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A Price-Based Approach for Controlling Networked Distributed Energy Resources

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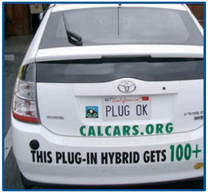
Outline

Motivation





- Distributed Energy Resources (DERs) can potentially be utilized to provide ancillary services
  - Power electronics grid interfaces commonly used in DERs can provide reactive power support for voltage control
  - Plug-in-hybrid vehicles (PHEVs) can provide active power for up and down regulation

Control and Coordination of DERs

- Effective control of DERs is key for enabling their utilization in providing ancillary services

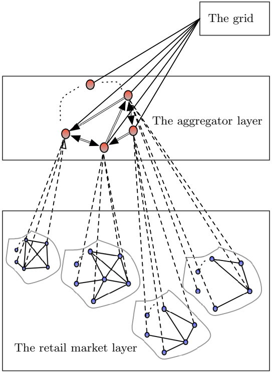
- Potential solution: centralized control (where each DER is commanded from a centralized decision maker)
  - Requires a communication network connecting the central controller with each distributed resource
  - Requires up-to-date knowledge of distributed resource availability on the distribution side

- Alternative approach: utilize distributed strategies for control and coordination of DERs, which offer several potential advantages
  - Easy and affordable deployment (no requirement for communication infrastructure between centralized controller and various devices)
  - Ability to handle incomplete global knowledge of DER availability
  - Potential resiliency to faults and/or unpredictable DER behavior

General Overview

- Consider a set of entities, referred to as aggregators, that through some market-clearing mechanism, are requested to provide certain amount of energy over some period of time

- Each aggregator can influence the energy provision/consumption of a group of DERs by offering them a pricing strategy



Objective: to incentivize the DERs to provide or consume energy, as appropriate, so as to allocate among them the amount of energy that the aggregator has been asked to provide

Outline

Distributed Energy Resources

- Each DER is a decision maker and can freely choose to participate after receiving a request from its aggregator
- DER actions include idle, provide, or consume energy
- DER decisions depends on its own utility function, along with the pricing strategy (for provision/absorption) designed by the aggregator
- DERs are price anticipating, i.e., they are aware that the aggregator designs the pricing as a function of the average energy available
- DERs are able to collect information from “neighboring” DERs with which they can exchange information

Problem Formulation

- Let $V = \{v_1, \dots, v_n\}$, $n \in \mathbb{Z}_{\geq 1}$ denote the set of DERs
- Let $x_i(t) \in [0, 1]$ denote the energy level of DER v_i at time $t \in \mathbb{R}_{\geq 0}$
- Let $\mathcal{X} \in \mathbb{R}$ be the amount of energy that the aggregator has contracted to provide/consume over some period of time:
  - when $\mathcal{X} \in \mathbb{R}_{<0}$, the aggregator needs to encourage the DERs to provide energy
  - when $\mathcal{X} \in \mathbb{R}_{>0}$, the aggregator needs to encourage the DERs to consume energy

Pricing Functions

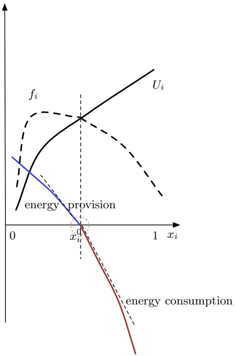
- The aggregator incentivizes the DERs via pricing to provide or consume energy within a time interval $[0, T]$
- Price per unit of energy provided/consumed is set for a time interval $[0, T]$ based on the DER average energy level, $\overline{x} = \sum_{i=1}^{n} x_i / n$, at the end of the time interval
  - Each DER decides its x_i at the beginning of the time interval
- The quantities $P_{\mathsf{c}}(\overline{x})$ and $P_{\mathsf{p}}(\overline{x})$, which are obtained as the outputs of some mappings

$$P_{\mathsf{c}} : [0,1] \to \mathbb{R}_{\geq 0} \qquad \text{and} \qquad P_{\mathsf{p}} : [0,1] \to \mathbb{R}_{\geq 0},$$

give, respectively, the price per unit of energy that DERs pay when consuming energy and receive when providing

DERs Payoff Functions

- Let $U_i : [0,1] \to \mathbb{R}_{\geq 0}$ denote the utility function of DER v_i
- Assume that each U_i is an increasing function of the available energy:
  - at no cost, it is beneficial to keep as much energy as possible



- Similar to other resource allocation problems [Johari, Tsitsiklis, '06], each DER wishes to maximize a payoff function of the form

$$f_i(x_i, x_{-i}, P_{\mathsf{c}}, P_{\mathsf{p}}) = \begin{cases} U_i(x_i) - (x_i - x_i^0) P_{\mathsf{c}}(\overline{x}), & x_i > x_i^0, \\ U_i(x_i) - (x_i - x_i^0) P_{\mathsf{p}}(\overline{x}), & x_i \leq x_i^0, \end{cases}$$

where (x_i^0, x_{-i}^0) denotes the initial energy profile of all DERs

Objective of the Aggregator

- Ensure that the DERs collectively provide $\mathcal{X} \in X_{\mathsf{agg}}$ units of energy; thus it wishes to maximize

$$f_{\mathsf{agg}}(x, P_{\mathsf{c}}, P_{\mathsf{p}}) = -|\mathcal{X} - \sum_{i=1}^{n} \alpha_i (x_i - x_i^0)|,$$

where $\alpha_i \in \mathbb{R}_{>0}$, for all $i \in \{1, \ldots, n\}$

- Based on the description given thus far, the aggregator and the DERs define a game

$$\mathbf{G}_{\mathsf{DERs\text{-}AGG}} = (V \cup \{v^{\mathsf{agg}}\}, [0,1]^n \times X_{\mathsf{agg}}, f_1 \times \ldots \times f_n \times f_{\mathsf{agg}}),$$

where players wish to maximize their payoff functions

Some Problem Statements

(a) **[Existence of equilibria]** Given the pricing strategies of the aggregator P_{c} and P_{p}, does there exist a Nash equilibrium solution to the DER game $\mathbf{G}_{\mathsf{DERs}}$ as defined below?

$$\mathbf{G}_{\mathsf{DERs}} = (V,\ [0,1]^n,\ f_1 \times \ldots \times f_n)$$

If so, is the Nash equilibrium unique?

(b) **[Distributed equilibria seeking]** If the answers to both parts of (a) are positive, can the DERs use a (distributed) strategy to seek the Nash equilibrium, after the pricing strategy is fixed?

(c) **[Optimal pricing]** If the answer to the existence question is positive, does there exists pricing strategies P_{c} and P_{p} such that

$$x^* \in \{z \in X \mid z = \text{argmax}_x f_{\mathsf{agg}}(x, P_{\mathsf{c}}, P_{\mathsf{p}})\},$$

where x^* is the Nash equilibrium of the DER game $\mathbf{G}_{\mathsf{DERs}}$?

Focus of this talk: problems (a) and (b), i.e., for a given prices strategy, we study the DER game $\mathbf{G}_{\text{DERs}}$ and propose a distributed algorithm that allows the DERs to seek for the Nash equilibrium when unique

Outline

Claim: under proper assumptions on the DER payoff functions $\mathbf{G}_{\text{DERs}}$ is a concave game

Casting $\mathbf{G}_{\text{DERs}}$ as a Concave Game

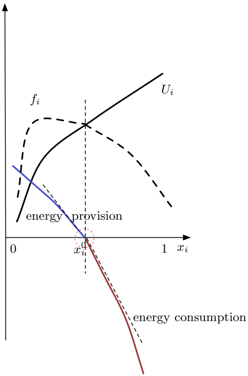
- A group of n players $\{v_1, \ldots, v_n\}$
  - In $\mathbf{G}_{\text{DERs}}$, v_i is the ith DER
- Player v_i takes action from $S_i \subset \mathbb{R}^{d_1}$, nonempty, convex and compact
  - In $\mathbf{G}_{\text{DERs}}$, $S_i = [0, 1],\ \forall i$
- Strategy set for all players is $S = S_1 \times \ldots \times S_n$
  - In $\mathbf{G}_{\text{DERs}}$, $S = [0, 1] \times \cdots \times [0, 1] = [0, 1]^n$ [no shared constraints]
- When players take actions according to $\boldsymbol{x} = (x_1, \ldots, x_n)$, $x_i \in \mathbb{R}^d$, the payoff function $f_i : S \to \mathbb{R}$ of the ith player is $f_i(\boldsymbol{x})$
  - In $\mathbf{G}_{\text{DERs}}$:

$$f_i(x_i, x_{-i}, P_{\mathsf{c}}, P_{\mathsf{p}}) = \begin{cases} U_i(x_i) - (x_i - x_i^0) P_{\mathsf{c}}(\overline{x}), & x_i > x_i^0, \\ U_i(x_i) - (x_i - x_i^0) P_{\mathsf{p}}(\overline{x}), & x_i \leq x_i^0, \end{cases}$$

- f_i is a locally Lipschitz concave mapping in its ith argument
  - What assumptions do we need on the f_i's s of the $\mathbf{G}_{\text{DERs}}$ so the conditions above are satisfied?

DER Game Payoff Functions

- Assumptions:
  - The utility function U_i is concave, nondecreasing, and continuously differentiable, for all $i \in \{1, \ldots, n\}$
  - The function P_{c} is convex, twice differentiable, and nondecreasing
  - The function P_{p} is concave, twice differentiable, nondecreasing
  - $P_{\mathsf{c}}(\overline{x}) \geq P_{\mathsf{p}}(\overline{x})$, for all $\overline{x} \in [0, 1]$



- The payoff function f_i is not necessarily differentiable

Proposition: Under the assumptions above, the payoff function f_i of each DER is concave in its first argument

Existence and Uniqueness of Equilibrium Points

Using a classical result on concave games [Rosen, '65], we have the following result:

Theorem: Under the assumptions on the DER payoff functions, $\mathbf{G}_{\mathsf{DERs}}$ has a Nash equilibrium

What about uniqueness?

- Under an additional condition [diagonally strict concavity], an extension of the result by Rosen to concave games with
  - non-smooth payoff functions, and
  - no shared constraints

  can be applied to guarantee uniqueness [Carlson, '01]

- We assume uniqueness for the remainder of the talk

Outline

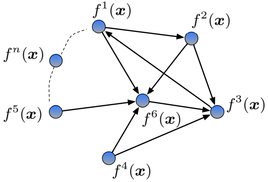
Distributed Nash Equilibrium Seeking

- At the Nash equilibrium $x^* \in S$, for all i,

$$f_i(x^*) = \max_{y_i} \{ f_i(x_1^*, \ldots, x_{i-1}^*, y_i, x_{i+1}^*, \ldots, x_n^*) \mid y_i \in S_i \}$$

  i.e., no player can improve its payoff by unilaterally deviating from x^*

Objective: can an equilibrium be found, collaboratively, in spite of partial access to information?



Assumptions:

- Each DER can only communicate with its neighboring DERs
- Each DER has access to its own payoff function only

Main Idea for Achieving Distributed Nash Seeking

- For simplicity of exposition, consider the unconstrained version of $\mathbf{G}_{\text{DERs}}$, i.e., $x_i \in \mathbb{R}, \ \forall i$; then, the fix points of the function

$$\Phi(x, y) = \sum_{i=1}^{n} f_i(y_1, \ldots, y_{i-1}, x_i, y_{i+1} \ldots, y_n),$$

  restricted to the subset where $y = x$, correspond to the Nash equilibrium of $\mathbf{G}_{\text{DERs}}$

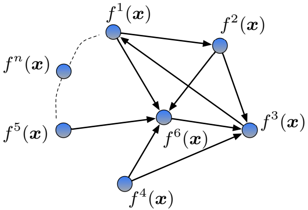
- Then, finding x^* boils down to designing a distributed algorithm that allows the DERs to compute the fix points of $\Phi(x, y)$

- The distributed algorithm we propose is inspired by
  - Continuous-time distributed protocols for optimization problems [Wang and Elia '10; Gharesifard and Cortés, '12]
  - Nash-seeking strategies for noncooperative games [Frihauf, Krstic, and Başar, '12]

Discrete-Time Distributed Nash-Seeking Dynamics

- Consider a network of n DERs $\{v_1, \ldots, v_n\}$

- The exchange of information between DERs is described by a connected graph, denoted by $\mathcal{G}$



- Let $x^* \in X$, $X = [0,1]^n$, denote the unique Nash equilibrium of the $\mathbf{G}_{\mathsf{DERs}} = (V,\ [0,1]^n,\ f_1 \times \ldots \times f_n)$

- Let $x^i \in \mathbb{R}^n$ denote the estimate of DER v_i about x^*

- Define $\boldsymbol{x}^T = ((x^1)^T, \ldots, (x^n)^T) \in \mathbb{R}^{n^2}$

- Let $\mathsf{L} \in \mathbb{R}^{n \times n}$ denote the Laplacian of $\mathcal{G}$ and define $\mathbf{L} = \mathsf{L} \otimes \mathsf{I}_n \in \mathbb{R}^{n^2 \times n^2}$, where $\otimes$ denotes the Kronecker product

Discrete-Time Distributed Nash-Seeking Dynamics

- Due to the lack of differentiability of the payoff functions, we need to formulate the algorithm as a set-valued dynamical system
- Define

$$\Psi(\boldsymbol{x}, \boldsymbol{z}) = \{\big(-\mathbf{L}\boldsymbol{x} - \mathbf{L}\boldsymbol{z} + \boldsymbol{s}_{\boldsymbol{x}}, \mathbf{L}\boldsymbol{x}\big) \mid \boldsymbol{s}_{\boldsymbol{x}} \in \mathcal{D}_{\boldsymbol{x}}\},$$

with

$$\mathcal{D}_{\boldsymbol{x}} = \{u \mid u = (\underbrace{\eta_1, 0, \ldots, 0}_{\text{computed by } v_1}, \ldots, \underbrace{0, \ldots, 0, \eta_n}_{\text{computed by } v_n})^T, \eta_i \in \partial_{x_i} f_i(x^i)\},$$

- The distributed Nash-seeking dynamics is given by

$$\boldsymbol{x}(k+1) \in \mathrm{P}\big(x(k) - \delta(\mathbf{L}\boldsymbol{x}(k) + \mathbf{L}\boldsymbol{z}(k) - \mathcal{D}_{\boldsymbol{x}(k)})\big),$$
$$\boldsymbol{z}(k+1) = z(k) + \delta\mathbf{L}\boldsymbol{x}(k),$$

with $\delta > 0$, and $\mathrm{P} = \Pi_{i=1}^{n^2}\mathrm{P}_i$, where $\mathrm{P}_i : \mathbb{R} \to [0,1]$, $i \in \{1, \ldots, n^2\}$, is the projection onto $[0,1]$

Convergence Results [Gharesifard, D-G, and Başar, '13]

Lemma: When the graph $\mathcal{G}$ is connected, the distributed Nash-seeking dynamics has at least one fixed point. Moreover, $(\boldsymbol{x}^*, \boldsymbol{z}^*)$ is a fixed point if and only if $\boldsymbol{x}^* = \mathbf{1}_n \otimes x^*$, where $x^* \in X$ is the Nash equilibrium of the DER game $\mathbf{G}_{\mathsf{DERs}}$

Theorem: When the graph $\mathcal{G}$ is connected, the distributed Nash-seeking dynamics is asymptotically convergent. Moreover, the projection onto the first component of its trajectory converges to $\boldsymbol{x}^* = \mathbf{1}_n \otimes x^*$, where $x^* \in \mathbb{R}^n$ is the Nash equilibrium of $\mathbf{G}_{\mathsf{DERs}}$

Outline

6-DER Example

- Consider a set of DERs $\{v_1, \ldots, v_6\}$ with the adjacency matrix of the communication graph $\mathcal{G}$ given by

$$A = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

- The utility function $U_i : [0,1] \to \mathbb{R}_{\geq 0}$, $i \in \{1, \ldots, 6\}$ of each DER is

$$U_i(x_i) = u_i^1 \log(1 + x_i) + u_i^2 x_i,$$

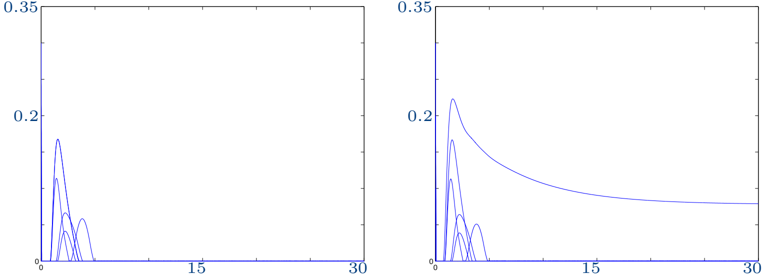
  where U_i is normalized so that $u_i^1, u_i^2 \in (0,1]$
- Assume linear pricings:

$$P_{\mathsf{c}}(\overline{x}) = c_1 \overline{x} + c_2,$$
$$P_{\mathsf{p}}(\overline{x}) = d_1 \overline{x} + d_2,$$

  with P_{c} and P_{p} normalized so that $c_1, d_1 \in (0,1]$ and $c_2, d_2 \in [0,1]$

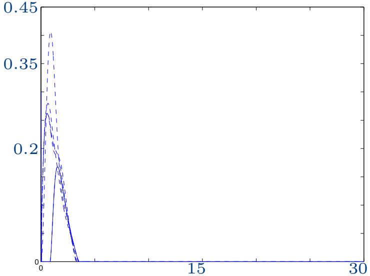
High Price for Consumption; Provision is Encouraged

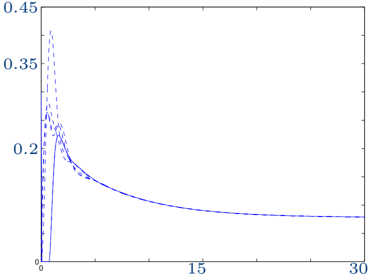
- Consider a scenario in which the aggregators need to encourage the DERs to provide energy
- The aggregator chooses the pricing parameters as $c_1 = 0.9$, $c_2 = 0.9$, $d_1 = 0.8$, and $d_2 = 0.8$
- Consider two cases:
  - **Case-1:** all DERs have low initial available energy and no incentive for consuming energy
  - **Case-2:** all DERs have low initial energy available; the only DER with incentive for consuming energy is v_5



High Price for Consumption; Provision is not Encouraged

- Consider a scenario in which the aggregator increases the price for consuming energy when the average energy available is high
- The aggregator chooses the pricing parameters as $c_1 = 0.7, c_2 = 0.1, d_1 = 0.1, d_2 = 0.1$
- Consider two cases:
  - **Case-3:** all DERs have low initial energy available in the DERs and no incentive for providing energy
  - **Case-4:** all DERs have low initial energy available in the DERs; the only DER with incentive for consuming energy is v_5





Outline

Summary

- We proposed a framework for controlling DER energy provision and consumption via pricing strategies
  - A group of aggregators is responsible for providing a certain amount of energy predetermined by some market-clearing mechanism
  - The DERs are assumed to be price anticipating and also have their own individual utility functions

- We formulated the problem as a two-layer game in which the aggregator sets prices for energy consumption/provision

- For fixed pricing, we give conditions under which the DER-layer game is concave and conditions under which the equilibrium is unique

- We propose a discrete-time algorithm which allows the DERs to compute the Nash equilibrium when unique

Future Work

- Characterization of optimal pricing strategies for the aggregator in the context of mechanism design
- Extension of the convergence results to communication networks described by directed graphs
- Study groups of aggregators and their interconnections with the retail market layer
- Robustness and resilience in pricing strategies