The Reachback Channel in Wireless Sensor Networks

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Outline

- The Problem of Reachback Communication in Sensor Networks.
  - Problem Definition, Applications, Challenges.
- Reachback Capacity with Non-Interfering Nodes:
  - Model of the Communications System, Problem Setup.
  - A Network Source/Channel Separation Theorem, Proof Outline.
  - The Region of Achievable Rates for *"Very Dumb"* Nodes, Proof Outline.
- The Case of Source Entropy Exceeding Reachback Capacity:
  - The Classical Multiterminal Source Coding Problem.
  - The Berger-Tung Inner/Outer Bounds on the Rate/Distortion Region.
  - Breaking the Long Chain I: Time Sharing of Berger-Yeung Codes, Proof Outline.
  - Breaking the Long Chain II: A Heuristic Form of Duality, Proof Outline.
- Summary and Future Work.

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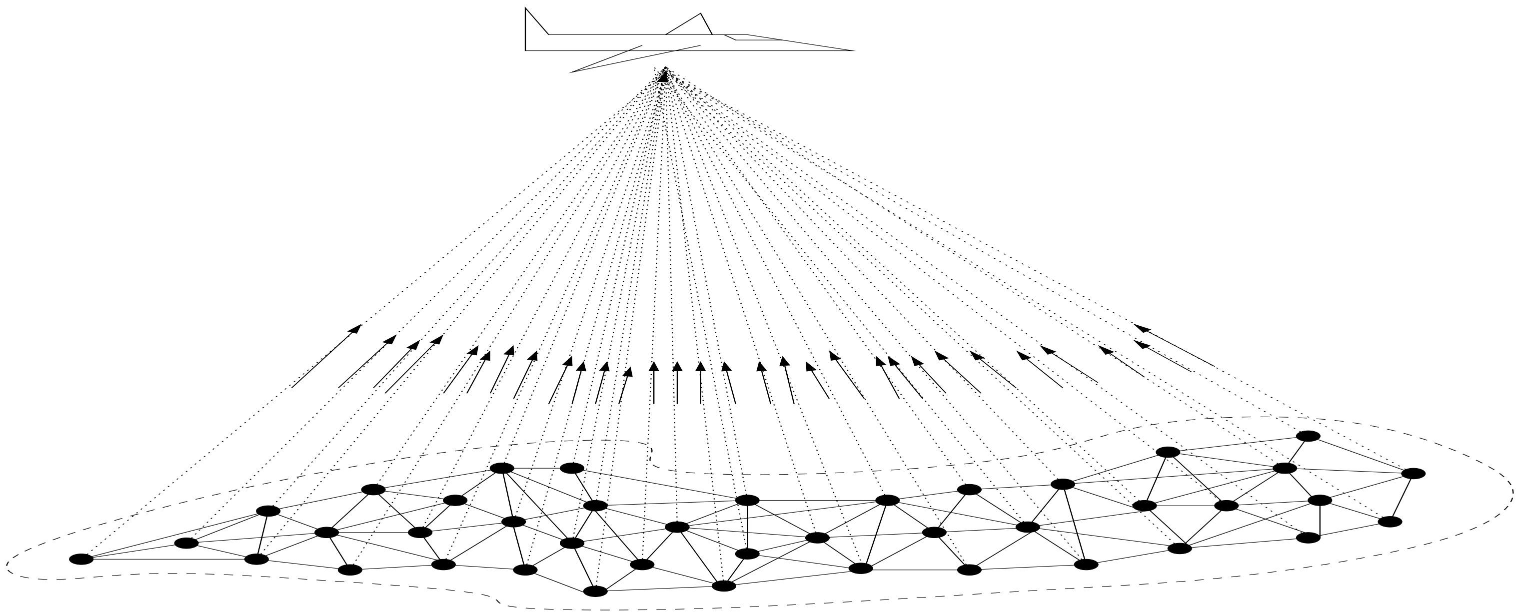
A Class of Sensor Networks—Main Characteristics

- Nodes operate under severe power constraints.
- Density and number of nodes is large.
- Nodes do not move (or move over very long time scales).
- Nodes switch between ON and OFF states randomly.
- Communication between nodes is over a wireless interface.
- Nodes can generate local data to feed into the network, or take data out of the network, or act as relay nodes, or simultaneously do all.

Reachback Communication: Problem Statement

Goal: to move data *out* of a sensor network.



Applications: disaster relief, disaster causing, environmental monitoring, data collection under health hazards, etc.

Reachback Communication: Challenges

- *The Information Theory View:*
  - *What are appropriate notions of capacity and rate/distortion for reachback?*
- The Computer Science View:
  - How do we route messages under extreme complexity constraints?
  - How much flow can be carried by these networks?
  - How do we build a distributed software radio for the uplink?

    $\longrightarrow$ *special lecture at* ACM SENSYS 2003.
- The Distributed Signal Processing and Communications View:
  - How do we solve basic signal processing tasks in distributed environments?
  - What are good codes to communicate reliably over this channel?
- The Physical Layer / Hardware View:
  - How do we detect information bearing signals generated by a distributed radio?
  - *How do we build hardware such that all of the above makes sense???*

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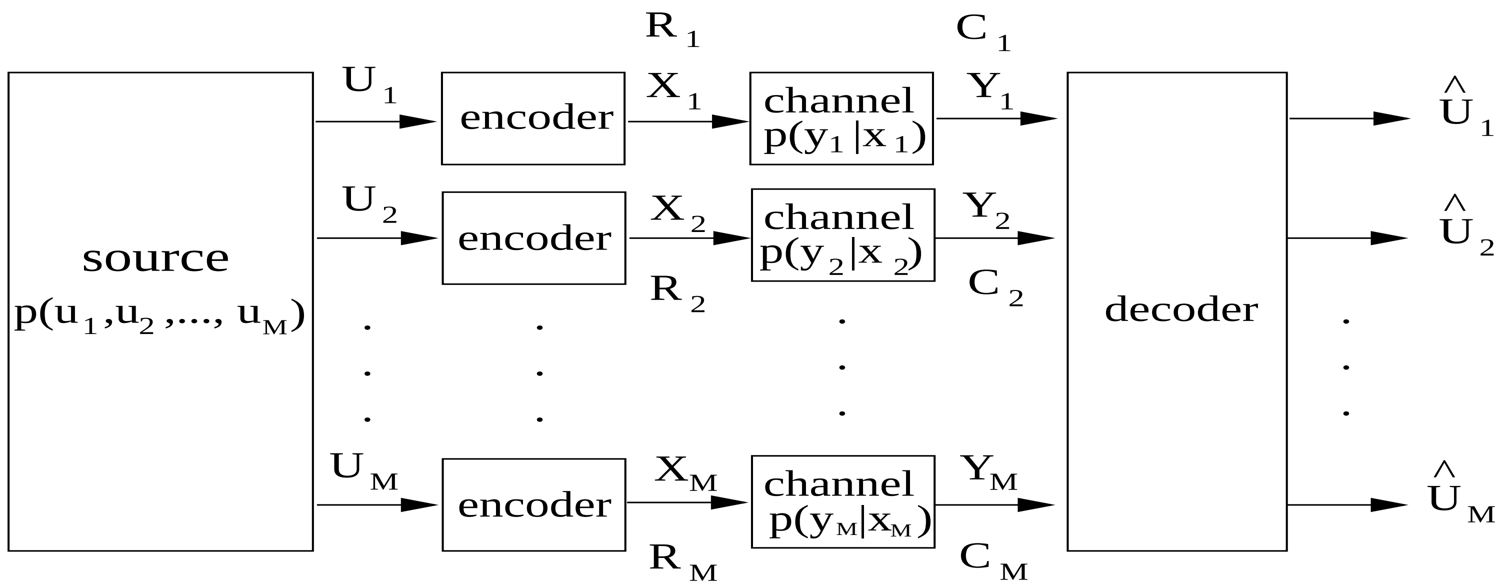
Reachback with Non-Interfering Nodes

Modeling assumption: an ideal MAC protocol is capable of eliminating all interference among nodes—perfect channel “slicing”.



Perfectly reasonable if nodes have some data to send all the time...

Some Previous Related Work

On the general problem of correlated sources over multiple access channels:

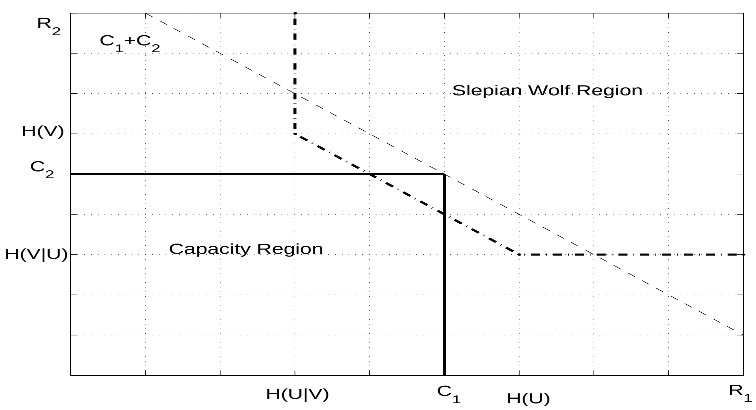
- T. M. Cover, A. A. El-Gamal, M. Salehi. *Multiple Access Channels with Arbitrarily Correlated Sources*. IEEE Trans. Inform. Theory, 26(6):648-657, 1980.
- G. Dueck. *A Note on the Multiple Access Channel with Correlated Sources*. IEEE Trans. Inform. Theory, 27(2):232-235, 1981.

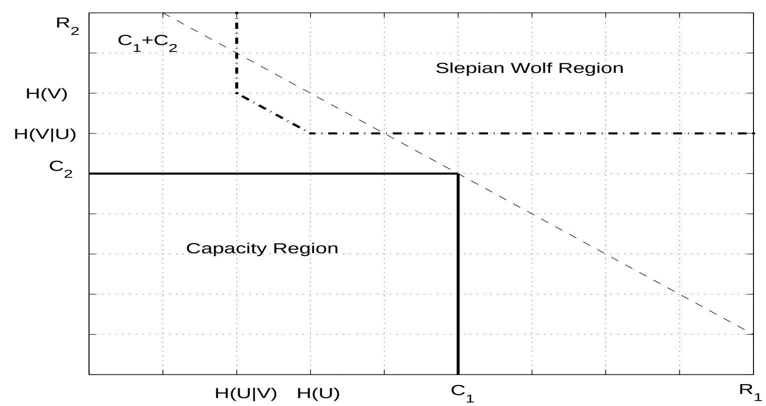
In a more general setup, these papers present only achievability results.

The capacity of an array of independent channels fed with correlated sources still remained an open problem.

A Visualization of the Regions of Interest

- What if $H(U) < C_1$ and $H(V) < C_2$?
  $\longrightarrow$ ok, even independent encoders work.
- What if $H(U) > C_1$ and $H(V) > C_2$?
  $\longrightarrow$ if $H(U,V) > C_1 + C_2$, not even genie; else, intersection $\neq \emptyset$.
- What if $H(U) < C_1$ and $H(V) > C_2$? (or $H(U) > C_1$ and $H(V) < C_2$)
  $\longrightarrow$ intersection $\neq \emptyset$ iff $H(V|U) < C_2$ (or $H(U|V) < C_1$).



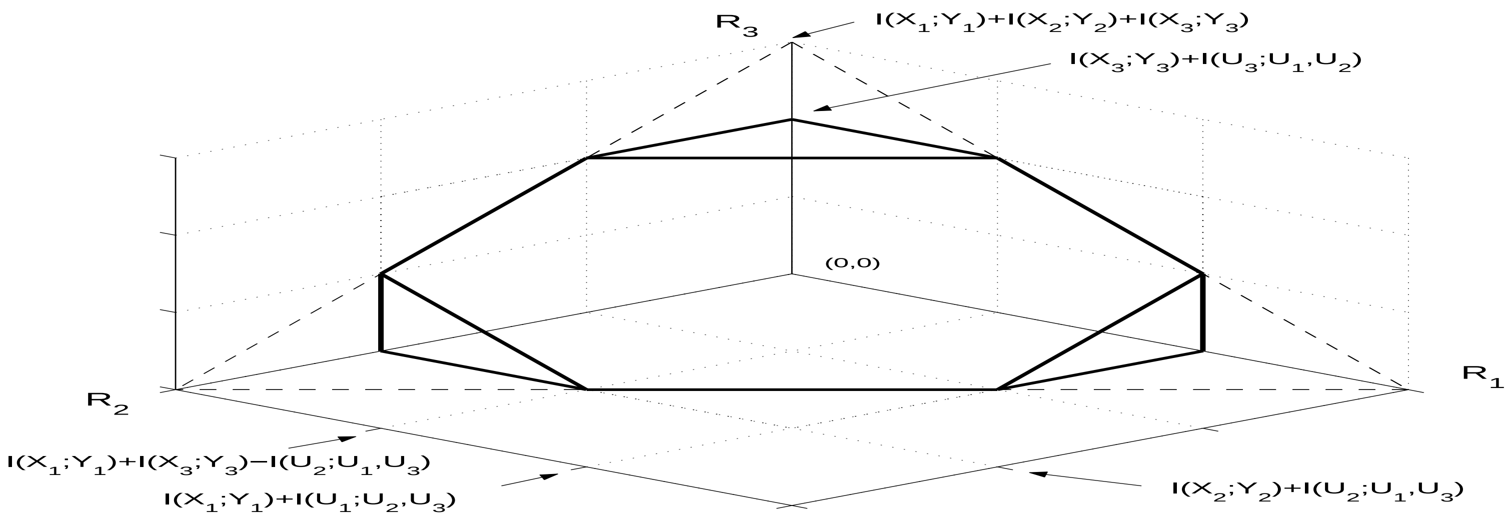


A Source/Channel Separation Theorem—Statement

Exact reconstruction of U, V is possible iff

$H(U|V) < I(X_1; Y_1)$ (equiv., $H(U) < I(U; V) + I(X_1; Y_1)$)
$H(V|U) < I(X_2; Y_2)$ (equiv., $H(V) < I(U; V) + I(X_2; Y_2)$)
$H(U, V) < I(X_1; Y_1) + I(X_2; Y_2)$

i.e., Slepian-Wolf source codes + capacity attaining channel codes.



J. Barros, S. D. Servetto. *Reachback Capacity with Non-Interfering Nodes*. In Proc. ISIT 2003.

A Source/Channel Separation Theorem—Proof Outline

The achievability part is trivial. For the converse:

- Fix $p(u, v, x_1, x_2, y_1, y_2) = p(u, v)p(x_1|u)p(x_2|v)p(y_1|x_1)p(y_2|x_2)$. Then $Y_1 - X_1 - U - V - X_2 - Y_2$ forms a (long) Markov chain.
- Take a block of n iid samples (U^n, V^n) of the source $p(u, v)$:
  - $X_1^n(U^n)$ and $X_2^n(V^n)$ are (block) encodings of the source.
  - Y_1^n, Y_2^n are the channel outputs with inputs X_1^n, X_2^n.
- $p(U^n, V^n, X_1^n, X_2^n, Y_1^n, Y_2^n)$ is well defined, and (long) Markov.
- Bound $H(U^n, V^n|Y_1^n, Y_2^n)$ using Fano's inequality, simplify.

If the channels were not independent, Y_1(Y_2) would not be independent of all else given X_1(X_2)—some simplifications would not be possible.

Achievable Rates for "Very Dumb" Sensors

What if... each sensor only knew its marginal distribution $p(x_i)$?

- Reliable communication is possible iff:

$$
\begin{aligned}
H(U|V) &< I(X_1;Y_1|V) \\
H(V|U) &< I(X_2;Y_2|U) \\
H(U,V) &< I(X_1;Y_1) + I(X_2;Y_2)
\end{aligned}
$$

- Proof outline:
  - Fix $p(x_1|u)$ and $p(x_2|v)$.
  - For all U^n, generate $X_1^n(U^n)$ by taking n iid samples of $p(x_1|u)$ (same for $X_2^n(V^n)$, with $p(x_2|v)$).
  - Decoder: look for $(U^n, V^n, X_1(U^n), X_2(V^n), Y_1^n, Y_2^n)$ jointly typical.
  - Write down error events, simplify.

Note: codebooks depend only on statistics of locally observed data...

The Penalty for Not Knowing Global Statistics

- The “dumb region” is contained in the region with global knowledge:

$$I(X_1;Y_1) - I(X_1;Y_1|V) = ... = I(Y_1;V) \geq 0$$
$$I(X_2;Y_2) - I(X_2;Y_2|U) = ... = I(Y_2;U) \geq 0$$

- ... and contains the region for independent encoders:

$$H(U) < I(U;V) + I(X_1;Y_1|V) = I(U,V) + I(X_1;Y_1) - I(Y_1;V)$$
$$H(V) < I(U;V) + I(X_2;Y_2|U) = I(U,V) + I(X_2;Y_2) - I(Y_2;U),$$

but from $Y_1 - X_1 - U - V - X_2 - Y_2$, we have that the upper bound on $H(U)$ is larger than $I(X_1,Y_1)$ (and similarly for $H(V)$ and $I(X_2;Y_2)$).

Summary on Reachback Capacity

- If sensors DO know global statistics, problem is solved:
  - A network source/channel separation theorem.
  - Slepian-Wolf codes followed by point-to-point capacity attaining codes is an optimal coding strategy.
- If sensors DO NOT know global statistics, problem is solved too:
  - Presented the region of achievable rates under the given constraints on the encoders.
  - Strict improvement over independent encoders and decoders.
  - Performance hit compared to when global statistics are known.
  - Improvement is due to exploiting correlations at the decoder.

J. Barros, S. D. Servetto. *Reachback Capacity with Non-Interfering Nodes*. In Proc. ISIT 2003.

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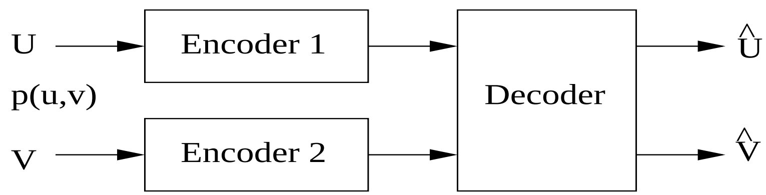
A Rate/Distortion Problem with Separate Encoders

But what if... sources do not admit a matching of Slepian-Wolf rates to the capacities of the channels?

Then the best we can hope for is to reconstruct some approximation *of the original message at the receiver...*



This is the classical *Multiterminal Source Coding* problem.

T. Berger. *The Information Theory Approach to Communications* (G. Longo, ed). Chapter on Multiterminal Source Coding.

The Berger-Tung Inner and Outer Bounds

Let (U, V) be drawn i.i.d. $\sim p(u, v)$. Let W and Z be two auxiliary random variables, such that there exist $\hat{U}(W, Z)$ and $\hat{V}(W, Z)$, for which $D_1 \geq Ed_U(U, \hat{U})$ and $D_2 \geq Ed_V(V, \hat{V})$. For a given (D_1, D_2), then:

$$\begin{aligned} R_1 &\geq I(UV; W|Z) \\ R_2 &\geq I(UV; Z|U) \\ R_1 + R_2 &\geq I(UV; WZ), \end{aligned}$$

- If $W - U - V - Z$ forms a long Markov chain, R(D) codes *do exist within* this region of (R_1, R_2)—the *BT inner* bound.
- If $W - U - V$ and $U - V - Z$ form two short Markov chains, R(D) codes *do not exist outside* this region of (R_1, R_2)—the *BT outer* bound.

S.-Y. Tung. *Multiterminal Source Coding*. PhD Thesis, Cornell University, 1978.

Conclusion: we need a coding strategy that breaks the long chain...

Breaking the Long Chain I: Berger-Yeung Codes

The Berger-Yeung Problem:

A special case of the general multiterminal source coding problem, in which we seek to determine the region of achievable tuples of the form $(R_1, R_2, D_1, 0)$.

Main result: $(R_1, R_2, D_1, 0)$ is achievable iff

$$\begin{aligned} R_1 &\geq I(U;W|V) \\ R_2 &\geq H(V|W) \\ R_1 + R_2 &\geq H(V) + I(U;W|V), \end{aligned}$$

where W is an auxiliary variable such that $W - U - V$ forms a Markov chain, and there exists a function $\hat{U}(V,W)$ such that $Ed(U,\hat{U}) \leq D_1$.

T. Berger, R. W. Yeung. *Multiterminal Source Encoding with One Distortion Criterion*. IEEE Trans. Inform. Theory, 35(2):228-236, 1989.

Breaking the Long Chain I: Time-Sharing of B-Y Codes

The Berger-Yeung region for $(R_1, R_2, D_1, 0)$ *requires the same constraints on its auxiliary variable as one of the constraints in the Berger-Tung outer bound, so... what do we get by time-sharing?*

(R_1, R_2, D_1, D_2) is achievable by time-sharing of Berger-Yeung codes iff

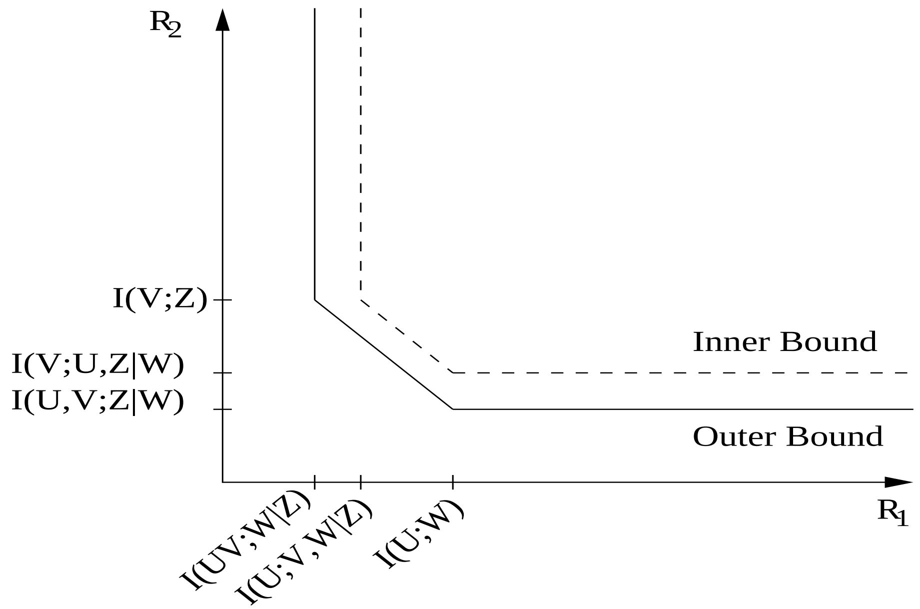
$$\begin{aligned} R_1 &\geq I(U;VW|Z) \\ R_2 &\geq I(V;UZ|W) \\ R_1 + R_2 &\geq H(UV) - H(U|VW) - H(Z|UV), \end{aligned}$$

where W, Z are auxiliary random variables such that $W - U - V$ and $U - V - Z$ form two Markov chains, and there exist functions $\hat{U}(V, W)$ and $\hat{V}(U, Z)$ such that $Ed(U, \hat{U}) \leq D_1$ and $Ed(V, \hat{V}) \leq D_2$.

J. Barros, S. D. Servetto. *On the Rate/Distortion Region for Separate Encoding of Correlated Sources*. In Proc. ISIT 2003.

Breaking the Long Chain I: Time-Sharing Leaves a Gap

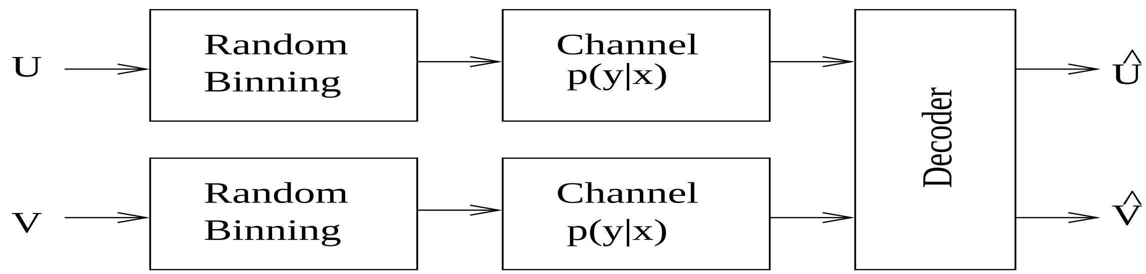
Comparing this region against the Berger-Tung outer bound, we find all faces are strictly inside that region:



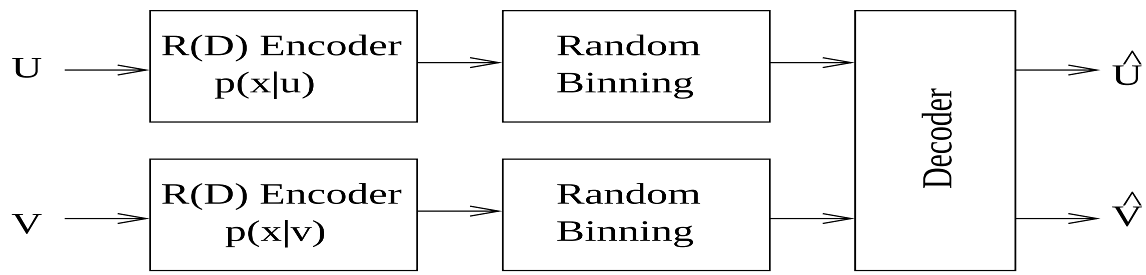
J. Barros, S. D. Servetto. *On the Rate/Distortion Region for Separate Encoding of Correlated Sources*. In Proc. ISIT 2003.

Breaking the Long Chain II: A Heuristic Form of Duality

If this is an optimal architecture for the capacity problem ...



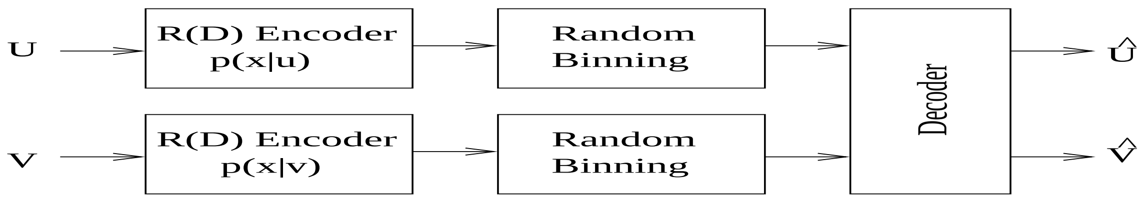
... would this be an optimal architecture for the rate/distortion problem?



We rely informally on the duality between capacity and rate/distortion.

Breaking the Long Chain II: Elements of the Analysis

Key idea: *replace* $p(u, v)$ *by a quantized source*.



- Two stage process: (a) quantize blocks of data, (b) put blocks of quantization indices into bins.
- Distortion constraint guarantees provided by classical $R(D)$ theory.
- Challenge: showing that the entropies of quantization indices satisfy the Berger-Tung constraints...

Decoding by joint-typicality, and rates achieved, only depend on source statistics through $p(i, j) = \sum_{U^n \to i, V^n \to j} p(U^n, V^n)$*... i.e., no long chain.*

Breaking the Long Chain II: Two-Stage Performance

The tuple (R_1, R_2, D_1, D_2) is achievable by the two-stage process iff

$$\begin{aligned} R_1 &\geq I(UV; W|Z) \\ R_2 &\geq I(UV; Z|W) \\ R_1 + R_2 &\geq I(UV; WZ), \end{aligned}$$

- where (W, Z) is pair of random variables in a class $\mathcal{C}$, in which all pairs satisfy that $W - U - V$ and $U - V - Z$ form a Markov chain;
- and where there exist functions $\hat{U}(W, Z)$ and $\hat{V}(W, Z)$ such that $D_1 \geq Ed_1(U, \hat{U})$ and $D_2 \geq Ed_2(V, \hat{V})$.

J. Barros, S. D. Servetto. *An Inner Bound for the Rate/Distortion Region of the Multiterminal Source Coding Problem*. In Proc. 37th Annual Conference on Information Sciences and Systems (CISS), 2003.

Now we get our expressions to match the BT outer bound... but does this construction work for ALL possible pairs of short chains???

Summary on Multiterminal Source Coding

Goal: to develop a coding strategy capable of reaching the surface of the Berger-Tung outer bound, requiring only two short chains...

What we have done so far:

- By time-sharing of Berger-Yeung codes:
  - Have a strategy that works for all possible pairs of short chains.
  - Cannot reach the surface of the Berger-Tung outer bound.

  Barros/Servetto. *On the Rate/Distortion Region for Separate Encoding of Correlated Sources*. ISIT 2003.

- By a cascade of independent rate/distortion codes plus binning:
  - Have a strategy whose rates match the Berger-Tung outer bound.
  - We have not yet shown that it works for all possible short chains.

  Barros/Servetto. *An Inner Bound for the Rate/Distortion Region of Multiterminal Source Coding*. CISS 2003.

Still searching for a way to have our cake and eat it too...

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Summary and Future Work

Summary:

- We formulated the problem of communicating over the reachback channel, and (briefly) discussed its multiple facets.
- Presented capacity and rate/distortion results for one specific reachback configuration—the case of no interference.

Current and future work:

- *Studying from Csiszar&Körner. All work discussed above relies on joint typicality arguments only—can types help?*
- We need to work out examples (Gaussian/MSE, binary/Hamming, ...)
- Thinking about other forms of cooperation...
  - A.-S. Hu, S. D. Servetto. *Optimal Detection for a Distributed Transmission Array*. In Proc. ISIT 2003.
  - C. Peraki, S. D. Servetto. *On the Scaling Laws of Wireless Networks with Directional Antennas*. In Proc. ACM MobiHoc 2003.

Main Corollary...

http://people.ece.cornell.edu/servetto/