

DIMACS WORKSHOP on ALGORITHMIC
MATHEMATICAL ART

Is
Popularization
of
Polynomiography
Possible?

Bahman Kalantari
Department of Computer Science
Rutgers University

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Once there was nothing.

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There are endless images and discoveries waiting to be unveiled.

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A BLENDING of ART, MATH, SCIENCE

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A **BLENDING** of **ART**, **MATH**, **SCIENCE**

A **VEHICLE** to **ATTRACT KIDS** to **MATH** and **SCIENCE**

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A **MEDIUM** for **ART** and **MATH** and **SCIENCE** and **FUN**
and **GAMES**

A Sort of Pedagogical Analogy

PHOTOGRAPHY versus POLYNOMIOGRAPHY

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A Sort of Pedagogical Analogy

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Q and A

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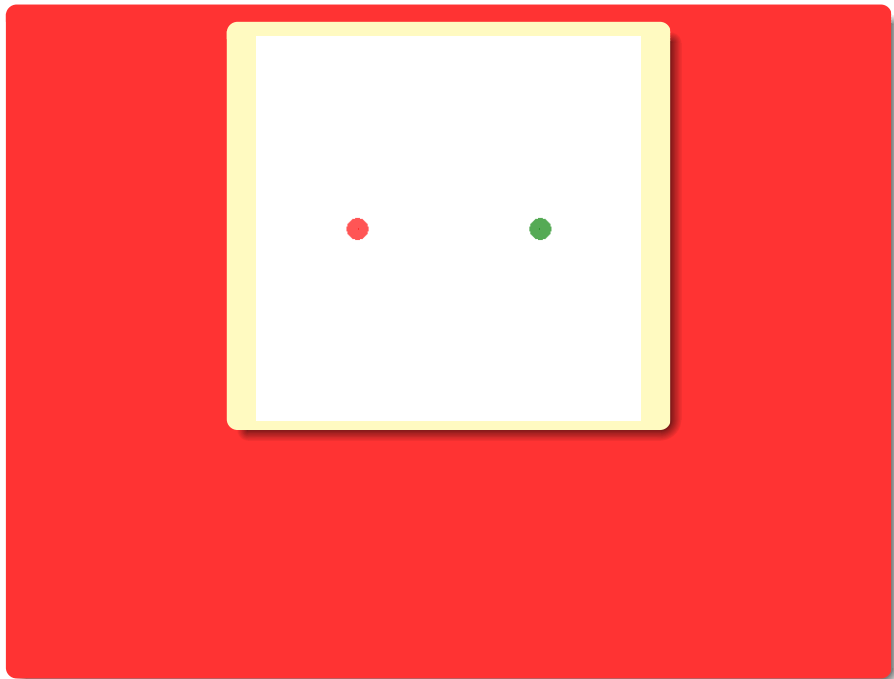
Q. Is there an age limit on playing this game?

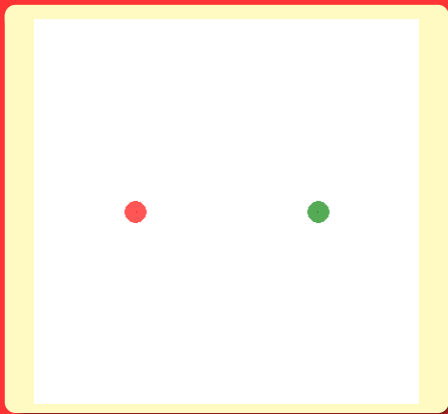
A. Yes, with appropriate development of the software the recommended age is between 5 and 125.

Let's Play Hide-and-Seek

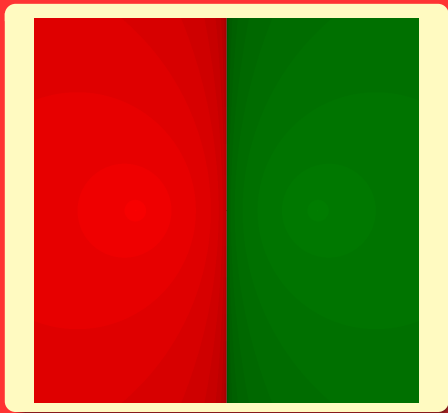
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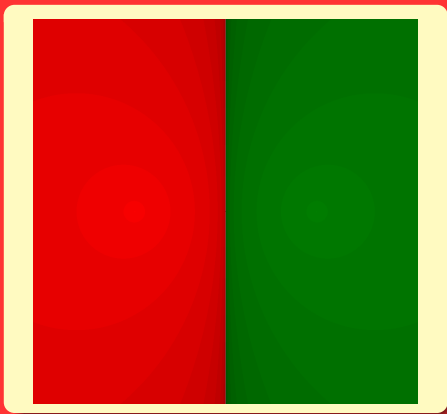
Let's Take Two Dots



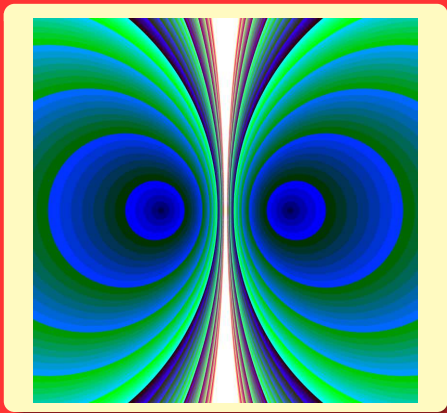


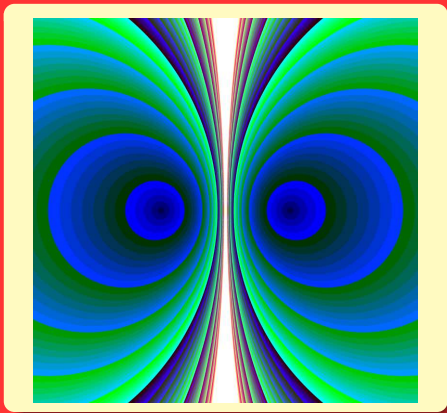
Hiding Equation: $x^2 - 2 = 0$.





A *polynomiograph* of the equation $x^2 - 2 = 0$.



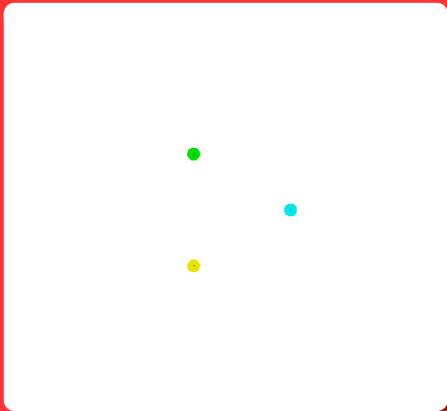


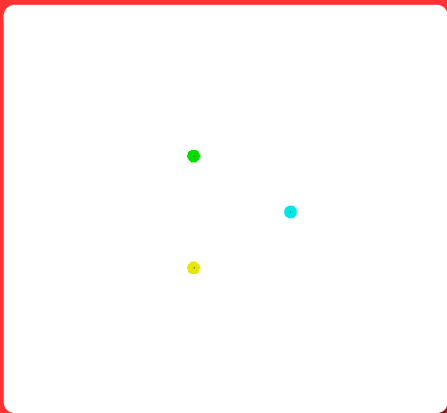
Another **polynomiograph** of the equation $x^2 - 2 = 0$.

Let's Play Hide-and-Seek
Again

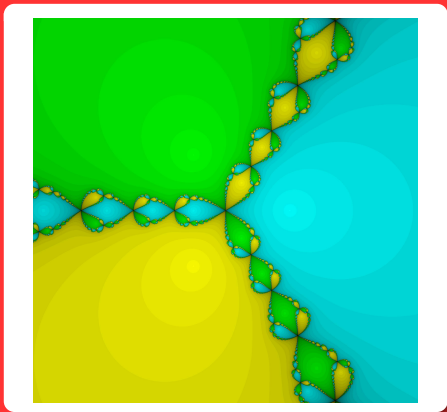
Let's Play Hide-and-Seek
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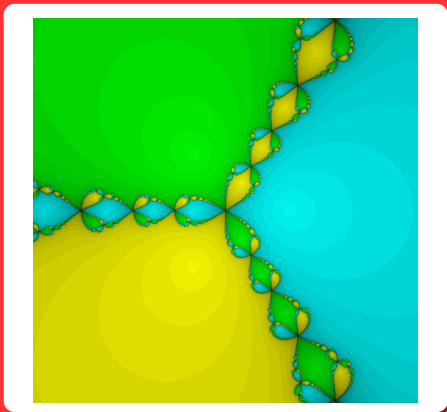
Let's Take Three Dots



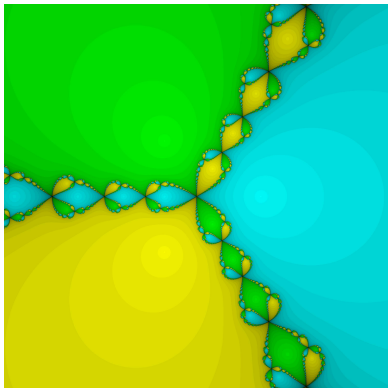


Polynomial Equation: $x^3 - 1 = 0$.

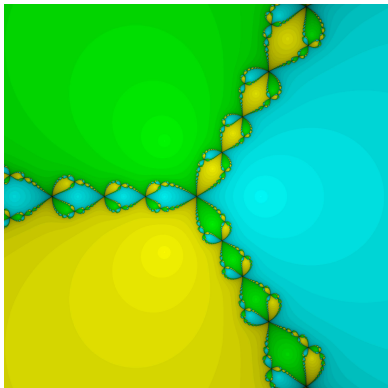




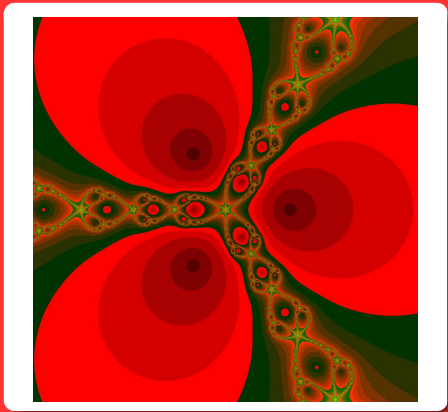
A polynomiograph of $x^3 - 1 = 0$.

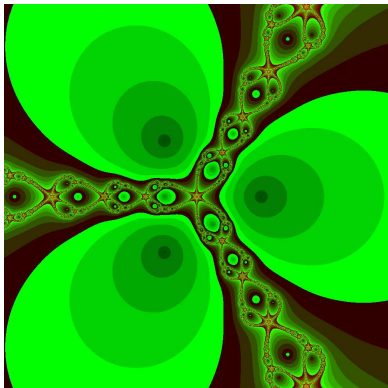


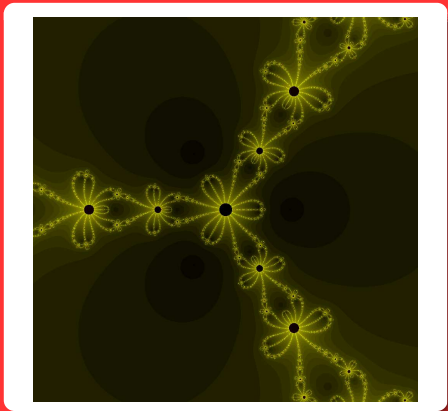
A polynomiograph of $x^3 - 1 = 0$.
Can we create other images from this equation?

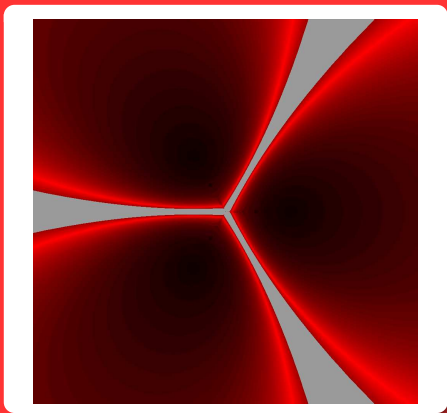


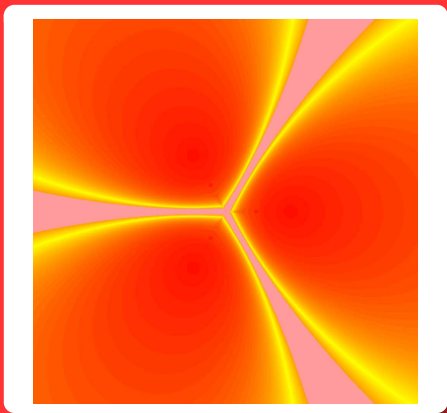
A polynomiograph of $x^3 - 1 = 0$.
Can we create other images from this equation?
Yes, many more ...

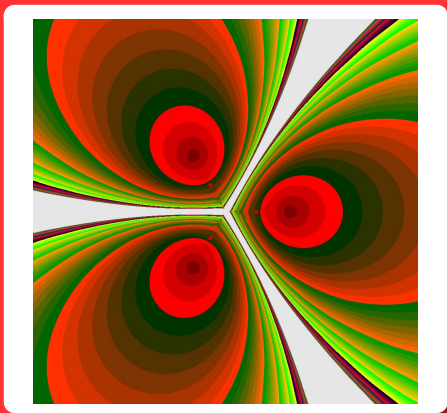


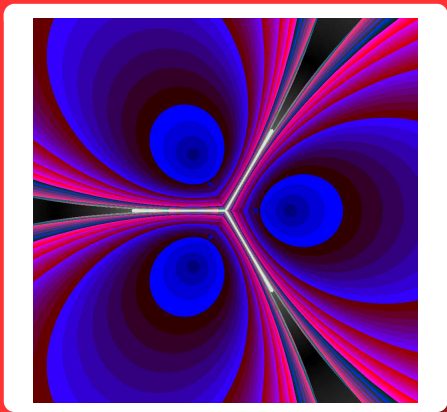


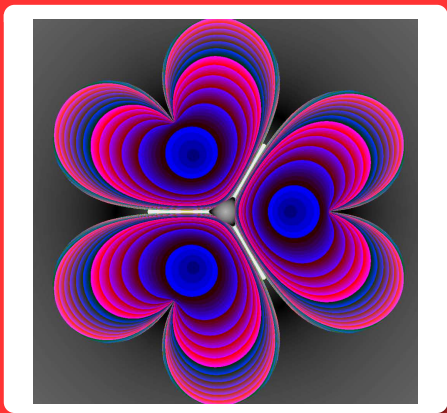


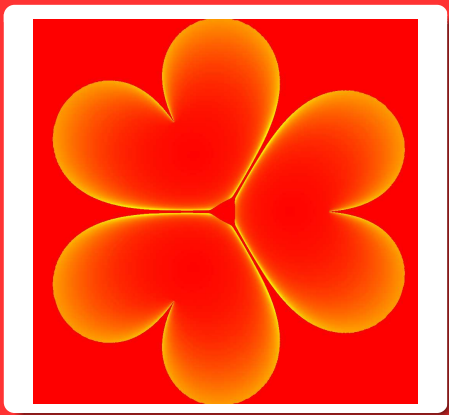


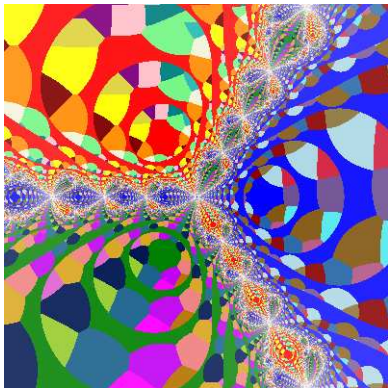


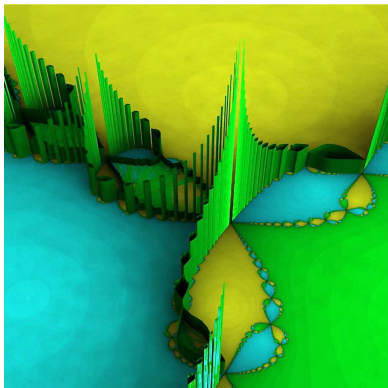


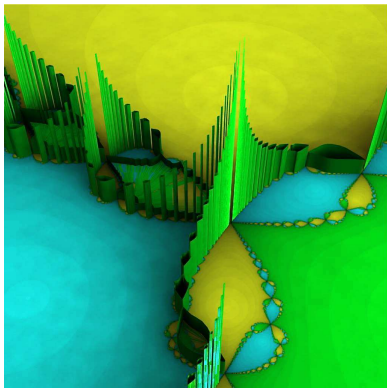












For a 3D animation based on $x^3 - 1$ see YouTube:
"Rise of Polynomials" at www.polynomiography.com

Polynomial Equation

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A polynomial is a linear combination of integral powers of a variable z ,

Polynomial Equation

A polynomial is a linear combination of integral powers of a variable z , an expression of the form

$$p(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0 = 0$$

where $a_n, a_{n-1}, \dots, a_1, a_0$ are numbers called *coefficients*. $a_n \neq 0$, and n is called the *degree* of the polynomial.

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$$p(z) = z^2 - 2 = 0.$$

Polynomial Equation

A polynomial is a linear combination of integral powers of a variable z , an expression of the form

$$p(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0 = 0$$

where $a_n, a_{n-1}, \dots, a_1, a_0$ are numbers called *coefficients*. $a_n \neq 0$, and n is called the *degree* of the polynomial.

Examples.

$$p(z) = z - 5 = 0.$$

$$p(z) = z^2 - 2 = 0.$$

$$p(z) = 7z^{45} - 25z^{36} + 43z^8 - 5z^6 + 2z - 9 = 0$$

Solution to Polynomial Equation ("root," or "zero")

Example.

$$p(z) = z^2 - 2 = 0.$$

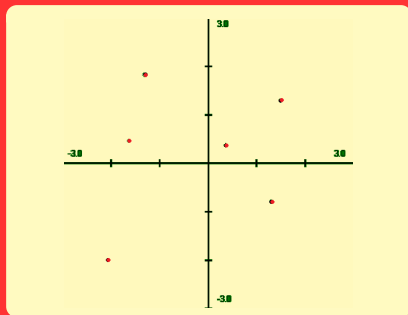
Solutions (root, zero):

$$z = \sqrt{2} = 1.414\dots$$

$$z = -\sqrt{2} = -1.414\dots$$

Zero of Polynomial has Dual Nature

It is a **NUMBER** and it is a **POINT** in the plane.



History of Polynomials in a Flash



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- ▶ A point on the plane has coordinates (a, b) .

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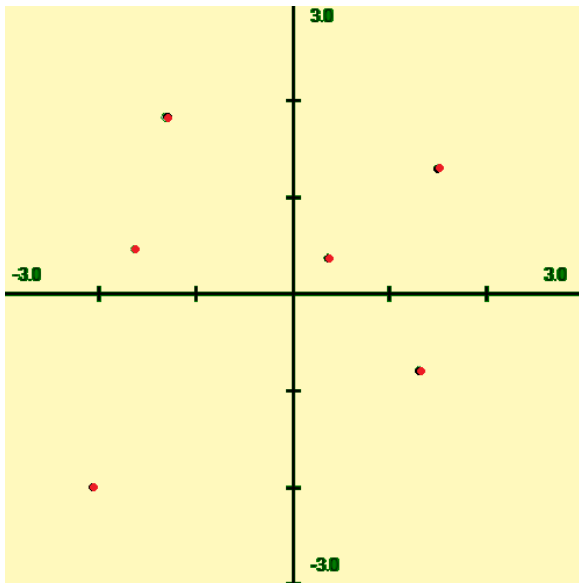
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- ▶ These discoveries took a few centuries!



Polynomial Equation:

$$\begin{aligned} p(z) = & z^6 + (1.92 - 1.2i)z^5 + (-.2362 - 4.2545i)z^4 + \\ & (-11.4353 - 1.7346i)z^3 + (8.638 + 11.8811i)z^2 + \\ & (32.5353 - 13.6329i)z + (-14.3137 - 8.9646i) \\ & = 0. \end{aligned}$$

Fundamental Theorem of Algebra



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Number of roots = Degree of Polynomial

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$p(z) = 5z^{17} - 4z^3 - 21$ has 17 roots.

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Example.

$p(z) = 5z^{17} - 4z^3 - 21$ has 17 roots.

$p(z) = z^3 - 1$ has 3 roots.

Polynomial Root-Finding is a Historic Problem

Many have investigated the root-finding problem:

The **Sumerians**, the **Babylonians**, the famous Persian mathematician and poet **Omar Khayyam**, the most famous mathematicians of the past such as **Euler**, **Newton**, **Lagrange**, **Descartes**, **Galois**, **Abel**, **Gauss**, **Caley**, **Shröder**; and the great contemporary mathematicians such as **Hermann Weyl**, and **Steve Smale**.

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It has even inspired the study of Newton's method in the complex plane and more general rational functions by **Fatou**, and **Julia** whose work in turn inspired **Mandelbrot** who coined the famous term **fractal**.

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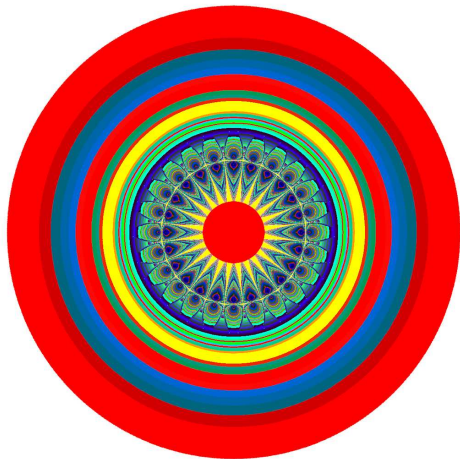
And for degree five and higher there is no GENERAL formula!

Really, even the quadratic formula is not useful if we are trying to approximate square-root of numbers such as square-root of 2.

$$10z^{48} - 11z^{24} + 1 = 0$$

Is this a nice polynomial?

A Polynomiograph of $10z^{48} - 11z^{24} + 1 = 0$



Newton's Method

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$$N(z) = z - \frac{p(z)}{p'(z)} = z - \frac{a_n z^n + a_{n-1} z^{n-1} + \dots + a_1 z + a_0}{n a_n z^{n-1} + (n-1) a_{n-1} z^{n-2} + \dots + a_1}$$

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θ is a root of $p(z)$ if and only if it is a *fixed point* of $N(z)$, i.e.

$$N(\theta) = \theta.$$

Example. If $p(z) = z^2 - 2$, Newton's iteration is

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Suppose $z_0 = 1 + 2i$, where $i = \sqrt{-1}$, can we still compute Newton's iterations?

Computing Newton's Iteration for Complex Input

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$$z_1 = \frac{z_0^2 + 2}{2z_0} = \frac{(1 + 2i)^2 + 2}{2 * (1 + 2i)} = \frac{1 + 4i + 4i^2 + 2}{2 + 4i}$$

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Take an object.

Take an object.

Do something with it.

Take an object.

Do something with it.

Then do something else.

Take an object.

Do something with it.

Then do something else.

Jasper Johns

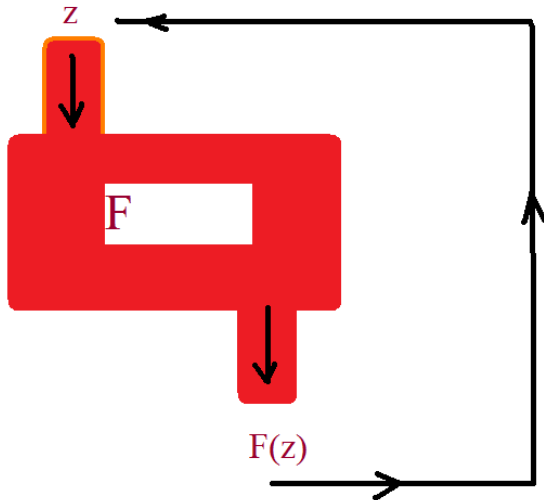


Figure: General Iteration Function, $F(z)$.

Take a tiny dot on the canvas.

Take a tiny dot on the canvas.

Move it somewhere else.

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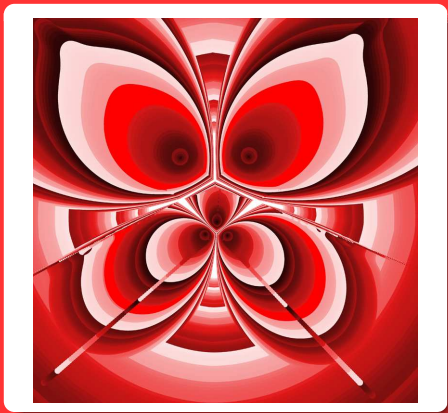
Then somewhere else.

Then somewhere else ...

More Points and Artistry:

Butterfly







Symmetry









Cathedral







Some Quotes

“Lose your fear of math with computer graphics that displays the beauty and symmetry hidden within algebraic equations.”

DISCOVER Magazine ... *on polynomiography*

“Over the centuries, mathematicians have developed a variety of methods of solving equations. Bahman Kalantari of Rutgers University has developed visualization software that brings the process of finding the roots of a polynomial equation into the realm of design and art.”

Ivars Peterson *SCIENCE NEWS*

“Professor Kalantari’s work combines in a very striking way mathematics and visual arts. His work on ‘polynomiography’ is very original and pretty.”

Cumrun Vafa *Professor of Physics, Harvard*

“Bahman Kalantari’s work on Polynomiography is visually striking and provides profound insight into root finding algorithms.

In future generations, I expect that visualization of mathematical algorithms will become an expected part of mathematical research.

Bahman Kalantari’s skills are here now and we can enjoy the beautiful results as he has applied them to Polynomiography.”

Cliff Reiter *Professor of Mathematics Lafayette College, Pennsylvania*

“The visual results are often elegant. This method has led [Kalantari] to develop a new and powerful method of artistic creation, ..., a playful and instructive technique where mathematics helps art, which gratefully, comes to support mathematics.”

Claude Bruter *Professor of Mathematics, U. of Paris*

“Polynomiography ... has an enormous and fruitful field of applications in visual arts, education and scientific research...”

Vera W. de Spinadel *President of International Mathematics & Design Association, Argentina*

“POLYNOMIOGRAPHY!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

I LOVE IT!!!

It was so incredibly cool.

I just want to take it home and play on it the rest of the summer!”

Alexandria Munger (age 14, a middle schooler at Girls Plus Math Camp., Illinois)

"I didn't know math could make such beautiful images."

A 9 year old boy (*Rutgers Day, April 25th, 2009*)

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What is the **Reimann Hypothesis problem**? What is an open problem? Why is there a million dollar prize on it?

Our First Encounter With A Polynomial Equation

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Linear equation

Middle school word problems

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When a math problem asks you to find the value of x , the goal is to isolate x , by doing things to both sides of the equation –until x is all by itself on one side and there is a number on the other side - and that number is your answer.

Danica McKellar
DOESN'T SUCK

author of best selling book MATH

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Danica is right, but in most real-life problems we need to solve equations which are not linear...motion, business,

Why Is Solving Polynomial Equations Important?

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The very ideas of abstract thinking and using mathematical notation are largely due to the study of polynomial equations.

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The very ideas of abstract thinking and using mathematical notation are largely due to the study of polynomial equations.

Furthermore, solving polynomial equations has historically motivated the introduction of some fundamental concepts of mathematics ...

Victor Pan, an internationally recognized leader in the field of computer science

A Futuristic Love Story

A Futuristic Love Story

(Setting: A street in a city in the world)



WOMAN: Excuse me Sir, is there any post office around here?

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WOMAN: (After a pause) I cannot promise, but my favorite restaurant's location is a solution to this polynomial:

$$\begin{aligned} & x^{17} + (-0.99 + 2.865\sqrt{-1})x^{16} + (-4.0289 - 3.7579\sqrt{-1})x^{15} + \\ & (7.6906 - 8.1716\sqrt{-1})x^{14} + (10.1661 + 10.81\sqrt{-1})x^{13} + \\ & (-14.9238 + 14.8406\sqrt{-1})x^{12} + (-17.6515 - \\ & 21.7828\sqrt{-1})x^{11} + (19.445 - 19.7326\sqrt{-1})x^{10} + \\ & (26.861 + 26.7445\sqrt{-1})x^9 + (-4.5597 + 36.2215\sqrt{-1})x^8 + \\ & (-23.0202 + 22.955\sqrt{-1})x^7 + (-103.1901 + \\ & 18.1539\sqrt{-1})x^6 + (-56.6536 + 20.3531\sqrt{-1})x^5 + \\ & (-47.8313 + 228.2366\sqrt{-1})x^4 + (-208.2068 - \\ & 229.4483\sqrt{-1})x^3 + (-117.5589 - 248.0972\sqrt{-1})x^2 + \\ & (-1092.4738 - 7.5769\sqrt{-1})x + (103.3231 + 536.4582\sqrt{-1}) \end{aligned}$$

The woman leaves.

The woman leaves.

The man falls in love.

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The man falls in love.

He uses Polynomiography to create an art piece from it.

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In the process he discovers a unique shape in the solutions.

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And knowing the restaurant locations now he desperately searches for her and eventually finds her, sitting alone.

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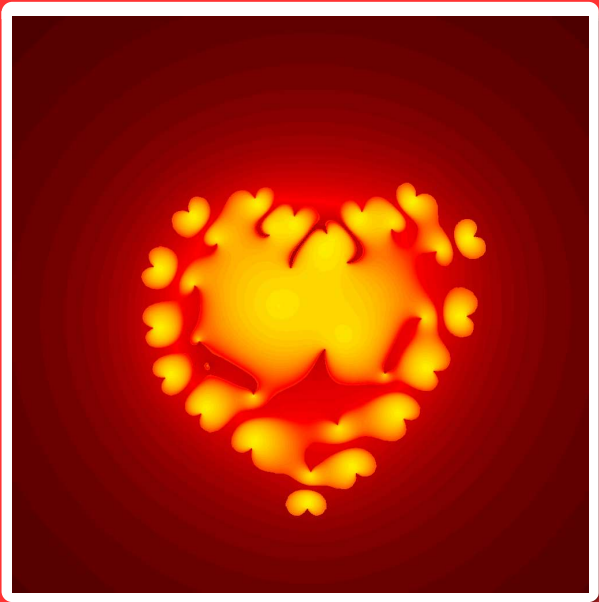
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He presents her the artwork:



She immediately falls in love.

She immediately falls in love.

They decide to get married.

She immediately falls in love.

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And have

She immediately falls in love.

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And have $(1 + 2\sqrt{-1}) \times (1 - 2\sqrt{-1}) = 5$ children.

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FIN

Moral of The Story:

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Second: solutions of a polynomial equation are like locations on a map.

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Fifth: solving polynomial equations could be a beautiful experience.

How Do I Select a Cool Polynomial?

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You can even convert an ordinary number into a polynomial:

Take any number, say **387624730**. You can convert it into a polynomial in many ways.

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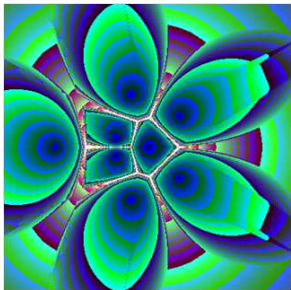
One can capture infinitely many polynomiographs of this single equation.

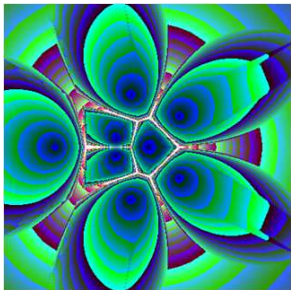
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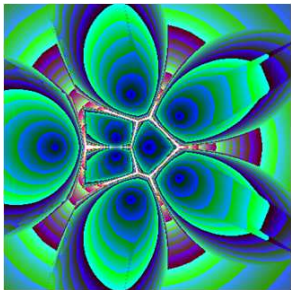
One can capture infinitely many polynomiographs of this single equation.

Here is one:





Could this lead to a futuristic ID number?



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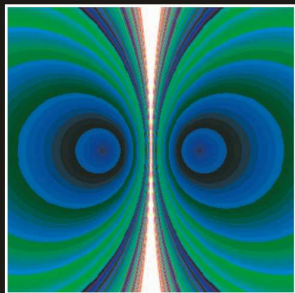
At the very least polynomiography's ability to encrypt ordinary numbers visually encourages the youth to take an interest in numbers and polynomials in the process of which they will learn and discover many properties of these and many other concepts of math.

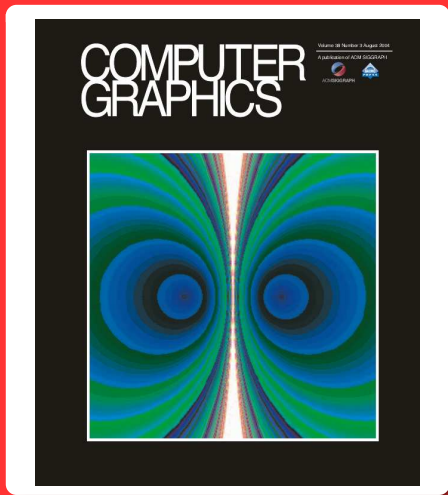
Polynomiography In Media

COMPUTER GRAPHICS

Volume 38 Number 3 August 2016

A publication of ACM SIGGRAPH



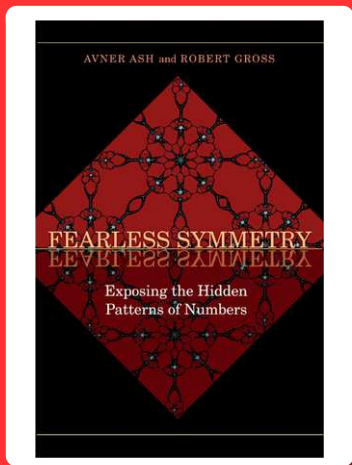


SIGGRAPH Quarterly (cover)

AVNER ASH and ROBERT GROSS

FEARLESS SYMMETRY
FEARLESS SYMMETRY

Exposing the Hidden
Patterns of Numbers



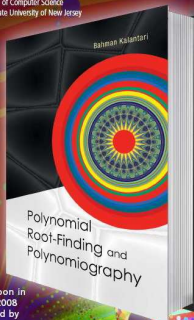
Princeton University Mathematics (cover)

POLYMIOMOGRAPHY = The Art of Roots The Roots of Art

The Fine Art and Science of Visualizing Roots of Polynomials

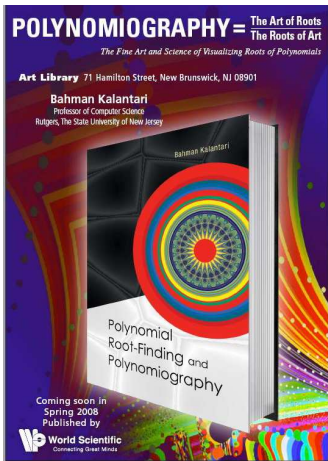
Art Library 71 Hamilton Street, New Brunswick, NJ 08901

Bahman Kalantari
Professor of Computer Science
Rutgers, The State University of New Jersey



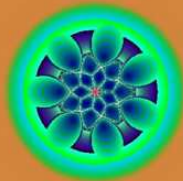
Coming soon in
Spring 2008
Published by

Wp World Scientific
Connecting Great Minds



World Scientific, "POLYNOMIAL ROOT-FINDING and
POLYNOMIOGRAPHY"

Meeting Alhambra



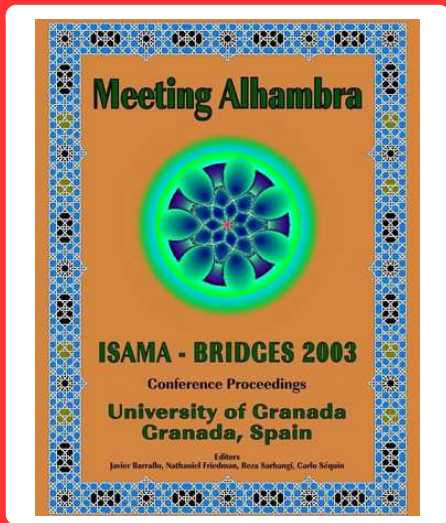
ISAMA - BRIDGES 2003

Conference Proceedings

**University of Granada
Granada, Spain**

Editors

Javier Barraló, Nathaniel Friedman, Reza Sarhangi, Carlo Séquin



Art-Math Proceedings (cover)

BEAUTIFUL MATTER



Symmetry is central to modern physics.
Source: Bahman Kalantari/Science Photo
Library

[Back to article](#)

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[Back to article](#)



Un visor infrarrojo facilita la conducción Nuevos ojos en la noche

El mismísimo coche luminático estadounidense se llega a sentir como si estuviera volando sobre las luces largas, pero con la ventaja de que no deslumbrará al resto de conductores, y que la longitud de onda de los infrarrojos no es perceptible a simple vista. El automovilista, al ver el obstáculo, la carretera con mayor intensidad, puede reaccionar antes

de un posible accidente. Los ingenieros de Mercedes-Benz son de los primeros que han instalado el aparato en sus vehículos, más concretamente en sus berlinas de lujo y en automóviles de la clase S, pero otros fabricantes no se han quedado atrás y también están equipando sus modelos de gama alta con este tipo de tecnología que puede evitar accidentes.

Con la luz de los faros no se ve bien el conchallo que está a la izquierda de la carretera. Si embargo, con la nueva cámara -amba- que se coloca en una de las lunas, se obtiene un claro detalle en el portabici del suscriptor -derecha-. El conductor entonces puede hacer la maniobra pertinente.



Matemáticas Arte con los números

Benjamín de la polimatemática, también invitado por el profesor de matemáticas de la Universidad Rutgers, EE.UU., Robert Kanier para describir cómo se puede hacer arte usando un ordenador y matemáticas polimáticas, las funciones matemáticas más feroces de nuestro tiempo de social. Es esencial, lo que hace Kanier es hacer las cosas de otros, incluso polímeros, ceros, hacer para qué valor de la función polimérica se hace cero. Una vez



Estas imágenes se llaman resoluciones poliméricas, esas funciones matemáticas con tiempo corto este: $10^{10} = 11^{10} + 23^{10} - 2$

identificados, se programa la función ya "definida" según el sentido artístico del matemático, más es, con los reducidos y colores que la selecciona. Así, las sucesiones numéricas se convierten en bellas imágenes.



INSOLITO... PERO CIERTO

¿Un pollo? Las 25.000 toneladas de carne congelada que se venden en EE.UU. convierten este pollo en el principal producto agrícola del país, según su valor en el mercado.

La salud del pez. El primer subido de los científicos fue la cruz. Ahora se sienta a hablar con un pez, con un grillo se dice la cruz. Con la palabra "bueno" el científico Louis Bruscia Thomas Hays Sider (Washington, de Ohio y NY, Sider) lo hace que pinta. La temperatura media en los océanos hace 3.500 millones de años era de 80 °C. Hoy es de 27 °C en la superficie.

Caliente supercaliente. Científicos del Instituto Pasteur en París, han descubierto en el salmón humano un proteoma antigénico, lo que significa que la proteína, según se ha demostrado en ratones, que viene el pollo... Además de su efecto a la cirugía estética, Cher y Michael Jackson comparten un cromosoma. Así se llama el pareto a las aves.

¿Una, piensan el hombre? El tumor genético, que solo afecta a perros, es el cáncer que se contagia. Los células malignas pasan de un animal a otro durante el acto sexual.

TIEDE

Suomen
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Tiede (popular science magazine of Finland)



LES MATHÉMATIQUES DU CŒUR

En faisant le pont entre la santé et le génie, des modèles mathématiques permettent d'imaginer, de simuler et de développer de nouveaux dispositifs pour traiter la défaillance cardiaque.

Accompagné de
N

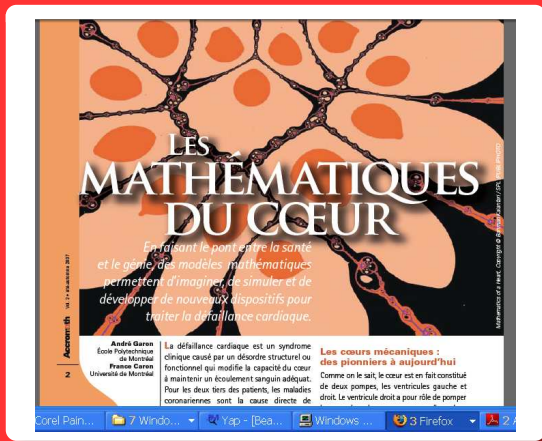
André Garon
École Polytechnique
de Montréal
François Garon
Université de Montréal

La défaillance cardiaque est un syndrome clinique causé par un désordre structurel ou fonctionnel qui modifie la capacité du cœur à maintenir un écoulement sanguin adéquat. Pour les deux tiers des patients, les maladies coronariennes sont la cause directe de

Les cours mécaniques : des pionniers à aujourd'hui

Comme on le sait, le cœur est en fait constitué de deux pompes, les ventricules gauche et droit. Le ventricule droit a pour rôle de pomper

Mathematics of the Heart, Copyright © 2007, Springer / IOP



Accromath (U. of Montreal University magazine)

SPRING 2003

BY LEE LUSARDI CONNOR

PEOPLE & PLACES



BEAUTY BY THE NUMBERS

What if you could use a computer to turn equations into dazzling, colorful designs? That's the kind of question only a computer scientist—a particularly creative computer scientist—would ask.

Enter Bahman Kalantari, an associate professor of computer science at Rutgers University in New Brunswick. His answer: "polynomigraphy"—a computer art form created by sampling polynomials, a fundamental algebraic function, into patterns. (Polynomials are defined as "linear combinations of integral powers of a variable," such as $x+1$.) "We can 'shoot pictures' of polynomials and then

color them using our own personal artistry," says Kalantari. "Just as with photography and painting, with practice new gets to be better and better at it."

Shown above is Kalantari's "Mathematics of a Heart." The possibilities are limitless, he says. "You can design images that would look wonderful as abstract painting, greeting cards, upholstery or any kind of decorative fabric."

Patents are now pending for software that will make polynomigraphy available to the public. In the meantime, check it out at www.asi.com/polomigraphy.com.

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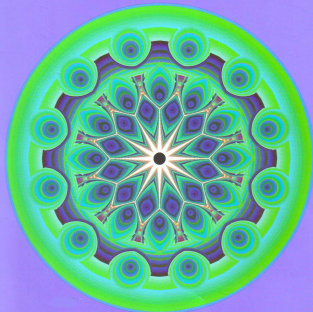
New Jersey Savvy-Living

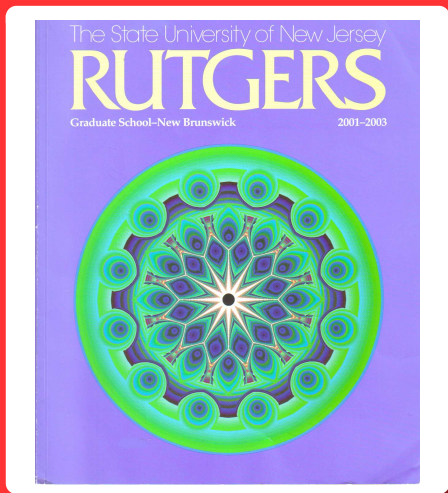
The State University of New Jersey

RUTGERS

Graduate School—New Brunswick

2001–2003

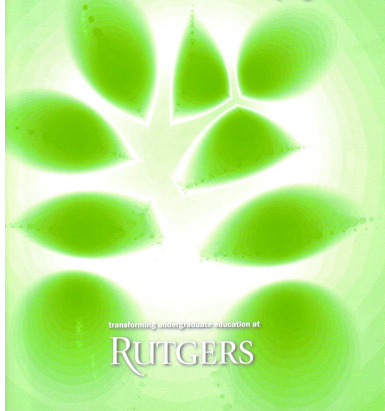




Rutgers Graduate Catalog (cover)

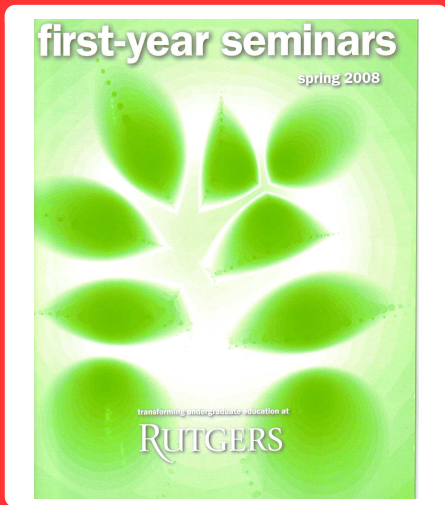
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transforming undergraduate education at

RUTGERS



Rutgers First-Year Seminars Catalog (cover)

Also featured in New Jersey
media and more

Polynomiography In Schools





First-Year Seminar Polynomiography students (and their cake)



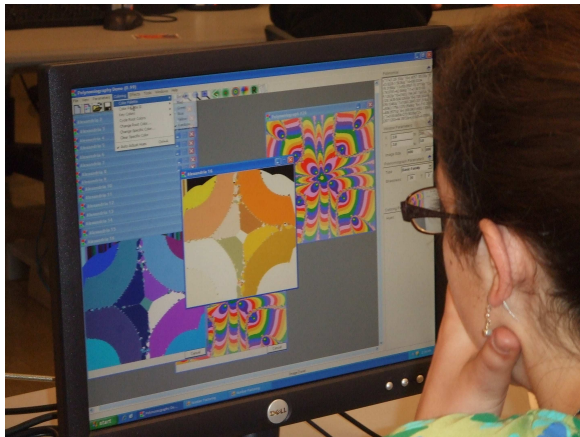


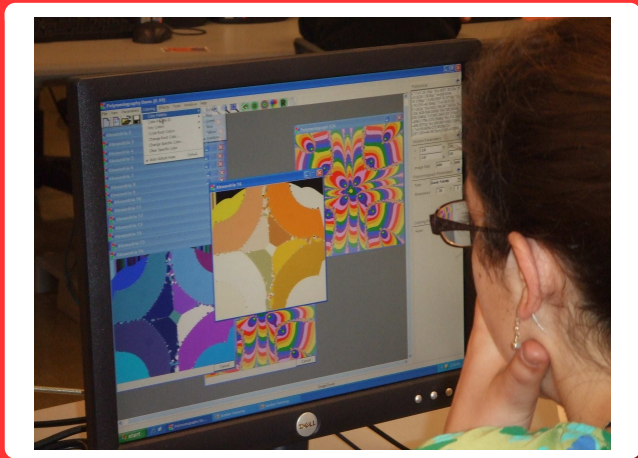
With New Jersey Randolph Middle School Students





Girls Plus Math Camp 6-8th graders (Western Illinois University, Macomb IL)





Young polynomiographer at work, discovering math and art.





Alexandia returns to the camp for second time. Her request last year was to raise the camper age limit to 14, otherwise she could not attend. First time camper are as enthusiastic.





A happy camper smiles as she has discovered much beauty behind math and its potentials...



What Are Kids Saying?

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"I want to know how all those numbers could make such cool pictures. It seems more interesting now."

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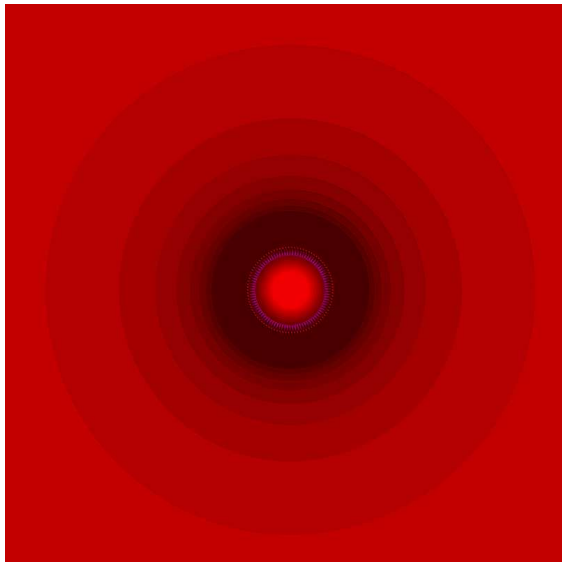
"Gives me ideas to pursue for Discrete Math Curriculum!"

Polynomiography In Art

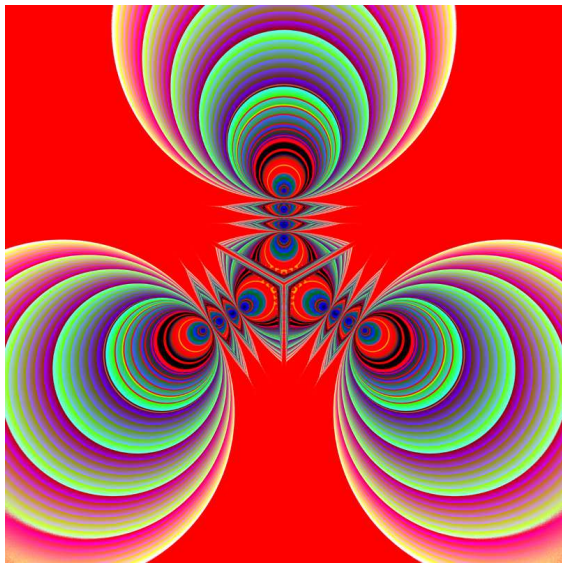
Polynomiography In Art

First A Gallery Tour

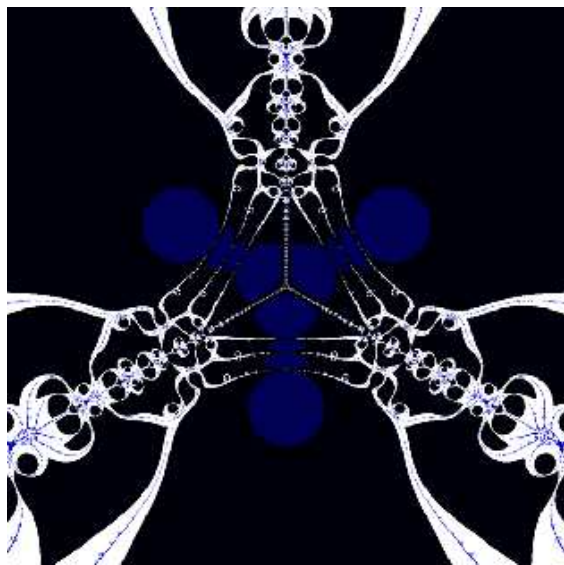




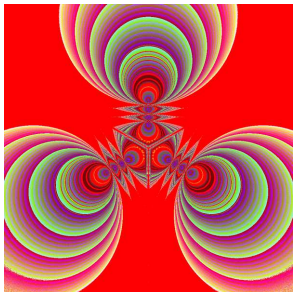
Hal

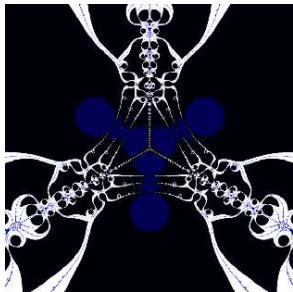
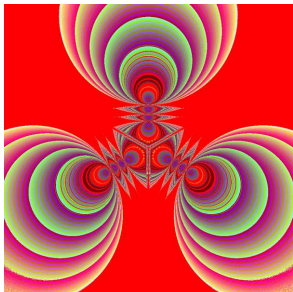


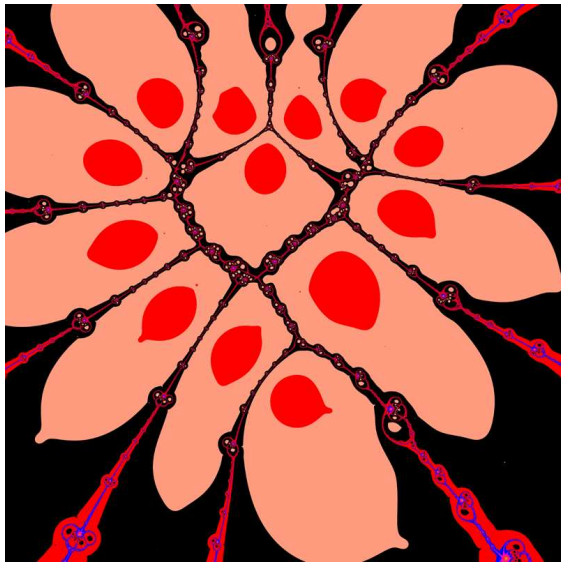
Life



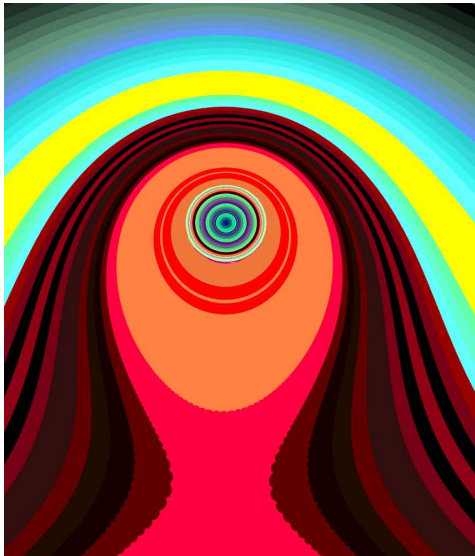
Death



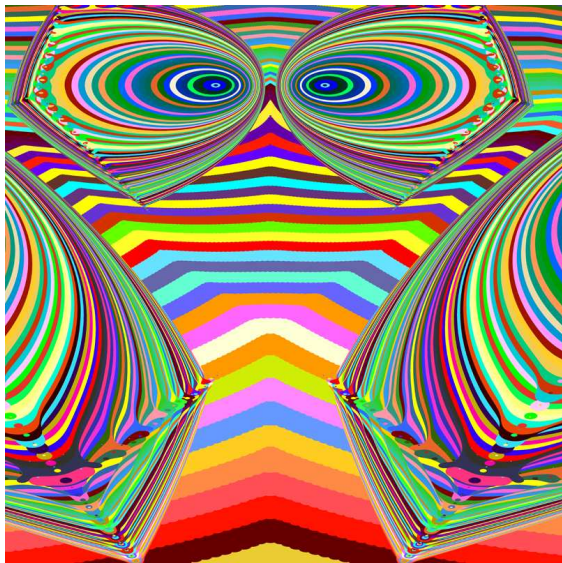


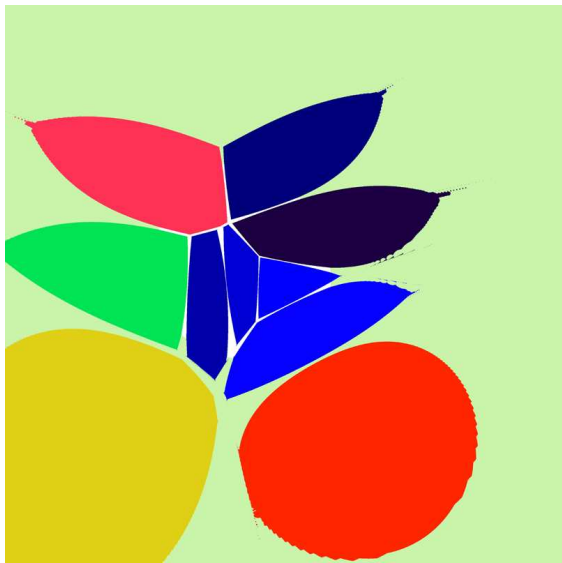


Mathematics of a Heart

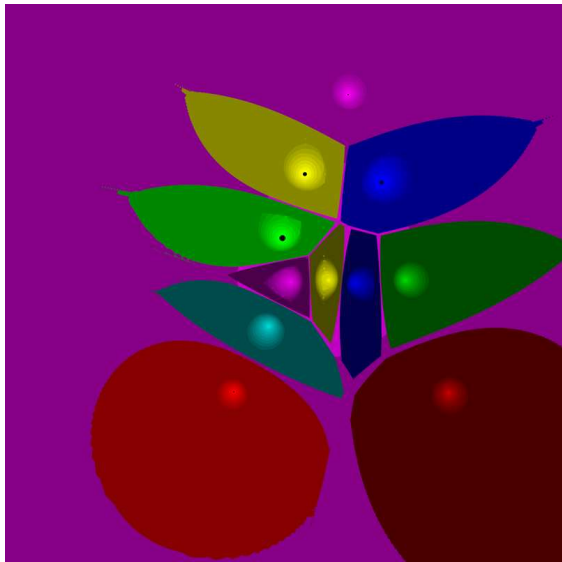


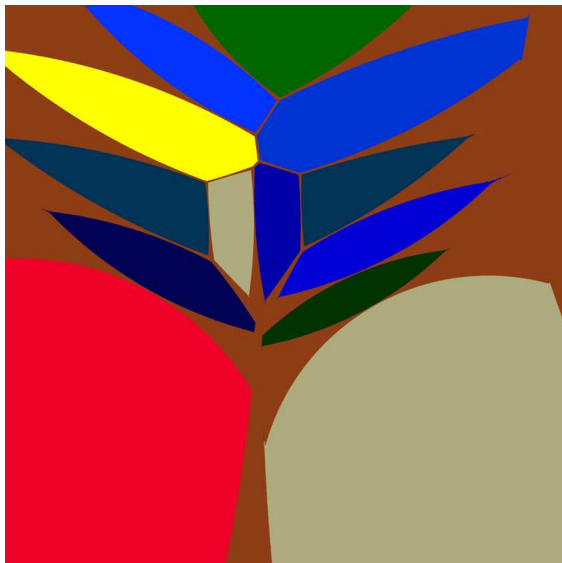
Mona Lisa in 2001

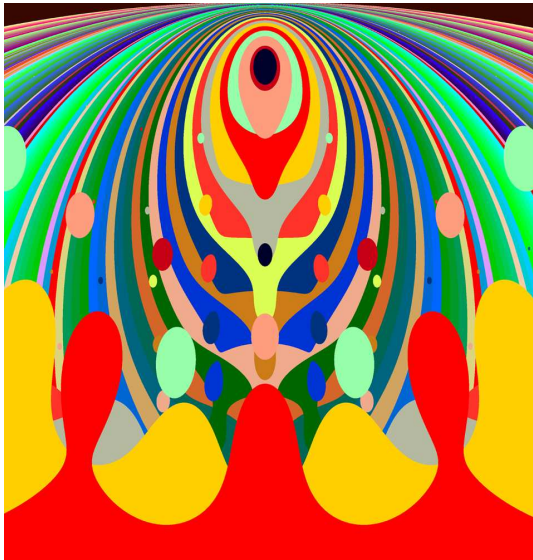




Summer



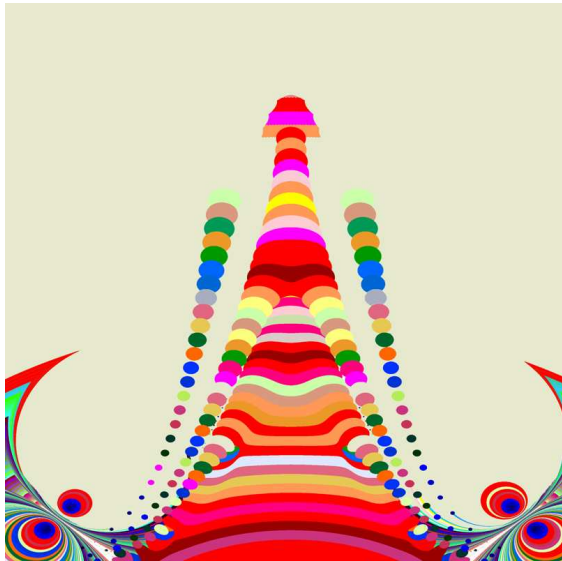




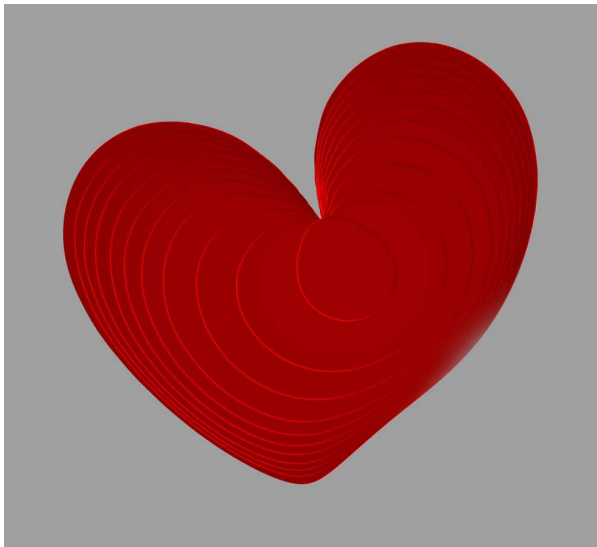
Symphony



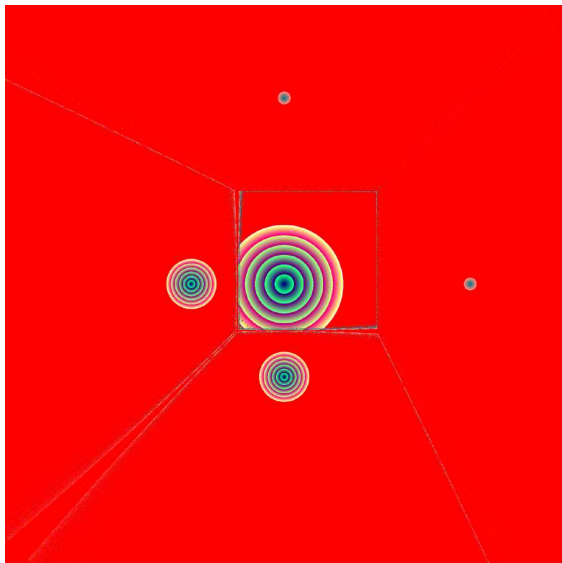
Waltz



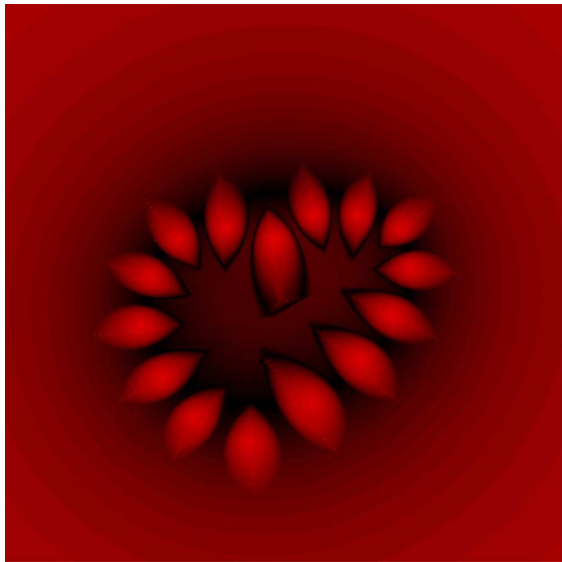
Eiffel Tower



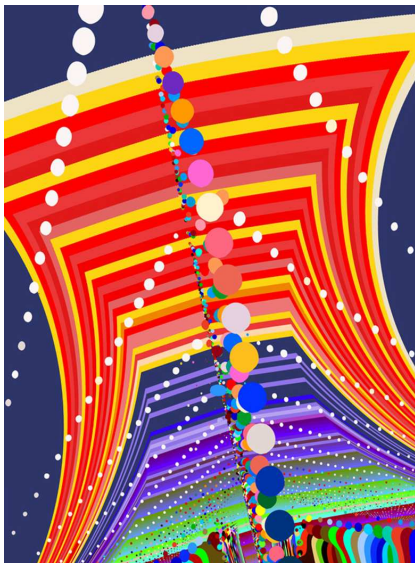
Valentine



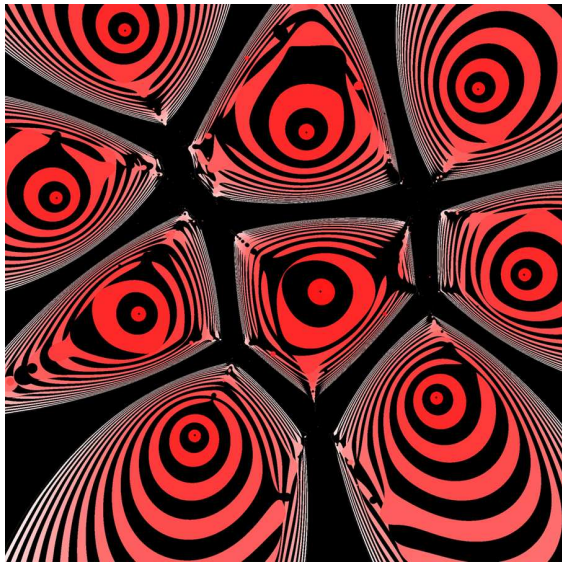
SquaringTheCircle

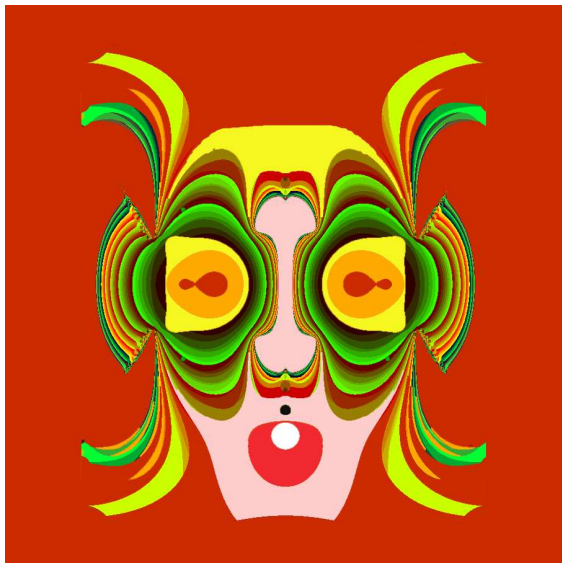


Shaping a Heart

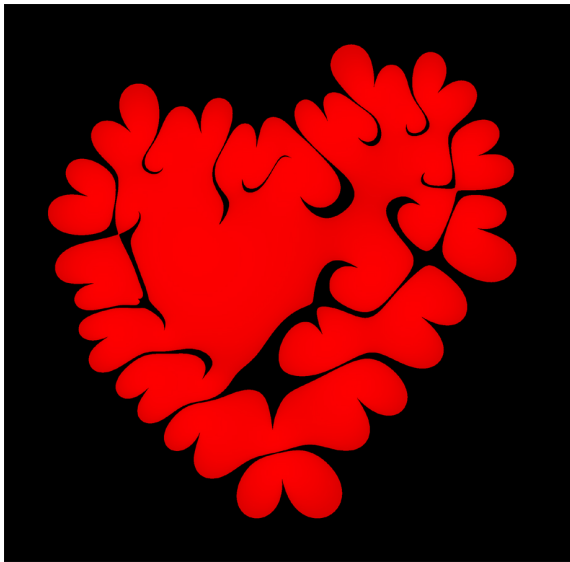


Party on the Brooklyn Bridge

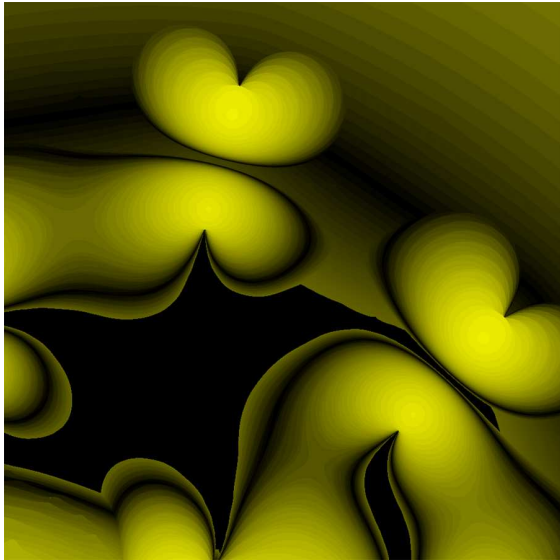




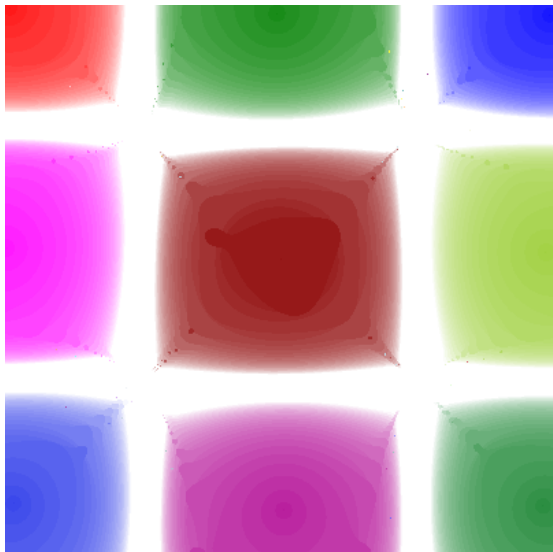
Masked Queen



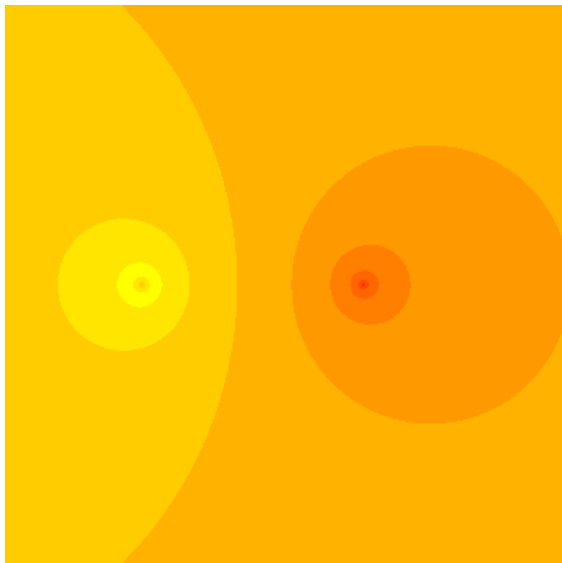
Hearts



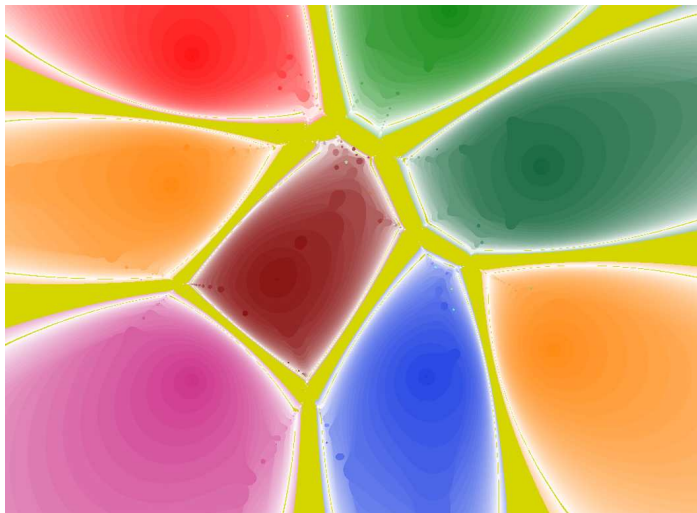
Abstract Hearts



Squares

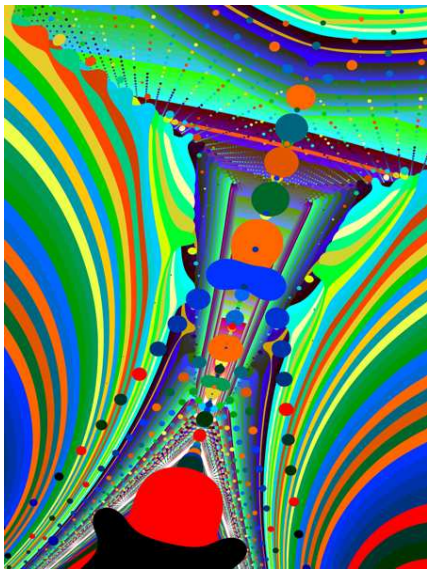


Circles

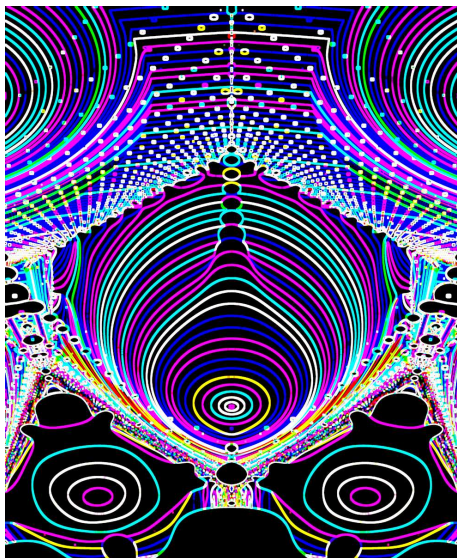




Acrobats in Paris



Circus



Times Square

Polynomiography In Exhibitions





Exhibition at Rutgers Art Library





Exhibition can also be viewed with 3D glasses





Rutgers' President McCormick visits The Exhibition





Polynomiography Artwork of Montgomery High School Students, Using a Demo Software



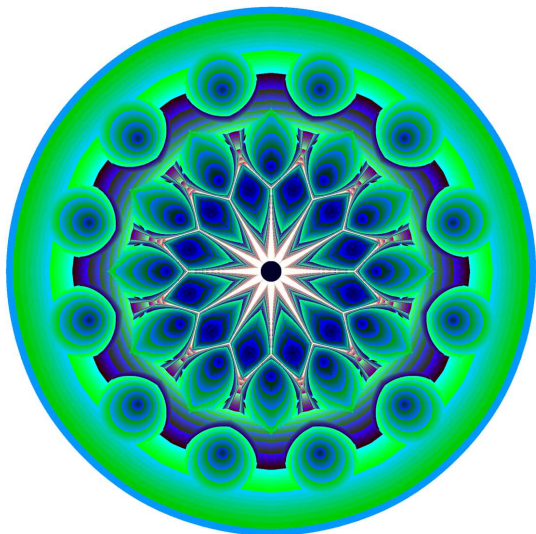


Polynomiography In Design

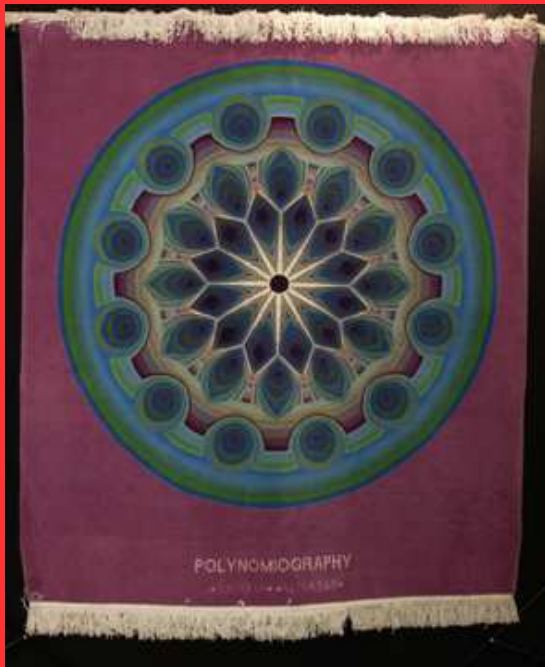
Polynomiography In Design

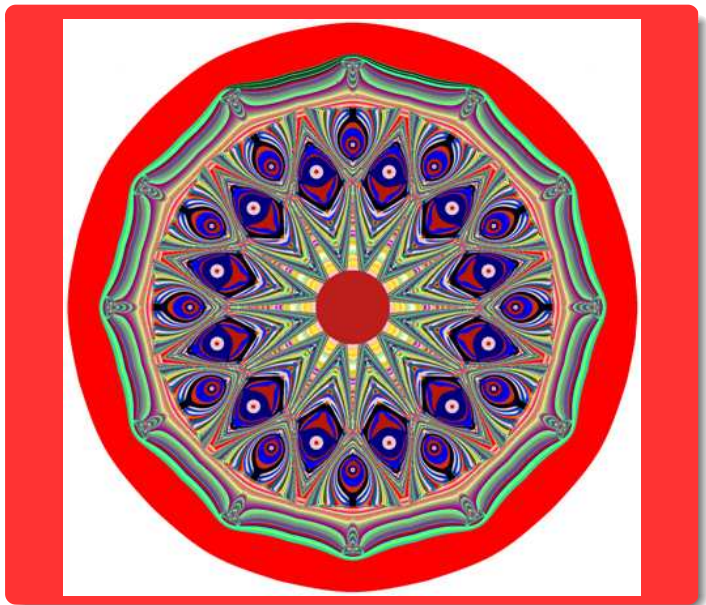
Designing A Carpet



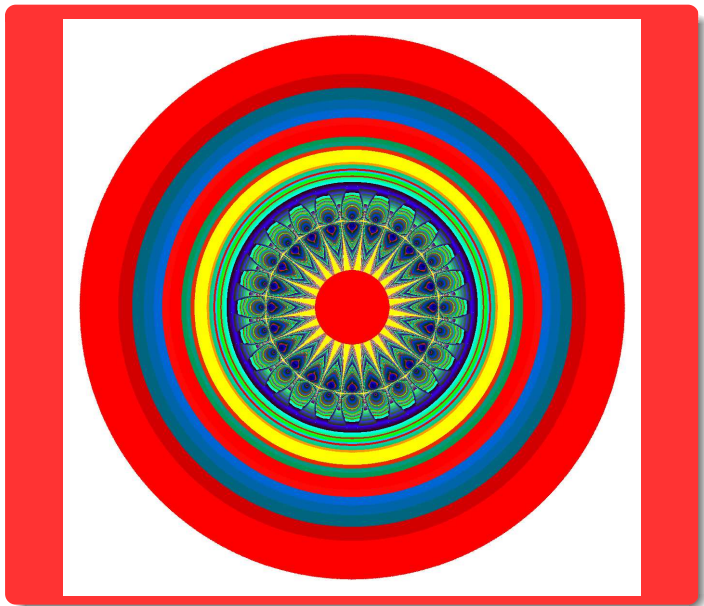


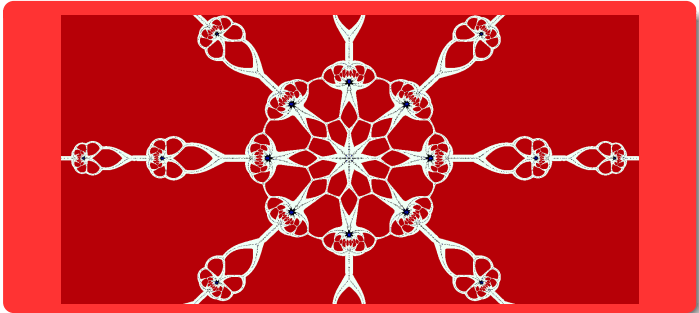


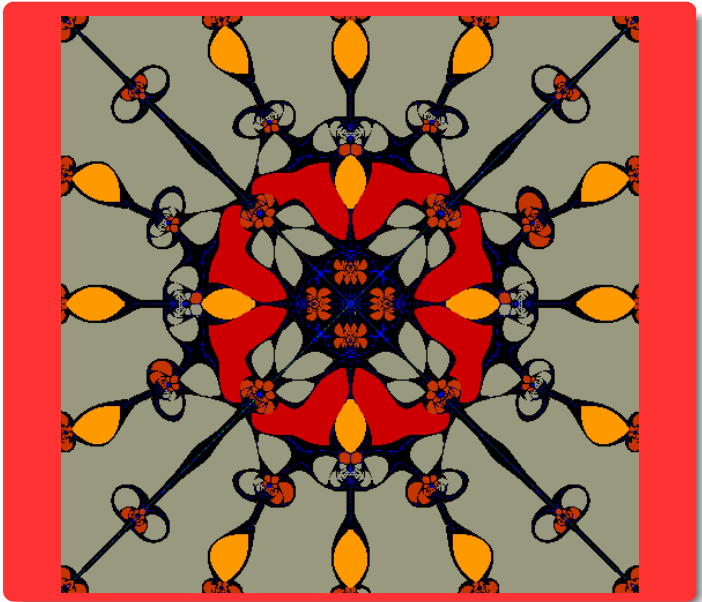


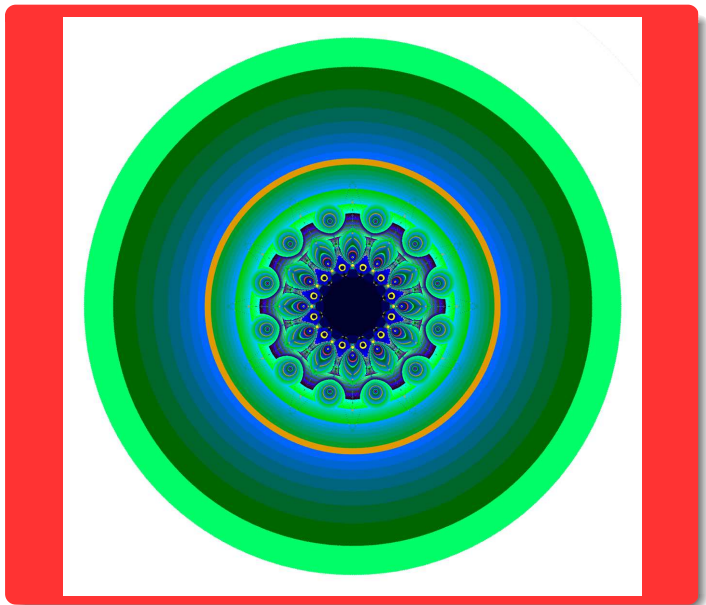


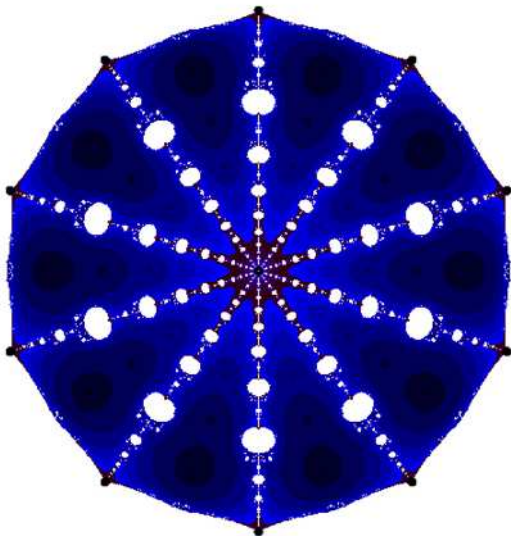


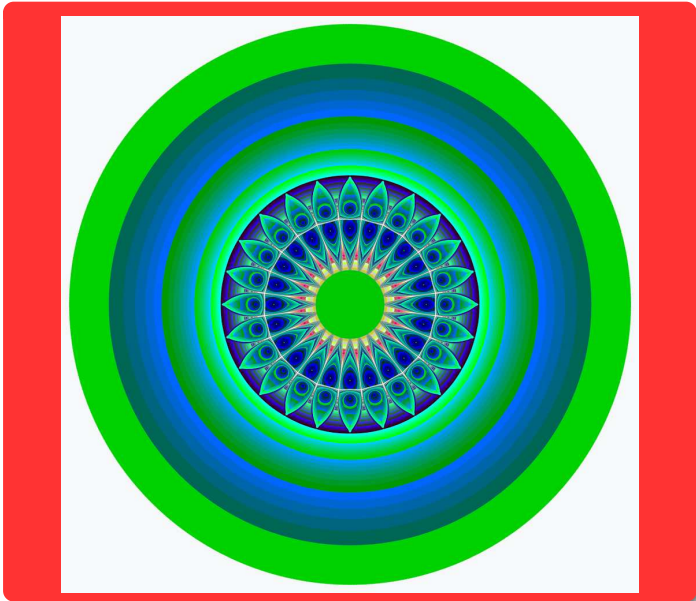


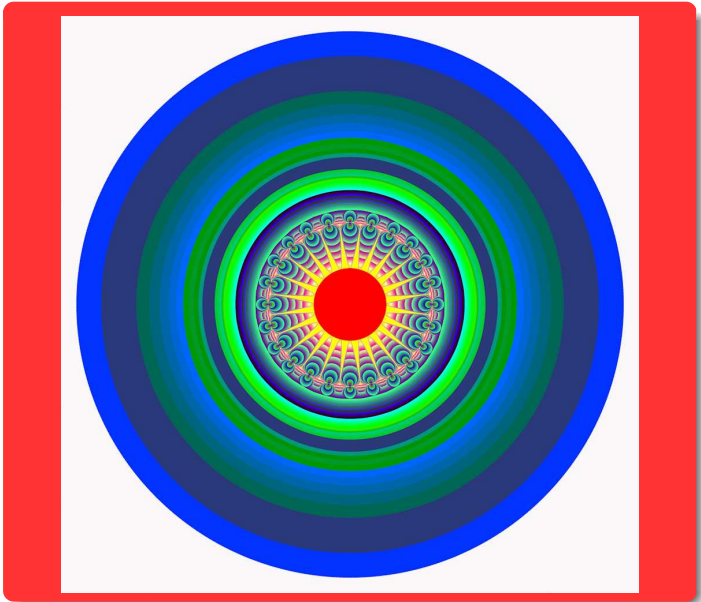




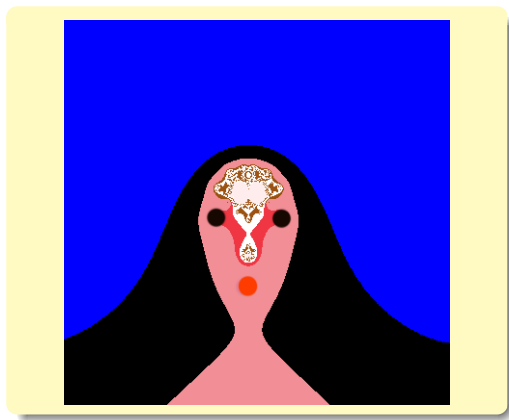


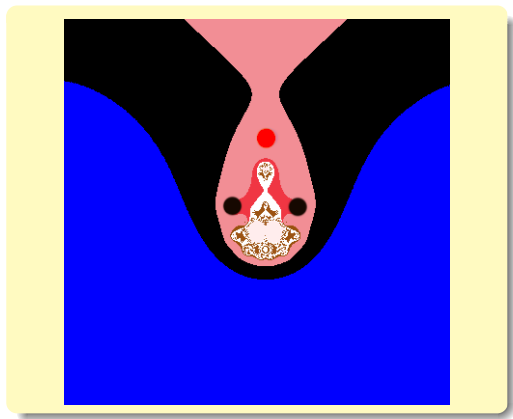






A Girl with a Secret



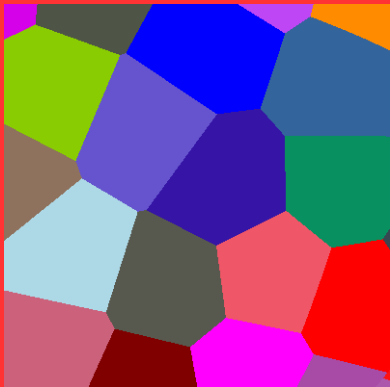


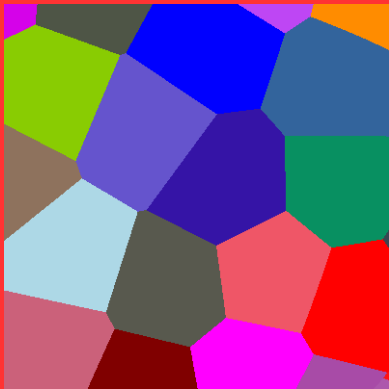
Artists and Polynomiography

Artists and Polynomiography

Can We Connect Artists with
Polynomiography?

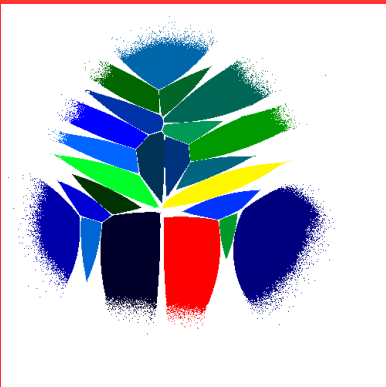


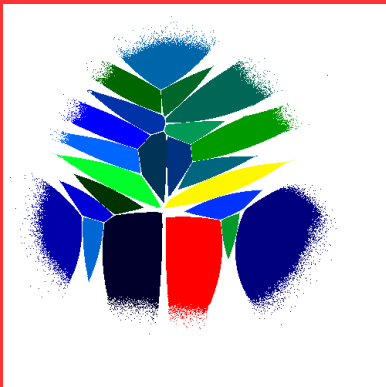




Klee and Polynomiography







Picasso and Polynomiography

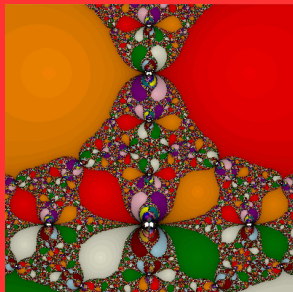
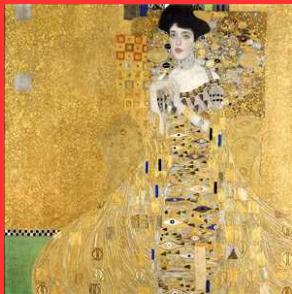


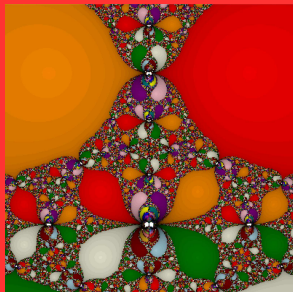




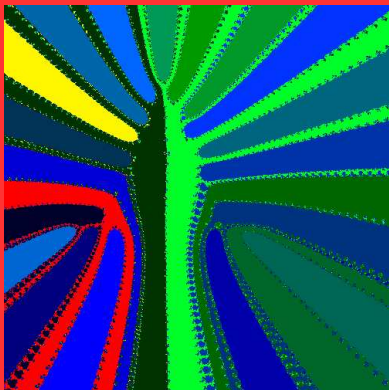
Lewitt and Polynomiography





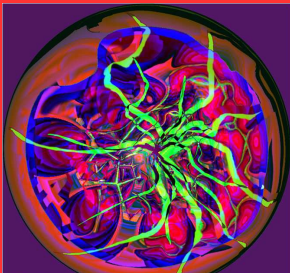


Klimt and Polynomiography

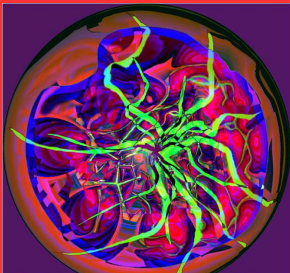




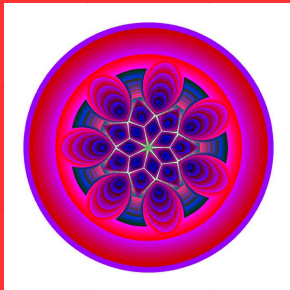


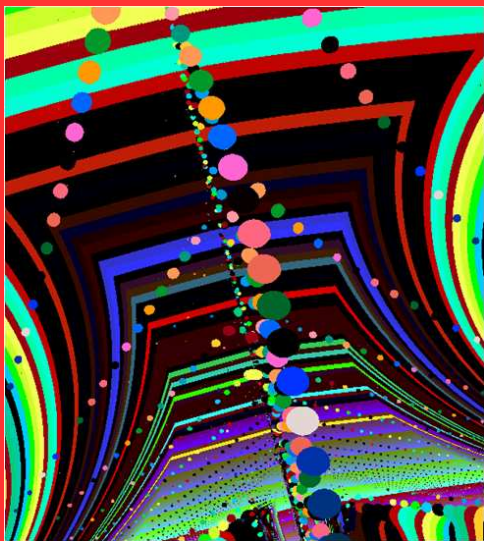


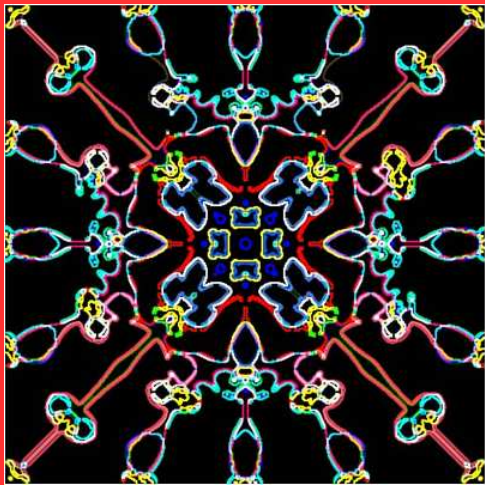
"Brain" Lillian F. Schwartz. Copyright 2003 Lillian F. Schwartz

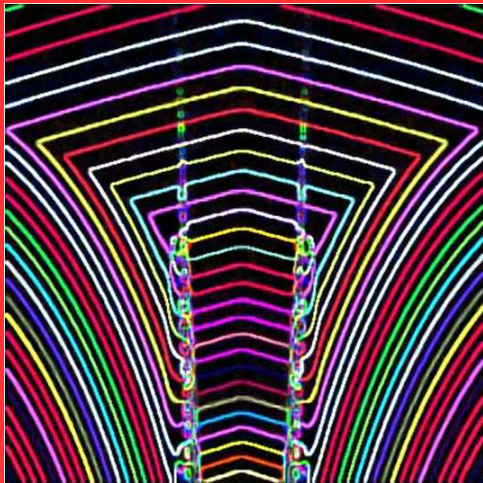


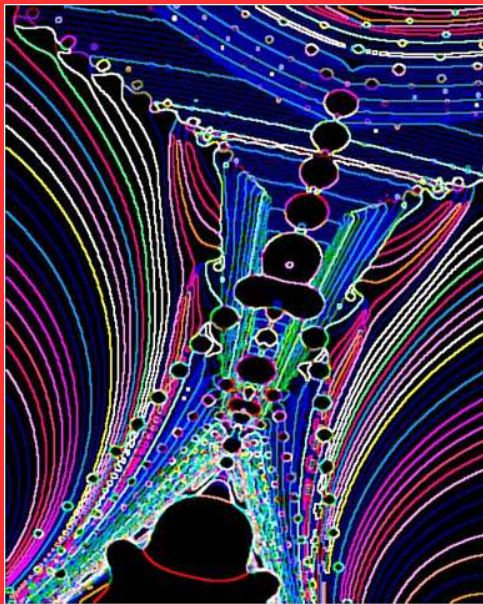
"Brain" Lillian F. Schwartz. Copyright 2003 Lillian F. Schwartz

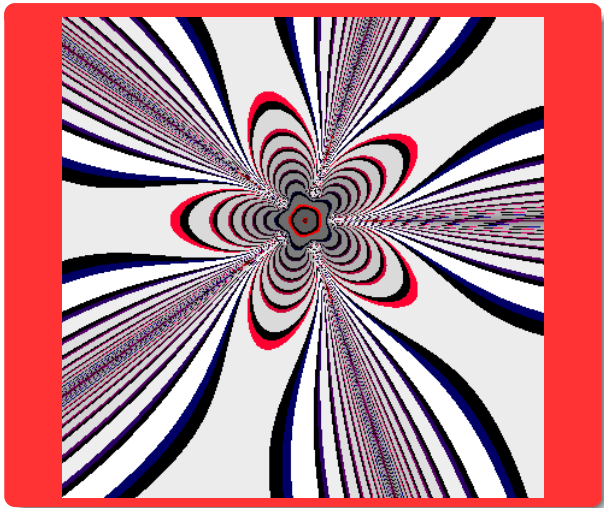


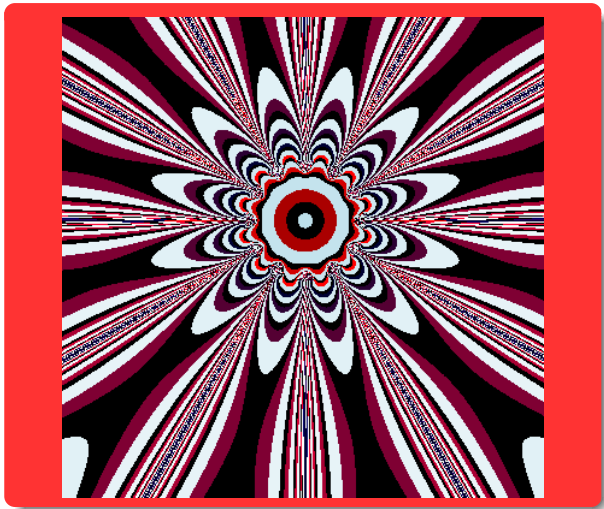


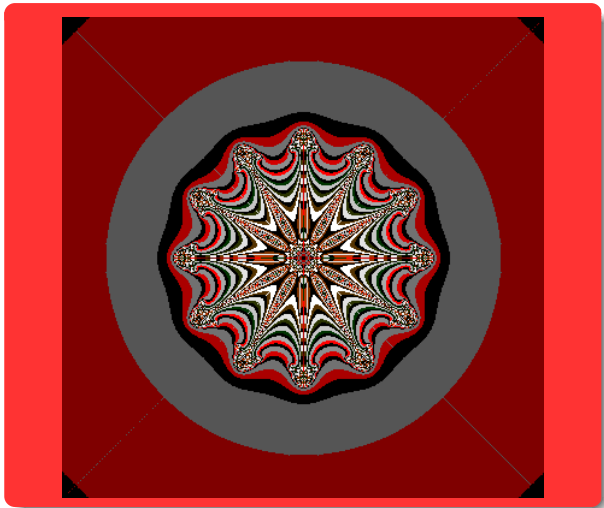


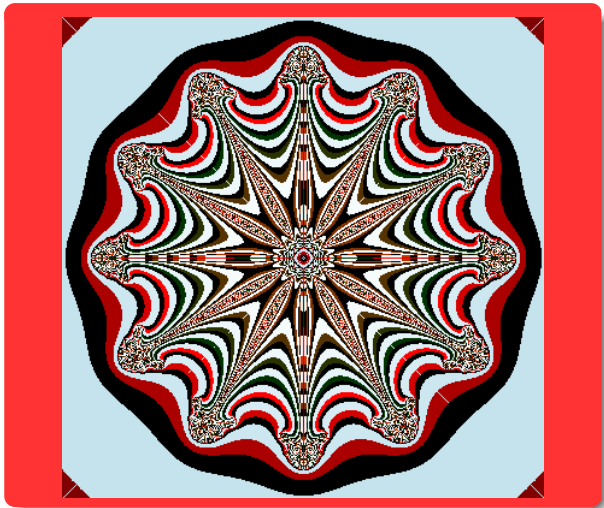


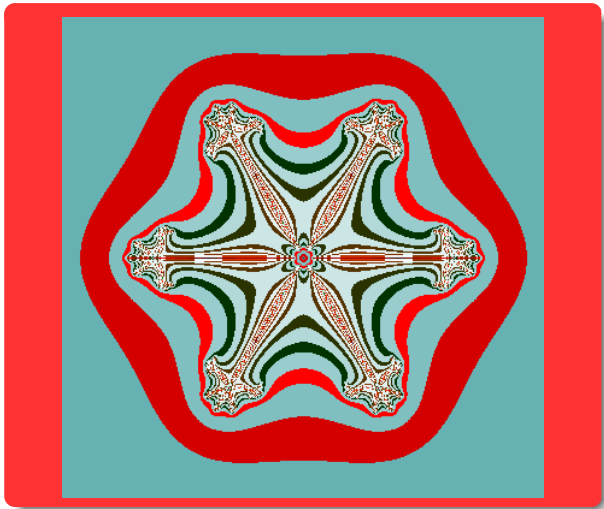


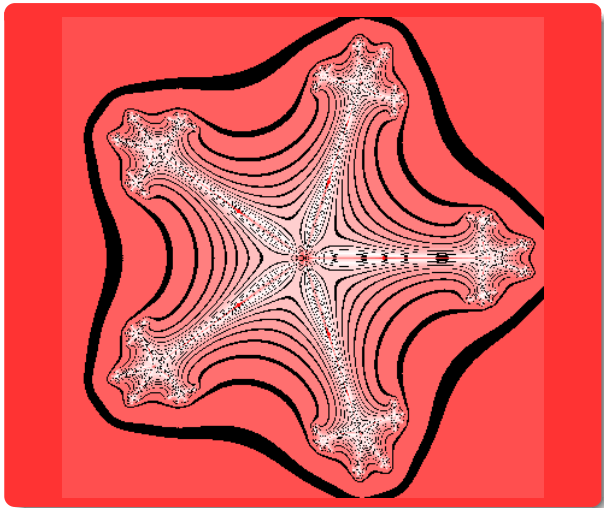


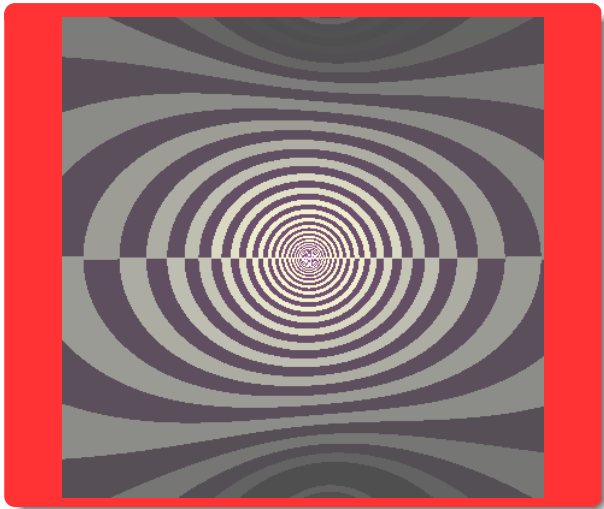


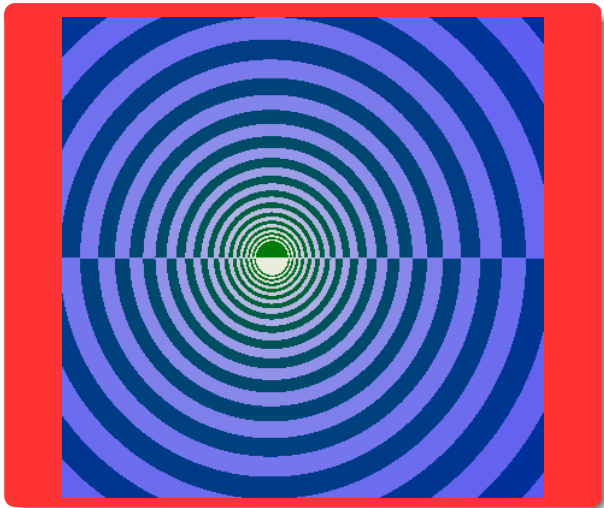


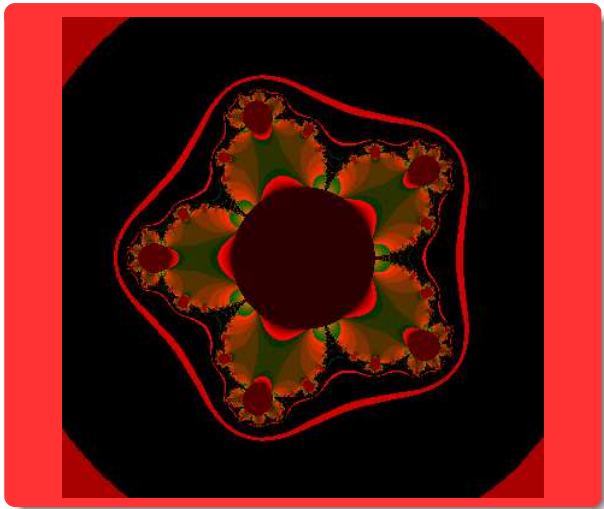


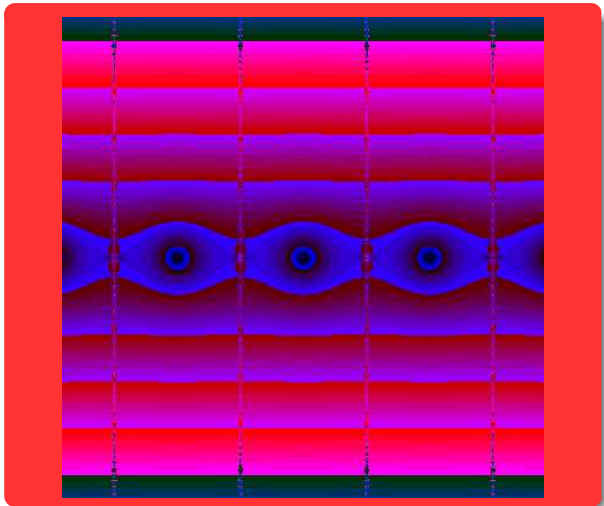


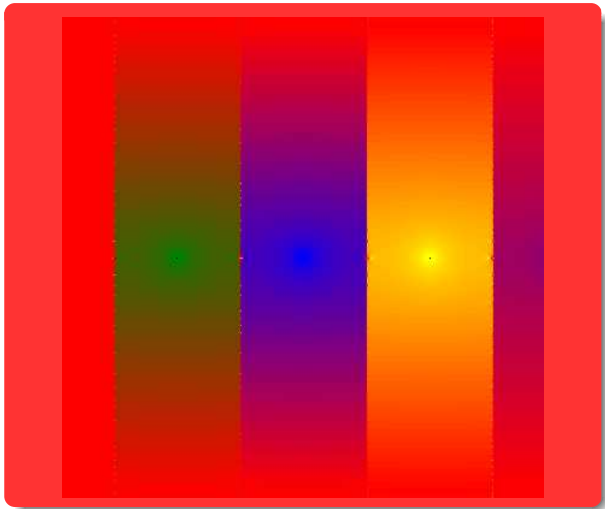






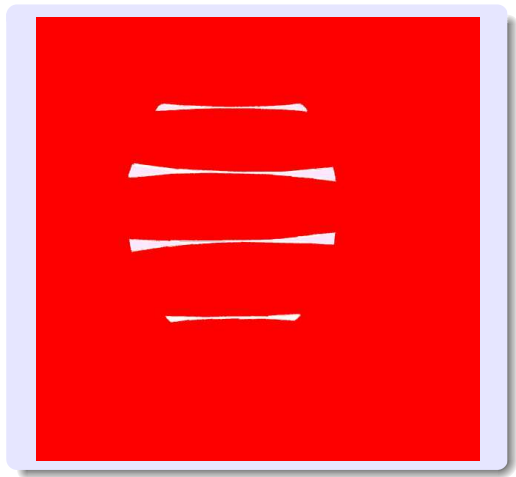


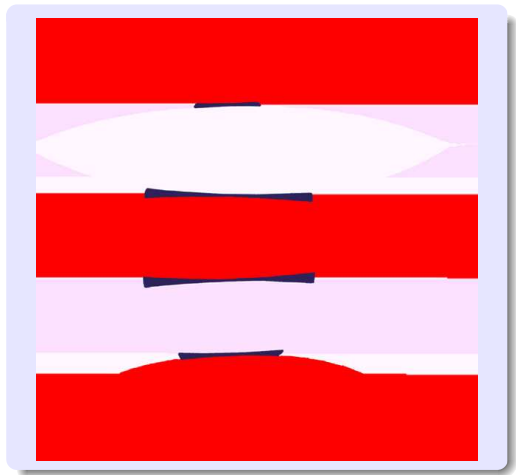


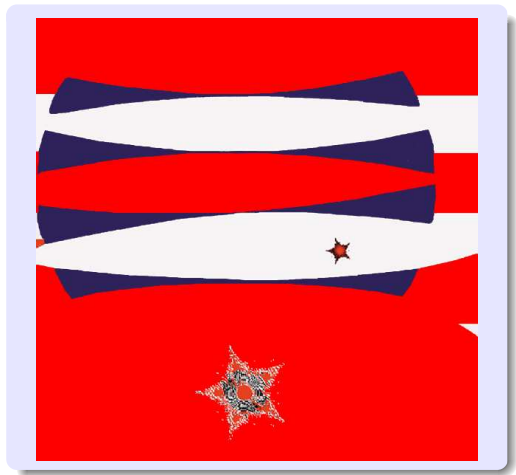


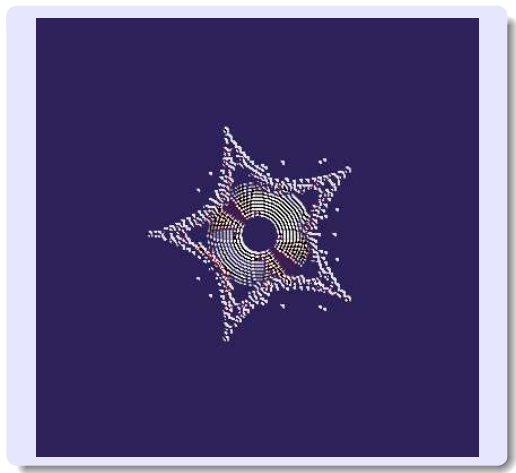


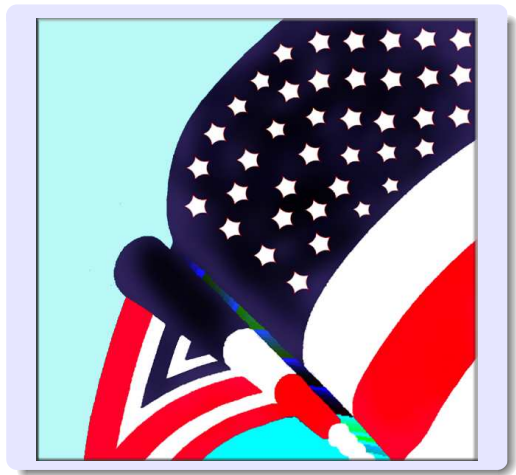
Making US Flag through Polynomiography- Inspired by Jasper Johns

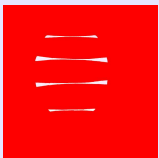


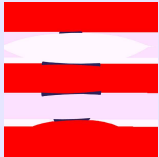
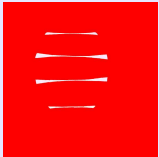


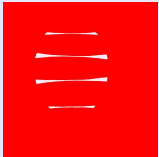


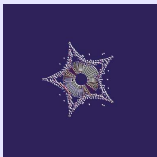
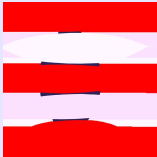
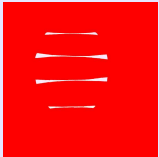


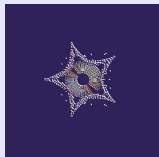
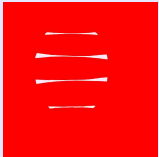


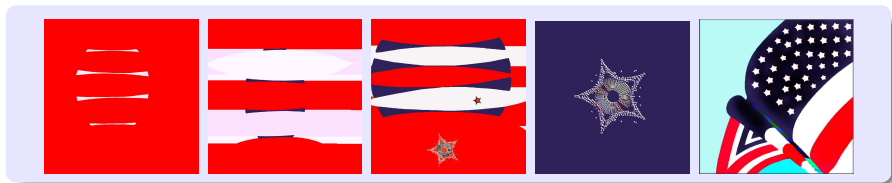










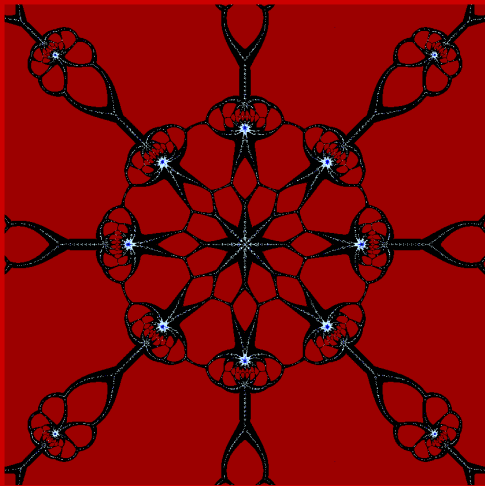


Evolution of Stars and Stripes

Endless Designs with a Single Polynomial

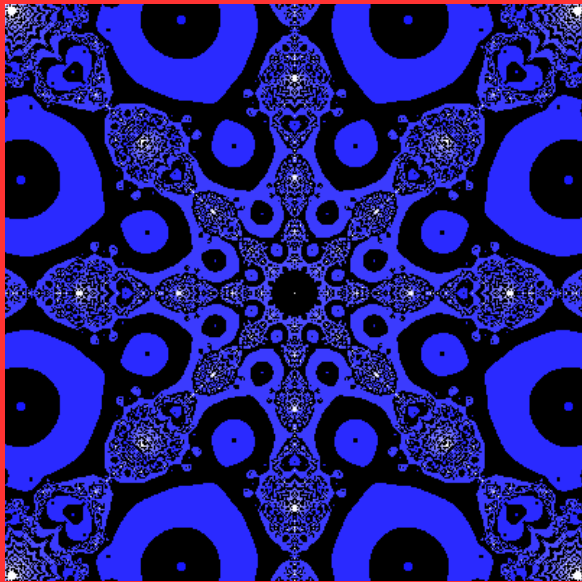
Endless Designs with a Single
Polynomial

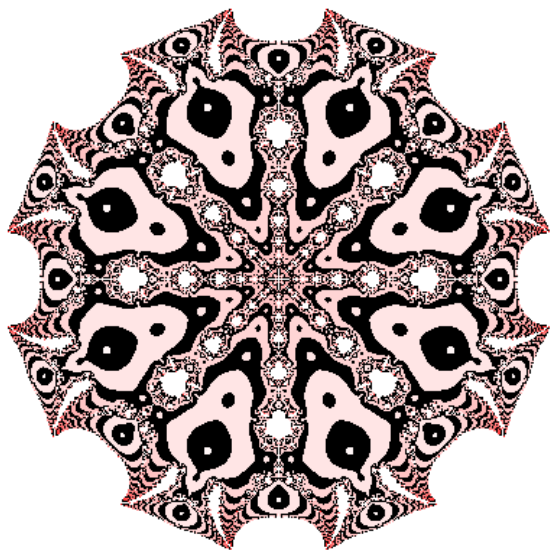
Acrobats

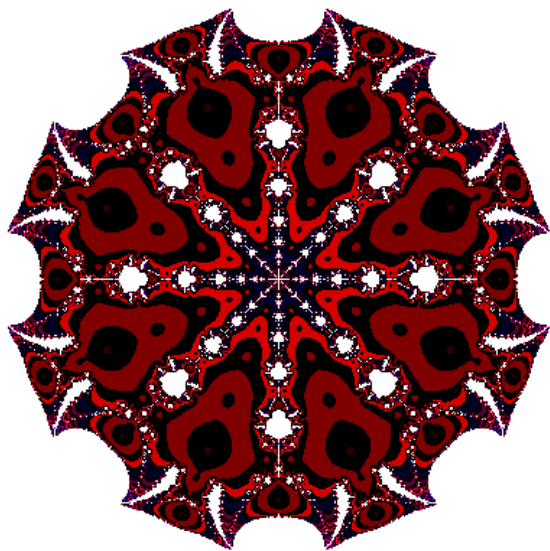


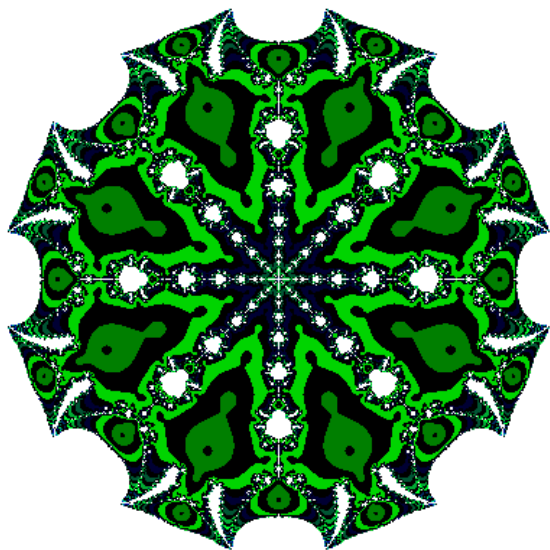


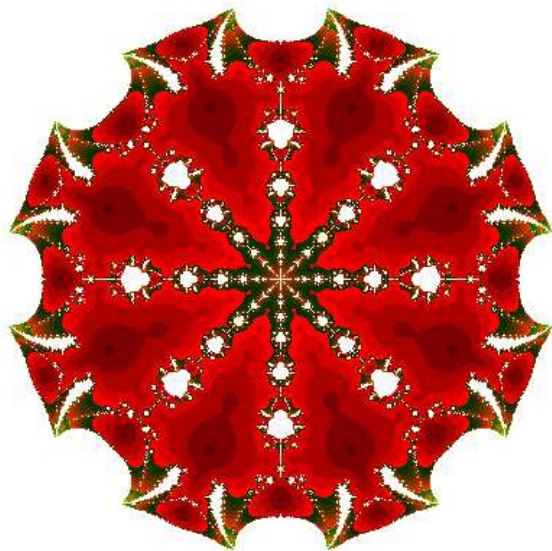


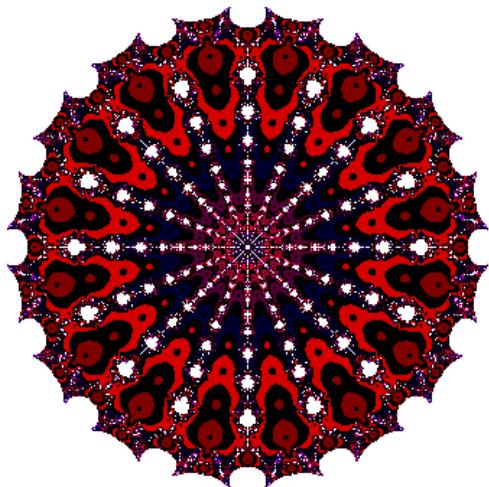


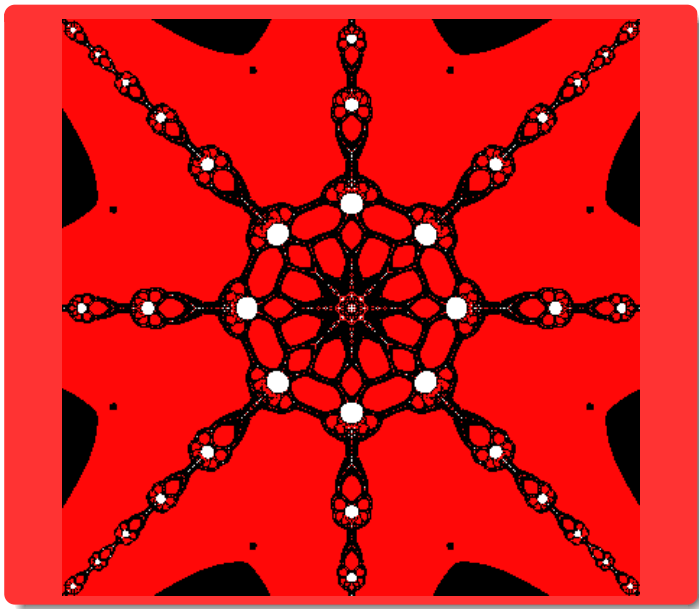


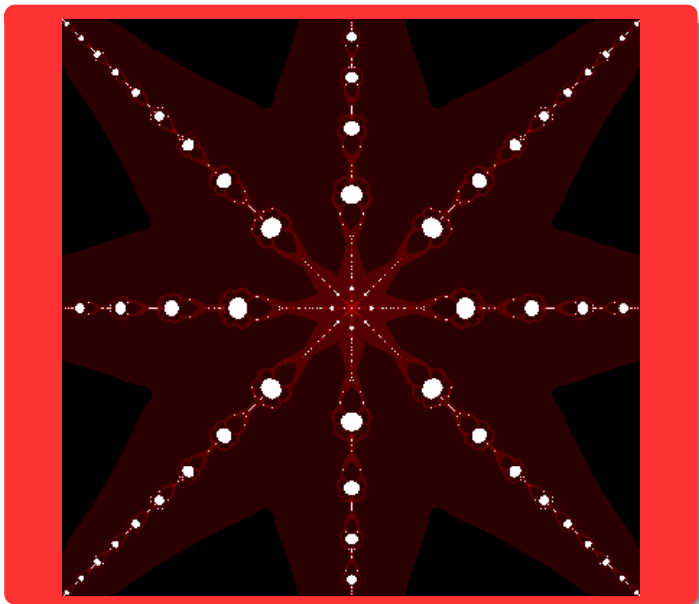


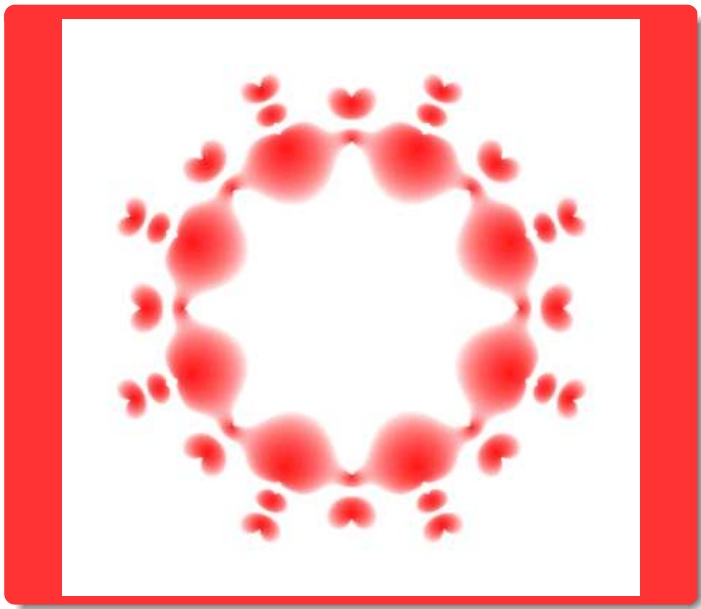


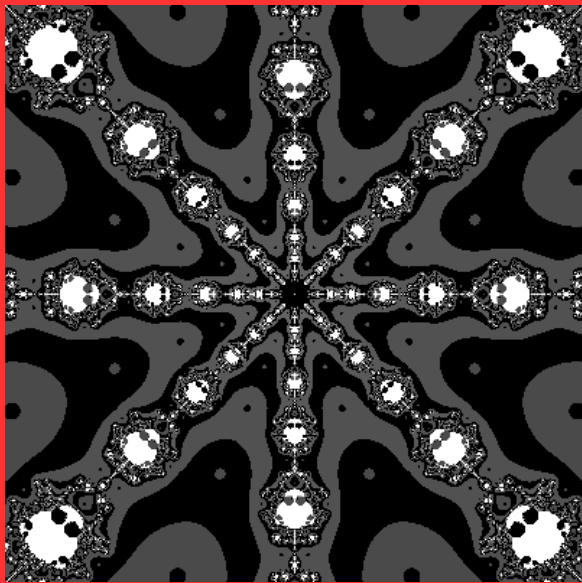














To Whom Can Polynomiography Appeal?

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Can it appeal to little kids?

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Can it appeal to little kids?

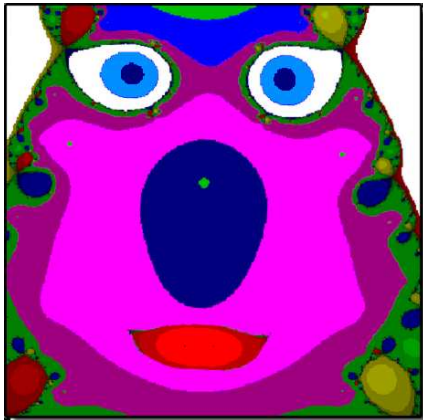
Can it appeal to Hollywood?

To Whom Can Polynomiography Appeal?

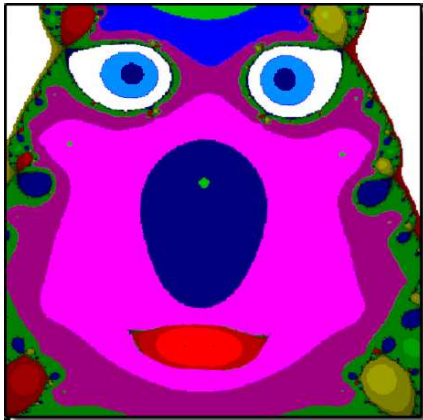
Can it appeal to little kids?

Can it appeal to Hollywood?

Can it lead to games?

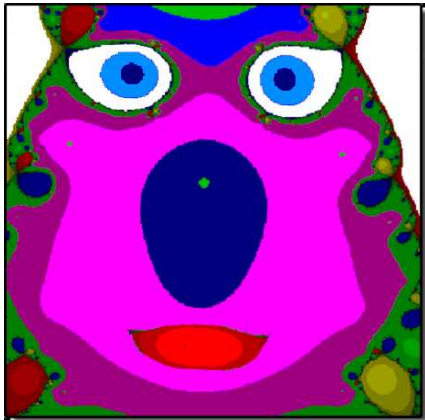


Ms. Poly



Ms. Poly

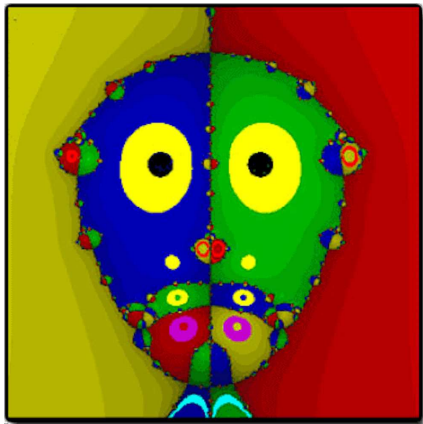
"Just give me a bunch of dots I will make Picasso jealous!"



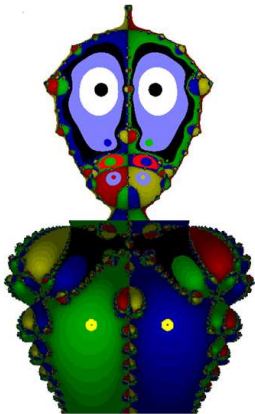
Ms. Poly

"Just give me a bunch of dots I will make Picasso jealous!"

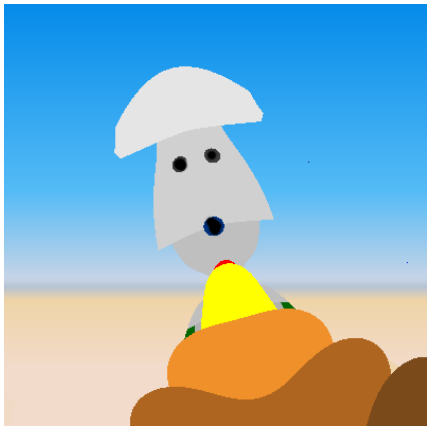
Ms. Poly (Master of Polynomiography)



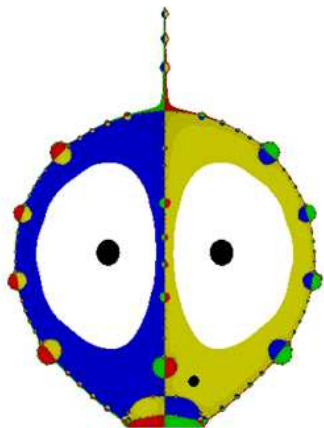
L3



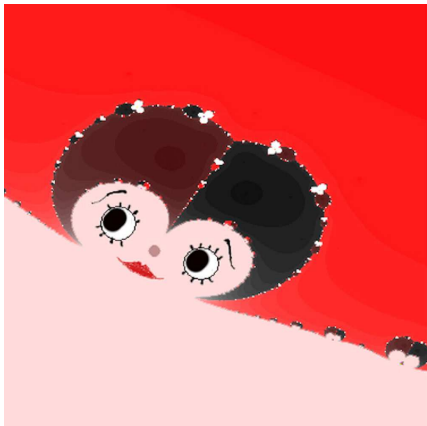
Don Quixote



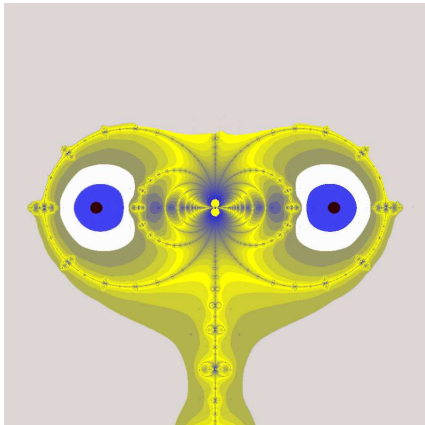
Snoopy on a ride

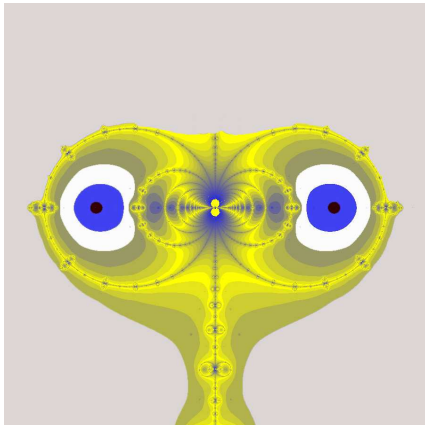


Hi!

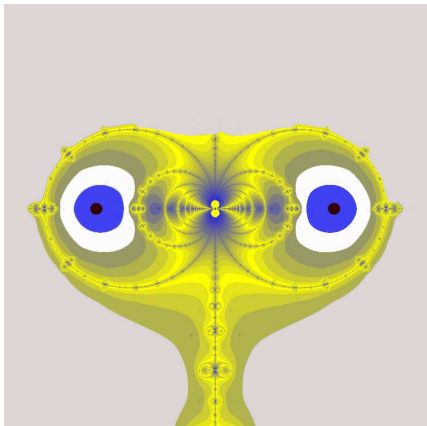


Pretty Betty





They call me Z.T.



They call me Z.T.
Is Popularization of Polynomiography
Possible?