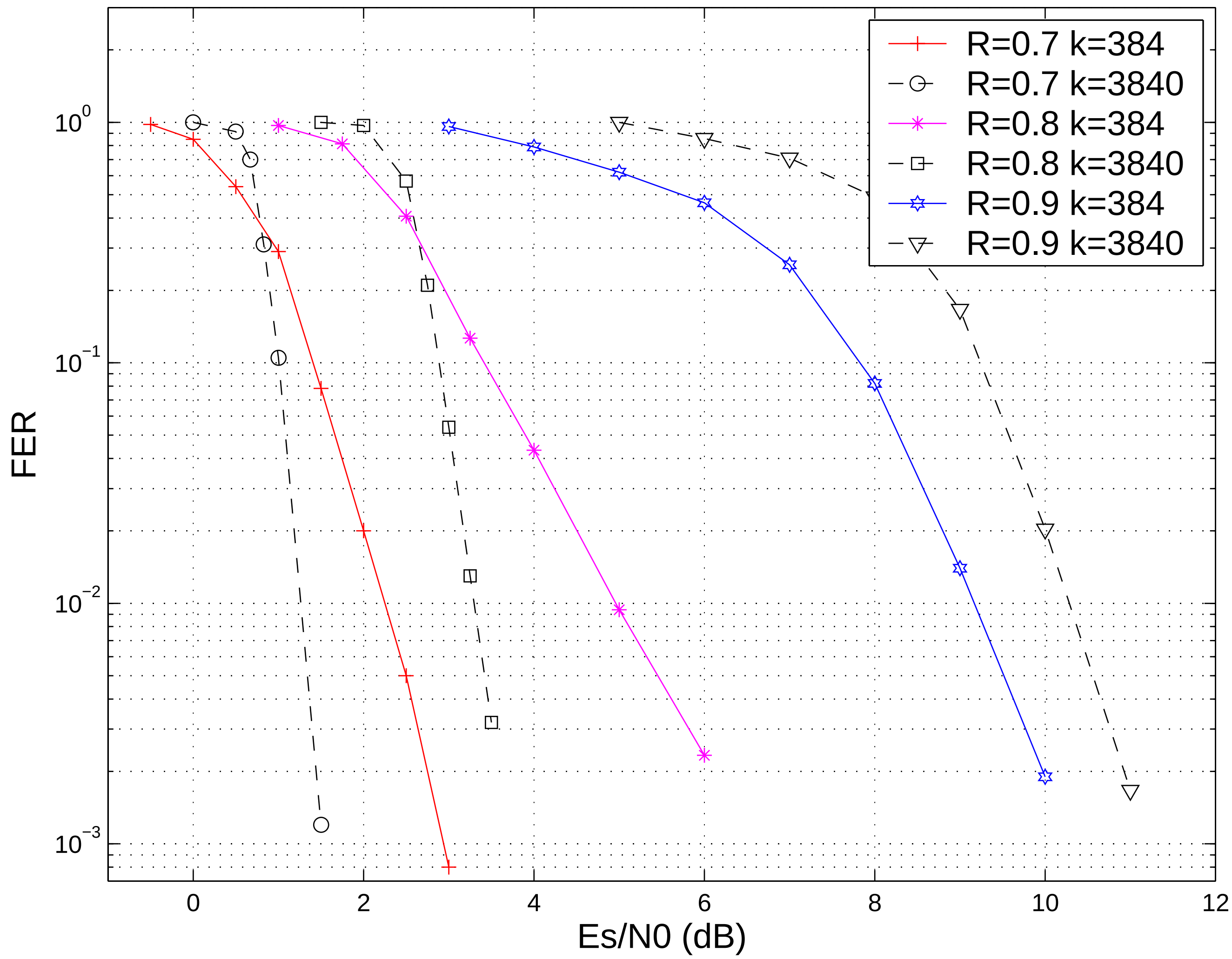

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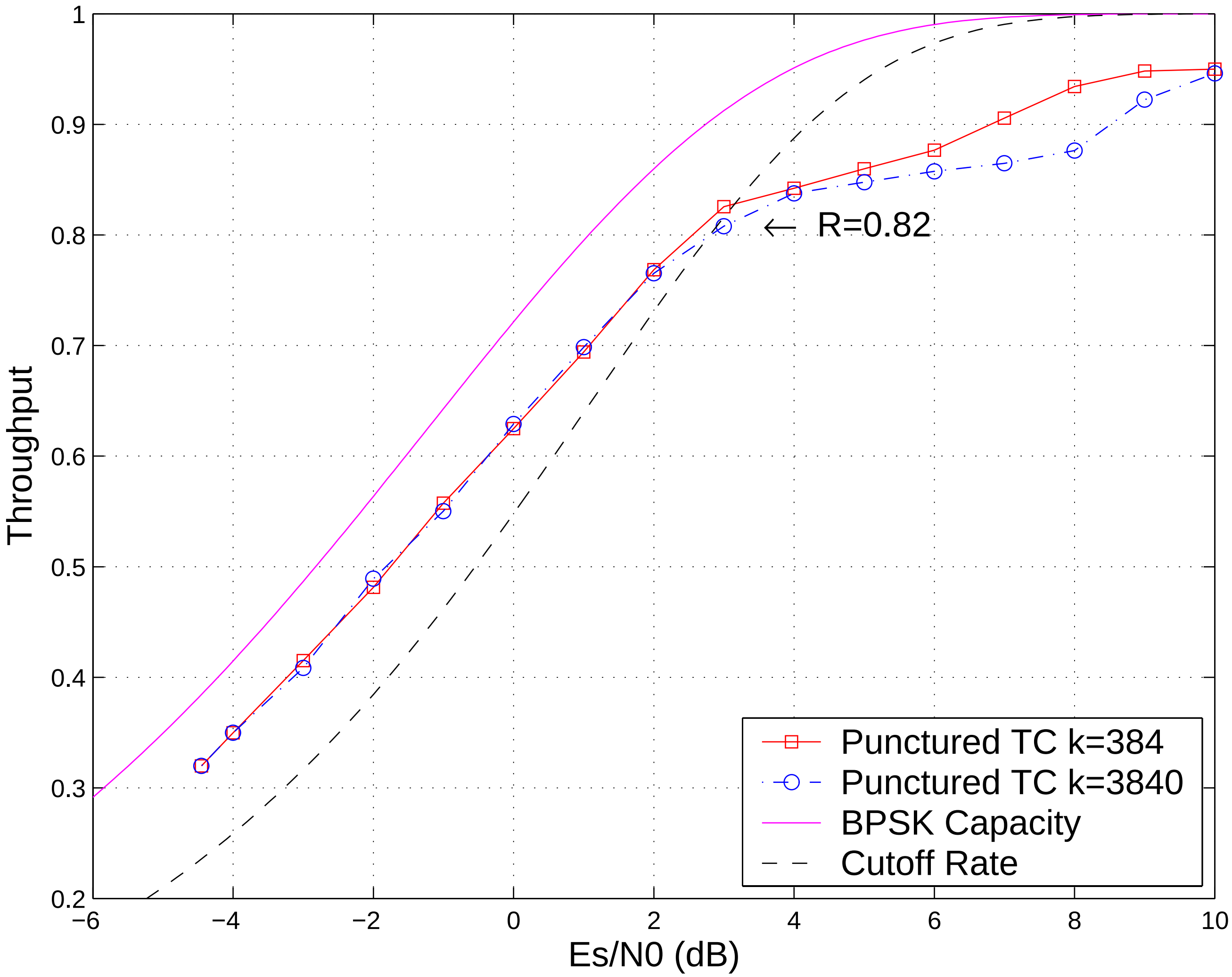

HARQ MODEL

- There are at most m transmissions.
- $\mathcal{I} = \{1, \ldots, n\}$ is the set indexing the bit positions in a codeword.
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- Bits at positions in $\mathcal{I}(j)$ are transmitted during j-th transmission.
- The channel remains constant during a single transmission:

$$\gamma_i = \gamma(j) \ \text{ for all } i \in \mathcal{I}(j).$$

PERFORMANCE MEASURE

Time Varying Channel

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P_e(\boldsymbol{x},\boldsymbol{x'}) &\leq \sum_{\boldsymbol{y}\in\mathcal{Y}^n} \sqrt{W^n(\boldsymbol{y}|\boldsymbol{x})W^n(\boldsymbol{y}|\boldsymbol{x'})} \\
&= \prod_{i=1}^{n} \Big(\sum_{b\in\mathcal{Y}} \sqrt{W_i(b|x_i)W_i(b|x_i')} \Big) \\
&\leq \prod_{i:x_i\neq x_i'} \gamma_i
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- $A_{d_1 \ldots d_m}$ is the number of codewords with weight d_j over $\mathcal{I}(j)$.
- The union bound on the ML decoder word error probability:

$$P \leq \sum_{d_1=1}^{|\mathcal{I}(1)|} \cdots \sum_{d_m=1}^{|\mathcal{I}(m)|} A_{d_1 \ldots d_m} \prod_{j=1}^{m} \gamma(j)^{d_j}$$

HARQ PERFORMANCE

Random Transmission Assignment

- A bit is assigned to transmission j with probability α_j.
- d is the weight of the original codeword.
- d_j is the weight of the d-th transmission sub-word.
- The probability that the sub-word weights are $d_1, d_2 \ldots, d_m$ is

$$\binom{d}{d_1}\binom{d-d_1}{d_2}\cdots\binom{d-d_1\cdots-d_{m-1}}{d_m}\alpha_1^{d_1}\alpha_2^{d_2}\ldots\alpha_m^{d_m}$$

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- The expected value of the union bound is

$$\sum_{d} A_d \left(\sum_{j=1}^{m} \gamma(j) \alpha_j \right)^h.$$

- The average Bhattacharyya noise parameter:

$$\overline{\gamma} = \sum_{j=1}^{m} \gamma(j) \alpha_j$$

A RANDOMLY PUNCTURED TURBO CODE
An Example of Random Transmission Assignment

- The puncturing probability is λ.
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- Requirement $-\log \overline{\gamma} > c_0^{[\mathcal{C}]}$

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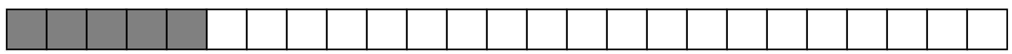
- The average noise parameter is $\overline{\gamma} = (1-\lambda)\gamma + \lambda$.
- Requirement $-\log \overline{\gamma} > c_0^{[\mathcal{C}]}$translates into

$$-\log \gamma > \log \left[\frac{1-\lambda}{\exp\left(-c_0^{[\mathcal{C}]}\right) - \lambda} \right].$$

INCREMENTAL REDUNDANCY

Concluding Remarks



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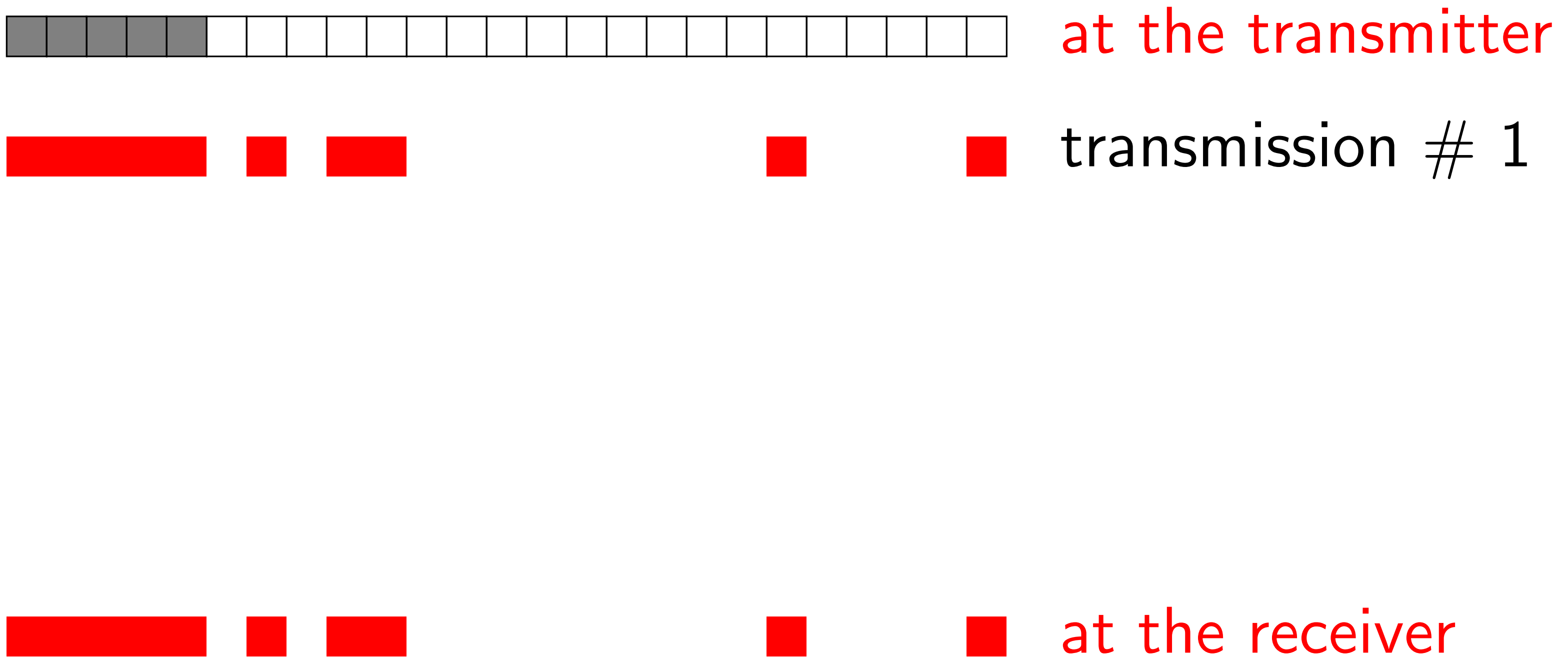

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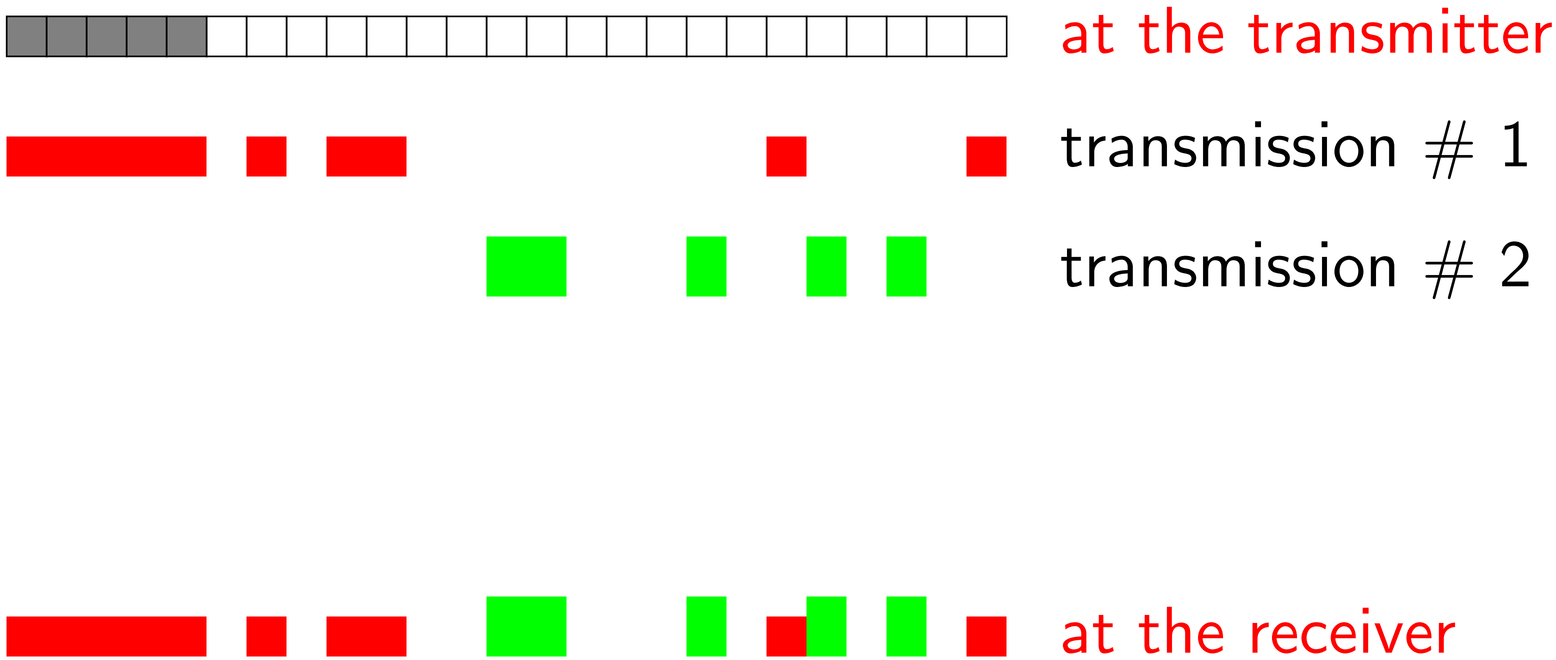

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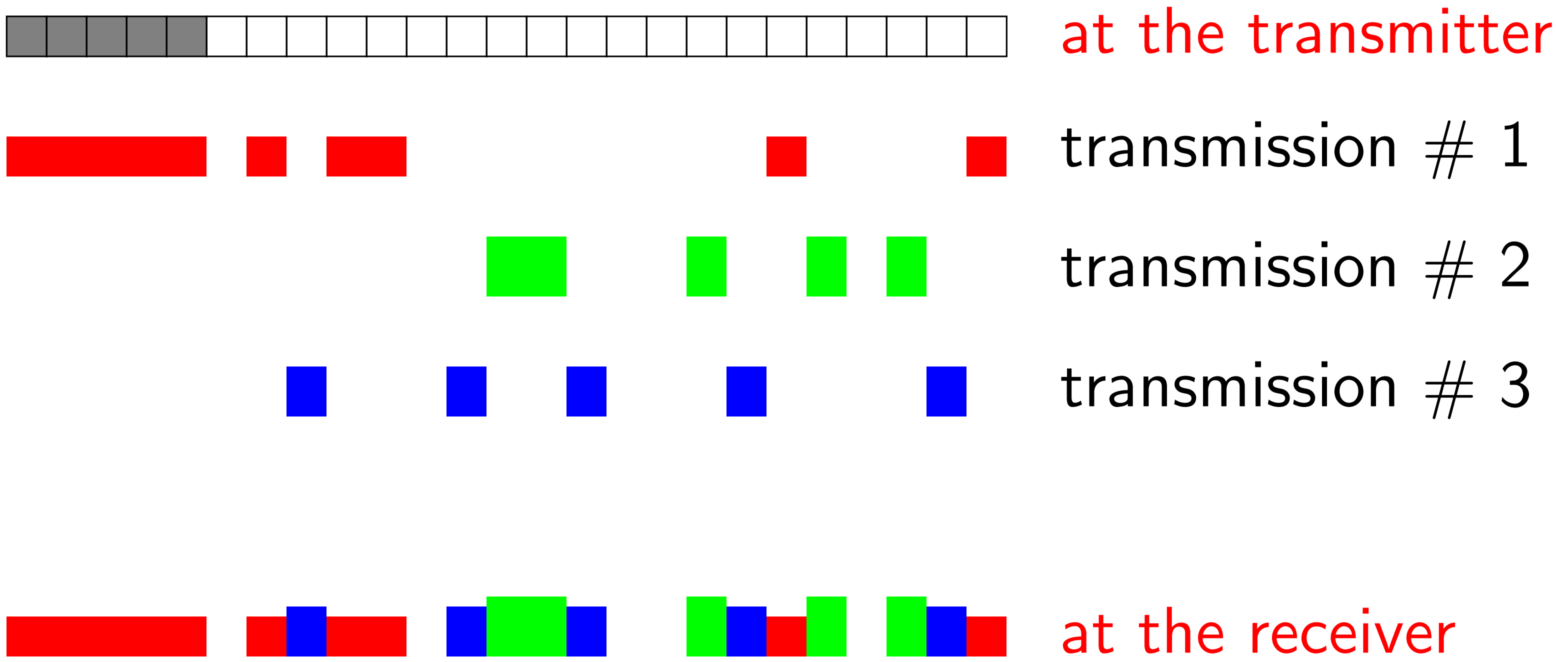

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