DIMACS WORKSHOP

Media for Play, Expression, Curiosity, and Learning:

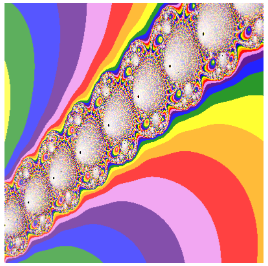
Mathematics through Polynomiography

• MAY 2009

.

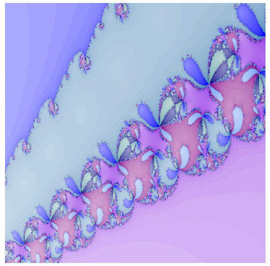
May 10, 2009

MY COORDINATES



- My Department's mission
- My Department's make up
- Examples of our activities (ICTM; Girls + Math; grants)
- Challenges our students face
- The importance of cooperation

OUR INFORMAL CHALLENGES



- Learning concepts (resides in imagination)
- Learning vocabulary (resides in semantics)
- Learning technical skills (resides in syntax)

- PS Do these simultaneously!

OUR FORMAL CHALLENGES

- Facing new plateaus (new material)
- Abstraction (illusive)
- The shift from Static to Dynamic (much research done)

EXAMPLES FOR ALL

Challenge: Learn a subject while learning the vocabulary and the language.

Examples:

- Literature in a foreign language
- Mathematics
- Music
- Visual Art

So we resort to ‘it is good for you’ and ‘rote’ drills.

EXAMPLES FOR HS & Me

- Functions

- Their shifts: $g(x) = f(x) + c \qquad g(x) = f(x - h)$

- Geometry

- Proofs

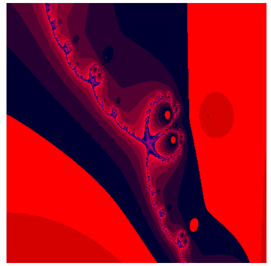
- Calculus

- Its applications

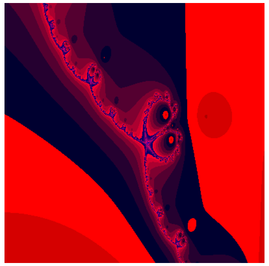
- Graduate level mathematics

- Its relevance

EXAMPLES FOR Mid Sch: **GRADE 3**



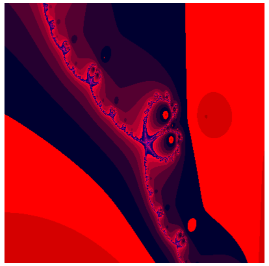
- Line Plots: Read, make and compare tallies & line plots
- Explore Pictographs: Explore reading and making pictographs
- Explore Bar graph: Explore reading and making bar graphs
- Coordinate Graphing, Pictures located: Use ordering pairs to identify and locate points on a grid
- Relate multiplication and addition: via skip counting and number-line
- Problem solving - Reading for Math: Coordinate map



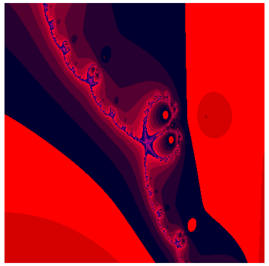
GRADE 4

- Data, Statistics, and Graphing Line plots,
- Explore Pictographs, Bar graphs, Coordinate Graphing, Explore Line Graphs
- Multiplication and division: Skip counting with number line
- Functions & Graphs: Use a table to explore functions & graph them (emphasis on tables)
- Measurement (line graphs)
- Geometry (Congruent and Similar - dots, Perimeter, Area, Problem Solving)

GRADE 5

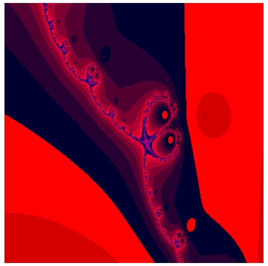


- Data, Statistics, and Graphs Line plots, Pictographs, Bar Graphs, Histograms, Line graphs, Stem-and -Leaf plots
- Mixed Numbers - Number Line; Estimate Sums and Differences of Mixed numbers - Number Line Measurement (line graphs)
- Integers (use of number lines)
- Graphing a Function
- Graph in Four Quadrants and Solve Problems Using Graphs
- Geometry (no number line reference)
- Perimeter, Area, and Volume (no number line reference)
- Percents (no number line reference)



GRADE 6

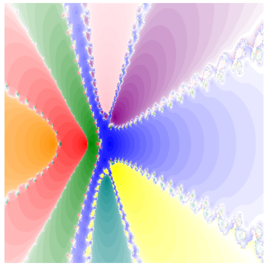
- Number Line - Compare and Order whole numbers & decimals
- Collect, organize and display data (line plots, pictographs)
- Range, Mean, Median, Mode
- Bar Graphs and Histograms, Line Graphs, Stem and Leaf Plots
- Graph a Function
- Use graphs to solve problems
- Algebra of functions
- Integers and the number line. Add and Subtract Integers
- Graph in Four Quadrants and Solve Problems Using Graphs
- Integers and Rational numbers
- Perimeter, Area, and Volume (no referance to Coord plane)



GRADE 7 AND ABOVE

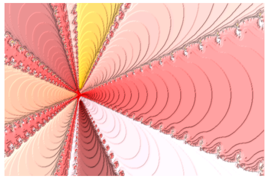
ALGEBRA I, ALGEBRA II, ... etc.

RECALL:



OUR FORMAL CHALLENGES

- Facing new plateaus (new material)
- Abstraction (illusive)
- The shift from Static to Dynamic (much research done)



NEW PLATEAUS

We meet that often!

ABSTRACTION

We meet that often!

What exactly is it?

We recognize it when we see it!

STATIC vs. DYNAMIC

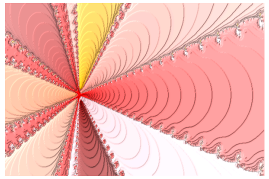
In learning math & sciences, we must, often, go from imagining the 'static' to the 'dynamic'.

He is 10 years old.	He was 10 years old in 1990.
Any row of a table	The whole table
Area is 3 x 5 = 15	Area is width times length
$y = f(x)$	$y = f(x) + c$
$y = f(x)$	Functionals

HOW DO WE SUCCEED?

- The van Hiele Model
- Contextual Learning
- ?Immersion in a Medium?

The van Hiele Model



- Dina and Pierre van Hiele: stressed the importance of **sequential** learning and an **activity** approach.

The model asserts that the learner moves sequentially through five **levels** of understanding.

CORD

- The Center for Occupational Research and Development (CORD) is a national nonprofit organization dedicated to leading change in education.

One of the models promoted is **Contextual learning; REACT**

Fasten Your Seatbelt, Please!

The Levels (vHM)

Level 0: **Visualization**

Students recognize figures

Level 1: **Analysis**

Students analyze component parts of the figures

Level 2: **Informal Deduction**

Students can establish interrelationships

Level 3: **Deduction**

The significance of deduction within an axiom system

Level 4: **Rigor**

Different axiom systems

The Specifics (vHM)

- A student must proceed through the levels in order.

- Each level has its own vocabulary and its own system of relations.

- They propose sequential phases of learning to help students move from one level to another.

The Phases (vHM)

Phase 1: **Inquiry/Information**
At this initial stage the teacher and the students engage in conversation and activity about the objects of study for this level. Observations are made, questions are raised, and level-specific vocabulary is introduced.

Phase 2: **Directed Orientation**
The students explore the topic through materials that the teacher has carefully sequenced. These activities should gradually reveal to the students the structures characteristic at this level.

Phase 3: **Explication**

Building on their previous experiences students express and exchange their emerging views about the structures that have been observed. Other than to assist the students in using accurate and appropriate vocabulary, the teacher's role is minimal. It is during this phase that the level's system of relations begins to become apparent.

Phase 4: **Free Orientation**

Students encounter more complex tasks - tasks with many steps, tasks that can be completed in more than one way, and open-ended tasks. They gain experience in resolving problems on their own and make explicit many relations among the objects of the structures being studied.

Phase 5: **Integration**
Students are able to internalize and unify relations into a new body of thought. The teacher can assist in the synthesis by giving "global surveys" of what students already have learned.

Reference: Teppo, Anne, *Van Hiele Levels of Geometric Thought Revisited,* **Mathematics Teacher**, March 1991, pg 210-221.

Mary L. Crowley's "The van Hiele Model of the Development of Geometric Thought."

Contextual Learning

Question: How do different students learn and how effective teachers teach?

Answer: Contextual Learning or scientific constructivism,

The approach incorporates these five teaching strategies:

- Relating,
- Experiencing,
- Applying,
- Cooperating, and
- Transferring.

(See: www.cord.org)

REACT (CL)

Relating: Linking the concept to be learned with something the student already knows.

Experiencing: Hands-on activities and teacher explanation allow students to discover new knowledge.

Applying: Students apply their knowledge to real-world.

Cooperating: Students solve problems as a team to reinforce knowledge and develop collaborative skills

Transferring: Students take what they have learnd and apply it to new situations and contexts.

Back to Normal Speed!

INTERMISSION

ENTRE'ACT

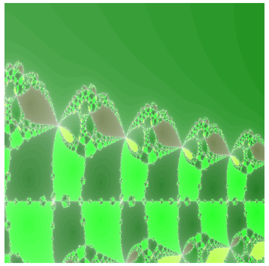
AN OBSERVATION

The Euclidean Plane is a structure to which **an earlier introduction** might be attainable in the middle school with success. Because

– It incorporates working with the ‘number line’ of course (and it does it directedly).

– The Euclidean Plane is physically modeled in real life with ease (easier than the number line).

– While learning about the E.P., the student can learn about **functions** and the associated concepts.



AN ARGUMENT (with hope for links to research)

Provide for students opportunities to absorb conceptual ideas neither through pure authority nor while they are being introduced to the technical development of the subject.

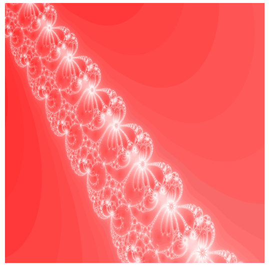
We do this in many experiences:

– we learn how to speak before we write

– we learn how to reason before we formally investigate

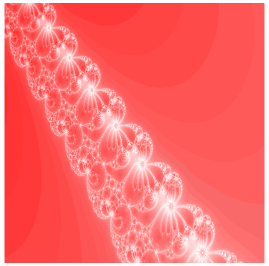
– we learn how to be wrong before distinguishing between right and wrong.

AN INVITATION



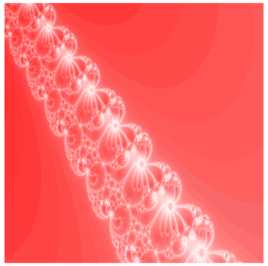
At various stages of their development, students can be encouraged to play, experiment, express and absorb while in contextual immersion for a smoother preparation for abstract subjects.

OPINION 0



OF COURSE,

- We must tell them ‘It is good for them;’
- We must follow disciplined curricular instruction;



OPINION 1

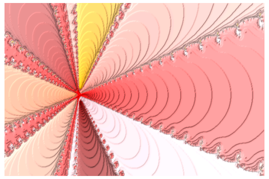
BUT

- Let's provide playful immersion into new media, land-scapes and plateaus.

OPINION 2

Stress 'abstraction' through 'role playing' (and other means).

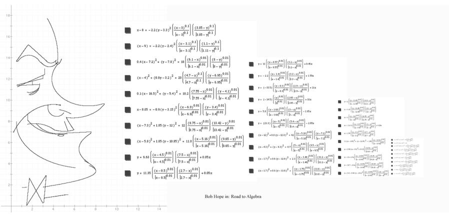
Research of Link(?)

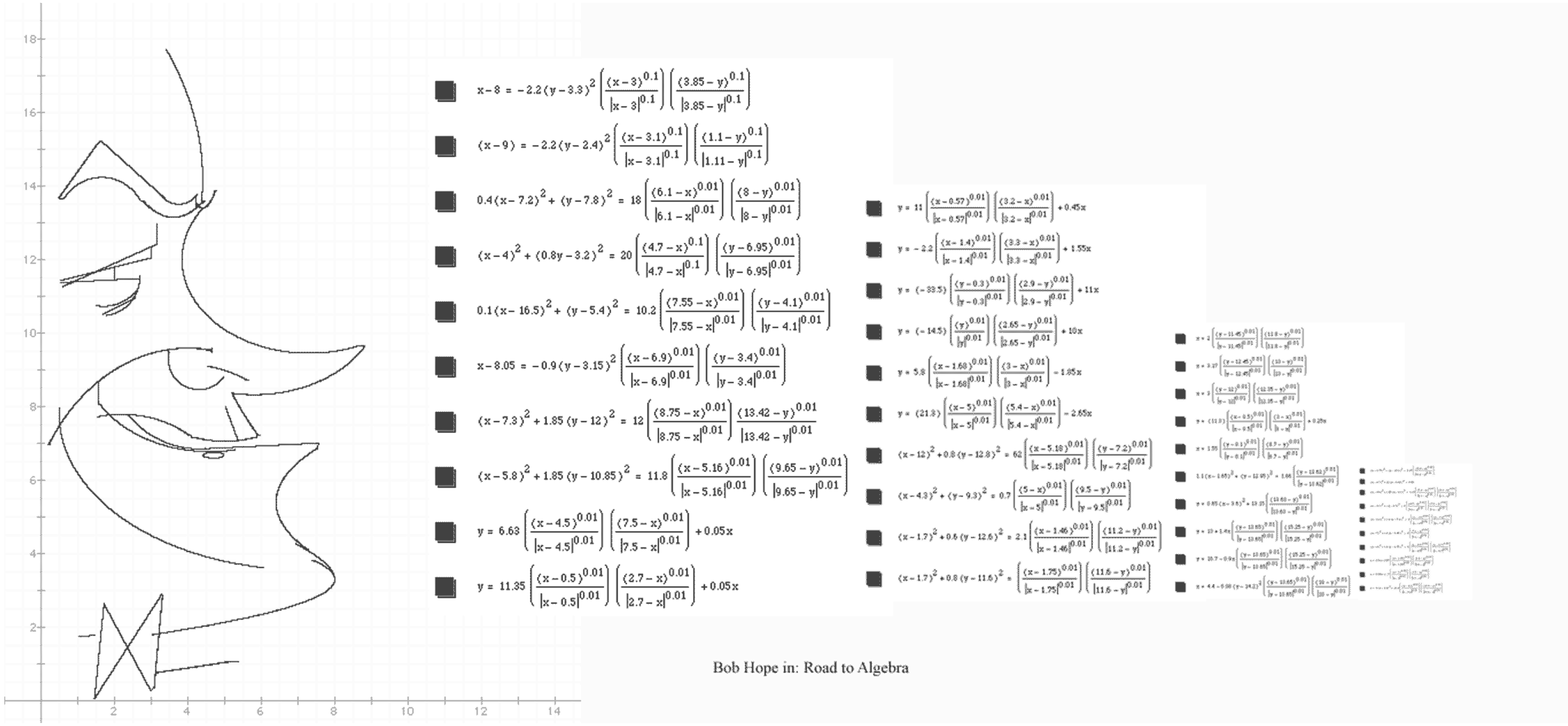


- Yackel & Hanna (2003) “They stress that in a supportive *environment*, all students, as early as elementary school, have the potential to ... participate ... .”
- *ibid* “... we are only beginning to understand

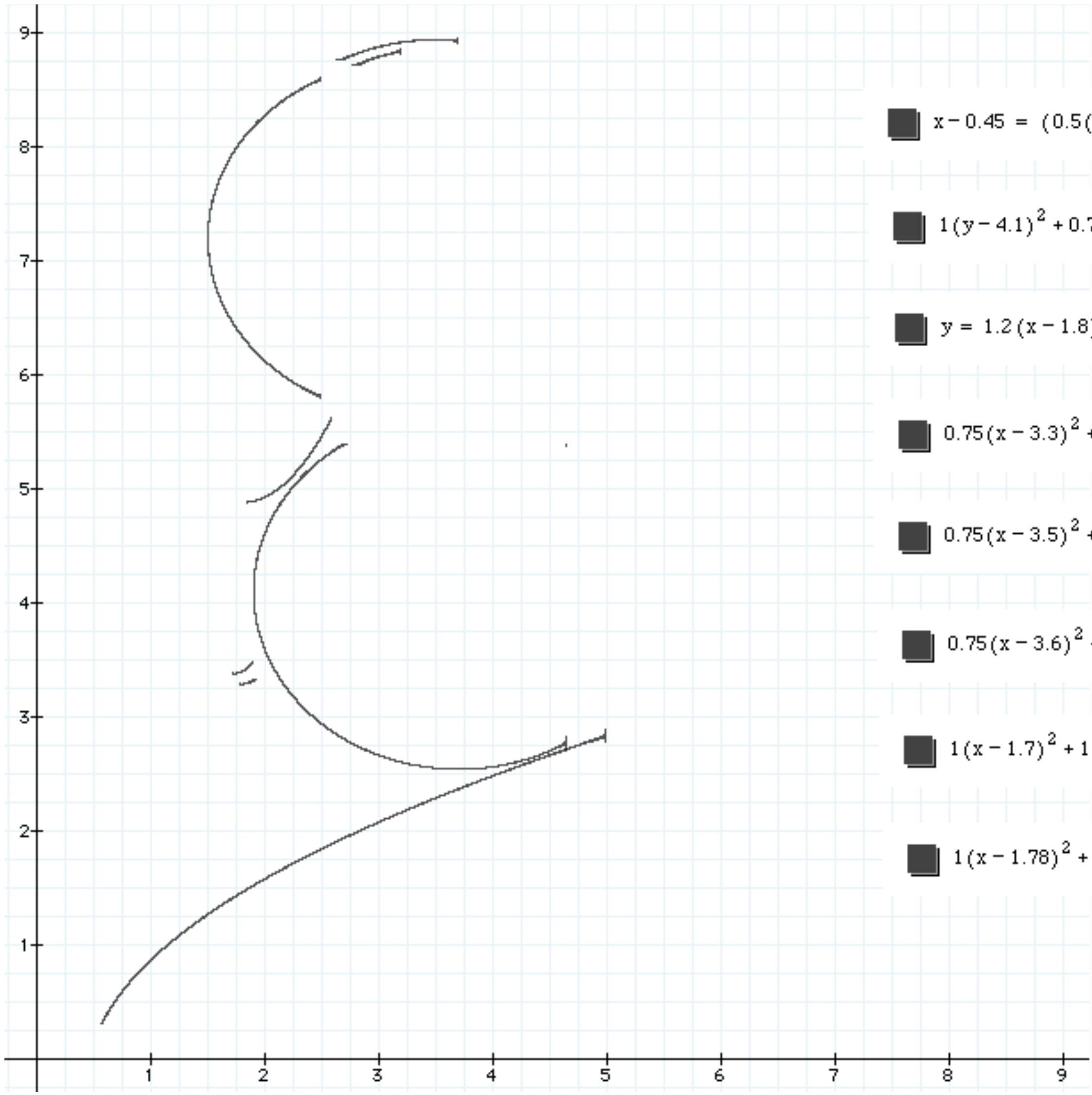
- Maher *et aila* (2009) “Our study addresses these issues.”
- Maher *et aila* (2009) “... results support the idea that given a supportive environment, all students can ... .”
- Maher *et aila* (2009) “Implications for this work suggest the importance of providing conditions and *context* for students to work together in building solutions to problems in actual classroom settings.”

A DETOUR: Drawing by Conics





Bob Hope in: Road to Algebra



$$x - 0.45 = (0.5(y + 0.18)^2)\frac{(5-x)^{0.1}}{|5-x|^{0.1}}\frac{(y-0.3)^{0.1}}{|y-0.3|^{0.1}}$$

$$1(y-4.1)^2 + 0.75(x-3.7)^2 = 2.43\frac{(4.65-x)^{0.05}}{|4.65-x|^{0.05}}\frac{(5.41-y)^{0.05}}{|5.41-y|^{0.05}}$$

$$y = 1.2(x-1.8)^2 + 4.88\frac{(2.6-x)^{0.05}}{|2.6-x|^{0.05}}\frac{(x-1.83)^{0.05}}{|x-1.83|^{0.05}}$$

$$0.75(x-3.3)^2 + 1(y-7.2)^2 = 2.43\frac{(2.5-x)^{0.05}}{|2.5-x|^{0.05}}$$

$$0.75(x-3.5)^2 + 1(y-7.38)^2 = 2.43\frac{(y-8.75)^{0.05}}{|y-8.75|^{0.05}}\frac{(3.7-x)^{0.05}}{|3.7-x|^{0.05}}$$

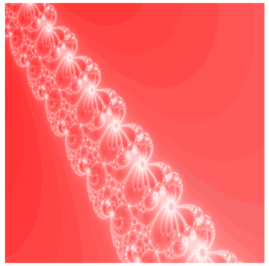
$$0.75(x-3.6)^2 + 1(y-7.32)^2 = 2.43\frac{(y-8.7)^{0.05}}{|y-8.7|^{0.05}}\frac{(3.2-x)^{0.05}}{|3.2-x|^{0.05}}$$

$$1(x-1.7)^2 + 1(y-3.6)^2 = 0.05\frac{(x-1.71)^{0.05}}{|x-1.71|^{0.05}}\frac{(3.5-y)^{0.05}}{|3.5-y|^{0.05}}$$

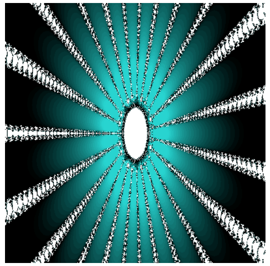
$$1(x-1.78)^2 + 1(y-3.51)^2 = 0.05\frac{(x-1.77)^{0.05}}{|x-1.77|^{0.05}}\frac{(3.35-y)^{0.05}}{|3.35-y|^{0.05}}$$

Hitchcock by conics

WHY POLYNOMIOGRAPHY?



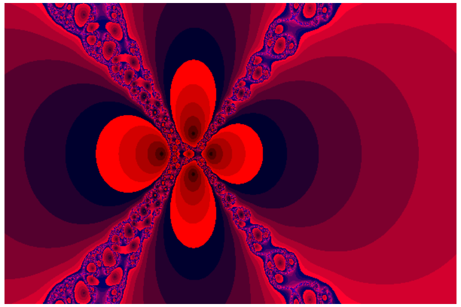
WHY POLYNOMIALS?



Because:

$$\frac{polynomials}{functions} \approx \frac{atoms}{molecules} \approx \frac{alphabet\ letters}{words}$$

That is, polynomials are critical in approximating critical functions, which are objects in mathematics and its critical application to real life.



WHY 'ROOTS'?

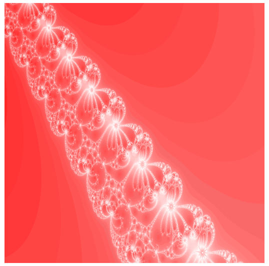
Because:

"When is it the right time to do this or that?"

For what t, is a formula about t = 'right amount'?

For what t, is $f(t) = r$?

For what t, is $f(t) - r = 0$?



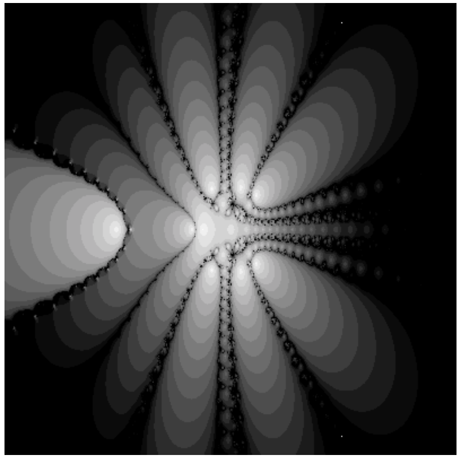
WHY 'ROOTS'?

Or

$$g(t) = 0$$

$$g(z) = 0$$

$$p(z) = 0$$



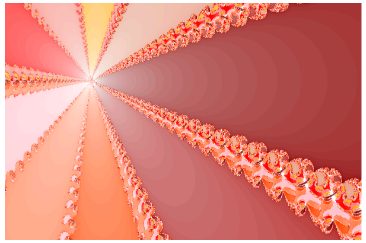
CONCLUSION

POSITION. Learning about polynomials and their roots should be a rudimentary element.

Whenever we can, we should provide a platform for introduction to polynomials.

THE ROLE OF

POLYNOMIOGRAPHY



- It is centered on polynomials.
- It introduces new ideas in Euclidean Plane.
- It can introduce the notion of ‘function’.
- It allows abstraction.
- It introduces contrast between statics and dynamics.
- It allows deeper (beyond HS & college) mathematical investigations.

A TRIBUTE:

1,2,3,4,5

a, b, c, d, e

6,7,8,9,10

f, g, h, i, j

11,12,13,14,15

k, l, m, n, o

16,17,18,19,20

p, q, r, s, t

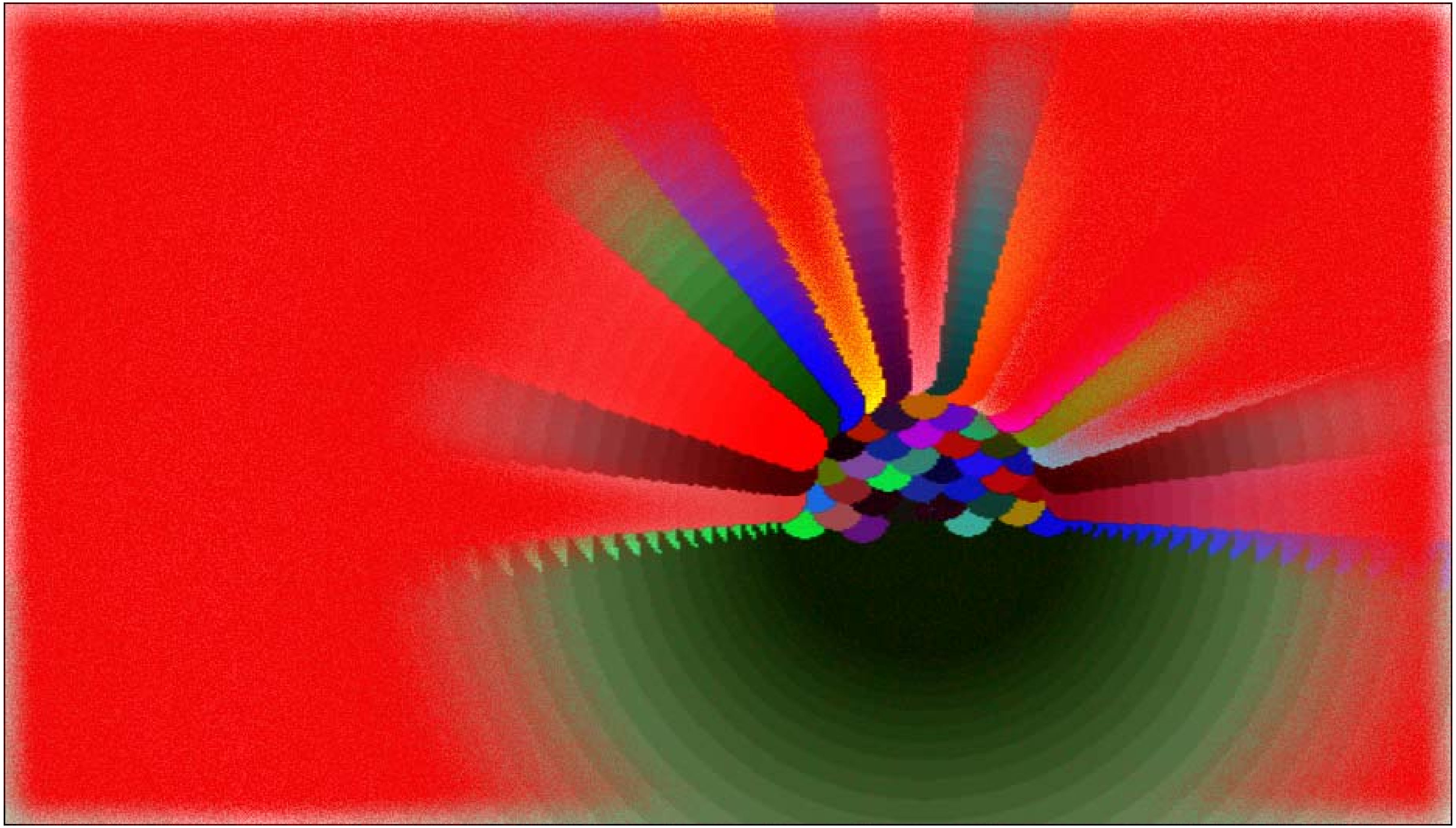
21,22,23,24,25

u, v, w, x, y

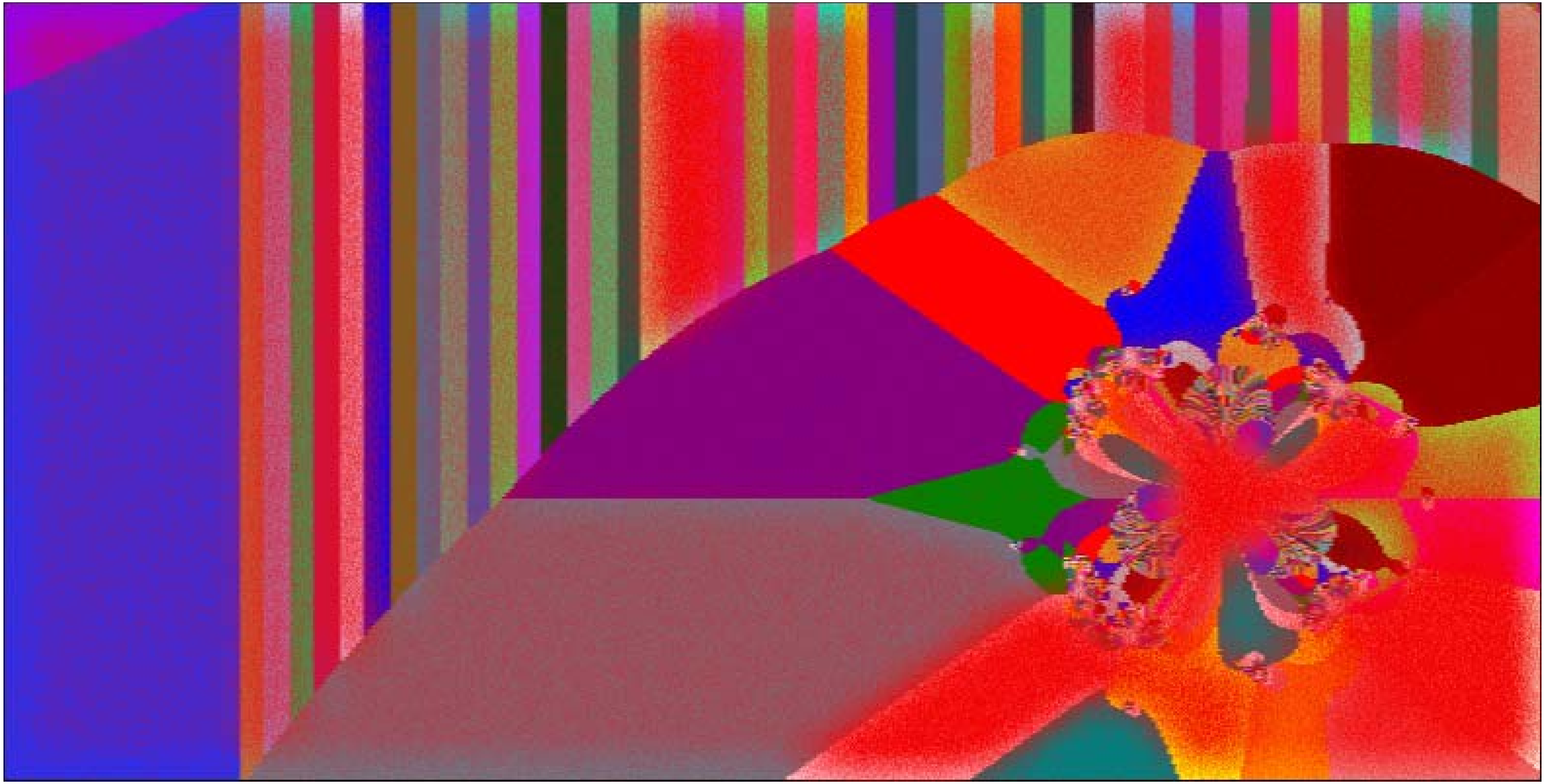
26

z

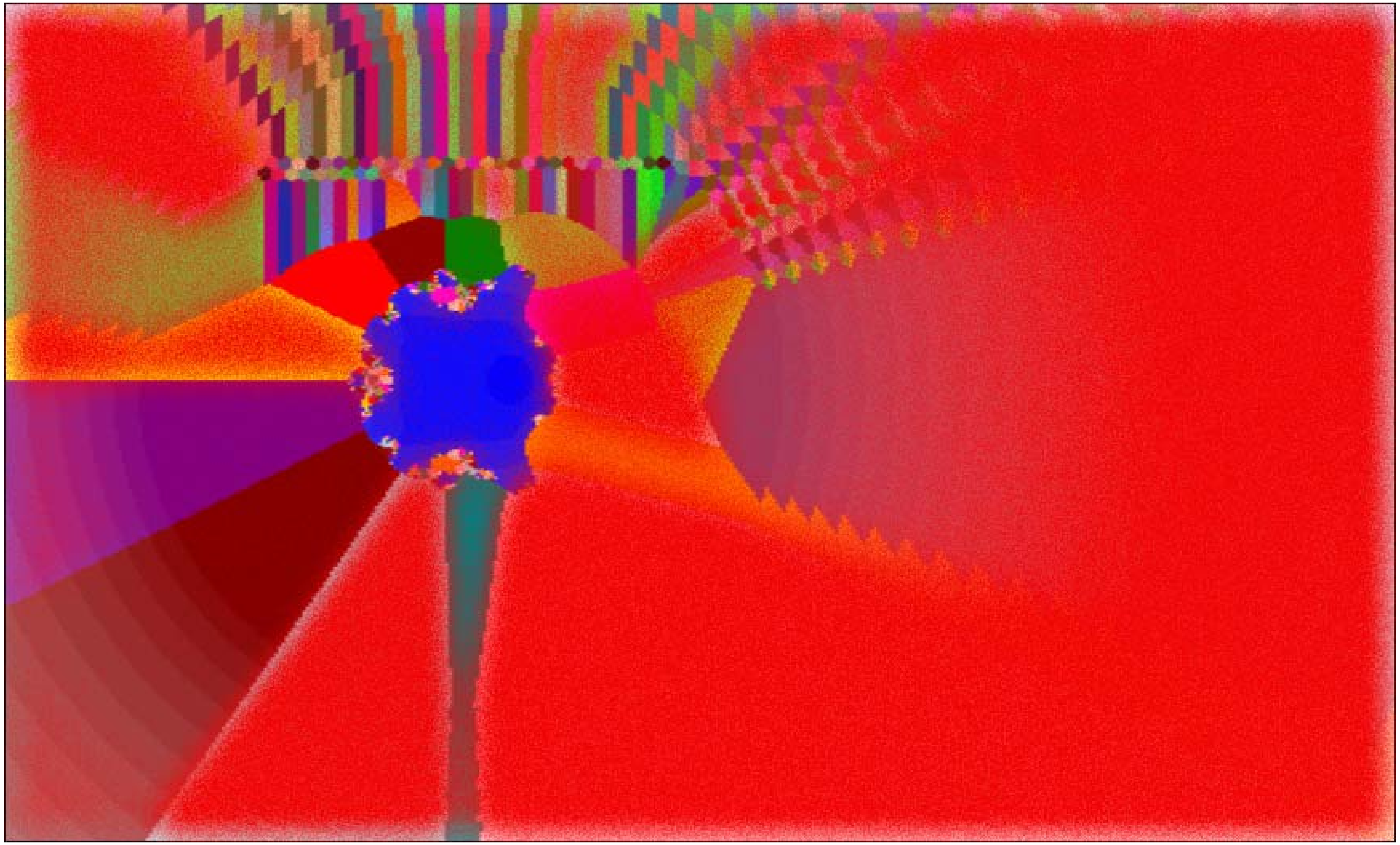
Bahman



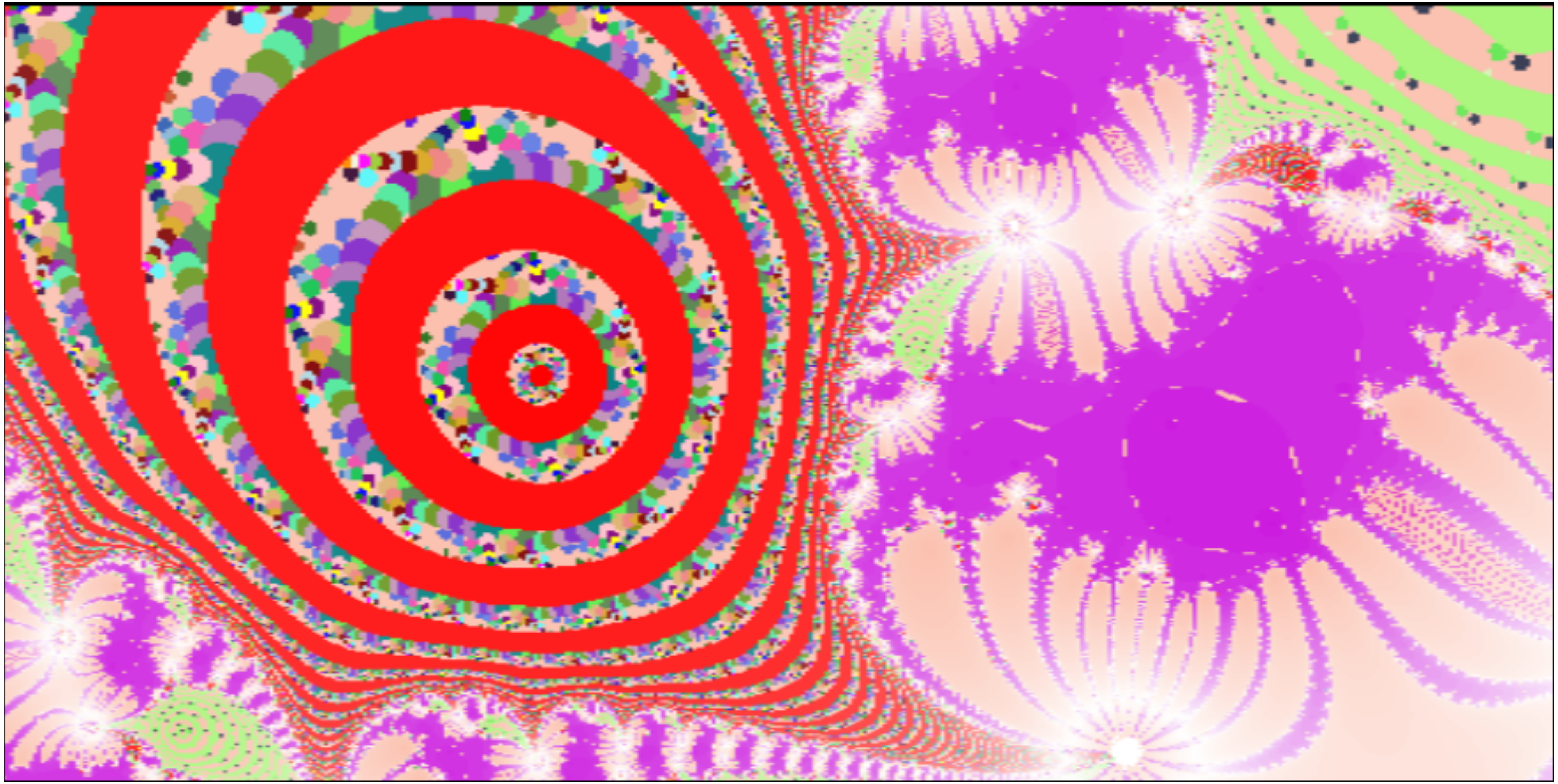
Dirk



Helaman



Radmila



Carolyn