Wireless Network Information Theory

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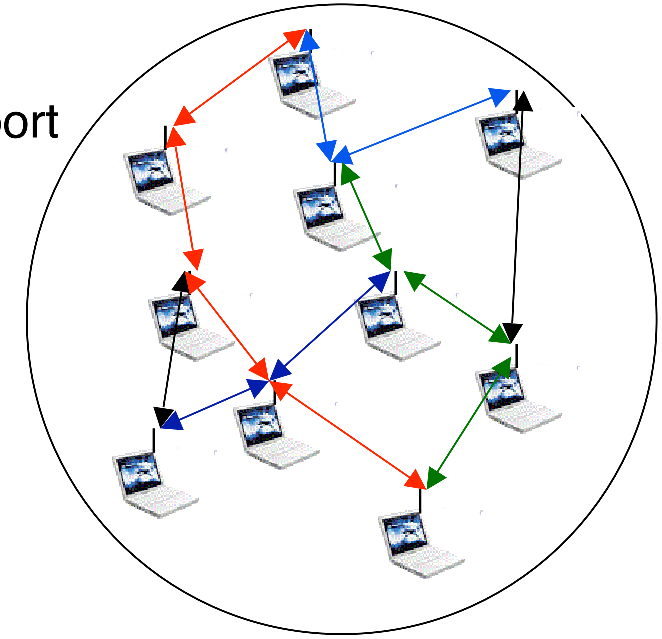
Wireless Networks

- Communication networks formed by nodes with radios

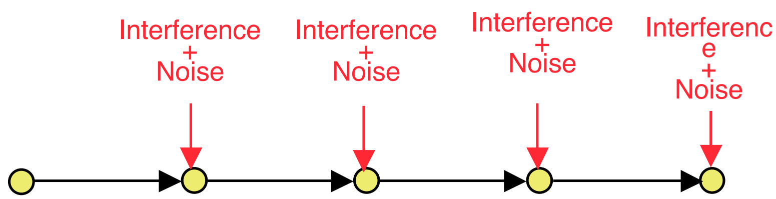
- Ad Hoc Networks
  - Current proposal for operation: Multi-hop transport
    - » Nodes relay packets until they reach their destinations
  - They should be spontaneously deployable anywhere
    - » On a campus
    - » On a network of automobiles on roads
    - » On a search and rescue mission
  - They should be able to adapt themselves to
    - » the number of nodes in the network
    - » the locations of the nodes
    - » the mobility of the nodes
    - » the traffic requirements of the nodes



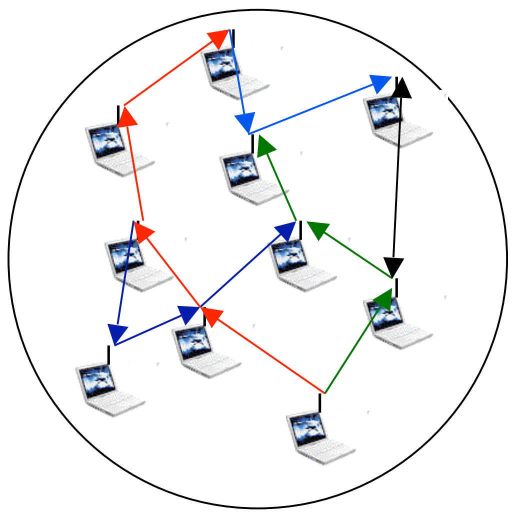
- Sensor webs

Current proposal for ad hoc networks

- Multi-hop transport
  - Packets are relayed from node to node
  - A packet is fully decoded at each hop
  - All interference from all other nodes is simply treated as noise



- Properties
  - Simple receivers
  - Simple multi-hop packet relaying scheme
  - Simple abstraction of "wires in space"



- This choice for the mode of operation gives rise to
  - Routing problem
  - Media access control problem
  - Power control problem
  - …..

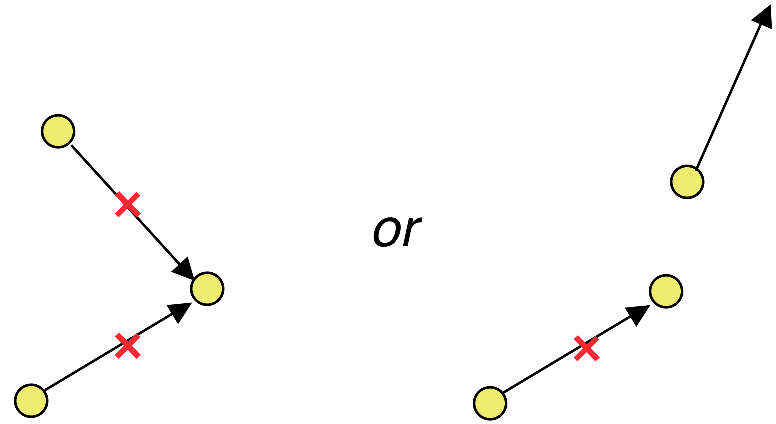
Three fundamental questions

- If all interference is treated as noise, then how much information can be transported over wireless networks?
- What is unconditionally the best mode of operation?
- What are the fundamental limits to information transfer?
- Allows us to answer questions such as
  - How far is current technology from the optimal?
  - When can we quit trying to do better?
    - E.g.. If "Telephone modems are near the Shannon capacity" then we can stop trying to build better telephone modems
  - What can wireless network designers hope to provide?
  - What protocols should be designed?

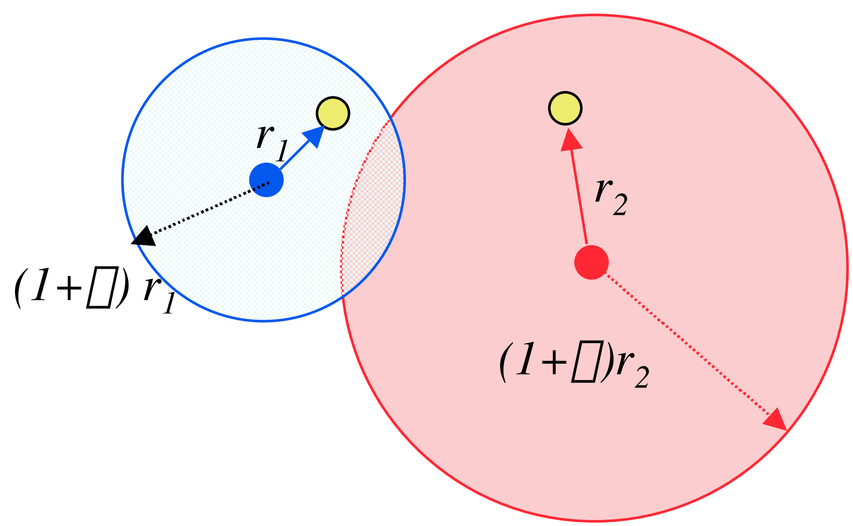
If interference is treated as noise ...

If all interference is regarded as noise …

- … then packets can collide destructively



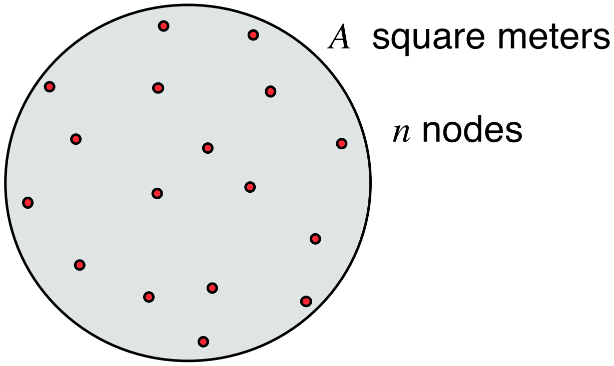
- <u>A Model for Collisions</u>
  - Reception is successful if Receiver not in vicinity of two transmissions



- <u>Alternative Models</u>
  - SINR ≥ □ for successful reception
  - Or Rate depends on SINR

Scaling laws under interference model

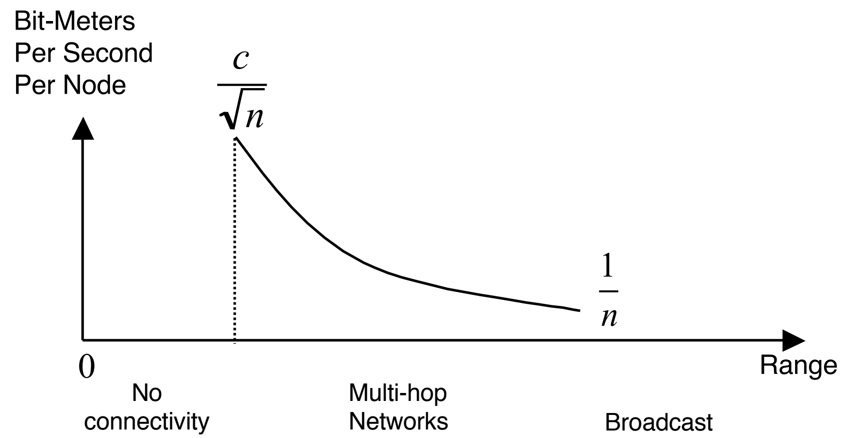
- Theorem (GK 2000)
  - Disk of area A square meters
  - n nodes
  - Each can transmit at W bits/sec



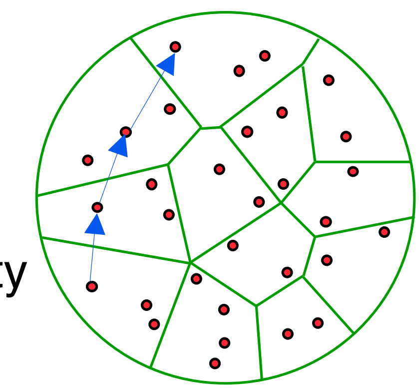
- Best Case: Network can transport $\Theta\left(W\sqrt{An}\right)$ bit-meters/second
- Square root law
  - Transport capacity doesn't increase linearly, but only like square-root
  - Each node gets $\frac{c}{\sqrt{n}}$ bit-meters/second
- Random case: Each node can obtain throughput of $\Theta\left(\frac{1}{\sqrt{n\log n}}\right)$ bits/second

Optimal operation under "collision" model

- Optimal operation is multi-hop
  - Transport packets over many hops of distance $\frac{c}{\sqrt{n}}$

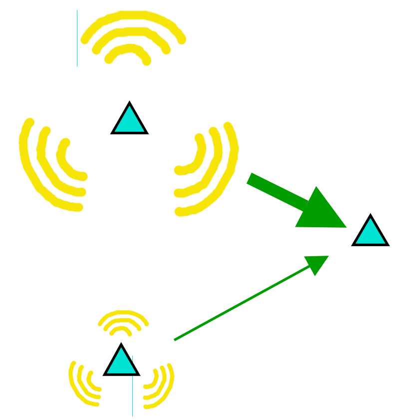


- Optimal architecture
  - Group nodes into cells of size about log n
  - Choose a common power level for all nodes
    - » Nearly optimal
  - Power should be just enough to guarantee network connectivity
    - » Sufficient to reach all points in neighboring cell
  - Route packets along nearly straight line path from cell to cell



But interference is not interference

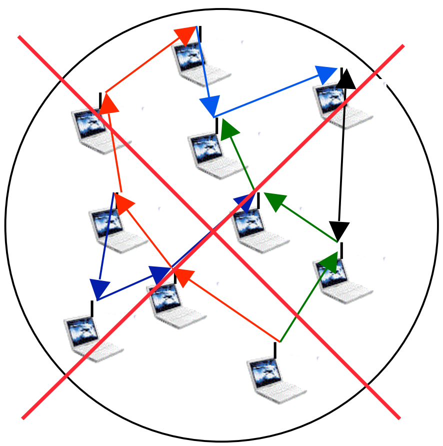
- Excessive interference can be good for you
  - Receiver can first decode loud signal perfectly
  - Then subtract loud signal
  - Then decode soft signal perfectly
  - So excessive interference can be good

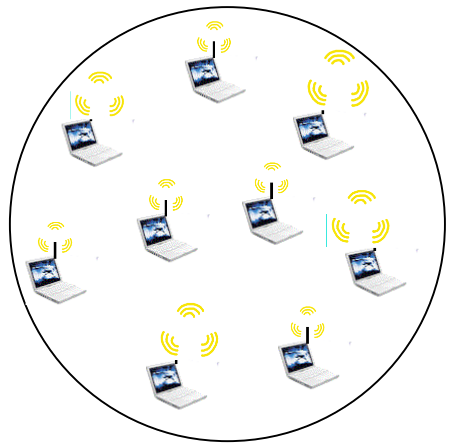


- Packets do <u>not</u> destructively collide
- Interference is information!
- So we need an information theory for networks to determine
  - How to operate wireless networks
  - How much information wireless networks can transport
  - The information theory should be able to handle general wireless networks

Towards fundamental limits in wireless networks

Wireless networks don't come with links

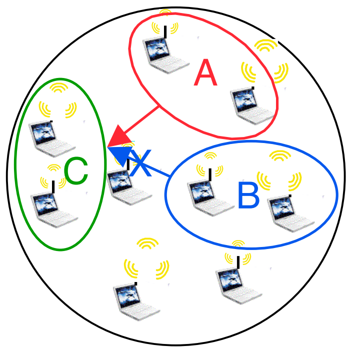




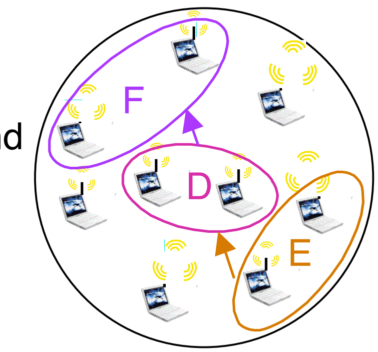
- They are formed by nodes with radios
  - There is no *a priori* notion of "links"
  - Nodes simply radiate energy

Nodes can cooperate in complex ways

Nodes in Group A cancel interference of Group B at Group C



… while Nodes in Group D amplify and forward packets from Group E to Group F



while ….

SINR = Signal / (Interference + Noise)

One strategy: Increase Signal for Receiver
Instead, why not: *Reduce Interference at Receiver*

One strategy: Decode and forward
Instead, why not: *Amplify and Forward*

…

How should nodes cooperate?

- Some obvious choices
  - Should nodes relay packets?
  - Should they amplify and forward?
  - Or should they decode and forward?
  - Should they cancel interference for other nodes?
  - Or should they boost each other's signals?
  - Should nodes simultaneously broadcast to a group of nodes?
  - Should those nodes then cooperatively broadcast to others?
  - What power should they use for any operation?
  - …
- Or should they use much more sophisticated unthought of strategies?
  - Tactics such as
    Decode and forward
    Amplify and Forward
    Interference cancellation
    ...
    may be too simplistic
  - Cooperation through
    Broadcast
    Multiple-access
    Relaying
    ...
    does not capture all possible modes of operation

“There are more things in heaven and earth, Horatio,
Than are dreamt of in your philosophy.”
— Hamlet

A plethora of choices

- The strategy space is infinite dimensional

- Problem has all the complexities of
  - team theory
  - partially observed systems

- We want Information Theory to tell us what the basic strategy should be

  - Then one can develop protocols to realize the strategy

Key Results: A dichotomy

- <u>If absorption in medium</u>
  - Transport capacity grows like $\Theta(n)$
    - » when nodes are separated by distance at least ρ_{min}
  - Square-root law is optimal
    - » $\Theta\left(\sqrt{An}\right) = \Theta(n)$
    - » Since area A grows like $\Theta(n)$
  - Multi-hop decode and forward is order optimal

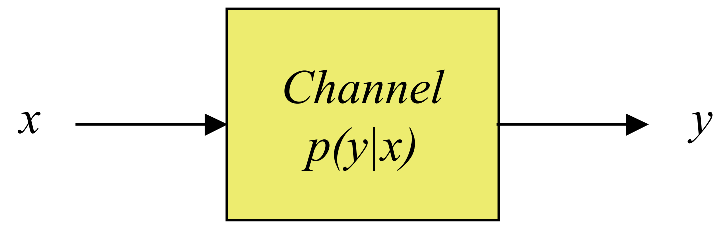
- <u>If there is no absorption, and attenuation is very small</u>
  - Transport capacity can grow superlinearly like $\Theta(n^{\theta})$ for $\theta > 1$
  - Coherent multi-stage relaying with interference cancellation can be optimal

- Along the way
  - Total power used by a network bounds the transport capacity
  - Or not
  - A feasible rate for Gaussian multiple relay channels

A quick review of information theory and networks

Shannon's Information Theory

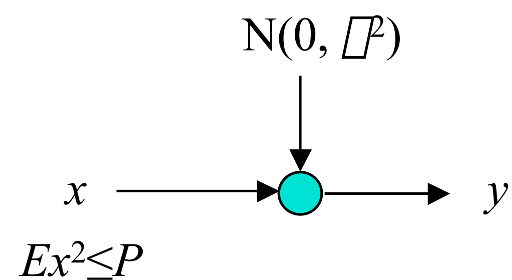
- <u>Shannon's Capacity Theorem</u>
  - Channel Model $p(y|x)$
    - » Discrete Memoryless Channel



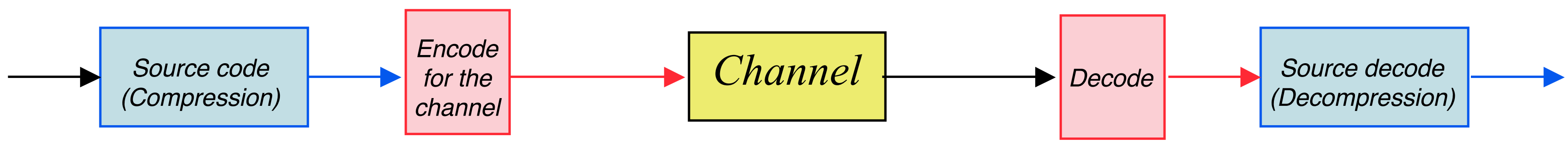
  - <u>Capacity</u> = $\text{Max}_{p(x)} I(X;Y)$ bits/channel use

$$I(X;Y) = \sum_{x,y} p(x,y) \log\left(\frac{p(X,Y)}{p(X)\,p(Y)}\right)$$

- <u>Additive White Gaussian Noise (AWGN) Channel</u>
  - Capacity = $S\left(\frac{P}{\sigma^2}\right)$, where $S(z) = \frac{1}{2}\log(1+z)$

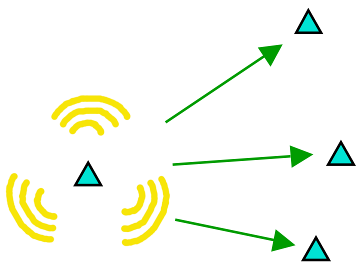


Shannon's architecture for communication

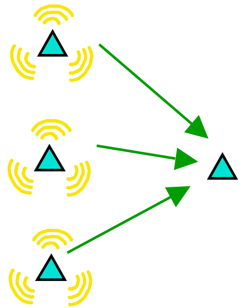


Network information theory: Some triumphs

- Gaussian scalar broadcast channel

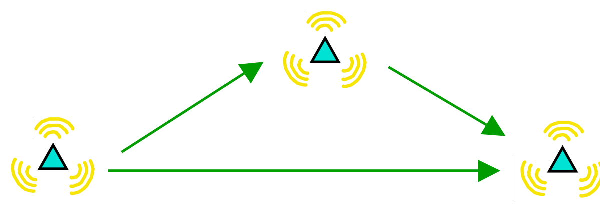


- Multiple access channel

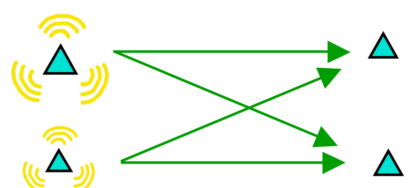


Network information theory: The unknowns

- The simplest relay channel



- The simplest interference channel

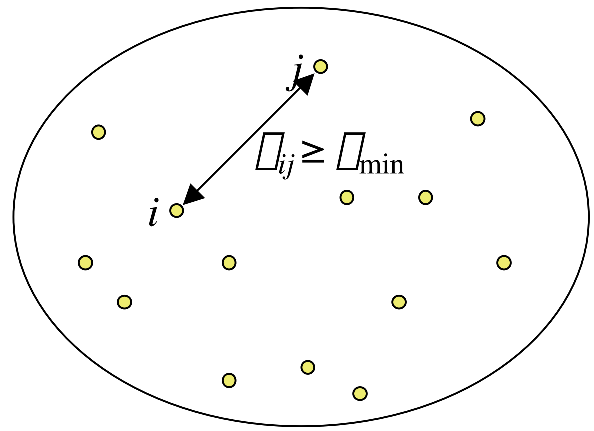


- Systems being built are much more complicated and the possible modes of cooperation can be much more sophisticated
  - How to analyze?
  - Need a general purpose information theory

The Model

Model of system: A planar network

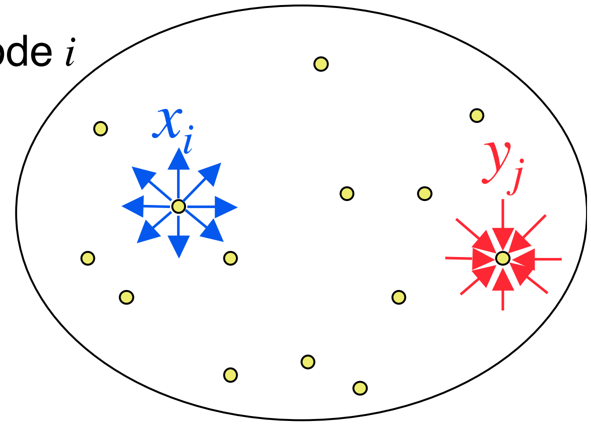
- n nodes in a plane
- ρ_{ij} = distance between nodes i and j
- Minimum distance ρ_{min} between nodes
- Signal attenuation with distance ρ: $\dfrac{e^{-\gamma\rho}}{\rho^{\delta}}$
  - $\gamma \geq 0$ is the absorption constant
    - » Generally $\gamma > 0$ since the medium is absorptive unless over a vacuum
    - » Corresponds to a loss of $20\gamma \log_{10} e$ db per meter
  - $\delta > 0$ is the path loss exponent
    - » $\delta = 1$ corresponds to inverse square law in free space



Transmitted and received signals

- W_i = symbol from some alphabet $\{1,2,3,\ldots,2^{TR_{ik}}\}$ to be sent by node i
- $x_i(t) = f_{i,t}(y_i^{t-1}, W_i)$ = signal transmitted by node i time t
- $y_j(t) = \sum_{\substack{i=1 \\ i \neq j}}^{n} \frac{e^{j\theta_{ij}}}{d_{ij}^{\delta/2}} x_i(t) + z_j(t)$ = signal received by node j at time t

  $z_j(t)$: $N(0,\sigma^2)$



- Destination j uses the decoder $\hat{W}_i = g_j(y_j^T, W_j)$
- Error if $\hat{W}_i \neq W_i$
- $(R_1, R_2, \ldots, R_l)$ is feasible rate vector if there is a sequence of codes with

$$\underset{W_1,W_2,\ldots,W_l}{Max} \Pr(\hat{W}_i \neq W_i \text{ for some } i \mid W_1,W_2,\ldots,W_l) \to 0 \text{ as } T \to \infty$$

- Individual power constraint $P_i \leq P_{ind}$ for all nodes i

  Or Total power constraint $\sum_{i=1}^{n} P_i \leq P_{total}$

The Transport Capacity: Definition

- Source-Destination pairs
  - $(s_1, d_1), (s_2, d_2), (s_3, d_3), \ldots, (s_{n(n-1)}, d_{n(n-1)})$
- Distances
  - $\square_1, \square_2, \square_3, \ldots, \square_{n(n-1)}$ distances between the sources and destinations
- Feasible Rates
  - $(R_1, R_2, R_3, \ldots, R_{n(n-1)})$ feasible rate vector for these source-destination pairs
- Distance-weighted sum of rates
  - $\square_i R_i \square_i$
- Transport Capacity

$$C_T = \sup_{(R_1, R_2, \ldots, R_{n(n\square 1)})} \square_{i=1}^{n(n\square 1)} R_i \cdot \square_i$$

bit-meters/second or bit-meters/slot

The Transport Capacity

- $C_T = \sup \sum_i R_i \rho_i$
  - Measured in bit-meters/second or bit-meters/slot
  - Analogous to man-miles/year considered by airlines
  - Upper bound to what network can carry
    - » Irrespective of what nodes are sources or destinations, and their rates
  - Satisfies a scaling law
    - » Conservation law which restricts what network can provide
    - » Irrespective of whether it is of prima facie interest
  - However it is of natural interest
    - » Allows us to compare apples with apples

The Results

When there is absorption or relatively large path loss

The total power bounds the transport capacity

- Theorem (XK 2002): Joules per bit-meter bound
  - Suppose $\gamma > 0$, there is some absorption,
  - Or $\delta > 3$, if there is no absorption at all
  - Then for all Planar Networks

$$C_T \le \frac{c_1(\delta,\gamma,\rho_{\min})}{\sigma^2} \cdot P_{total}$$

where

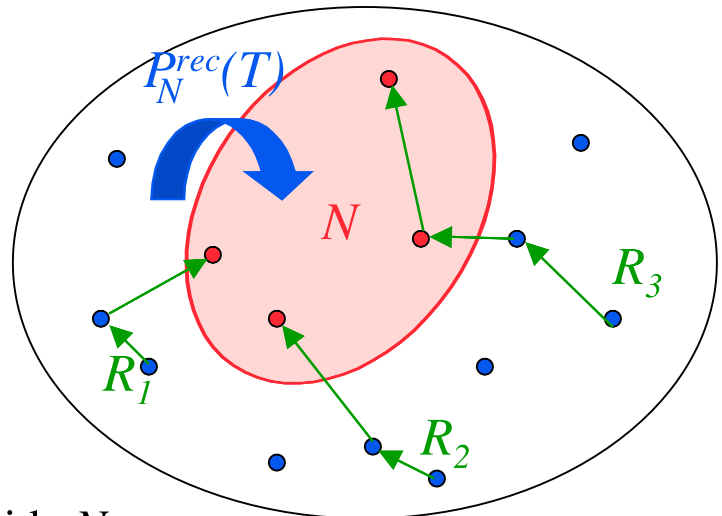
$$c_1(\delta,\gamma,\rho_{\min}) = \frac{2^{2\delta+7}}{\gamma^2 \rho_{\min}^{2\delta+1}} \frac{e^{-\gamma\rho_{\min}/2}(2 - e^{-\gamma\rho_{\min}/2})}{(1 - e^{-\gamma\rho_{\min}/2})} \quad \text{if } \gamma > 0$$

$$= \frac{2^{2\delta+5}(3\delta - 8)}{(\delta-2)^2(\delta-3)\rho_{\min}^{2\delta-1}} \quad \text{if } \gamma = 0 \text{ and } \delta > 3$$

Idea behind proof

- A Max-flow Min-cut Lemma

  – N = subset of nodes



  – $P_N^{rec}(T) =$ Power received by nodes in N from outside N

$$= \frac{1}{T}\sum_{t=1}^{T}\sum_{j \in N} E\left[\left|\sum_{i \notin N} \frac{x_i(t)}{\rho_{ij}^{\delta}}\right|^2\right]$$

  – Then

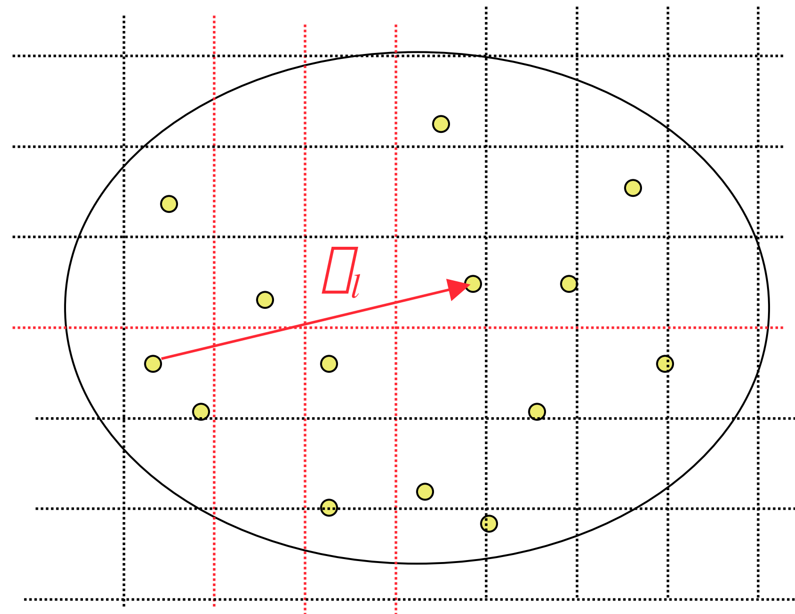
$$\sum_{\{l : d_l \in N \text{ but } s_l \notin N\}} R_l \le \frac{1}{2\sigma^2} \liminf_{T \to \infty} P_N^{rec}(T)$$

To obtain power bound on transport capacity

- Idea of proof

- Consider a number of cuts one meter apart

- Every source-destination pair (s_l, d_l) with source at a distance ρ_l is cut by about ρ_l cuts



- Thus

$$\sum_l R_l \rho_l \le c \sum_{N_k} \sum_{\{l \text{ is cut by } N_k\}} R_l \le \frac{c}{2\Delta^2} \sum_{N_k} \liminf_{T\to\infty} P_{N_k}^{rec}(T) \le \frac{cP_{total}}{\Delta^2}$$

O(n) upper bound on Transport Capacity

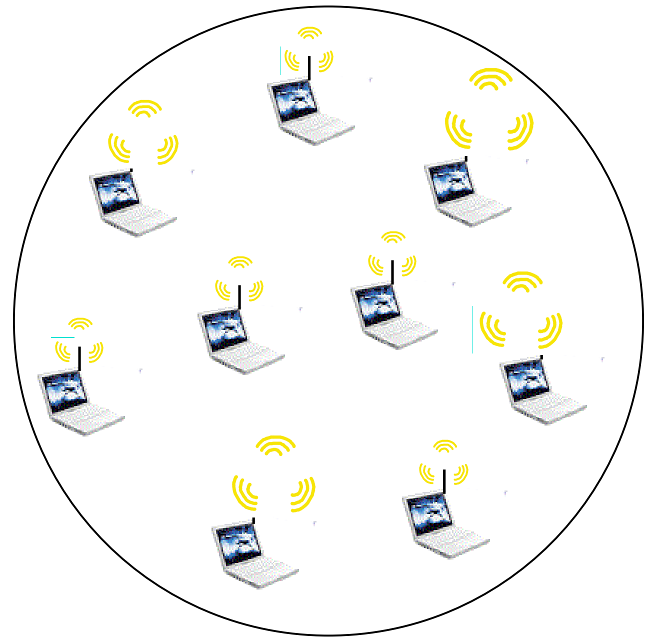
- Theorem
  - Suppose $\gamma > 0$, there is some absorption,
  - Or $\delta > 3$, if there is no absorption at all
  - Then for all Planar Networks

$$C_T \le \frac{c_1(\gamma, \delta, r_{min}) P_{ind}}{\sigma^2} \cdot n$$

where

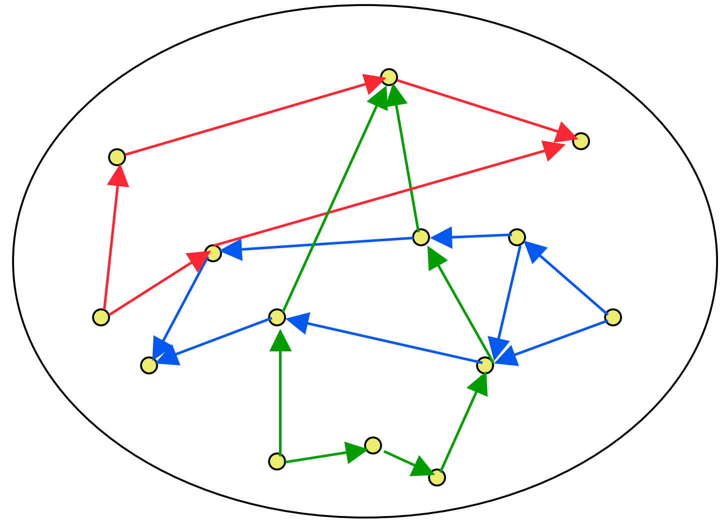
$$c_1(\gamma, \delta, r_{min}) = \frac{2^{2\delta+7}}{\gamma^2 r_{min}^{2\delta+1}} \frac{e^{-\gamma r_{min}/2}(2 - e^{-\gamma r_{min}/2})}{(1 - e^{-\gamma r_{min}/2})} \quad \text{if } \gamma > 0$$

$$= \frac{2^{2\delta+5}(3\delta - 8)}{(\delta - 2)^2 (\delta - 3) r_{min}^{2\delta-1}} \quad \text{if } \gamma = 0 \text{ and } \delta > 3$$



Feasibility of a rate vector

- <u>Theorem</u>



  - A set of rates $(R_1, R_2, \ldots, R_l)$ can be supported by multi-hop transport if
  - Traffic can be routed, possibly over many paths, such that
  - No node has to relay more than $S\left(\dfrac{e^{\square 2\square\bar{\square}} P_{ind} / \bar{\square}^{2\square}}{c_3(\square,\square,\square_{\min}) P_{ind} + \square^2}\right)$
  - where $\bar{\square}$ is the longest distance of a hop

    and $c_3(\square,\square,\square_{\min}) = \dfrac{2^{3+2\square} e^{\square\square_{\min}}}{\square\square_{\min}^{1+2\square}}$ if $\square > 0$

    $= \dfrac{2^{2+2\square}}{\square_{\min}^{2\square}(\square\square 1)}$ if $\square = 0$ and $\square > 1$

Multihop transport can achieve $\Theta(n)$

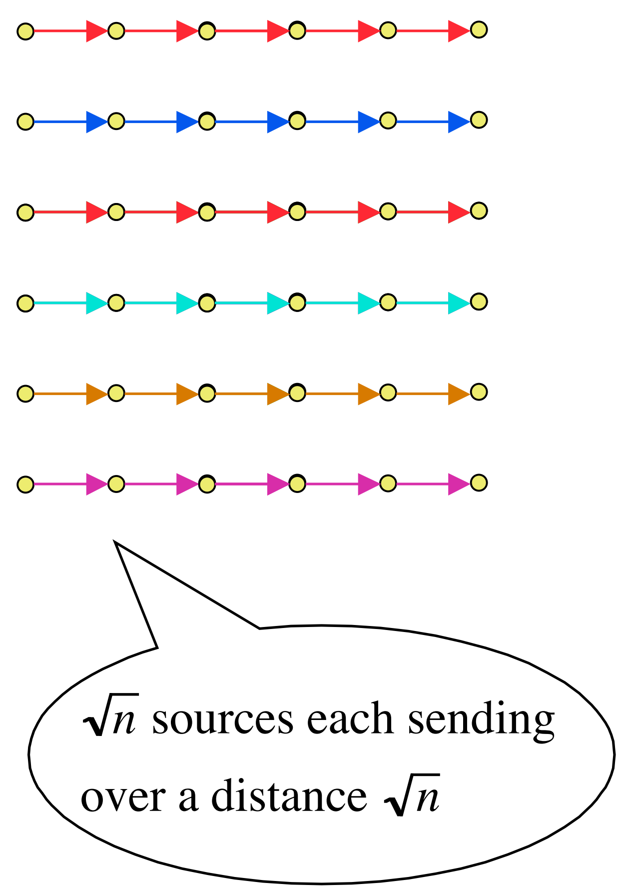
- Theorem
  - Suppose $\gamma > 0$, there is some absorption,
  - Or $\delta > 1$, if there is no absorption at all
  - Then in a regular planar network

$$C_T \geq S\left(\frac{e^{-2\gamma} P_{ind}}{c_2(\gamma,\delta) P_{ind} + \sigma^2}\right) \cdot n$$

where

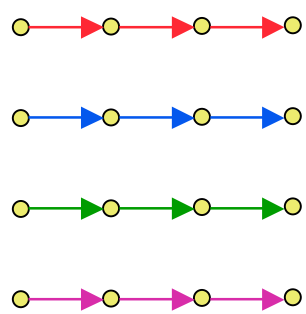
$$c_2(\gamma,\delta) = \frac{4(1+4\gamma)e^{-2\gamma} - 4e^{-4\gamma}}{2\gamma(1-e^{-2\gamma})} \quad \text{if } \gamma > 0$$

$$= \frac{16\delta^2 + (2\pi - 16)\delta - \pi}{(\delta-1)(2\delta-1)} \quad \text{if } \gamma = 0 \text{ and } \delta > 1$$



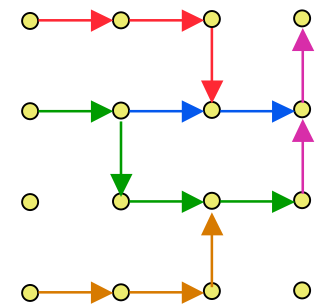
Optimality of multi-hop transport

- <u>Corollary</u>
  - So if $\square > 0$ or $\square > 3$
  - And multi-hop achieves $\square(n)$
  - Then it is optimal with respect to the transport capacity up to order
- Example



Multi-hop is almost optimal in a random network

- Theorem

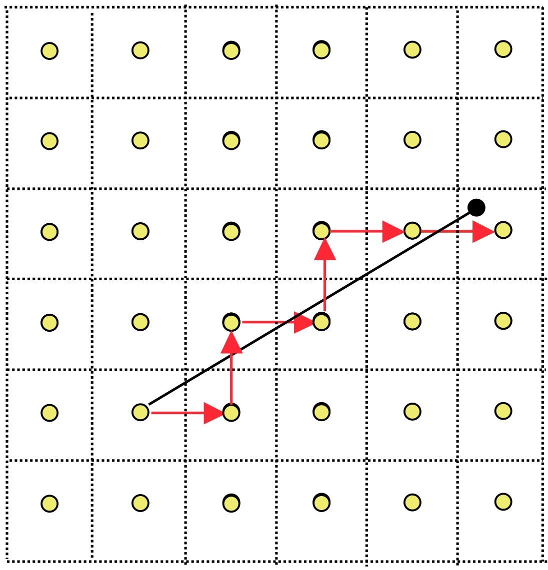


  - Consider a regular planar network
  - Suppose each node randomly chooses a destination
    - » Choose a node nearest to a random point in the square
  - Suppose □> 0 or □> 1
  - Then multihop can provide $\square\left(\frac{1}{\sqrt{n \log n}}\right)$ bits/time-unit for every source with probability □ 1 as the number of nodes n □

- Corollary
  - Nearly optimal since transport capacity achieved is $\square\left(\frac{n}{\sqrt{\log n}}\right)$

Idea of proof for random source - destination pairs

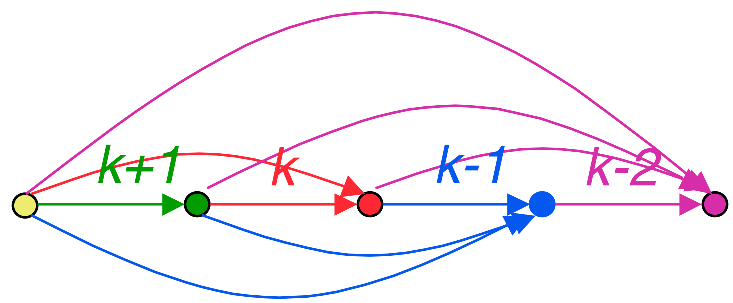
- Simpler than GK since cells are square and contain one node each
- A cell has to relay traffic if a random straight line passes through it
- How many random straight lines pass through cell?
- Use Vapnik-Chervonenkis theory to guarantee that no cell is overloaded



What happens when the attenuation is very low?

Another strategy emerges as of interest ...

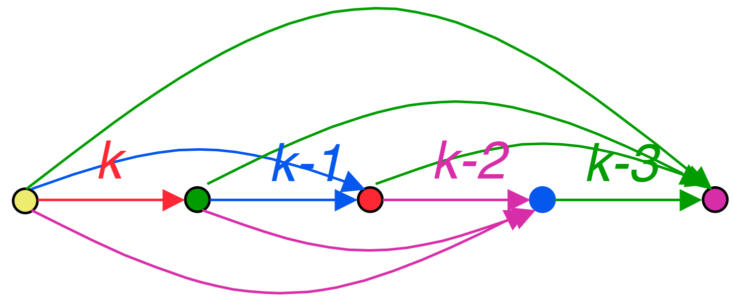
- Coherent multi-stage relaying with interference cancellation (CRIC)

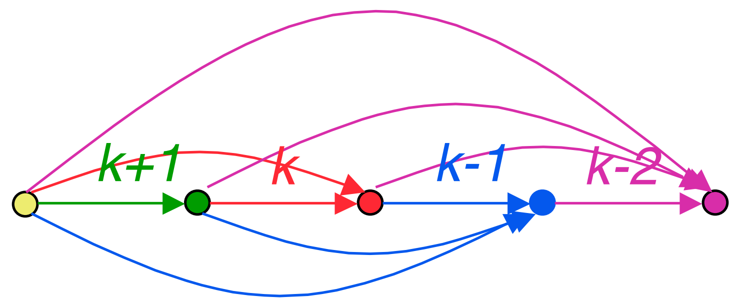


- All upstream nodes coherently cooperate to send a packet to the next node
- A node cancels all the interference caused by all transmissions to its downstream nodes

Decoding

- Coherent multi-stage relaying with interference cancellation (CRIC)

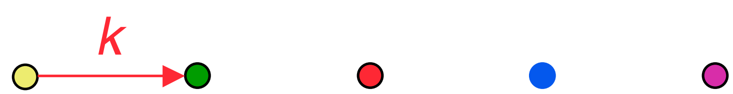


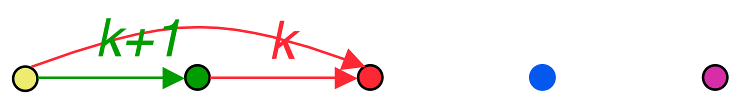


- All upstream nodes coherently cooperate to send a packet to the next node
- A node cancels all the interference caused by all transmissions to its downstream nodes

Interference cancellation

- Coherent multi-stage relaying with interference cancellation (COMSRIC)





- All upstream nodes coherently cooperate to send a packet to the next node
- A node cancels all the interference caused by all transmissions to its downstream nodes

A feasible rate for the Gaussian multiple-relay channel

- Theorem

  – Suppose γ_{ij} = attenuation from i to j

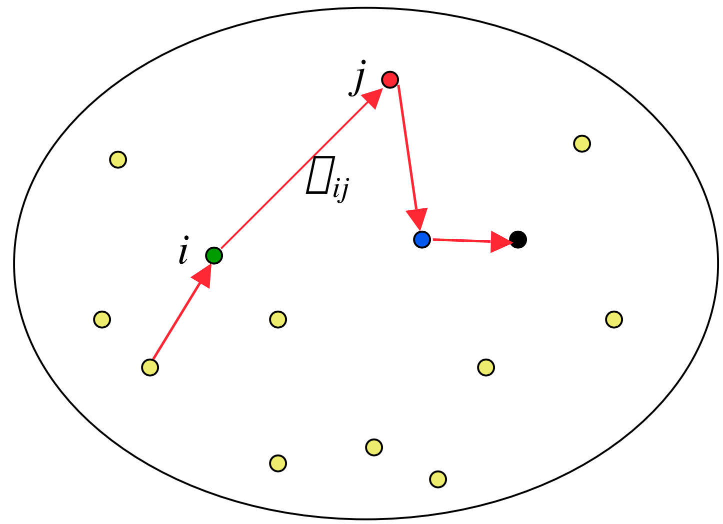
  – Choose power P_{ik} = power used by i intended directly for node k

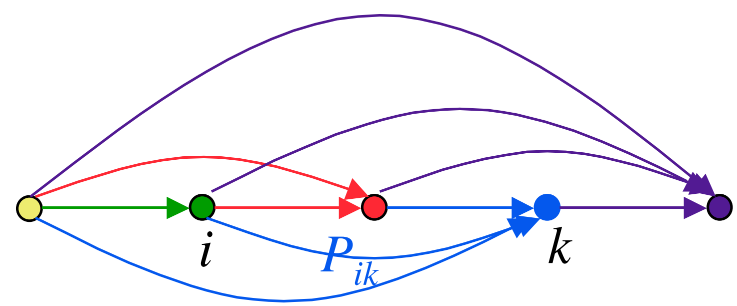
  – where $\sum_{k \neq i}^{M} P_{ik} \leq P_i$

  – Then

$$R < \min_{1 \leq j \leq n} S\left(\frac{1}{\sigma^2} \sum_{k=1}^{j} \left(\sum_{i=0}^{k-1} \gamma_{ij} \sqrt{P_{ik}} \right)^2 \right)$$

  is feasible

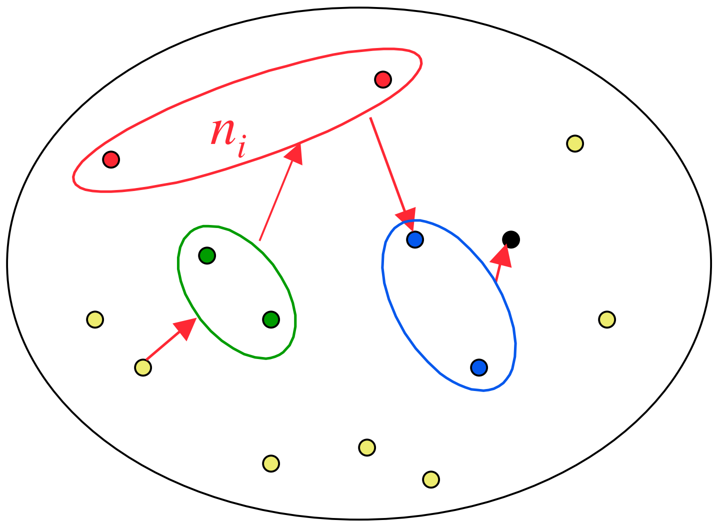




A group relaying version

- Theorem

  – A feasible rate for group relaying



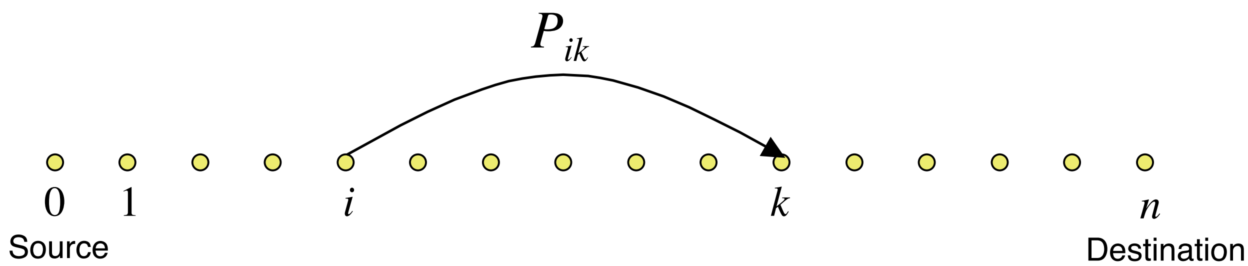
  – $R < \min_{1 \le j \le M} S\left(\frac{1}{\sigma^2} \sum_{k=1}^{j} \left(\sum_{i=0}^{k-1} \beta_{N_i N_j} \sqrt{P_{ik} / n_i} \cdot n_i \right)^2 \right)$

Unbounded transport capacity can be obtained for fixed total power

- Theorem
  - Suppose □= 0, there is no absorption at all,
  - And □< 3/2
  - Then C_T can be unbounded in regular planar networks even for fixed P_{total}
- Theorem
  - If □= 0 and □< 1 in regular planar networks
  - Then no matter how many nodes there are
  - No matter how far apart the source and destination are chosen
  - A fixed rate R_{min} can be provided for the single-source destination pair

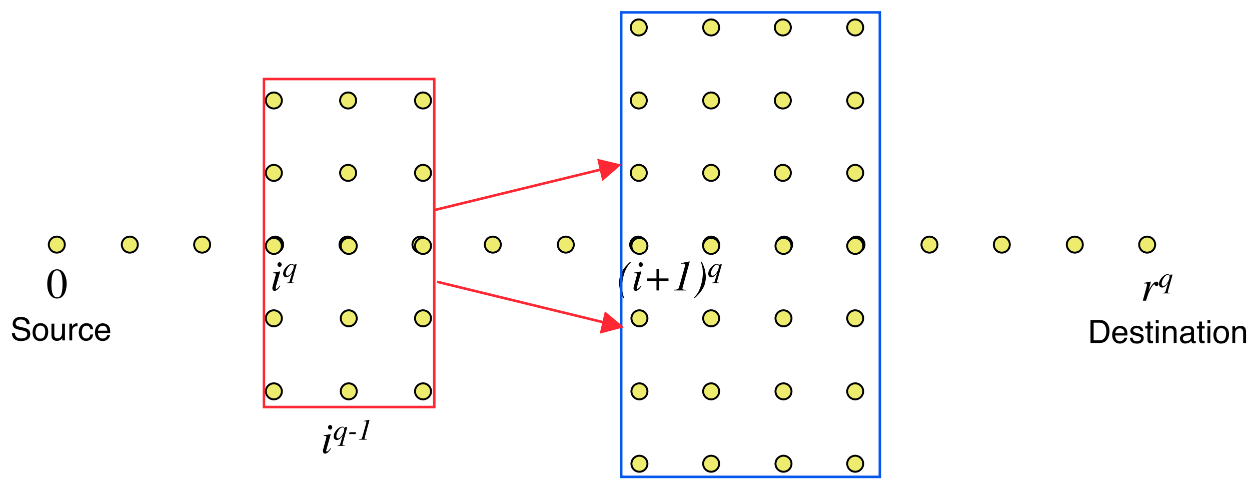
Idea of proof of unboundedness

- Linear case: Source at 0, destination at *n*



- Choose $P_{ik} = \frac{P}{(k \square i)^{\square} k^{\square}}$

- Planar case



Superlinear transport capacity □$(n^{□})$

- <u>Theorem</u>

  – Suppose □= 0

  – For every 1/2 < □< 1, and 1 < □ < 1/□

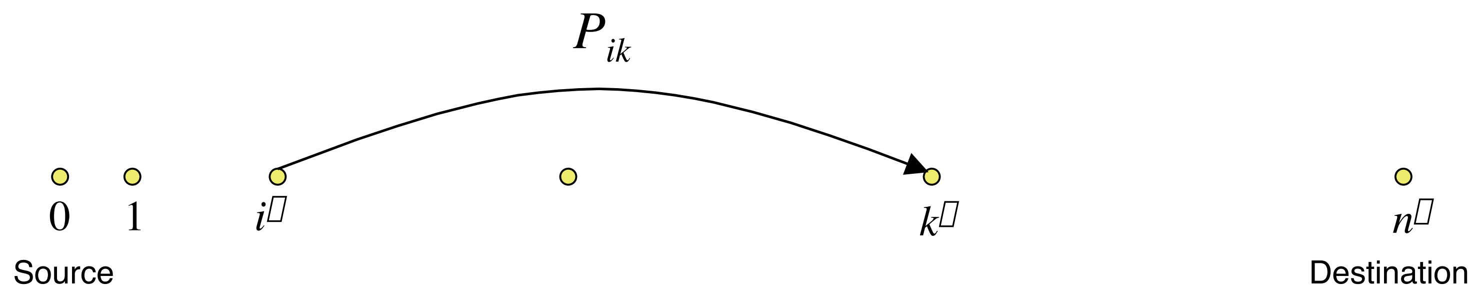
  – There is a family of linear networks with

  $$C_T = □(n^{□})$$

  – The optimal strategy is coherent multi-stage relaying with interference cancellation

Idea of proof

- Consider a linear network



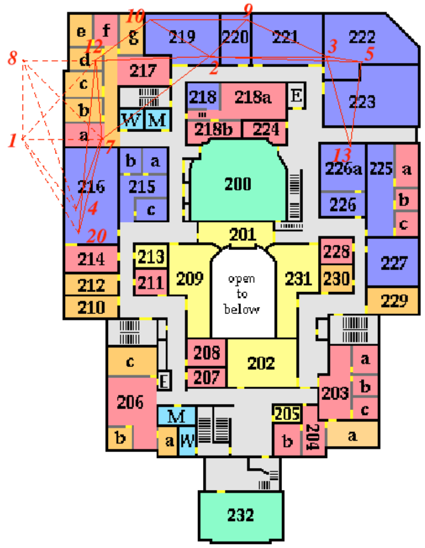
- Choose

$$P_{ik} = \frac{P}{(k \,□\, i)^{□}} \quad \text{where } 1 < □ < 3 \,□\, 2□□$$

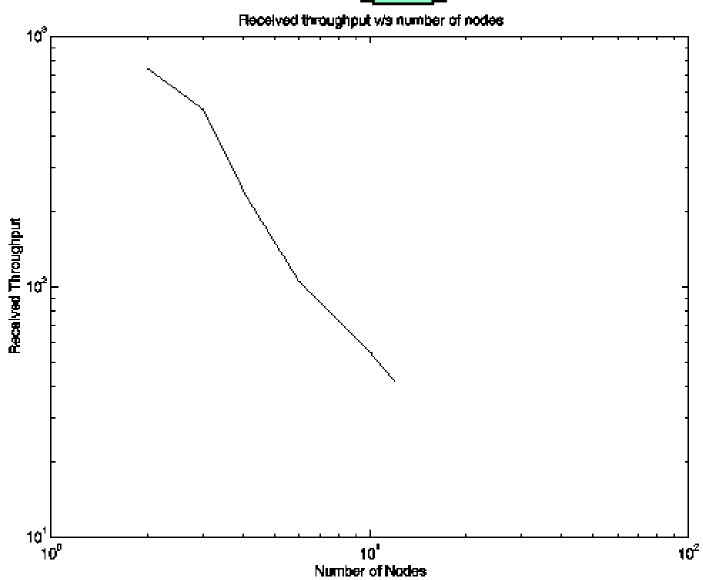
- A positive rate is feasible from source to destination for all n
  - By using coherent multi-stage relaying with interference cancellation

- To show upper bound
  - Sum of power received by all other nodes from any node j is bounded
  - Source destination distance is at most n□

Experimental scaling law

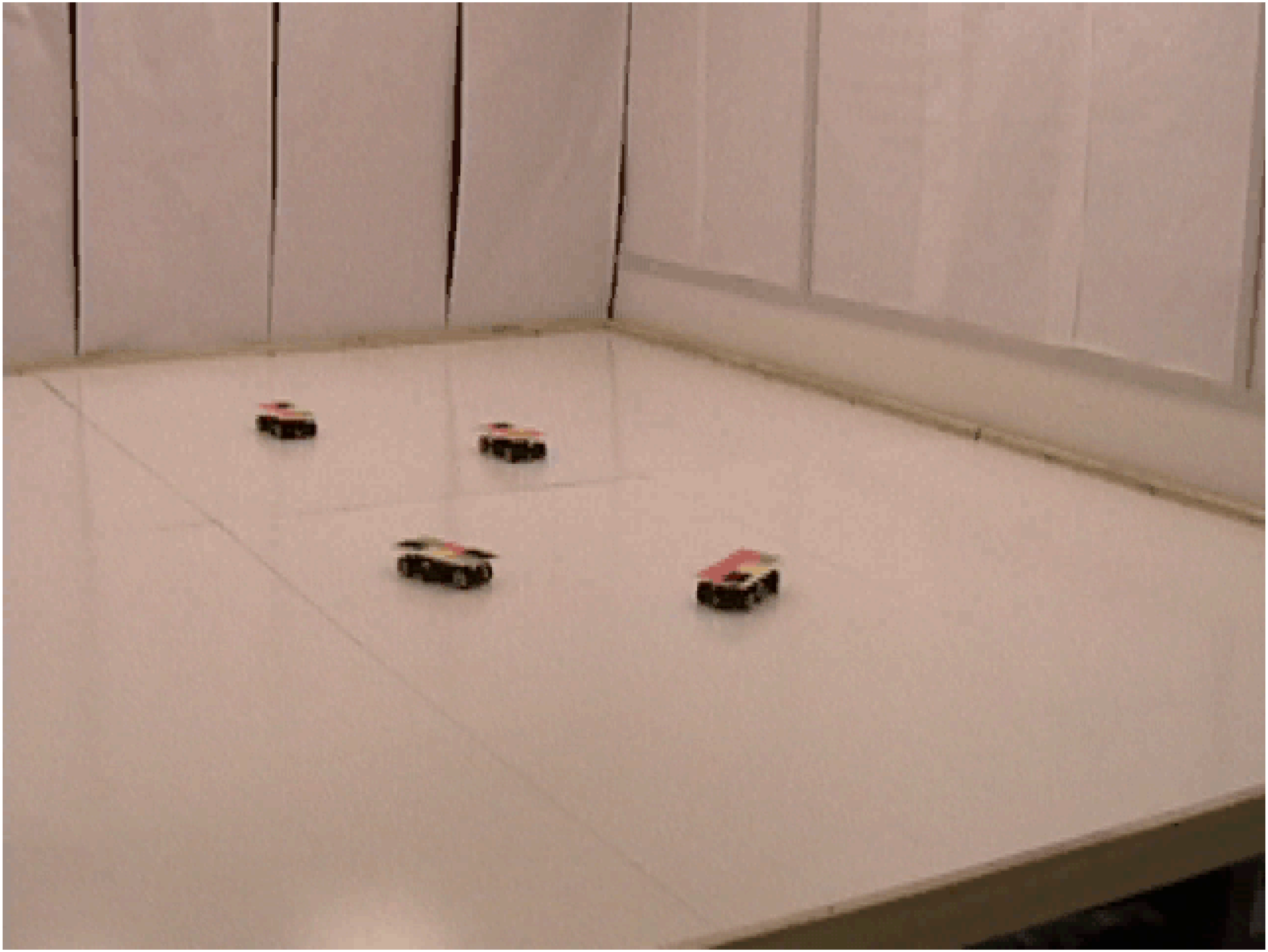


Log(Thpt)



Log(Number of Nodes)

- Throughput = $2.6/n^{1.68}$ Mbps per node
  - No mobility
  - No routing protocol overhead
    - Routing tables hardwired
  - No TCP overhead
    - UDP
  - IEEE 802.11
- Why $1/n^{1.68}$?
  - Much worse than optimal capacity = $c/n^{1/2}$
  - Worse even than $1/n$ timesharing
  - Perhaps overhead of MAC layer?



Concluding Remarks

- Studied networks with arbitrary numbers of nodes
  - Explicitly incorporated distance in model
    - Distances between nodes
    - Attenuation as a function of distance
    - Distance is also used to measure transport capacity
- Make progress by asking for less
  - Instead of studying capacity region, study the transport capacity
  - Instead of asking for exact results, study the scaling laws
    - The exponent is more important
    - The preconstant is also important but is secondary - so bound it
  - Draw some broad conclusions
    - Optimality of multi-hop when absorption or large path loss
    - Optimality of coherent multi-stage relaying with interference cancellation when no absorption and very low path loss
- Open problems abound
  - What happens for intermediate path loss when there is no absorption
  - The channel model is simplistic - fading, multi-path, Doppler, …...
  - …..

To obtain papers

- Papers can be downloaded from

  http://black.csl.uiuc.edu/~prkumar

- For hard copy send email to

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