NETWORK CODING:
A SOURCE CODING PERSPECTIVE

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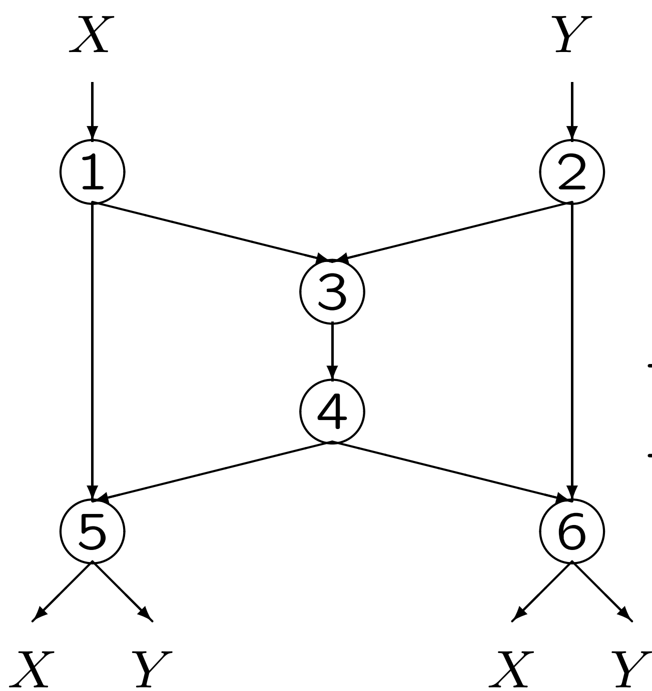
A DISTRIBUTED LOSSLESS SOURCE CODING PROBLEM



$(X_i, Y_i) \sim$ i.i.d. $p(x, y)$

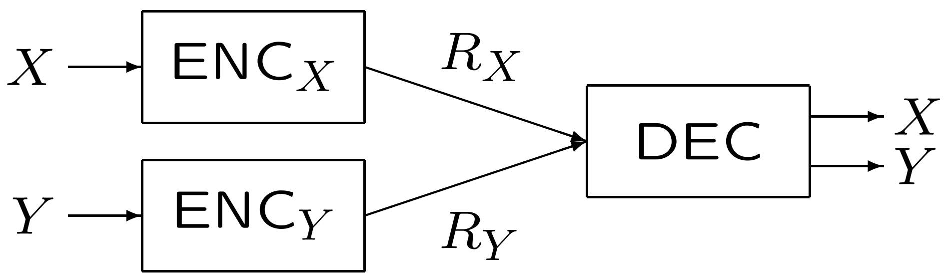
All links are lossless

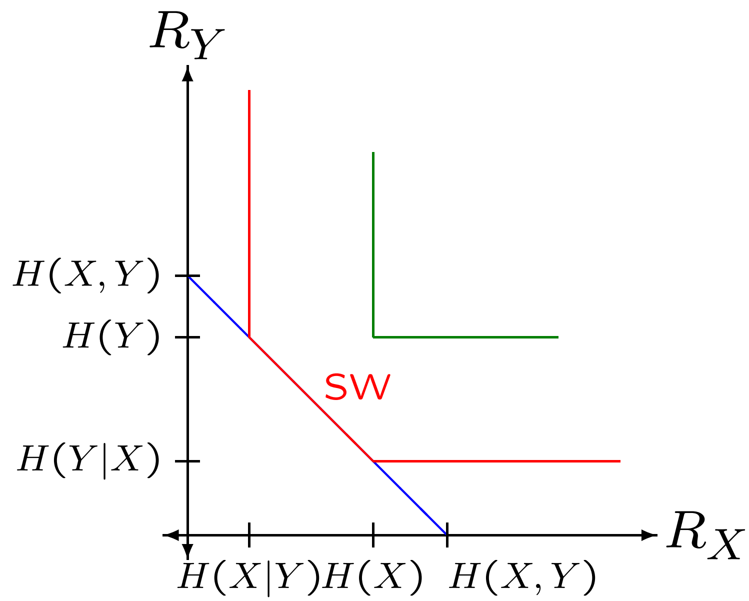
Two encoders: nodes 1 and 2

Two decoders: nodes 5 and 6

RELATED WORK

[Slepian & Wolf 1973],[Csiszar 1982]

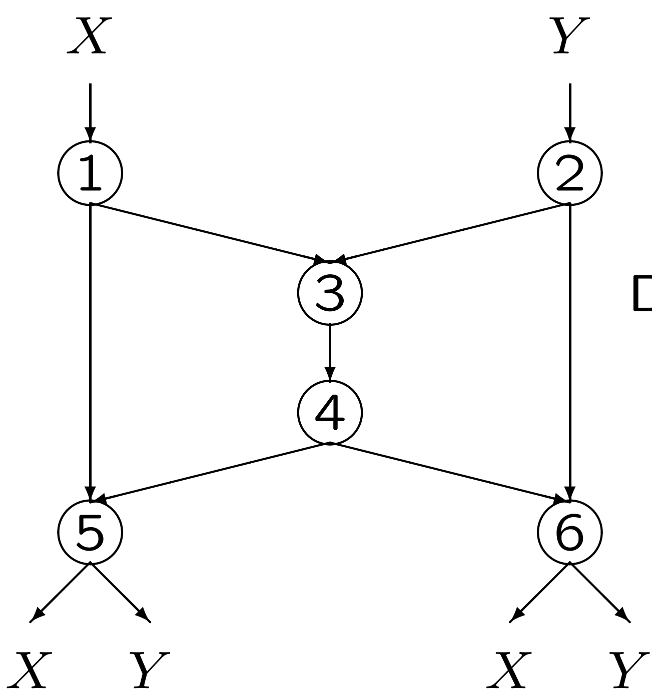




Optimal error exponents achievable with:
Linear encoders and *minimal entropy decoders* or *maximal a posteriori probability decoders* achieve optimal performance.

Randomized design

RELATED WORK



IF (X, Y) UNCOMPRESSIBLE

[Ahlswede, Cai, Li, and Yeung 2000]

Demands can be met in a multicast network

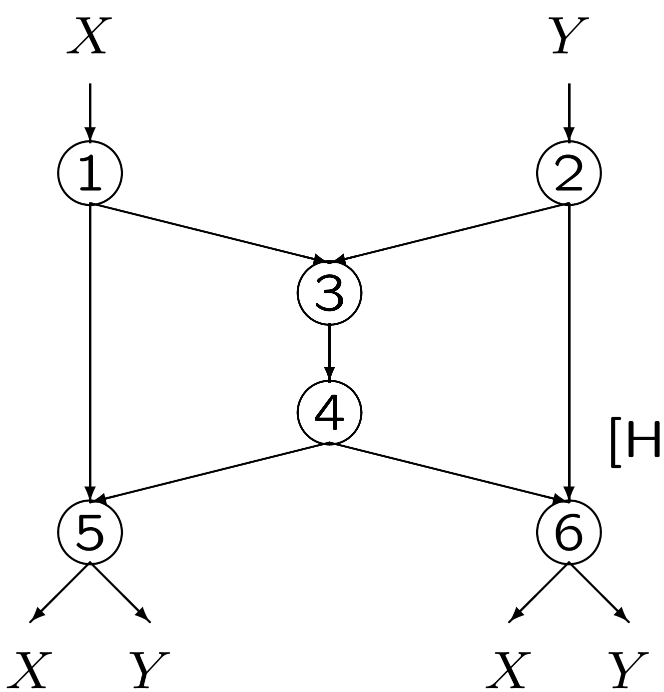
if and only if

min cut $= H(X) + H(Y) = H(X, Y)$

[Li, Yeung, and Cai 2003]

Linear codes suffice.

RELATED WORK



IF (X, Y) UNCOMPRESSIBLE

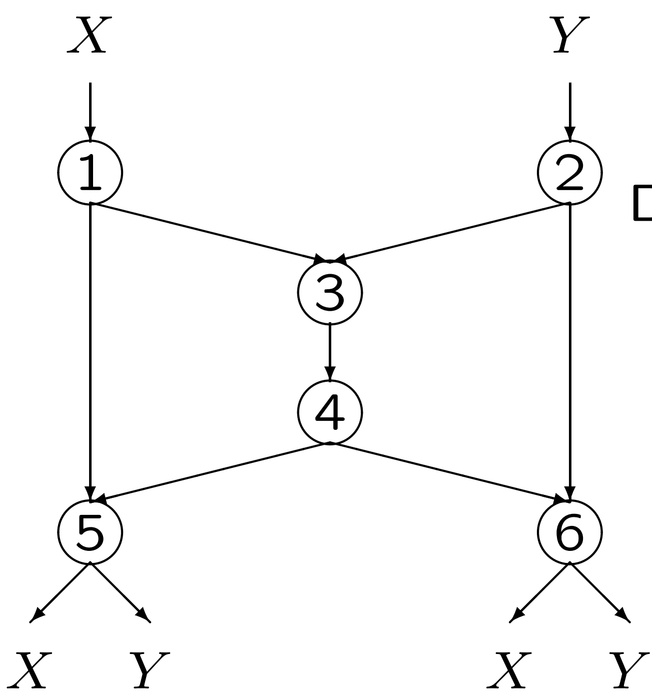
[Koetter & Médard 2002]

Algebraic framework

[Ho, Koetter, Médard, Karger, & Effros 2003]

Distributed randomized design

SOLUTION:
(NETWORK CODING W/COMPRESSIBLE SOURCES)



[Song & Yeung 2001]

[Ho, Médard, Effros & Koetter 2004]

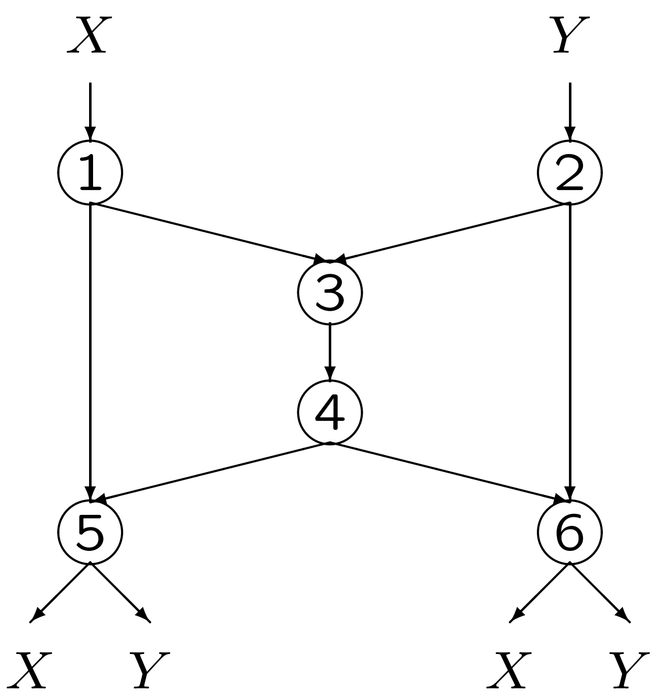
Demands can be met in a multicast network

if and only if

min cut from each transmitter
exceeds conditional entropy

min cut from all transmitters
exceeds joint entropy

SOLUTION:
(NETWORK CODING W/COMPRESSIBLE SOURCES)



Linear codes

Randomized code design

Minimal entropy or MAP decoding

Optimal error exponents

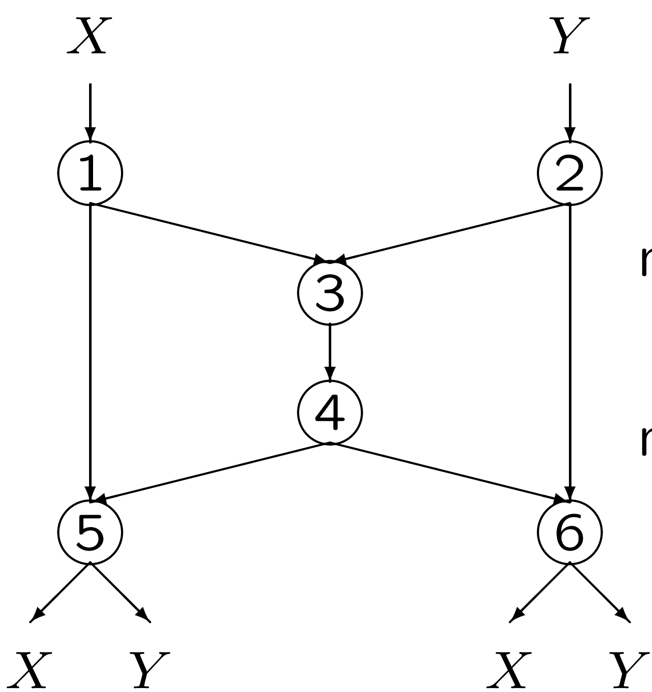
NOTICE

- Redundancy is removed or added in different parts of the network depending on available capacity

- Achieved without knowledge of source entropy rates at interior network nodes

- For the special case of a Slepian-Wolf source network consisting of a link from each source to the receiver, the network coding error exponents reduce to known error exponents for linear Slepian-Wolf coding [Csi82]

EXAMPLE 1



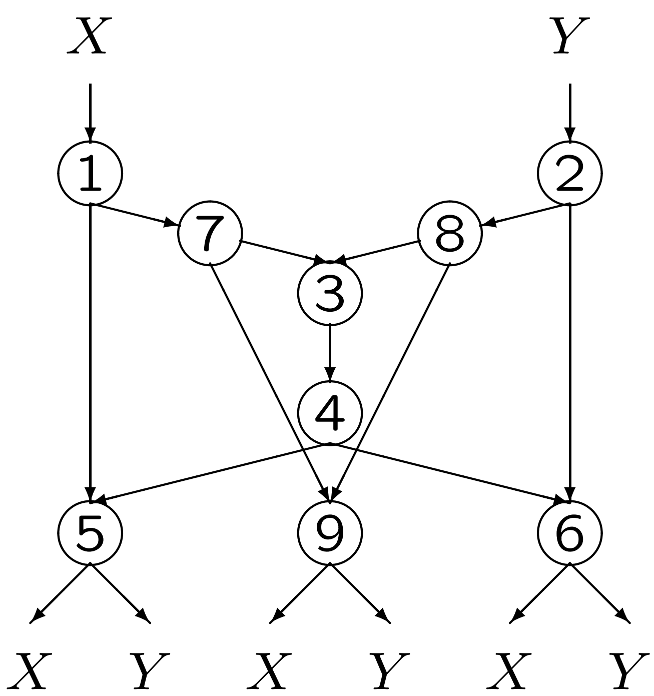
$$\min\{C_{13}, C_{34}, C_{46}, C_{15} + C_{45}\} \geq H(X|Y)$$

$$\min\{C_{23}, C_{34}, C_{45}, C_{26} + C_{46}\} \geq H(Y|X)$$

$$\min\{C_{15} + C_{13} + C_{23}, C_{15} + C_{34}, C_{15} + C_{45}\} \geq H(X,Y)$$

$$\min\{C_{13} + C_{23} + C_{26}, C_{34} + C_{26}, C_{46} + C_{26}\} \geq H(X,Y)$$

EXAMPLE 2



$$\min\{C_{17}, C_{73}, C_{34}, C_{46}, C_{79}, C_{15} + C_{45}\} \geq H(X|Y)$$

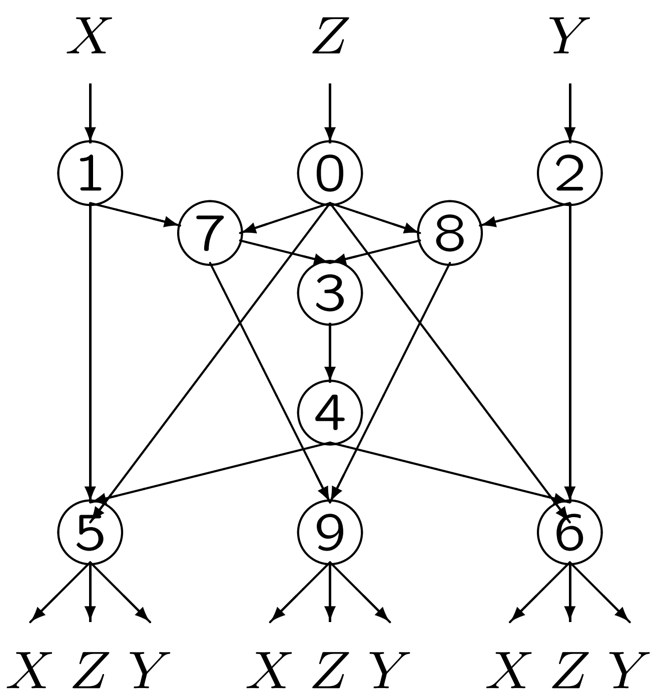
$$\min\{C_{28}, C_{83}, C_{34}, C_{45}, C_{89}, C_{26} + C_{46}\} \geq H(Y|X)$$

$$\min\{C_{15} + \min\{C_{17}, C_{73}\} + \min\{C_{28}, C_{83}\}, C_{15} + C_{34}, C_{15} + C_{45}\} \geq H(X,Y)$$

$$\min\{\min\{C_{17}, C_{73}\} + \min\{C_{28}, C_{83}\} + C_{26}, C_{34} + C_{26}, C_{46} + C_{26}\} \geq H(X,Y)$$

$$\min\{C_{17}, C_{79}\} + \min\{C_{28}, C_{89}\} \geq H(X,Y)$$

EXAMPLE 3



min cut from $X \geq H(X|Y,Z)$
min cut from $Y \geq H(Y|X,Z)$
min cut from $Z \geq H(Z|X,Y)$

min cut from $(X,Y) \geq H(X,Y|Z)$
min cut from $(Y,Z) \geq H(Y,Z|X)$
min cut from $(X,Z) \geq H(X,Z|Y)$
min cut from $(X,Y) \geq H(X,Y|Z)$

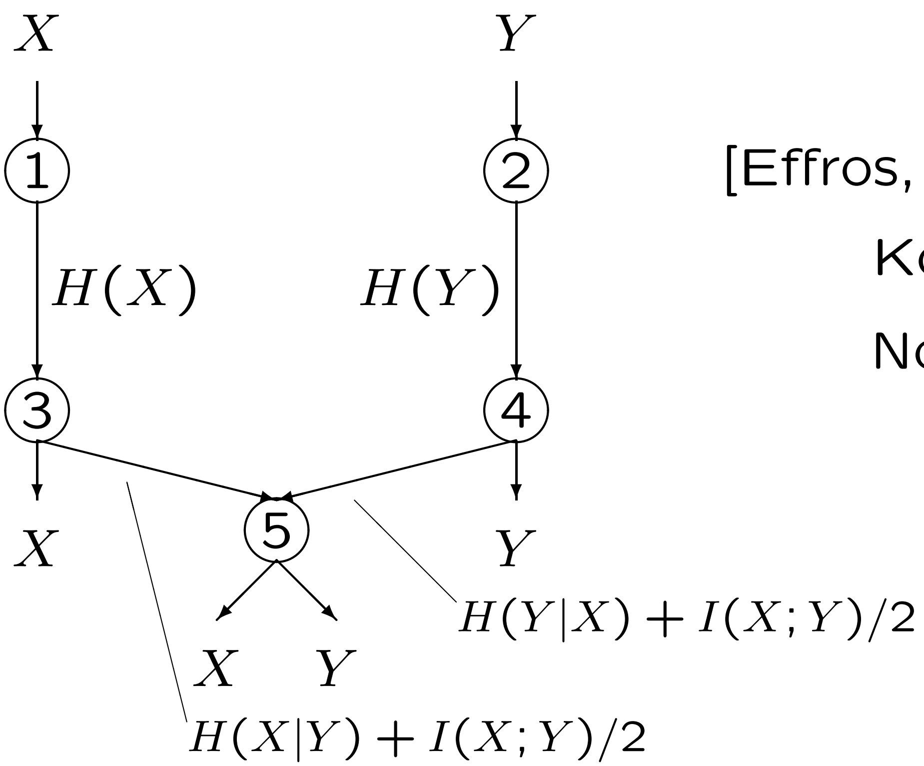
min cut from $(X,Y,Z) \geq H(X,Y,Z)$

SEPARATION

In *separate* source and network coding the source and network code agree only on the rates from each transmitter to each receiver.

(Note: The rates may differ from receiver to receiver even in a multicast network.)

QUESTION:
WHAT DO WE LOSE BY SEPARATING SOURCE AND NETWORK CODING?



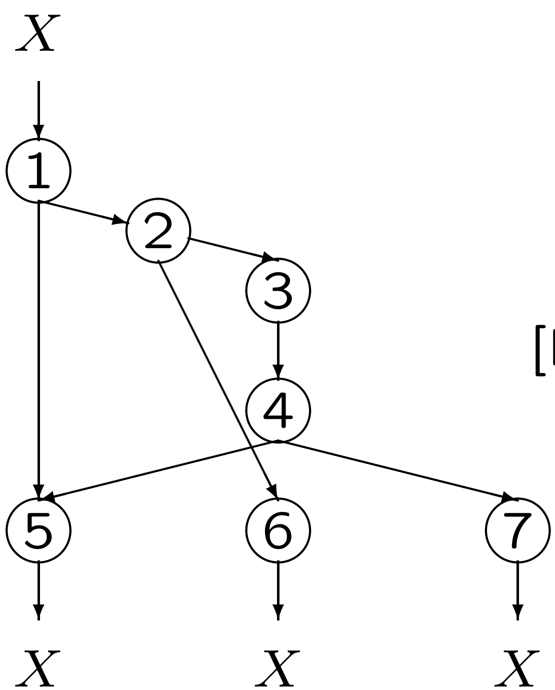
ACHIEVABILITY!

[Effros, Médard, Ho, Ray, Karger, Koetter, Hassibi 2003]

Non-multicast network

Separation FAILS!

QUESTION:
CAN SEPARATION FAIL
IN A MULTICAST NETWORK?

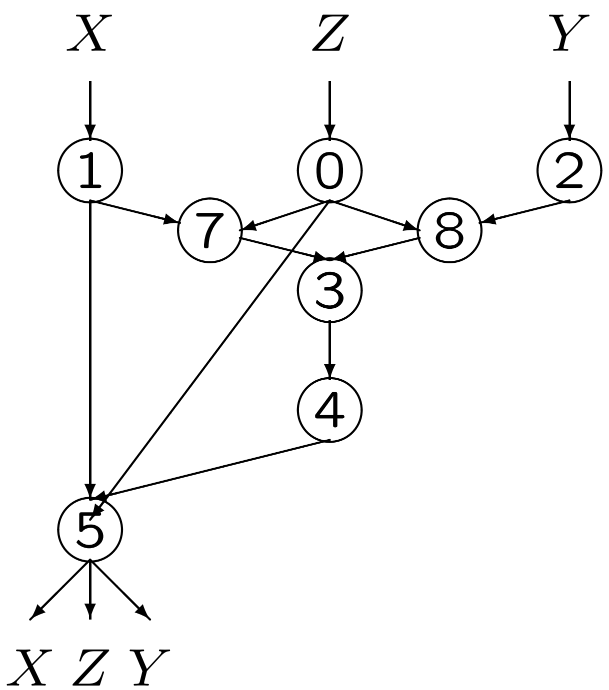


If all information begins in one place,

then no...

[Ramamoorthy, Jain, Chou, Effros 2004]

QUESTION:
CAN SEPARATION FAIL
IN A MULTICAST NETWORK?

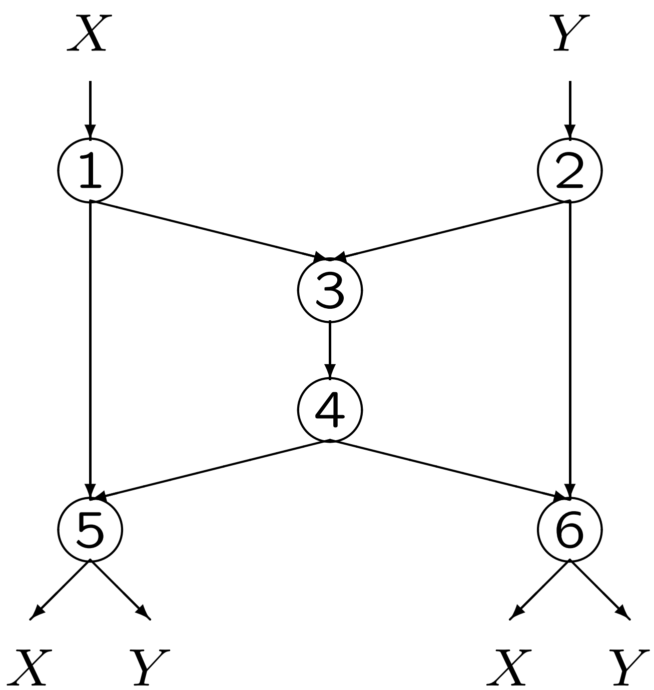


If only one node makes demands,

then no...

[Ramamoorthy, Jain, Chou, Effros 2004]

QUESTION: CAN SEPARATION FAIL IN A MULTICAST NETWORK?

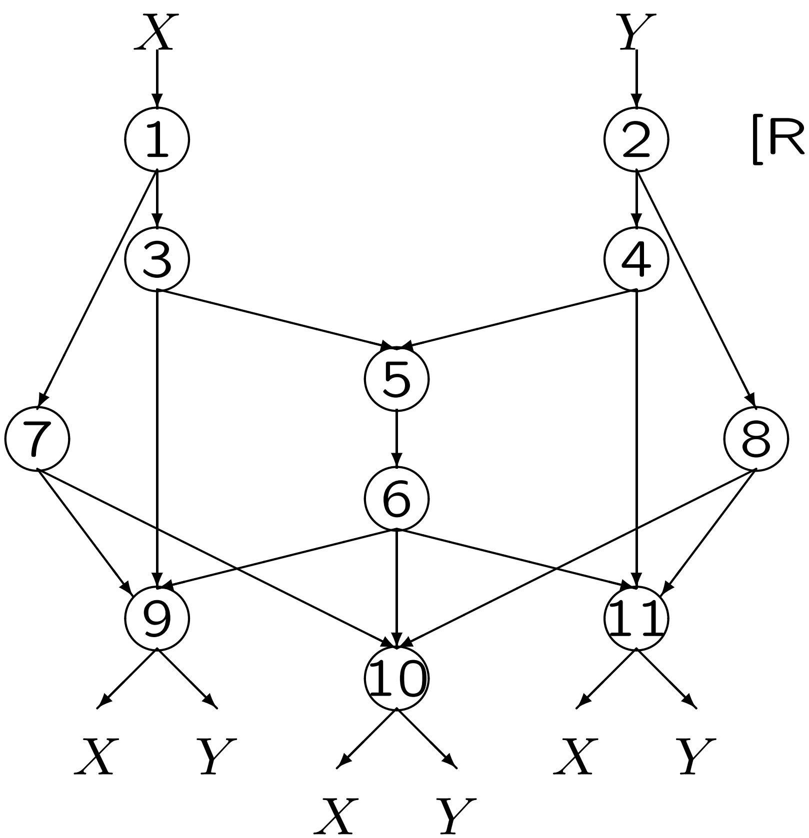


If information originates at two nodes and two nodes make demands then no!

[Ramamoorthy, Jain, Chou, Effros 2004]

QUESTION: CAN SEPARATION FAIL IN A MULTICAST NETWORK?

$H(X) = H(Y) = 2$, $H(X,Y) = 3$

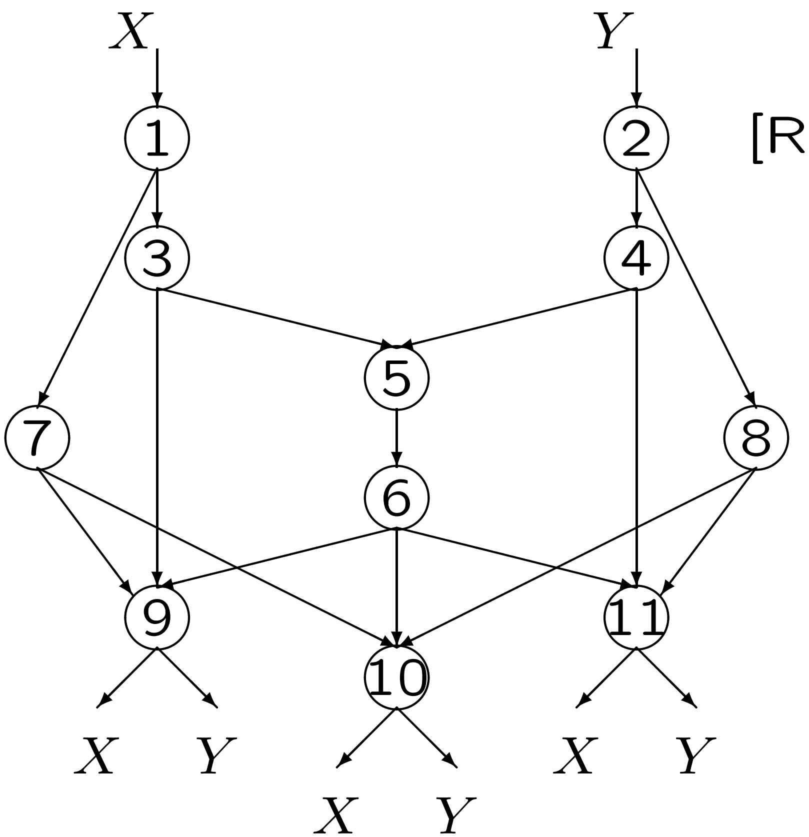


YES!

[Ramamoorthy, Jain, Chou, Effros 2004]

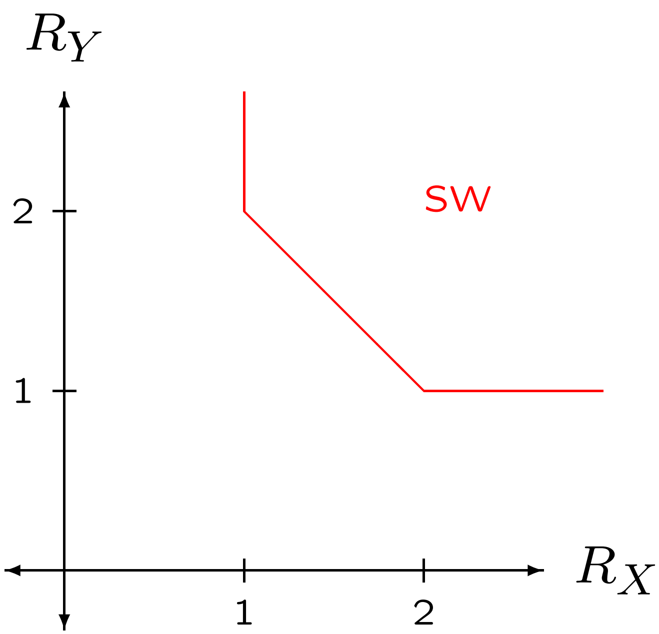
IDEA

$H(X) = H(Y) = 2$, $H(X,Y) = 3$



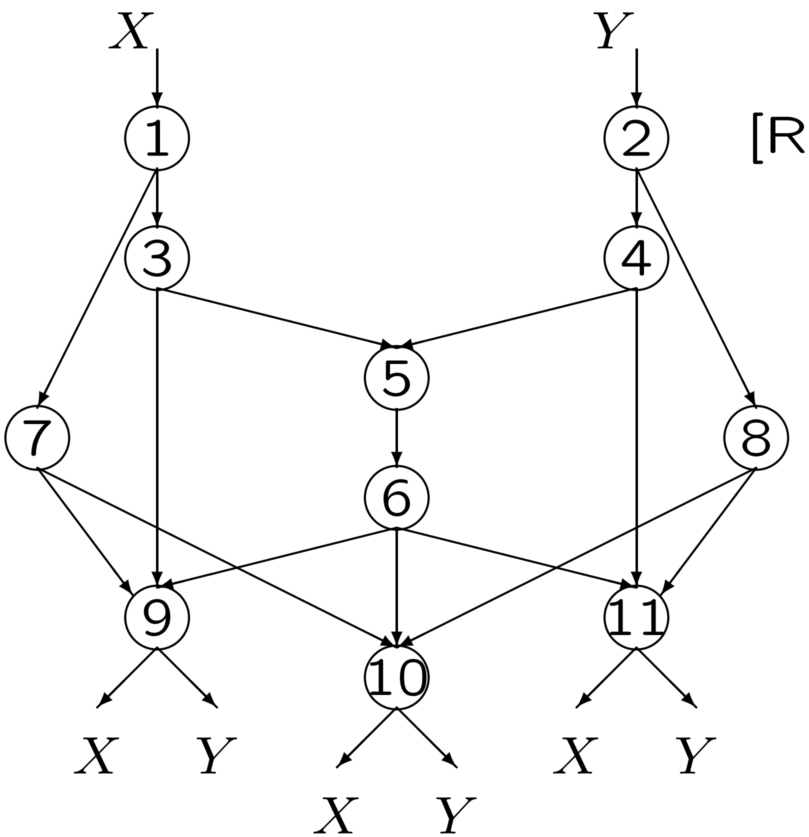
YES!

[Ramamoorthy, Jain, Chou, Effros 2004]



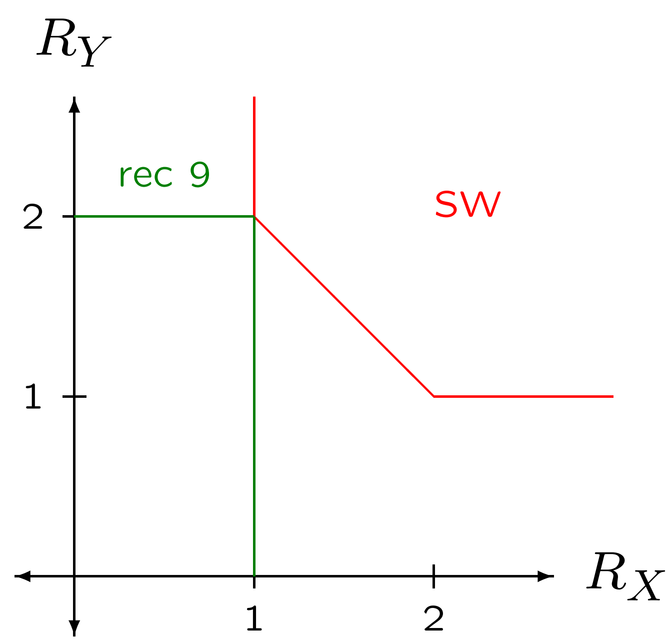
IDEA

$H(X) = H(Y) = 2,\ H(X,Y) = 3$



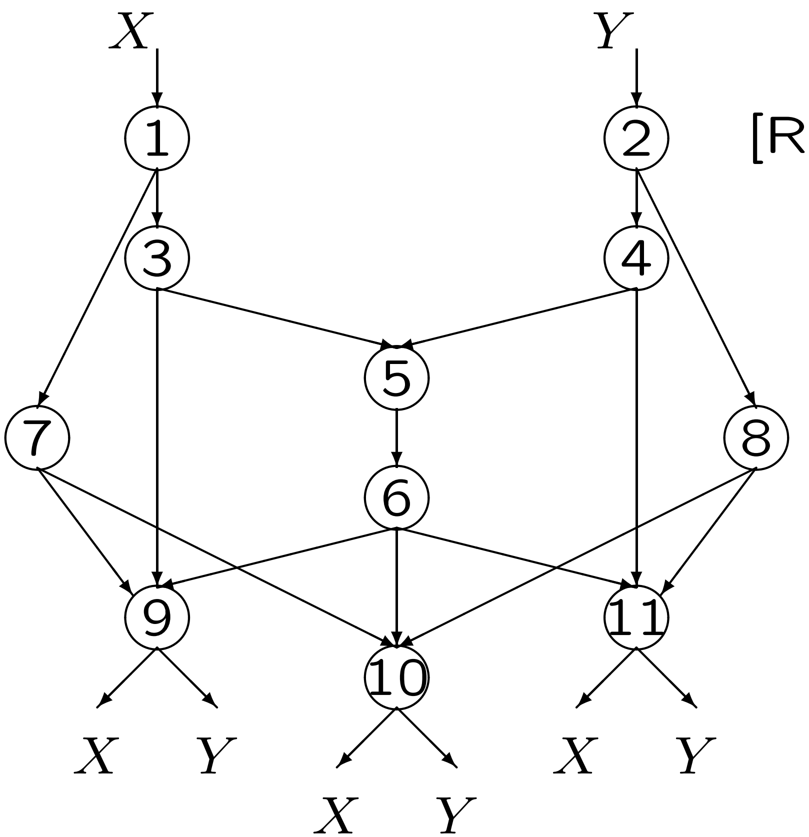
YES!

[Ramamoorthy, Jain, Chou, Effros 2004]



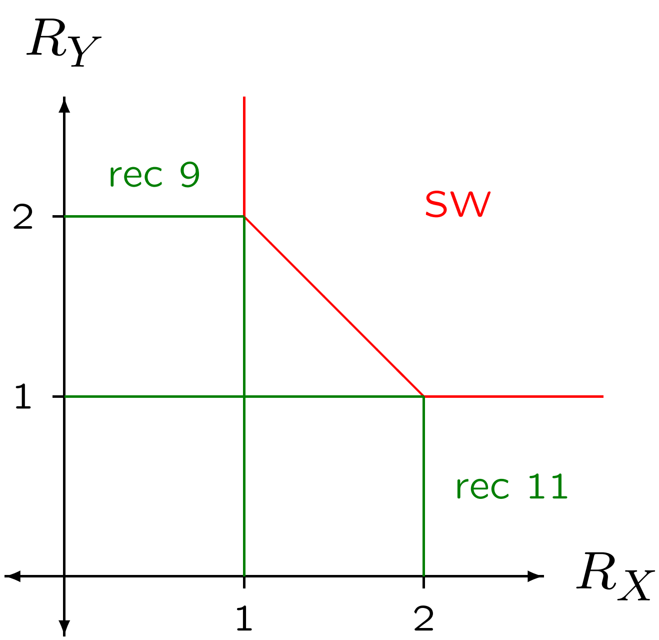
IDEA

$H(X) = H(Y) = 2$, $H(X, Y) = 3$



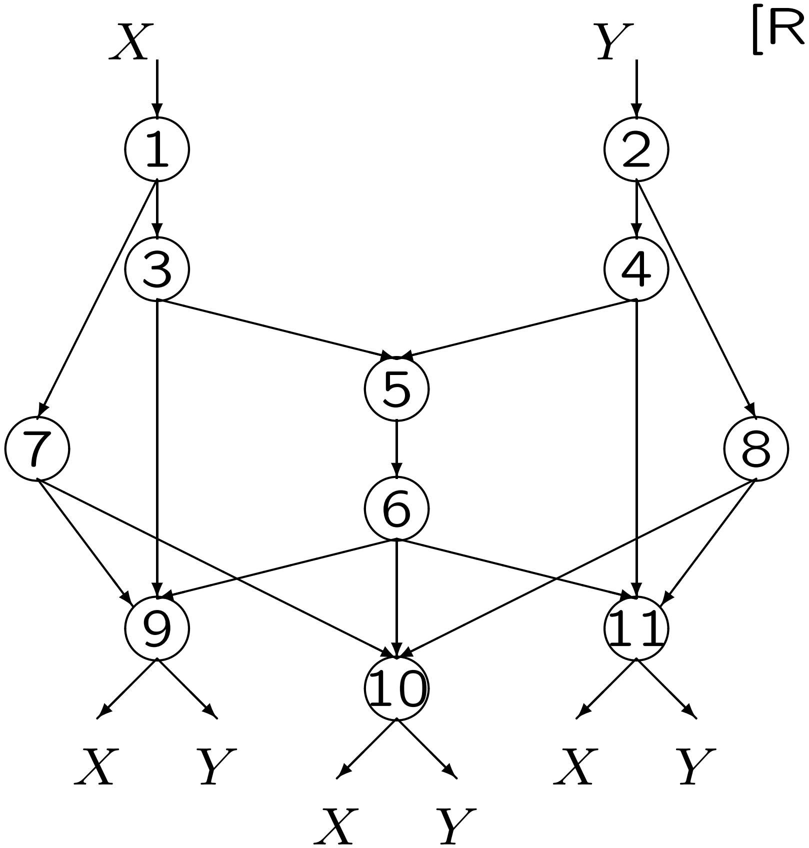
YES!

[Ramamoorthy, Jain, Chou, Effros 2004]



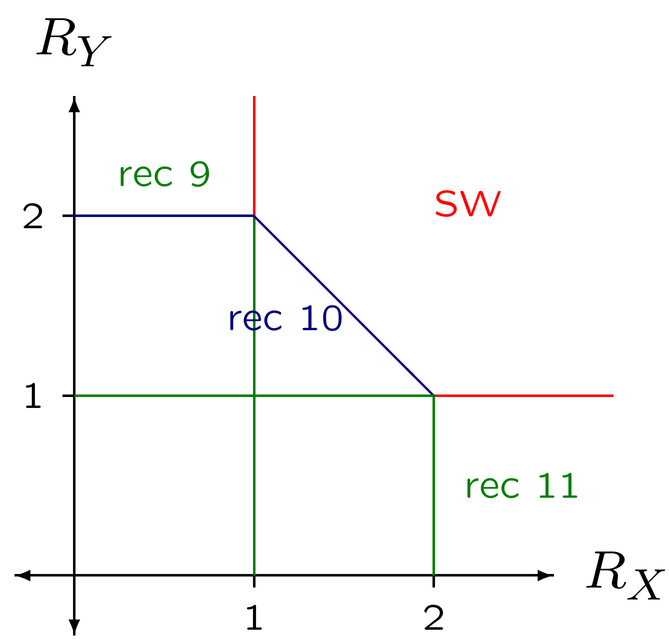
IDEA

$H(X) = H(Y) = 2$, $H(X, Y) = 3$



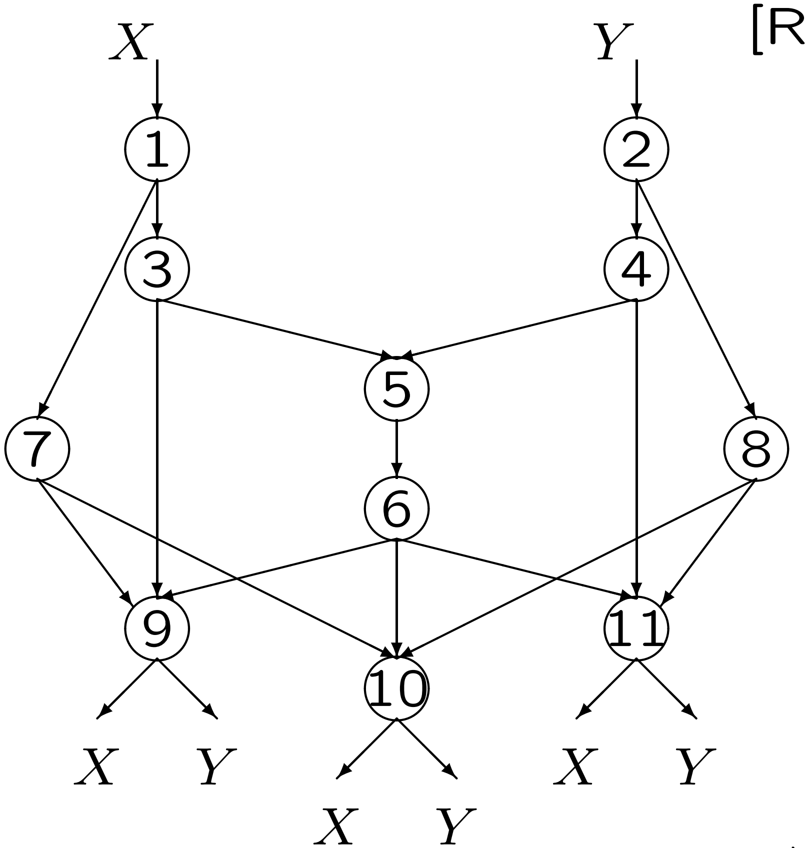
YES!

[Ramamoorthy, Jain, Chou, Effros 2004]



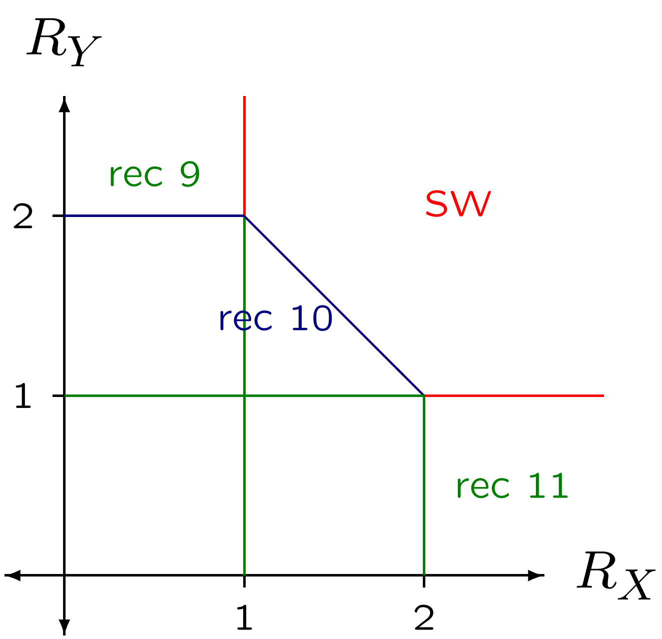
IDEA

$H(X) = H(Y) = 2,\ H(X, Y) = 3$



YES!

[Ramamoorthy, Jain, Chou, Effros 2004]

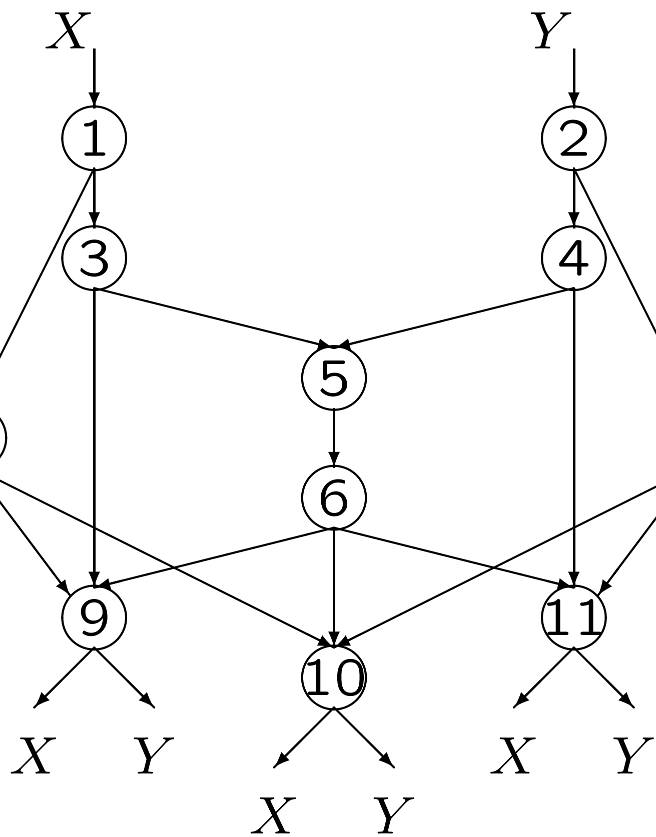


Cap regions intersect SW

$\Rightarrow$ Reliable communication possible

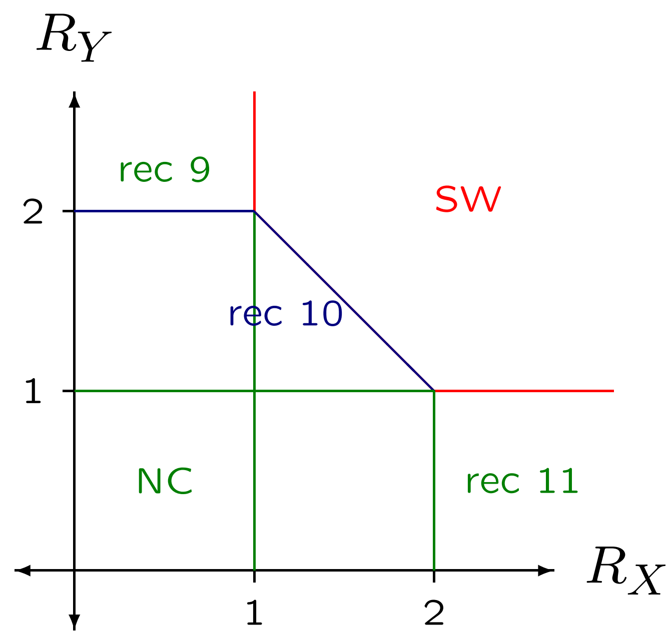
IDEA

$H(X) = H(Y) = 2$, $H(X,Y) = 3$



YES!

[Ramamoorthy, Jain, Chou, Effros 2004]

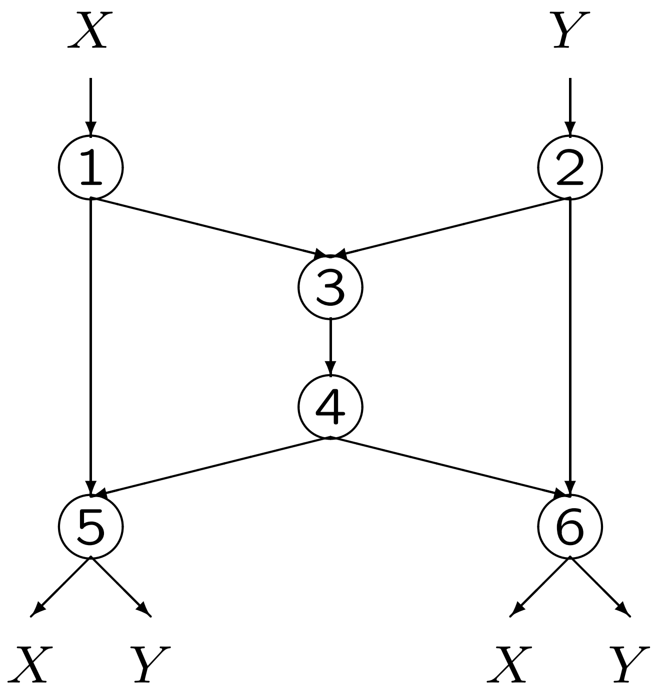


NC and SW regions don't overlap

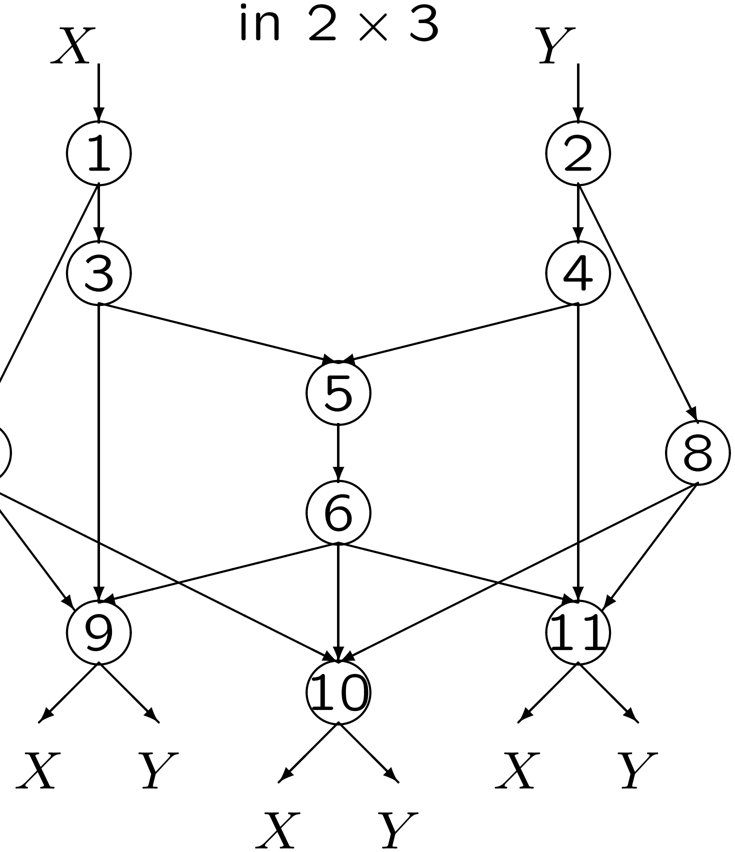
No separate solution.

SO...

Separation succeeds in 2×2

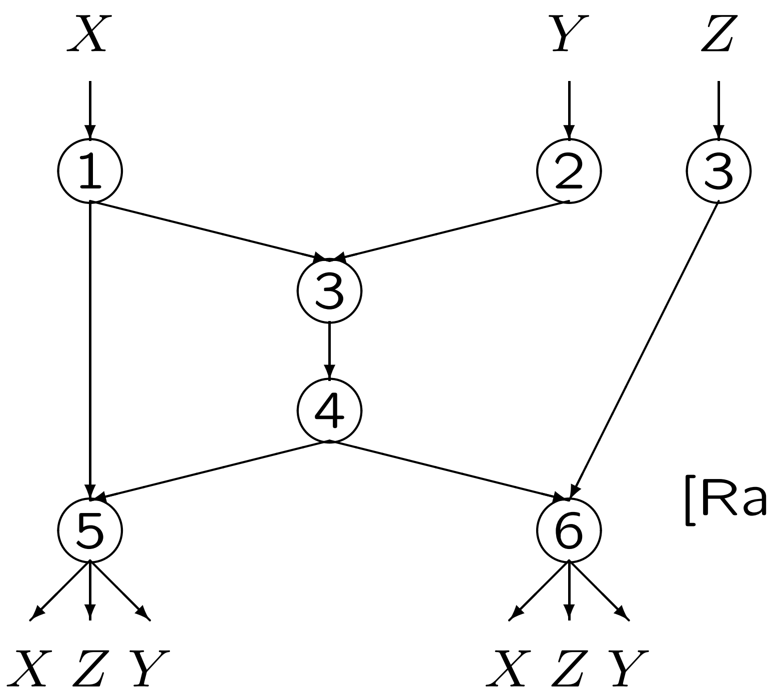


Separation can fail in 2×3



WHAT ABOUT 3×2**??**

A 3×2 **EXAMPLE**



$$H(X) = H(Y) = H(Z) = 1$$

$$H(X, Y) = 2,\ Y = Z$$

Separation fails.

[Ramamoorthy, Jain, Chou, Effros 2004]

SUMMARY

Separation Between Source and Network Coding

	1	2	> 2
1	+	+	+
2	+	+	-
> 2	+	-	-

SHOULD YOU EVER USE SEPARATE CODES?

maybe

ISSUE # 1: COMPLEXITY

- Decoding the source code is not a matrix multiplication

- Decoder complexity depends on density of encoding matrix

- Efficient source codes use low density encoding matrices

ISSUE # 1: COMPLEXITY

- Decoding separate source & network codes:
  matrix multiplication + low density source code decoding

- Distributed randomized joint code design
  fails to maintain the low density structure.

- Decoding joint source &network codes:
  (likely high density) joint decoding

ISSUE # 2: FREQUENCY

- Separation *can* fail in most network classes $\not\Rightarrow$ separation *does* fail in most networks.

- On the one hand:
  If network coding is not required, then separation cannot fail.

- On the other hand:
  Distributed, randomized network code design can cause separate decoding to fail even when network coding is not needed for capacity.

CONCLUSIONS

- Distributed randomized network coding can achieve distributed compression of correlated sources.

- Error exponents generalize results for linear Slepian Wolf coding.

- Separate source and network codes may have complexity advantages, but they can fail.