# Urban Link Travel Time Estimation Using Large-scale Taxi Data with Partial Information 

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## Outline

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- Link Travel Time Estimation Model
- Base Model
- Probabilistic Model
- Numerical Results
- Conclusion
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## Introduction

- New York City has the largest market for taxis in North America:
- 12,779 yellow medallion (2006)
- Industrial revenue $\$ 1.82$ billion (2005)
- Serving 240 million passengers per year
- $71 \%$ of all Manhattan residents' trips
- GPS devices are installed in each taxicab

- Taxi data recorded by New York Taxi and Limousine Commission (NYTLC)
- Massive amount of data!
- 450,000 to 550,000 daily trip records
- More than 180 milliion taxi trips a year
- Providing a lot of opportunities!


## Introduction

$\square$ Taxi trips in NYC


Trip Origin


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## Introduction

## Estimating urban link travel times

- Traditional approaches:
- Loop detector data
- Automatic Vehicle Identification tags
- Video camera data
- Remote microwave traffic sensors
- Why taxicab data?

- Novel large-scale data sources
- Ideal probes monitoring traffic condition
- Large coverage
- Do not need fixed sensors
- Cheap!


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## Introduction

$\square$ The data

- NYTLC records taxi GPS trajectory data, but not public
- Only trip basis data available
- Contains only OD coordinate, trip travel time and distance, etc.
- Path information not available
- Large-scale data with partial informationThe problem
- Given large-scale taxi OD trip data, estimate urban link travel times
- Sub-problems to solve:
- Map data to the network
- Path inference
- Estimate link travel time based on OD data


## Study Region

- $1370 \times 1600 \mathrm{~m}$ rectangle area in Midtown Manhattan
- Data records fall within the region are subtracted



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## Study Region

$\square$ Test network

- Network contains:
- 193 nodes
- 381 directed links


## PuRDUE



## Study Region

$\square$ Number of observations in the study region

- Day 1: Weekday (2010/03/15, Monday)
- Day 2: Weekend (2010/03/20, Saturday)

Histogram for day 1


Histogram for day 6


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## Base Model

## Base link travel time estimation model

- Hourly average link travel time estimations
- Direct optimization approach
- Overall framework: four phases


Link Travel Time Estimation

Estimate link travel times by solving an optimization problem

* Zhan, X., Hasan, S., Ukkusuri, S. V., \& Kamga, C. (2013). Urban link travel time estimation using large-scale taxi data with partial information. Transportation Research Part C: Emerging Technologies, 33, 37-49.


## Base Model

$\square$ Data mapping

- Mapping points to nearest links in the network
- Mapped point (blue) are used
- Identify intermediate origin/ destination nodes
- $\alpha_{1}, \alpha_{2}$ are defined as distance proportions from mapped points to the intermediate origin/destination node



## PuRDUE

## Base Model

$\square$ Construct reasonable path sets

- Number of possible paths could be huge!
- Need to shrink the size of possible path set
- Use trip distance to eliminate unreasonable paths
- K-shortest path algorithm ${ }^{*}(\mathrm{k}=20)$ is used to generate initial path sets
- Filter out unreasonable paths (threshold: weekday $15 \% \sim 25 \%$, weekend $50 \%$ )

[^0]
## Base Model

$\square$ Route choice model

- Assumption:
- Each driver wants to minimize both trip time and distance to make more trips thus make more revenue
- A MNL model based on utility maximization scheme

$$
P_{m}(\vec{t}, d, \theta)=\frac{e^{-\theta C_{m}\left(\vec{t}, d_{m}\right)}}{\sum_{j \in R_{i}} e^{-\theta C_{j}\left(\vec{t}, d_{j}\right)}}
$$

- Path cost measured as a function of trip travel time and distance

$$
\begin{array}{r}
C_{m}\left(\vec{t}, d_{m}\right)=\beta_{1} \cdot g_{m}(\vec{t})+\beta_{2} \cdot d_{m} \\
g_{m}(\vec{t})=\alpha_{1} t_{O}+\alpha_{2} t_{D}+\sum_{l \in L} \delta_{m l} t_{l}
\end{array}
$$

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## Base Model

$\square$ Link travel time estimation

- Minimizing the squared difference between expected $\left(E\left(Y_{i} \mid R_{i}\right)\right)$ and observed $\left(Y_{i}\right)$ path travel times

$$
\begin{aligned}
& E\left(Y_{i} \mid R_{i}\right)=\sum_{m \in R_{i}} g_{m}(\vec{t}) P_{m}(\vec{t}, d, \theta) \\
& \vec{t}=\underset{\vec{t}}{\arg \min } \sum_{i \in D}\left(y_{i}-E\left(Y_{i} \mid R_{i}\right)\right)^{2}
\end{aligned}
$$

- Solve using Levenberg-Marquardt (LM) method
- Parallelized codes developed to estimate the model
- Entire optimization solved within 10 minutes on an intel i7 laptop
- Numerical results show in later section


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## Probabilistic Model

$\square$ Limitations of the base model

- Point estimate of hourly average travel time
- Not incorporating variability of link travel times
- Not utilizing historical data
- Problems of compensation effect
- Less robust
$\square$ Solution: Adopt a probabilistic framework
- Accounting for variability in link travel times
- More robust
- Historical information can be incorporated as priors


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## Probabilistic Model

$\square$ Assumptions:

1. Link travel time: $x_{l} \sim \mathcal{N}\left(\mu_{l}, \sigma_{l}^{2}\right)$
2. Path travel time is the summation of a set of link travel times

$$
P\left(y_{i} \mid k, \boldsymbol{x}\right)=P\left(y_{i} \mid k, \boldsymbol{\mu}, \boldsymbol{\Sigma}\right)=N\left(\alpha_{1} \mu_{0}+\alpha_{2} \mu_{D}+\sum_{l \in k} \mu_{l},\left(\alpha_{1} \sigma_{o}\right)^{2}+\left(\alpha_{2} \sigma_{D}\right)^{2}+\sum_{l \in k} \sigma_{l}^{2}\right)
$$

3. Route choice based on the perceived mean link travel times and distance

$$
\pi_{k}^{i}\left(\boldsymbol{\mu}, \boldsymbol{\beta}, d_{i}\right)=\frac{\exp \left[-C_{k}^{i}\left(\boldsymbol{\mu}, \boldsymbol{\beta}, d_{i}\right)\right]}{\sum_{s \in R^{i}} \exp \left[-C_{S}^{i}\left(\boldsymbol{\mu}, \boldsymbol{\beta}, d_{i}\right)\right]}
$$

- where $\boldsymbol{x}, \boldsymbol{\mu}, \boldsymbol{\Sigma}$ are the vector of link travel times, their mean and variance


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## Probabilistic Model

$\square$ Mixture model

- A Mixture model is developed to model the posterior probability of the observed taxi trip travel times given link travel time parameters $\boldsymbol{\mu}, \boldsymbol{\Sigma}$

$$
H(\boldsymbol{y} \mid \boldsymbol{\mu}, \boldsymbol{\Sigma}, \boldsymbol{D})=\prod_{i=1}^{n} \sum_{k \in R^{i}} \pi_{k}^{i}\left(\boldsymbol{\mu}, \boldsymbol{\beta}, d_{i}\right) P\left(y_{i} \mid k, \boldsymbol{\mu}, \boldsymbol{\Sigma}\right)
$$

- Introducing $z_{k}^{i}$ as the latent variable indicating if path $k$ is used by observation $i$

Plate notation


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## Probabilistic Model

## Bayesian Mixture model

- Incorporating historical information:
- Prior on $\boldsymbol{\mu}$ :
$H(\boldsymbol{y} \mid \boldsymbol{\mu}, \boldsymbol{\Sigma}, \boldsymbol{D})=\prod_{i=1}^{n} \sum_{k \in R^{i}} \pi_{k}^{i}\left(\boldsymbol{\mu}, \boldsymbol{\beta}, d_{i}\right) P\left(y_{i} \mid k, \boldsymbol{\mu}, \boldsymbol{\Sigma}\right) \cdot \prod_{j \in L} p\left(\mu_{j}\right)$

- Priors on $\boldsymbol{\mu}$ and variance $\boldsymbol{\Sigma}$
$H(\boldsymbol{y} \mid \boldsymbol{\mu}, \boldsymbol{\Sigma}, \boldsymbol{D})=\prod_{i=1}^{n} \sum_{k \in R^{i}} \pi_{k}^{i}\left(\boldsymbol{\mu}, \boldsymbol{\beta}, d_{i}\right) P\left(y_{i} \mid k, \boldsymbol{\mu}, \boldsymbol{\Sigma}\right) \cdot \prod_{j \in L} p\left(\mu_{j}\right) p\left(\sigma_{j}^{2}\right)$


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## Probabilistic Model

$\square$ Solution approach

- An EM algorithm is proposed for estimation
- A iterative procedure of two steps:
- E-step:

$$
\mathbb{E}\left(z_{k}^{i}\right)=\frac{\sum_{z_{k}^{i}} z_{k}^{i}\left[\pi_{k}^{i}\left(\boldsymbol{\mu}, \boldsymbol{\beta}, d_{i}\right) P\left(y_{i} \mid k, \boldsymbol{\mu}, \boldsymbol{\Sigma}\right)\right]^{z_{k}^{i}}}{\sum_{z_{k}^{i}} \sum_{s \in R^{i}}\left[\pi_{s}^{i}\left(\boldsymbol{\mu}, \boldsymbol{\beta}, d_{i}\right) P\left(y_{i} \mid s, \boldsymbol{\mu}, \boldsymbol{\Sigma}\right)\right]^{z_{s}^{i}}}=\gamma\left(z_{k}^{i}\right)
$$

- M-step: Let $\tau_{l}=\sigma_{l}^{2}, \boldsymbol{\tau}=\boldsymbol{\Sigma}$,

$$
\begin{gathered}
Q(\boldsymbol{\mu}, \boldsymbol{\tau})=\mathbb{E}_{\boldsymbol{z}}[\ln P(\boldsymbol{y}, \mathbf{z} \mid \boldsymbol{\mu}, \boldsymbol{\tau})]=\sum_{i=1}^{n} \sum_{k \in R^{i}} \gamma\left(z_{k}^{i}\right)\left[\ln \pi_{k}^{i}\left(\boldsymbol{\mu}, \boldsymbol{\beta}, d_{i}\right)+\ln P\left(y_{i} \mid k, \boldsymbol{\mu}, \boldsymbol{\tau}\right)\right] \\
\left(\boldsymbol{\mu}^{n e w}, \boldsymbol{\tau}^{\text {new }}\right)=\underset{\boldsymbol{\mu}, \boldsymbol{\tau}}{\arg \max } Q(\boldsymbol{\mu}, \boldsymbol{\tau})
\end{gathered}
$$

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## Probabilistic Model

$\square$ Solving for large-scale data and large networks

- The M-step involves a large-scale optimization problem
- Our goal:
- Solve for large-scale data input
- Solve for large network
- Short term link travel time estimation (say 15min)
- Solution: parallelize the computation!
- Alternating Direction Method of Multiplier (ADMM) to decouple the problem into smaller sub-problems
- Solve decomposed sub-problems in parallel
- Deals with large size of network and data
- Faster model estimation


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## Numerical Results

$\square$ Model results for base model

- Validation metrics
- Root mean square error

$$
\text { RMSE }=\sqrt{\frac{1}{n} \sum_{i=1}^{n}\left(T_{i}^{P r}-T_{i}^{O b}\right)^{2}}
$$

- Mean absolute percentage error

$$
\text { MAPE }=\frac{1}{n} \sum_{i=1}^{n}\left|\frac{T_{i}^{P r}-T_{i}^{O b}}{T_{i}^{O b}}\right| \times 100 \%
$$

## Numerical Results

$\square$ Model results for base model

- Test data: 3/15/2010 ~ 3/21/2010

| Day | Error | $9: 00-10: 00$ | $13: 00-14: 00$ | $19: 00-20: 00$ | $21: 00-22: 00$ |
| :--- | :--- | :---: | :---: | :---: | :---: |
|  |  | 2.614 | 1.981 | 1.937 | 1.372 |
|  |  | $29.51 \%$ | $24.22 \%$ | $26.27 \%$ | $21.87 \%$ |
| Tuesday |  | 2.461 | 2.302 | 1.827 | 1.437 |
|  |  | $29.63 \%$ | $25.59 \%$ | $23.33 \%$ | $22.20 \%$ |
| Wednesday | RMSE (min) | $3.827^{*}$ | $3.216^{*}$ | 2.18 | 1.691 |
|  | MAPE | $41.32 \%^{*}$ | $34.97 \%^{*}$ | $28.73 \%$ | $24.40 \%$ |
| Thursday | RMSE (min) | 2.468 | 2.699 | 2.49 | 1.382 |
|  | MAPE | $27.28 \%$ | $27.92 \%$ | $28.54 \%$ | $21.05 \%$ |
| Friday | RMSE (min) | 2.26 | 2.179 | 1.692 | 1.334 |
|  | MAPE | $27.76 \%$ | $27.04 \%$ | $25.17 \%$ | $22.26 \%$ |
| Saturday | RMSE (min) | 1.034 | 1.69 | 1.839 | 1.584 |
|  | MAPE | $16.84 \%$ | $24.58 \%$ | $27.14 \%$ | $21.61 \%$ |
| Sunday | RMSE (min) | 2.041 | 1.518 | 1.395 | 1.16 |
|  | MAPE | $25.44 \%$ | $23.70 \%$ | $22.72 \%$ | $19.87 \%$ |

[^1]
## Numerical Results



## Conclusion

- Two new models are proposed to estimate urban link travel times
- Utilizing data with only partial information
- Efficiently estimation using base model with reasonable accuracy
- Mixture models are proposed to get more robust and accurate estimations
- Applicable to trajectory data, can provide more accurate estimations
$\square$ Future work
- Test the mixture models for larger network
- Efficient implementation using distributed computing technique
- Result validation


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## Q\&A

## Thank you!

## Questions / Comments ?

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[^0]:    * Y. Yen, Finding the K shortest loopless paths in a network, Management Science 17:712-716, 1971.

[^1]:    * Traffic disturbance caused by Patrick's Day Parade.

