The best-deterministic method for the stochastic unit commitment problem

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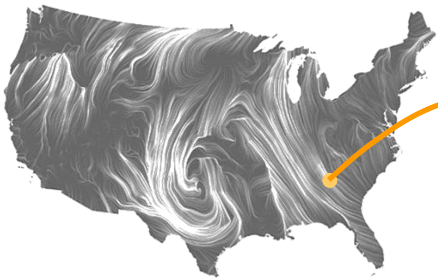
Joint work with Hugo P. Simao, Warren B. Powell

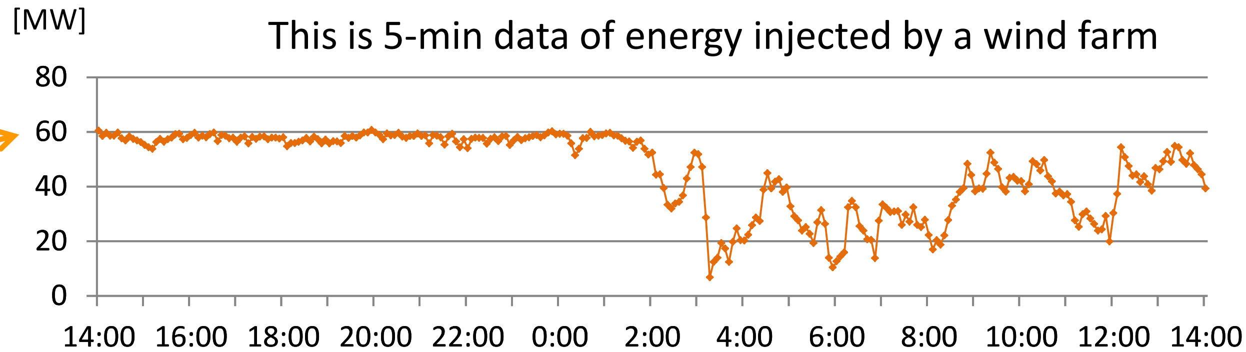
Feb 21, 2013

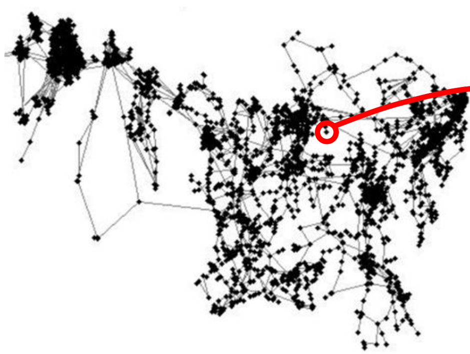
DIMACS Workshop on Energy Infrastructure: Designing for Stability and Resilience

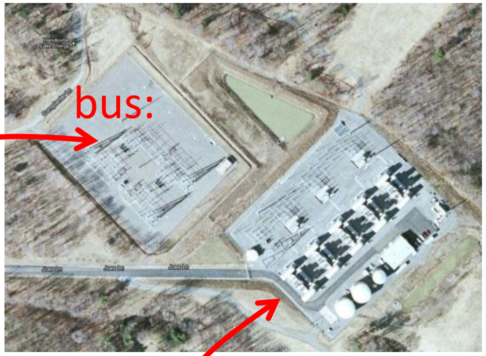
The challenge of using wind energy



Wind: complex to forecast;
high-dimensional process.







Net power injections at each bus (node) from generators and loads, including wind

$$\mathbf{f(v,\theta)=p} \quad \text{active power balance}$$

$$\mathbf{g(v,\theta)=q} \quad \text{reactive power balance}$$

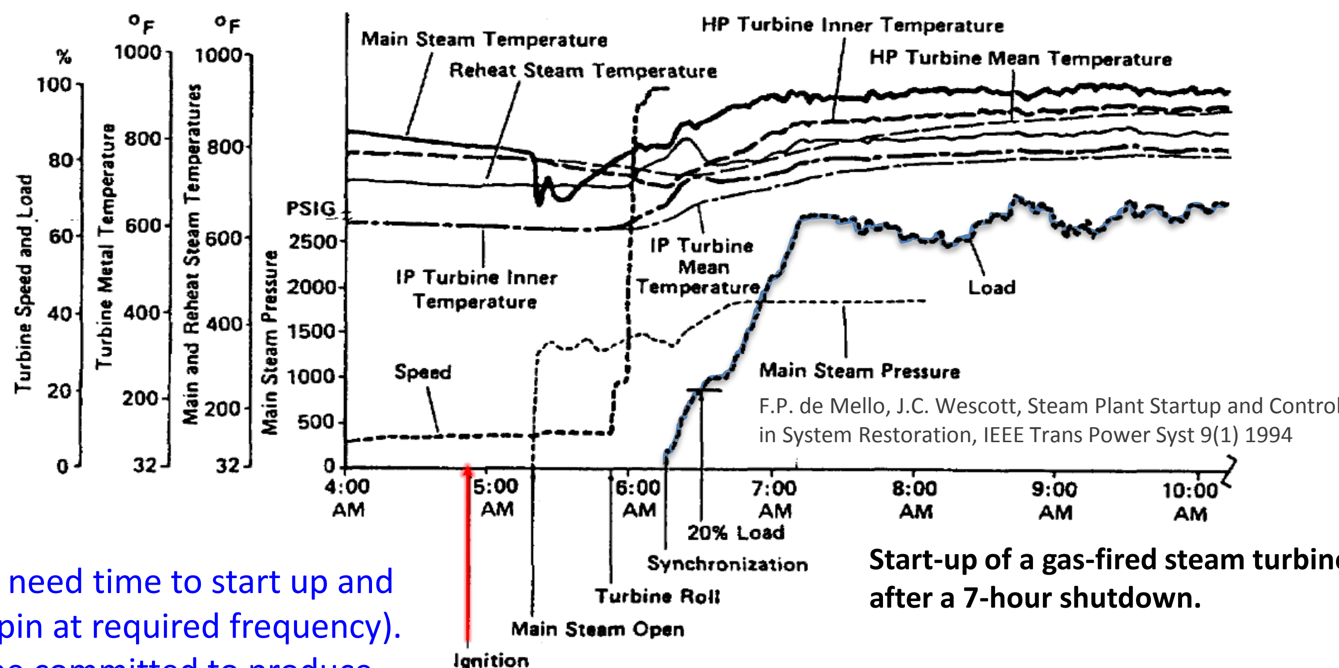
voltage magnitude and angle at each bus, assuming steady state @ 60Hz

Loads *Generators*

60.05 Hz **60 Hz** **59.95 Hz**

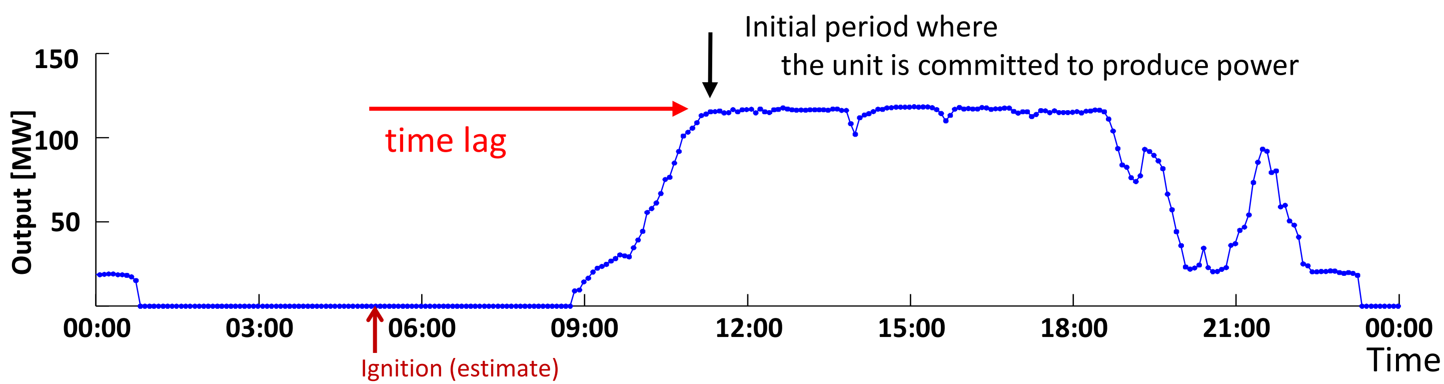
Generating units must balance variations from stochastic injections (load:- and wind:+) in real-time.
The control relies on frequency changes and on signals sent by the system operator.

Decision time lag for steam turbines



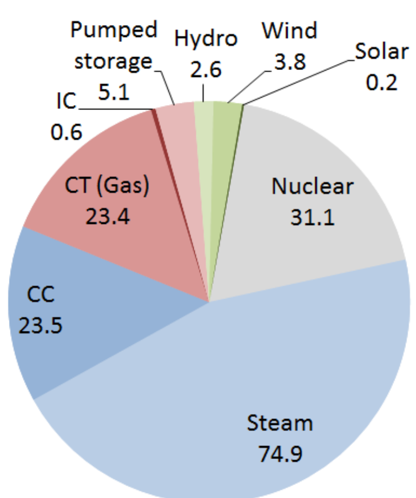
Start-up of a gas-fired steam turbine after a 7-hour shutdown.

Steam units need time to start up and be online (spin at required frequency). They must be committed to produce power in advance.

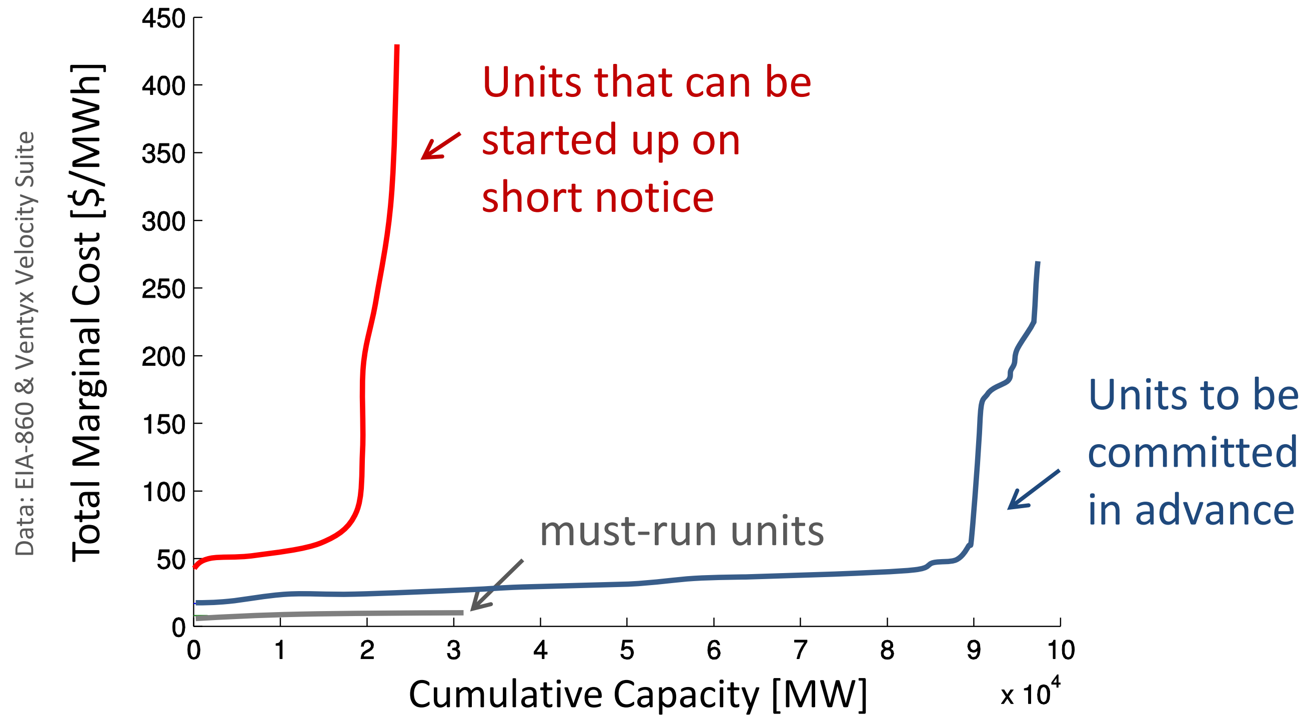


Aggregated cost curves say: Do not wait too long

Capacity [GW]



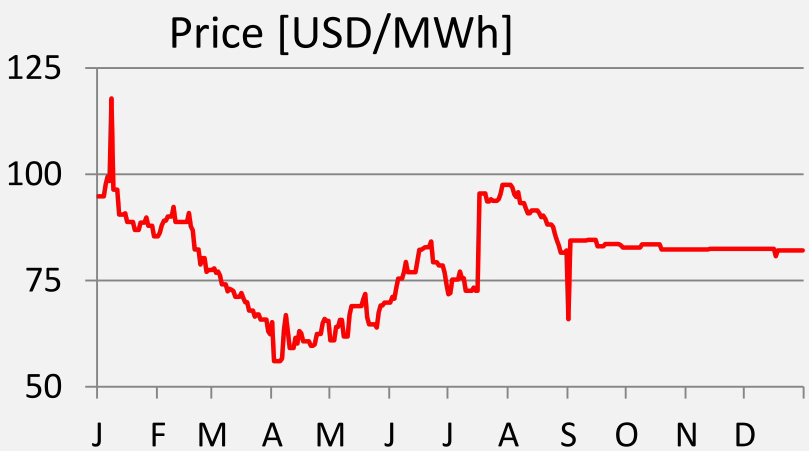
Cost-based offer curve of dispatchable units

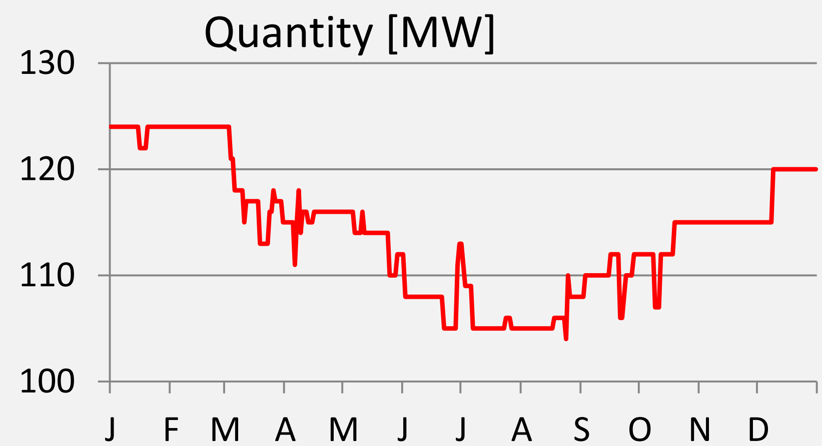


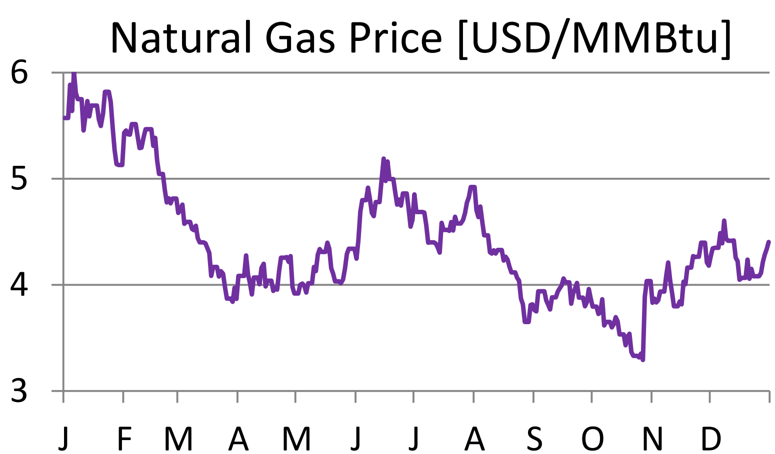
Assumptions for this graph:
No transmission constraints. No startup costs.
Not plotted: Pumped Storage , Hydro, Wind, Solar.
We are plotting curves from cost estimates, not bids.

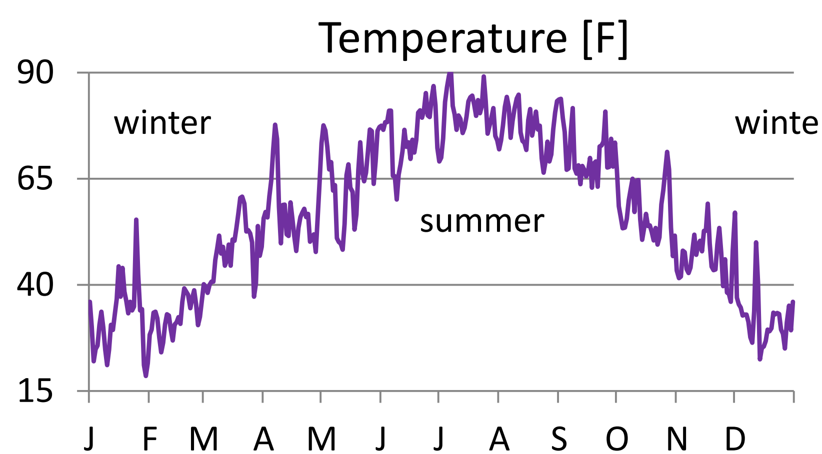
Offer dynamics for peaker units

daily bids of a combustion turbine bidding a single price-quantity block, year 2010









Multistage stochastic unit commitment

Stochastic formulation with **startup decision time lags δ_j** (12h, 6h, 3h, 1h,…)

given $\{W_{tj}\}$: random process for variable energy resource j in J^{VER}

$\{L_t\}$: random demand process

\# periods   # units   0-1 indicator of startup at time t   output of unit j

minimize $\mathbb{E}\{ \sum_{t=1}^{T} \sum_{j=1}^{J} c_{tj}^{start}\, v_{t-\delta_j,\, tj} + c_{tj} p_{ttj} \}$ startup & energy cost

startup cost   energy unit-cost

subject to $\sum_{j=1}^{J} p_{ttj} = L_t$ a.s., for each t energy balance

(assuming NO demand-side flexibility)

constraints for dispatchable $j \in J^D$, for each t:

decision for time t'

0-1 shutdown indicator

0-1 online state

$t\; t'\; j$ unit j

F_t-measurable

$v_{t-\delta_j,\, tj} - w_{t-\delta_j,\, tj} = u_{t-\delta_j,\, tj} - u_{t-\delta_j-1,\, t-1,\, j}$ lagged startup decisions

$u_{t-\delta_j,\, tj}$ $\underline{P}_j \le p_{ttj} \le u_{t-\delta_j,\, tj} \overline{P}_j$ capacity constraints

$-R_j^{down} \le p_{ttj} - p_{t-1,\, t-1,\, j} \le R_j^{up}$ ramping constraints

(simplified statement)

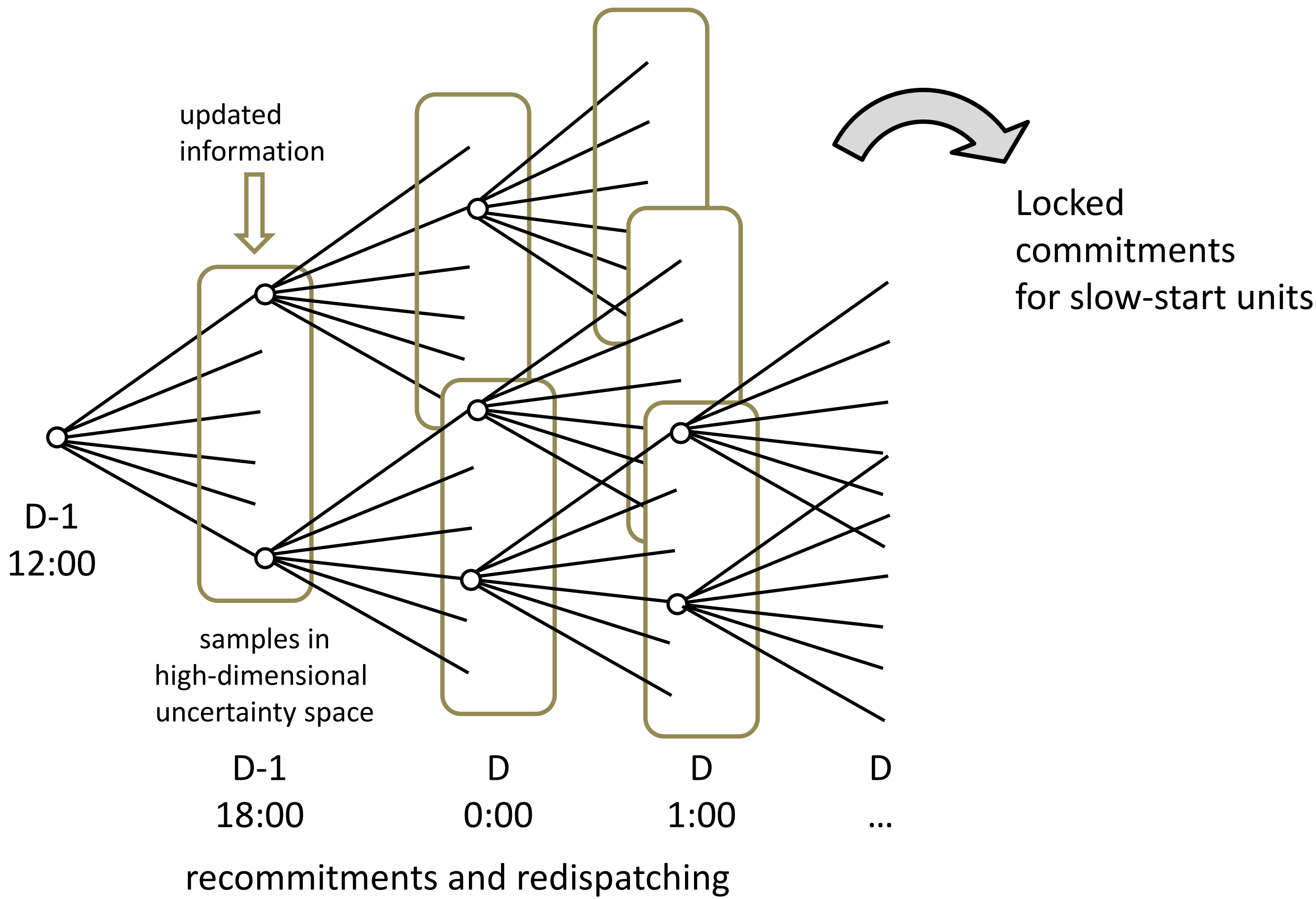
…

constraints for variable energy resources j in J^{VER}

$p_{ttj} \le u_{t-\delta_j,\, tj} W_{tj}$ a.s., for each t [curtailment]

$u_{t-\delta_j,\, tj}\, ,\, v_{t-\delta_j,\, tj}\, ,\, w_{t-\delta_j,\, tj} \in \{0,1\}.$

Multistage stochastic unit commitment



Two-stage stochastic unit commitment

Stochastic MILP formulation in the day-ahead paradigm:

Time lags δ_j valued in {12h, 0h} only (slow- and fast- start).

minimize $\mathbb{E}\{ \sum_{t=1}^{T} \sum_{j=1}^{J} c_{tj}^{start}\, v_{t-\delta_j,\, tj} + c_{tj} p_{t-\delta_j, tj} \}$

subject to $\sum_{j=1}^{J} p_{ttj} = L_t$ a.s., for each t

constraints for dispatchable $j \in J^D$:

$$\left.\begin{array}{l} v_{0tj} - w_{0tj} = u_{0tj} - u_{0,\, t-1,\, j} \\ u_{0tj} \underline{P}_j \le p_{ttj} \le u_{0tj} \overline{P}_j \end{array}\right\}$$

j in slow-start units: lock the day-ahead startups

$$\left.\begin{array}{l} v_{ttj} - w_{ttj} = u_{ttj} - u_{t-1,\, t-1,\, j} \\ u_{ttj} \underline{P}_j \le p_{ttj} \le u_{ttj} \overline{P}_j \end{array}\right\}$$

j in fast-start units: do not lock day-ahead startups

$$-R_j^{down} \le p_{ttj} - p_{t-1,\, t-1,\, j} \le R_j^{up}$$

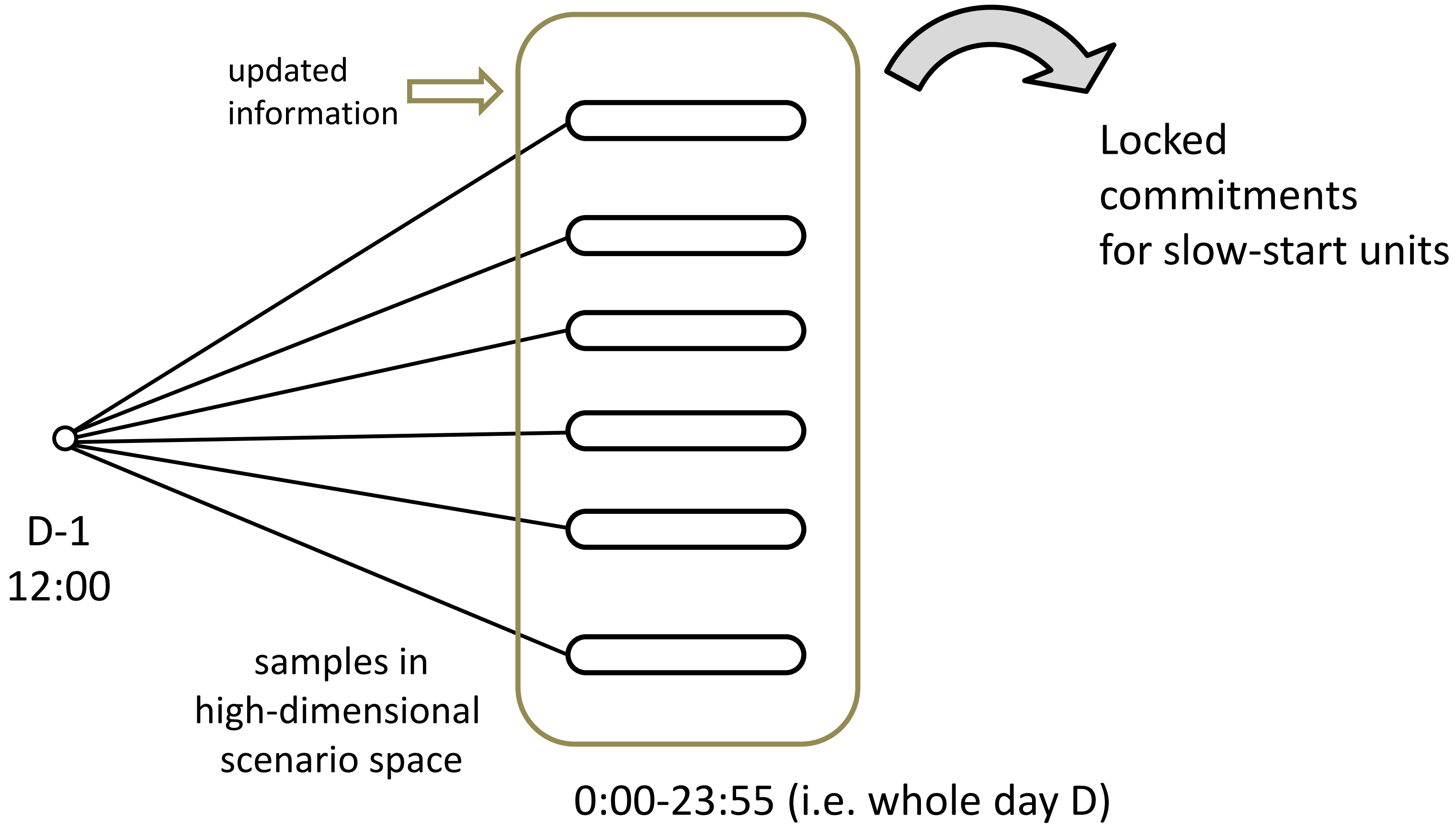
constraints for variable energy resources j in J^{VER}

$p_{ttj} \le u_{ttj} W_{tj}$ a.s., for each t

u_{0tj}, v_{0tj}, w_{0tj} (j slow), u_{ttj}, v_{ttj}, w_{ttj} (j fast) $\in \{0,1\}$.

Each u_{0tj} (j slow start) is implemented as a here-and-now decision.

Two-stage stochastic unit commitment



Perfect dispatch over day D
(since whole day is visible)

Deterministic unit commitment

output of variable energy source

load

W_{tj}, L_t are set to forecasts $\overline{W_{0tj}}$, $\overline{L_{0t}}$.

minimize $\quad \sum_{t=1}^{T} \sum_{j=1}^{J} c_{tj}^{start} v_{0tj} + c_{tj} p_{0tj}$

subject to $\quad \sum_{j=1}^{J} p_{0tj} = \overline{L_{0t}}$

$\sum_{j=1}^{J^D} (u_{0tj} \overline{P}_j - p_{0tj}) \geq \overline{S_{0t}}$ reserve requirements

constraints for dispatchable $j \in J^D$:

decision for time t

$0\ t\ j$ unit j

F_0-measurable

$v_{0tj} - w_{0tj} = u_{0tj} - u_{0,t-1,j}$

$u_{0tj} \underline{P}_j \leq p_{0tj} \leq u_{0tj} \overline{P}_j$

$-R_j^{down} \leq p_{0tj} - p_{0,t-1,j} \leq R_j^{up}$

constraints for variable energy resources j in J^{VER}

$p_{0tj} \leq u_{0tj} \overline{W_{0tj}}$

$u_{0tj}, v_{0tj}, w_{0tj} \in \{0,1\}$.

Each u_{0tj} (j slow start) is implemented as here-and-now decision.

Practical complexity of stochastic unit commitment

1968
Early stochastic mixed-integer linear programming (MILP) model for unit commitment

1990
parallel computing for solving stochastic programs

2006
Convex multistage stochastic programming is intractable (*)

2010
PJM completes a 6-year effort of deploying and integrating its security-constrained MILP unit commitment

J. Muckstadt and R. Wilson, "An application of mixed-integer programming duality to scheduling thermal generating systems," *IEEE Trans. Power Apparatus and Systems*, vol. 87, no. 12, pp. 1968–1978, 1968.

M. Avriel, G. Dantzig, and P. Glynn, "Decomposition and parallel processing for large-scale electric power system planning under uncertainty," in *Proc. NSF Workshop on Resource Planning Under Uncertainty*, 1990, pp. 3–34.

(*) For generic convex programs, using the sample average approximation

A. Shapiro, "On complexity of multistage stochastic programs," *Oper. Res. Let.*, vol. 34, no. 1, pp. 1–8, 2006.

A. Ott, "Experience with PJM market operation, system design, and implementation," *IEEE Trans. Power Systems*, vol. 18, no. 2, pp. 528–534, 2003.

Abstract idealized setup:

Dream: *solve the 2-stage MILP model*

1st-stage decision; probability of scenario k

SP: $\min f(\mathbf{x}) + \sum_{k=1}^{K} p_k\, g(\mathbf{x}, \mathbf{y}_k, \boldsymbol{\xi}_k)$
s.t. $\mathbf{x} \in \mathcal{X},\ \mathbf{y}_k \in \mathcal{Y}(\mathbf{x}, \boldsymbol{\xi}_k)$ k=1,…,K.

2nd-stage decisions; scenario k

Reality:
We have tools to reduce to 1-2% the optimality gap of the MILP

P(ξ): $\min f(\mathbf{x}) + g(\mathbf{x}, \mathbf{y}, \boldsymbol{\xi})$
s.t. $\mathbf{x} \in \mathcal{X},\ \mathbf{y} \in \mathcal{Y}(\mathbf{x}, \boldsymbol{\xi})$.

Best-Deterministic Approximation

- Let v^*, S be the optimal value and first-stage solution set of the stochastic program. Let $x^* \in S$.
- Let $v(x)$ be the optimal value of the stochastic program when the first-stage decision is fixed to x. We have $v(x^*)=v^*$ for all $x^* \in S$.

  $v(x)$ can be evaluated by optimizing separately over each scenario.
- Let $S'(\xi)$ be the optimal first-stage solution set of the stochastic program with its probability distribution degenerated to ξ. Let $x'(\xi) \in S'(\xi)$.
- Value of the Stochastic Solution [Birge 1982]:

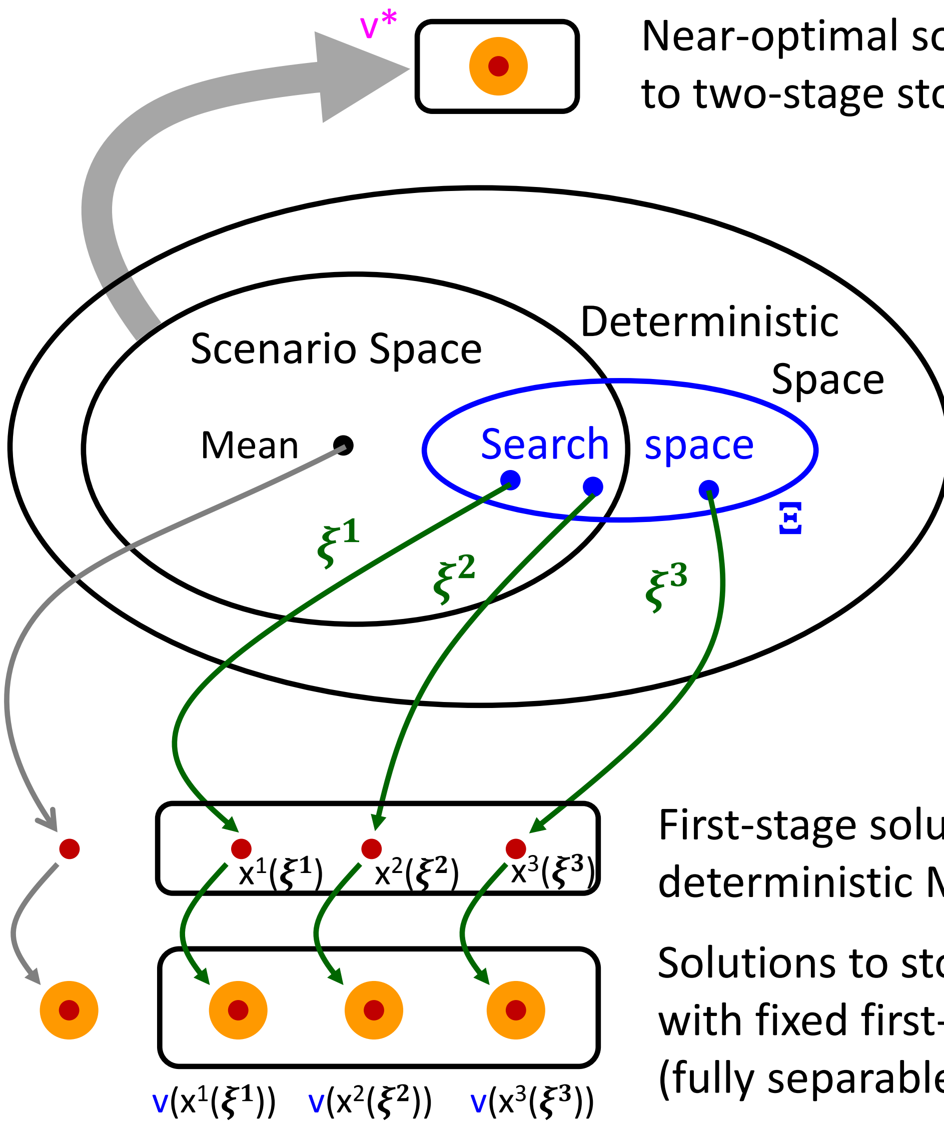
  $VSS = v(x'(\bar{\xi}))-v(x^*)$ where $\bar{\xi}= \sum_{k=1}^{K} p^k \xi^k$

  J.R. Birge, The value of the stochastic solution in stochastic linear programs with fixed recourse, Math. prog. 24, 314-325, 1982.
- Value of the stochastic solution over the best-deterministic solution:

  $VSS^{BD} = \inf_{\xi \in \Xi} [v(x'(\xi))- v(x^*)]$ for Ξ : space easy to cover.
- Best-deterministic approximation:

  Try to find $\xi^* \in \mathrm{argmin}_{\xi \in \Xi} v(x'(\bar{\xi}))$ and then implement $x'(\xi^*)$.

Pictorial representation for the VSS-BD



Near-optimal solution to two-stage stochastic MILP

Goal:

minimize VSS^{BD}

given search space, cpu time budget.

First-stage solution to deterministic MIP

Solutions to stochastic MILP with fixed first-stage decision (fully separable).

“Best-Deterministic” unit commitment

In our tests, we take quantiles of the predictive distributions

W_{tj} , L_t are set to planning forecasts $\widetilde{W_{0tj}}$, $\widetilde{L_{0t}}$.

minimize $\quad \sum_{t=1}^{T} \sum_{j=1}^{J} \boxed{\tilde{c}_{tj}^{\,\mathrm{start}}}\, v_{0tj} + c_{tj}\, p_{0tj}$

subject to $\quad \sum_{j=1}^{J} p_{0tj} = \boxed{\widetilde{L_{0t}}}$

$\sum_{j=1}^{J^D} (u_{0tj}\, \overline{P}_j - p_{0tj}) \geq \boxed{\widetilde{S_{0t}}}$

reserve needs may be added/modified.

constraints for dispatchable $j \in J^D$:

decision for time t

$0\ t\ j$ unit j

F_0-measurable

$v_{0tj} - w_{0tj} = u_{0tj} - u_{0,\,t-1,\,j}$

$u_{0tj}\ \ \underline{P}_j \leq p_{0tj} \leq u_{0tj} \overline{P}_j$

$-R_j^{down} \leq p_{0tj} - p_{0,\,t-1,j} \leq R_j^{up}$

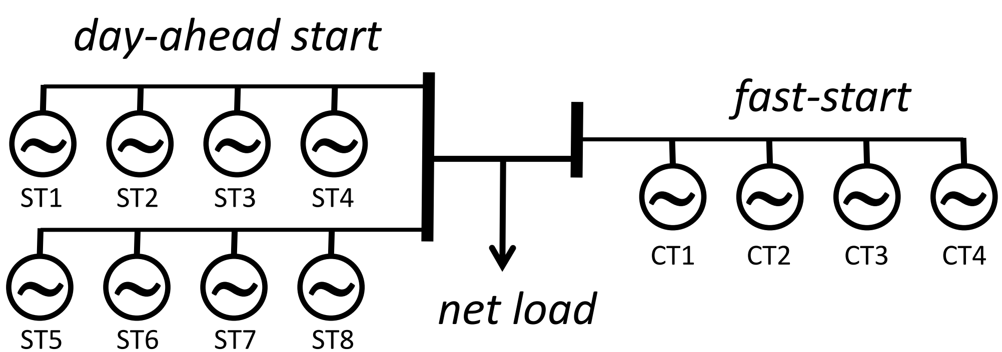
constraints for variable energy resources j in J^{VER}

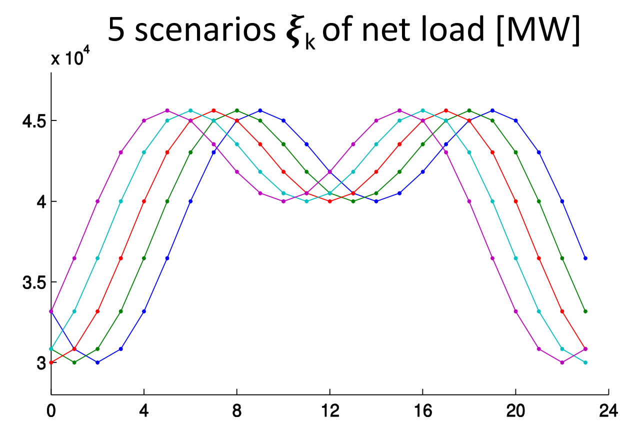
$p_{0tj} \leq u_{0tj} \boxed{\widetilde{W_{0tj}}}$ *planning forecast*

u_{0tj} , v_{0tj} , $w_{0tj} \in \{0,1\}$.

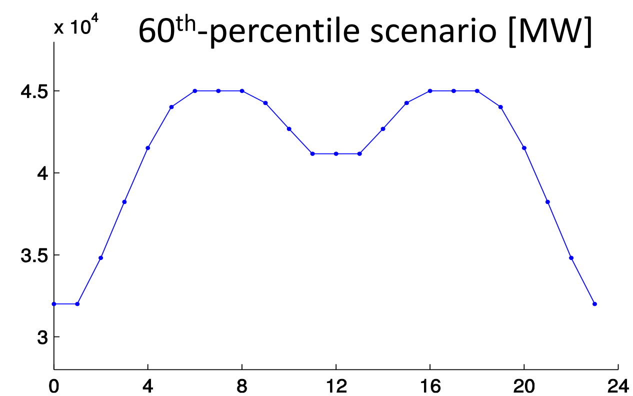
Each u_{0tj} (j slow start) is implemented as here-and-now decision.

VSS-BD for unit commitment (test 1)





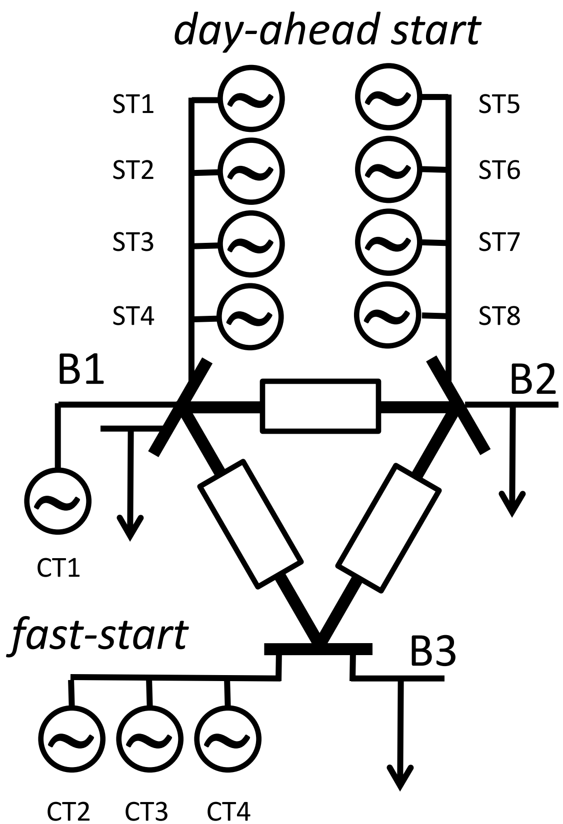
	Expected Cost	Time [s]	Gap [%]	Loss [%]
Stochastic MIP high-accuracy	**2.70335e+07**	**285.93**	**0.10**	**0.00**
Stochastic MIP	2.70501e+07	9.11	0.46	0.06
Middle scenario	2.78027e+07	2.90	0.48	2.85
Mean scenario	2.71157e+07	1.56	0.46	0.30
50-quantile	2.77531e+07	0.92	0.41	2.66
60-quantile	**2.70375e+07**	**0.75**	**0.48**	**0.01**
70-quantile	2.73184e+07	0.21	0.33	1.05



code: www.princeton.edu/~defourny/MIP_UC_example.m

VSS-BD for unit commitment (test 2)

Test with transmission constraints.



3 x 3 x 3 net load scenarios

	Expected Cost	Time [s]	Gap [%]	Loss [%]
Stochastic MIP	**2.00287e+07**	**19481.00**	**0.50**	**0.00**
Stochastic MIP low accuracy	**2.00552E+07**	**1657.00**	**0.85**	**0.13**
60-60-60 quantile	2.04341e+07	31.67	0.50	2.02
60-60-70 quantile	2.01821e+07	0.84	0.43	0.77
60-70-60 quantile	2.01821e+07	0.78	0.45	0.77
70-60-60 quantile	2.04505e+07	0.81	0.43	2.11
60-70-70 quantile	2.01514e+07	1.79	0.42	0.61
60-70-70 quantile high accuracy	**2.00866e+07**	**2.45**	**0.00**	**0.30**
70-70-60 quantile	2.01514e+07	1.51	0.47	0.61
70-70-60 quantile high accuracy	**2.00866e+07**	**2.15**	**0.10**	**0.30**
70-60-70 quantile	2.01514e+07	1.09	0.47	0.61
70-60-70 quantile high accuracy	**2.00866e+07**	**4.87**	**0.09**	**0.30**
70-70-70 quantile	2.06064e+07	3.24	0.48	2.88

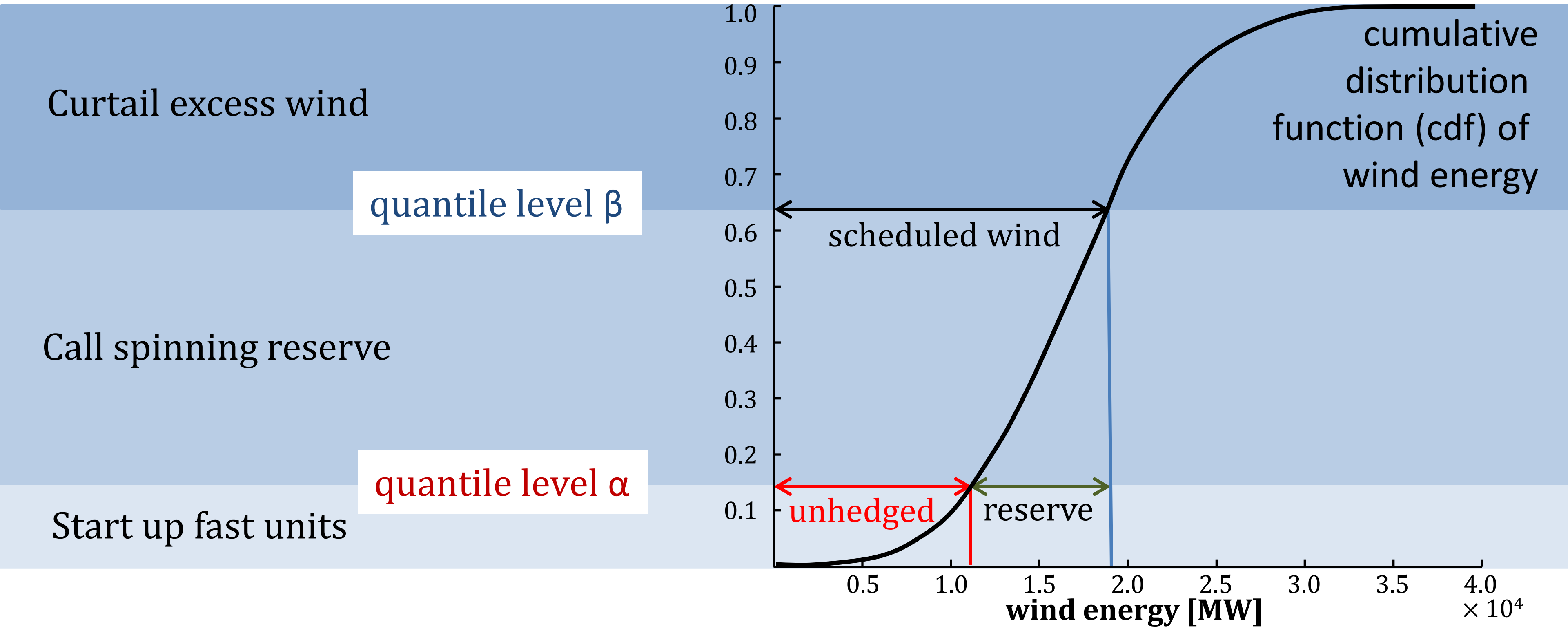
code: www.princeton.edu/~defourny/MIP_UC_3node.m

Guiding the search

Rather than finding a best-deterministic solution by direct search, we could compute a priori a single scenario (by stochastic programming).

- Stochastic optimization of wind forecasts and reserve requirements

Optimization of the wind that can be scheduled in day-ahead, along with various reserves for hedging against wind being lower than expected, using a very simplified expression of the costs and constraints.



Optimality of quantile solutions

Let us recall a textbook result:

The newsvendor problem

$$\text{Max} \quad -c\,x + \mathbb{E}\{\, p \;\min[x, D]\}$$

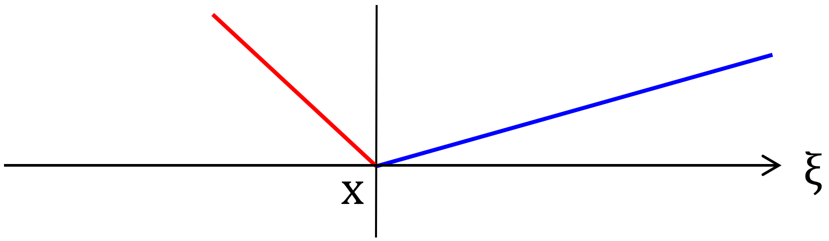
where $0 < c < p$, and D is a r.v. with cdf G (demand)

admits the optimal solution $\quad x = G^{-1}(\alpha), \quad \alpha = (p - c)/p$.

- $x = G^{-1}(\alpha)$ is a quantile of the distribution of ξ .
- The same problem can also be written as

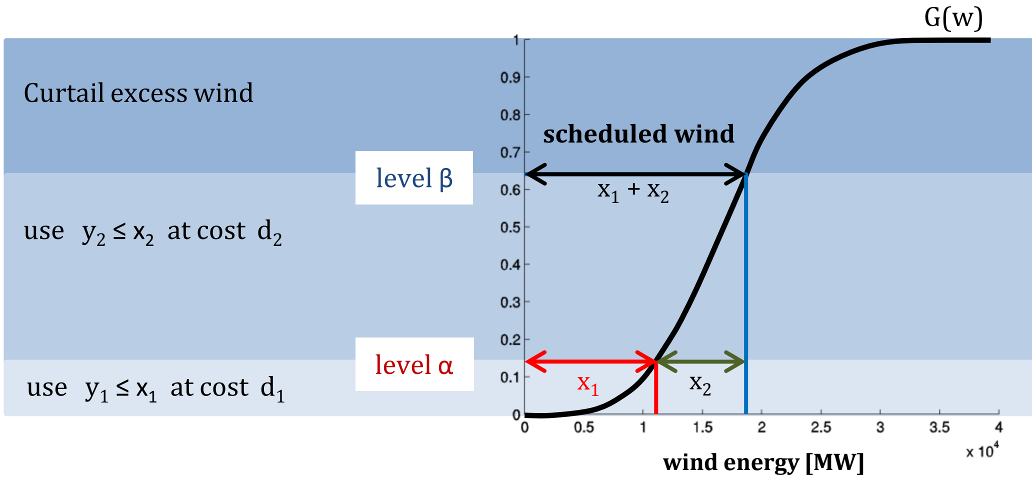
$$\text{Min}\ \mathbb{E}\{ \quad (c\text{-}p)\, D \quad + \quad c\,[\, x - D\,]^+ \quad + \quad (p\text{-}c)\,[\, D - x]^+ \,\} .$$

exogenous   overage cost   underage cost



Extension to multiple quantiles

Let $0 \le c_1 < c_2 < c_3 < d_2 < d_1$.
Let w (wind) be a positive,
abs. cont. r.v., with cdf *G*.
Let $L > 0$ (fixed load; dedicated reserve assumed to be in place.)



Proposition:

The stochastic program

$$\text{minimize} \quad c_1x_1 + c_2x_2 + c_3x_3 + E\{d_1y_1 + d_2y_2\}$$

$$\text{subject to} \quad x_1 + x_2 + x_3 = L, \quad x_3 \ge 0 \qquad \text{(day-ahead schedule meets load)}$$

$$w + y_1 + y_2 \ge x_1 + x_2 \quad \text{a.s.} \qquad \text{(compensation of missing wind)}$$

$$0 \le y_1 \le x_1, \; 0 \le y_2 \le x_2 \quad \text{a.s.} \qquad \text{(consequence of reserve choices)}$$

admits an optimal solution based on quantiles as long as $x_3 \ge 0$.

$x_1 + x_2$: total wind energy to be "scheduled" day-ahead.
x_3 : energy from dispatchable units committed in day-ahead (rarely < 0.)

Recursive algorithm

Function $(x_1, \ldots, x_n) = \textbf{SOLVE}(c_1, \ldots, c_n, d_1, \ldots, d_{n-1}, L;\ G)$

Step 1. Define $\alpha_i = (c_{i+1} - c_i)/(d_i - d_{i+1})$, $i = 1,\ldots, n-1$, where $d_n = 0$.

Quantile levels

Step 2. If $J = \{ i : \alpha_i < \alpha_{i-1} \}$ is empty, go to Step 3.

Otherwise: select $j = \inf J$.

Set $x_j = 0$.

Set $(x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_n)$

$= \textbf{SOLVE}(c_1, \ldots, c_{j-1}, c_{j+1}, \ldots, c_n,$

$d_1, \ldots, d_{j-1}, d_{j+1}, \ldots, d_{n-1}, L; G)$.

Return $(x_1, \ldots, x_n)$.

Recursive call on reduced input

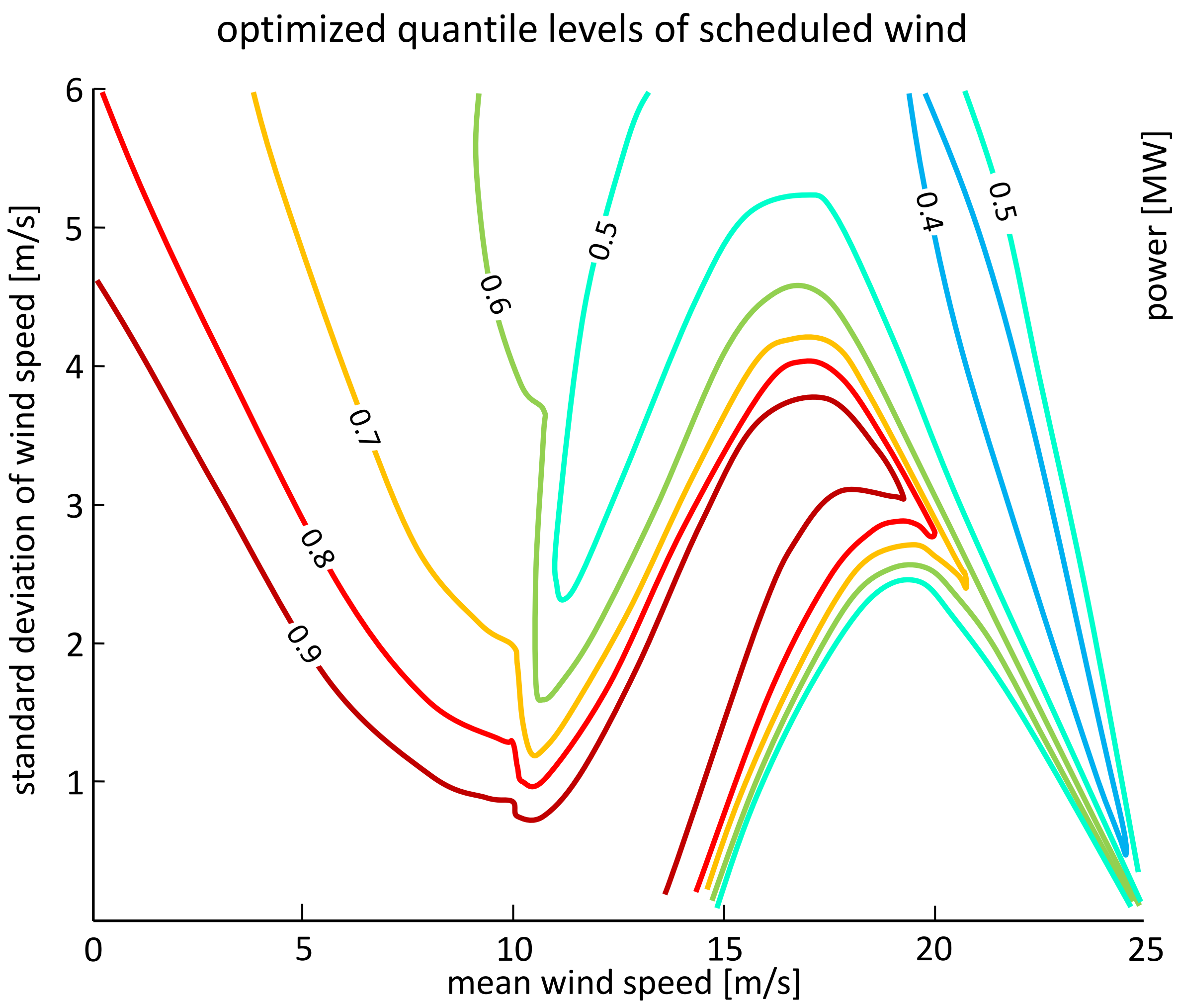
Step 3. Set $x_1 = G^{-1}(\alpha_1)$, $x_i = G^{-1}(\alpha_i) - G^{-1}(\alpha_{i-1})$,

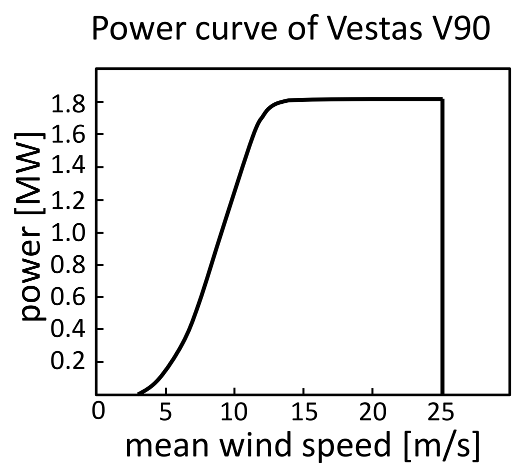
$x_n = L - (x_1 + \ldots + x_{n-1})$. Return $(x_1, \ldots, x_n)$.

Quantile solution

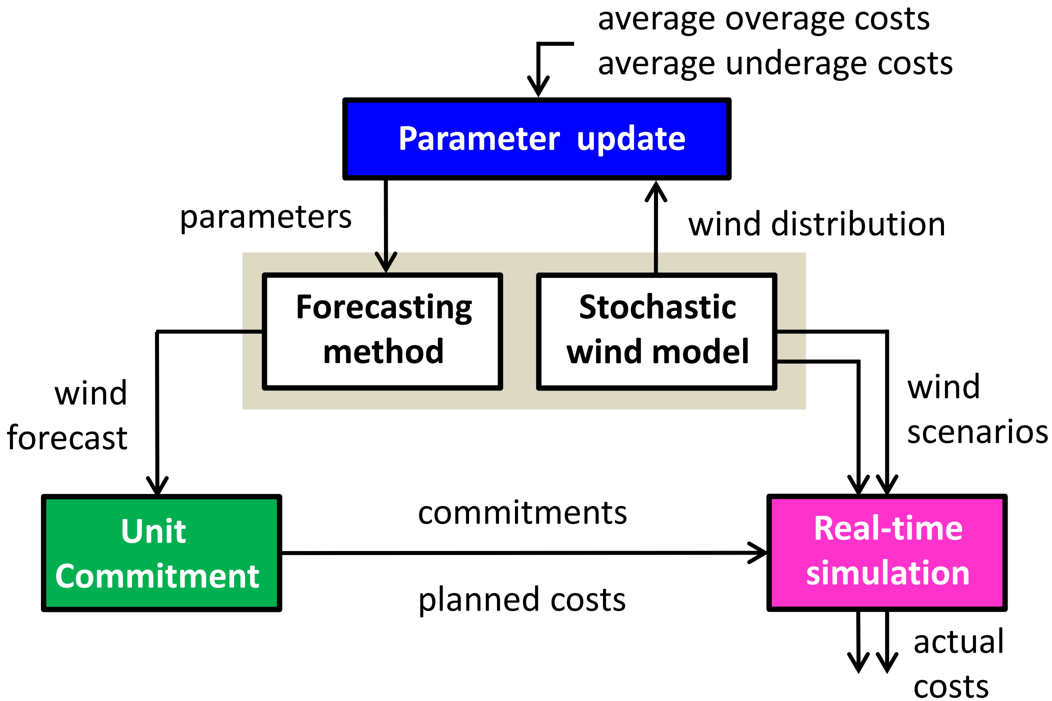
Shows that the optimal solution is formed of zeros and differences of quantiles.

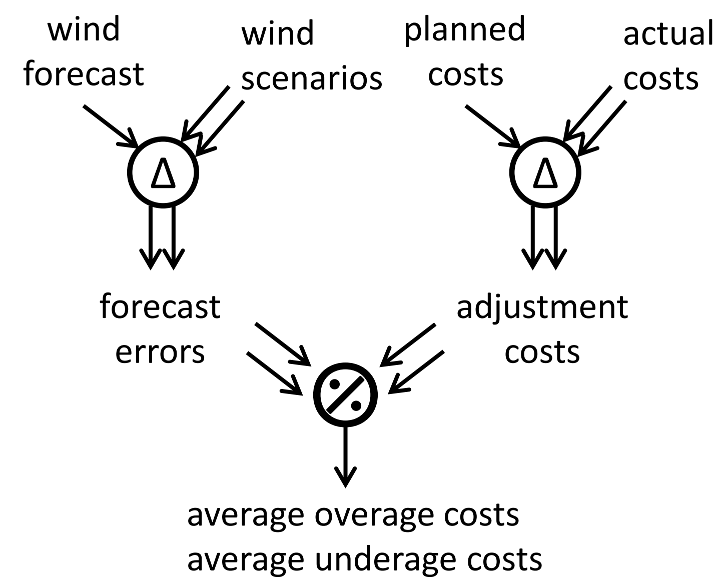
Quantile levels as a function of wind speed mean and standard deviation





Learning algorithm

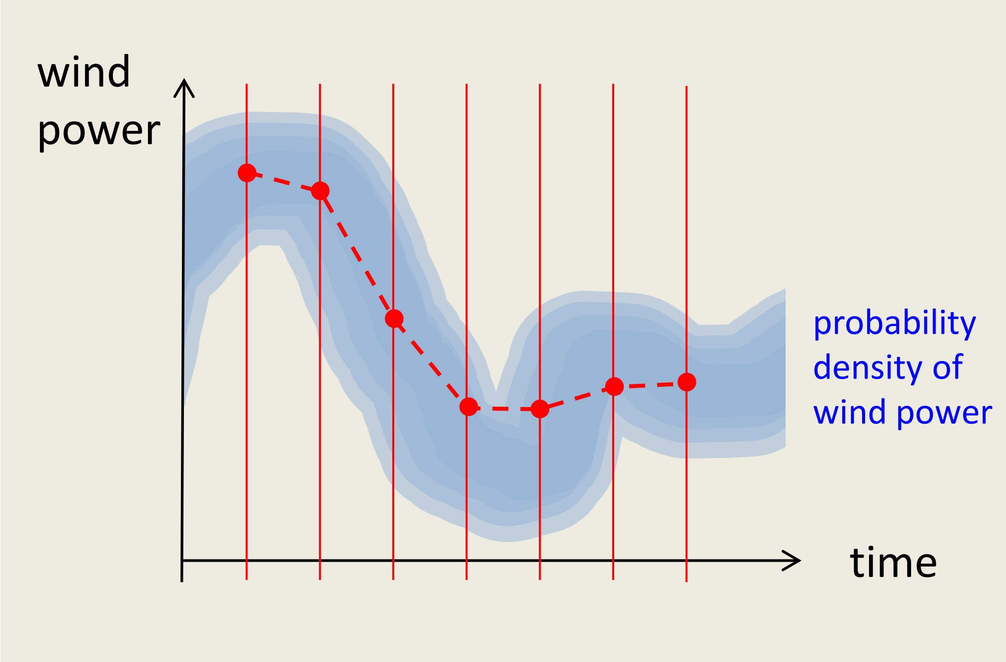




1. Start with some parameters for setting the wind forecast $\bar{Y}_1, \ldots, \bar{Y}_T$.
2. Solve the UC problem given the forecast.
3. Given simulations of forecast errors and adjustment costs,
   estimate average overage & underage costs $C_1^+, \ldots, C_T^+; \; C_1^-, \ldots, C_T^-$.
4. Update the parameters and go back to Step 2.

BD, H.P. Simao, W.B. Powell, "Robust forecasting for unit commitment with wind", Proc. *46th Hawaii International Conference on System Sciences*, Maui, HI, January 2013.

Forecast parameter update



quantile level

$$q_t = \frac{c_t^-}{c_t^+ + c_t^-}$$

If forecasting too much wind is relatively expensive, the quantile level will decrease.

forecasted wind

$$y_t = F_t^{-1}(q_t)$$

X-quantile forecasts

- Goal: explaining the successive quantile levels by other processes, such as the load. Let X_t be that process. Let $\bar{X}_t$ be its forecast.
- Justification: the cost of adjustments is influenced by the state of the grid (load, congestions, …)

quantile level

$$q_t = \frac{c_t^-}{c_t^+ + c_t^-}$$

quantile level function

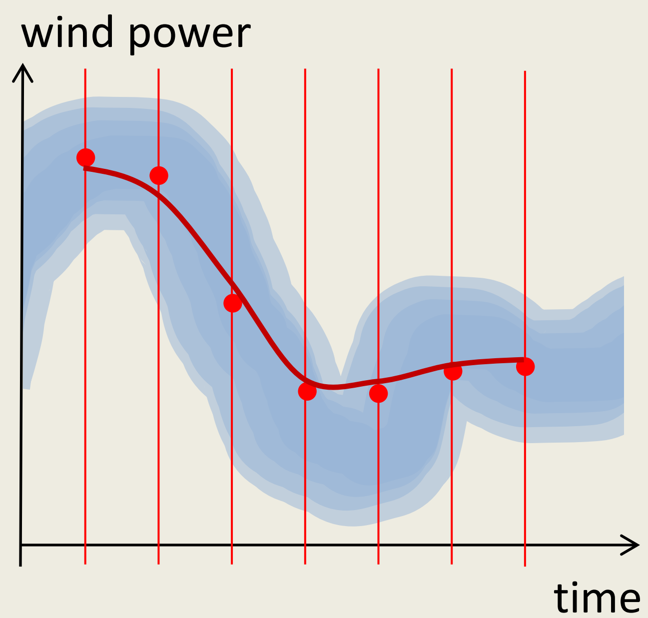
$$\rho(\bar{X}_t) \simeq q_t$$

regression model

$$\rho(\bar{X}_t) = \frac{1}{1+e^{-(\alpha+\beta\cdot\bar{X}_t)}}$$

forecast parameters

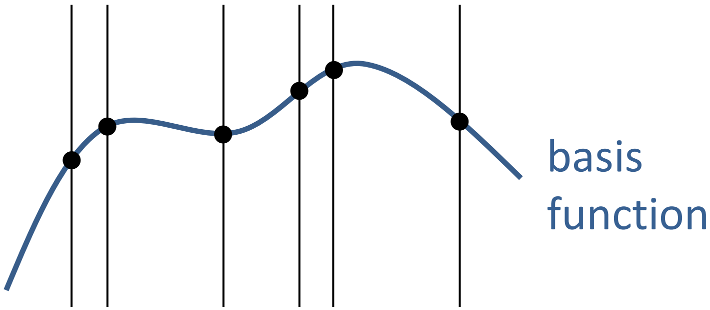
α, β



$$y_t = F_t^{-1}[\rho(\bar{X}_t)]$$

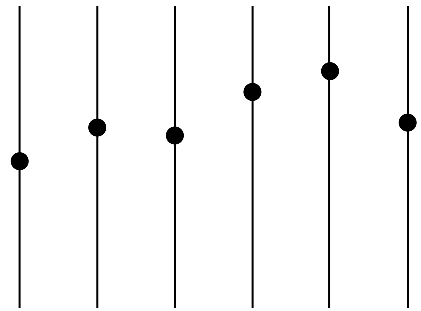
Numerical test: Stochastic processes

1. Sample N times uniformly in [0,T]

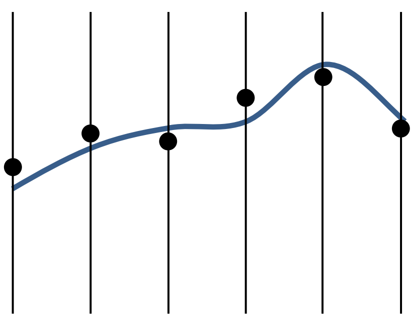


(sort the sampled times)

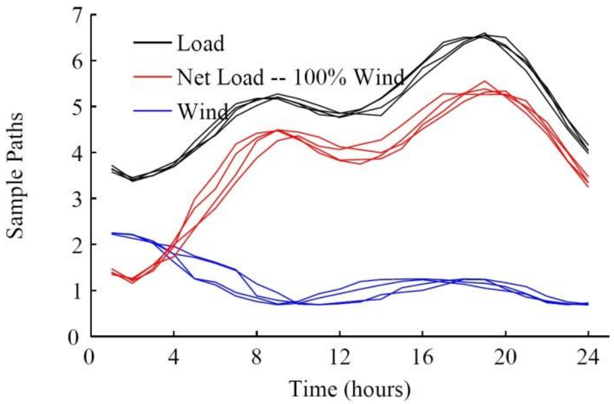
2. Use the N values of the function with uniform time increments



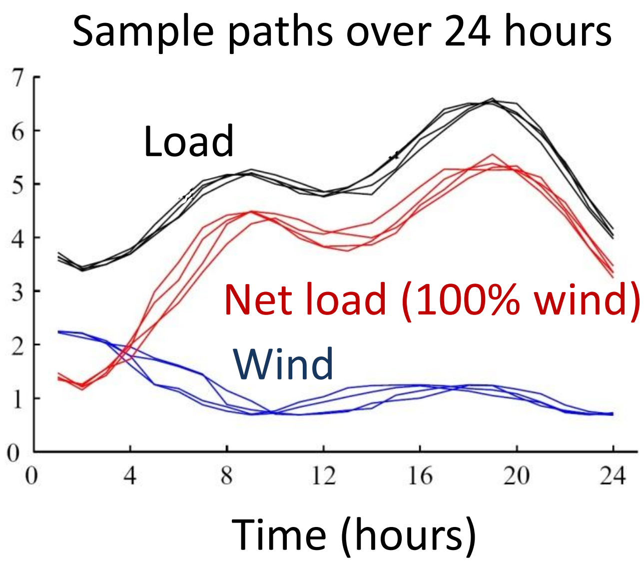
3. Add "vertical" noise

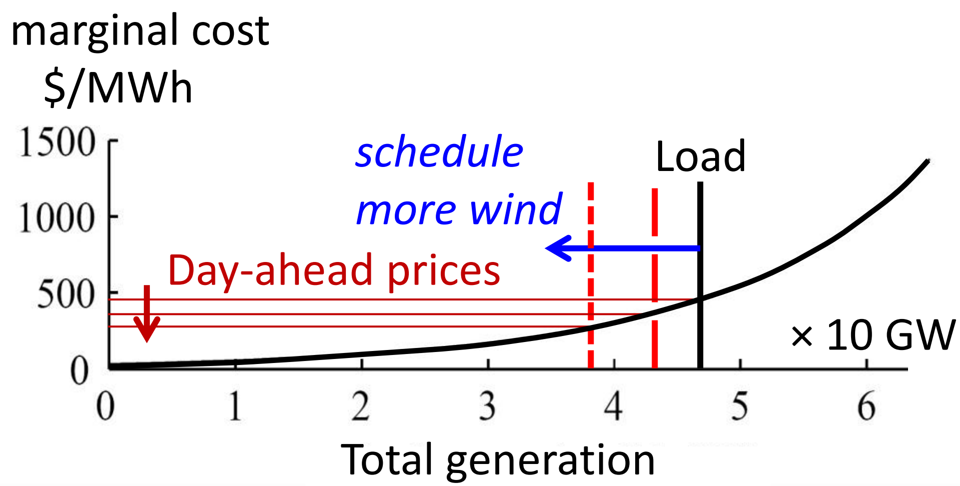


Processes with random time shifts and random magnitude shifts

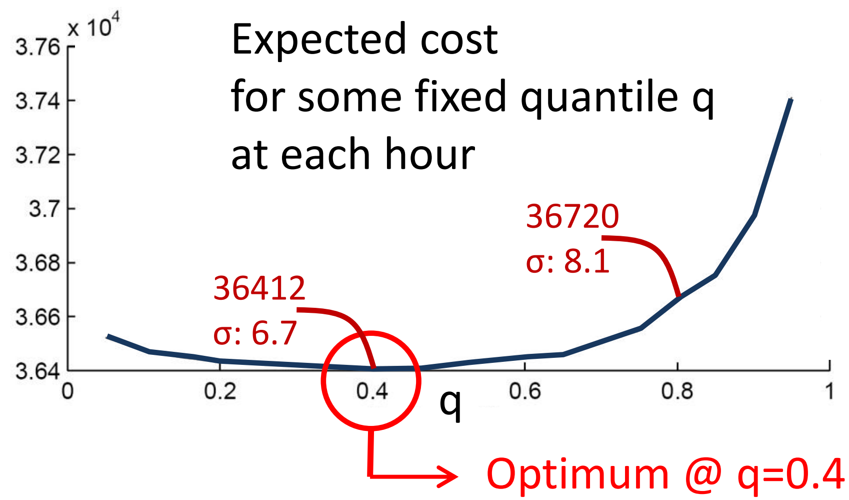


Direct quantile search

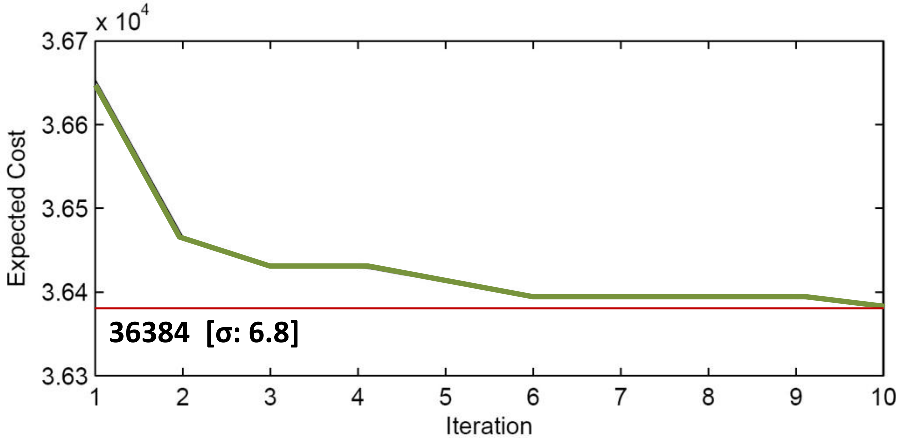




empirical
distributions
from sample paths

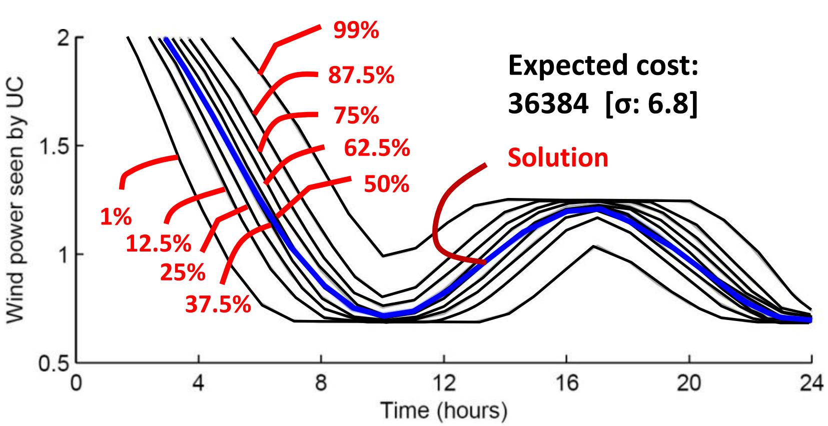


Learned time-varying quantiles

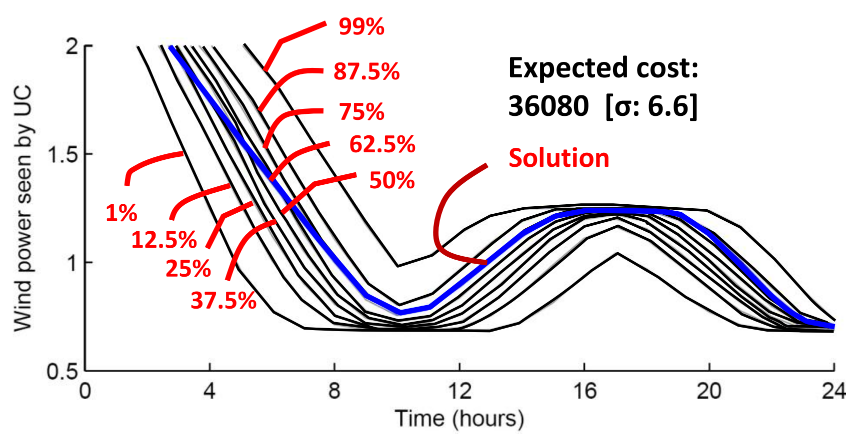


Wind forecast	Expected cost	Std error
q_t = 0.8 ∀ t	36720	8.1
q_t = 0.4 ∀ t	36412	6.7
q_t: Left	36384	6.8
q_t: Right	36080	6.6

Wind forecast @ iter 10



Wind forecast @ iter 10



Summary of the talk

- Value of the stochastic solution over the best-deterministic solution.
- Best-deterministic approximation presented as a particular algorithmic approach to two-stage stochastic unit commitment.
- Search space based on quantiles: the motivation is that quantile solutions can be optimal for wind and reserve scheduling without capacity constraints.

codes: www.princeton.edu/~defourny/MIP_UC_example.m
www.princeton.edu/~defourny/MIP_UC_3nodes.m

Thank you!

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