Update and Repair Efficient Codes for Distributed Storage

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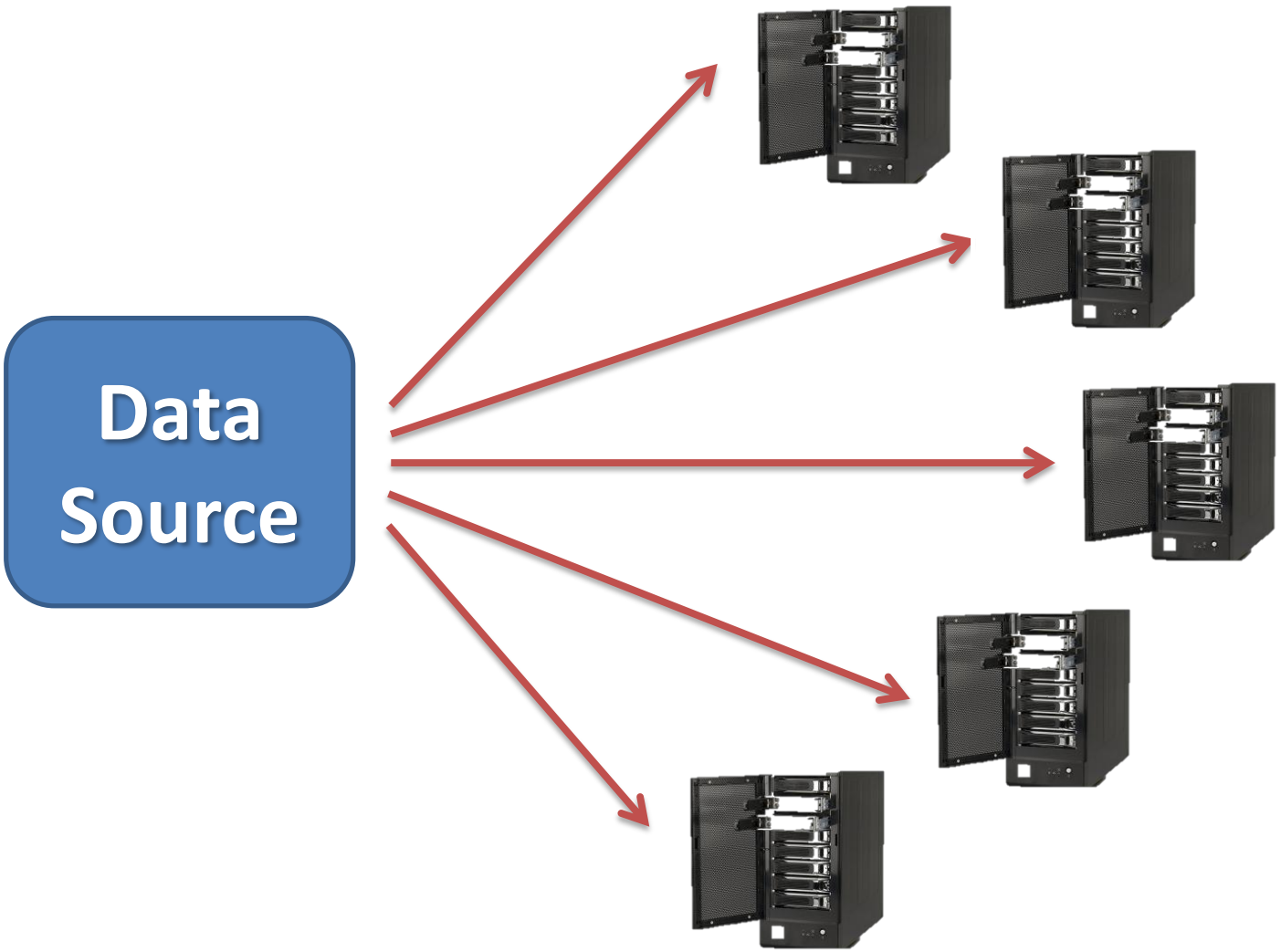
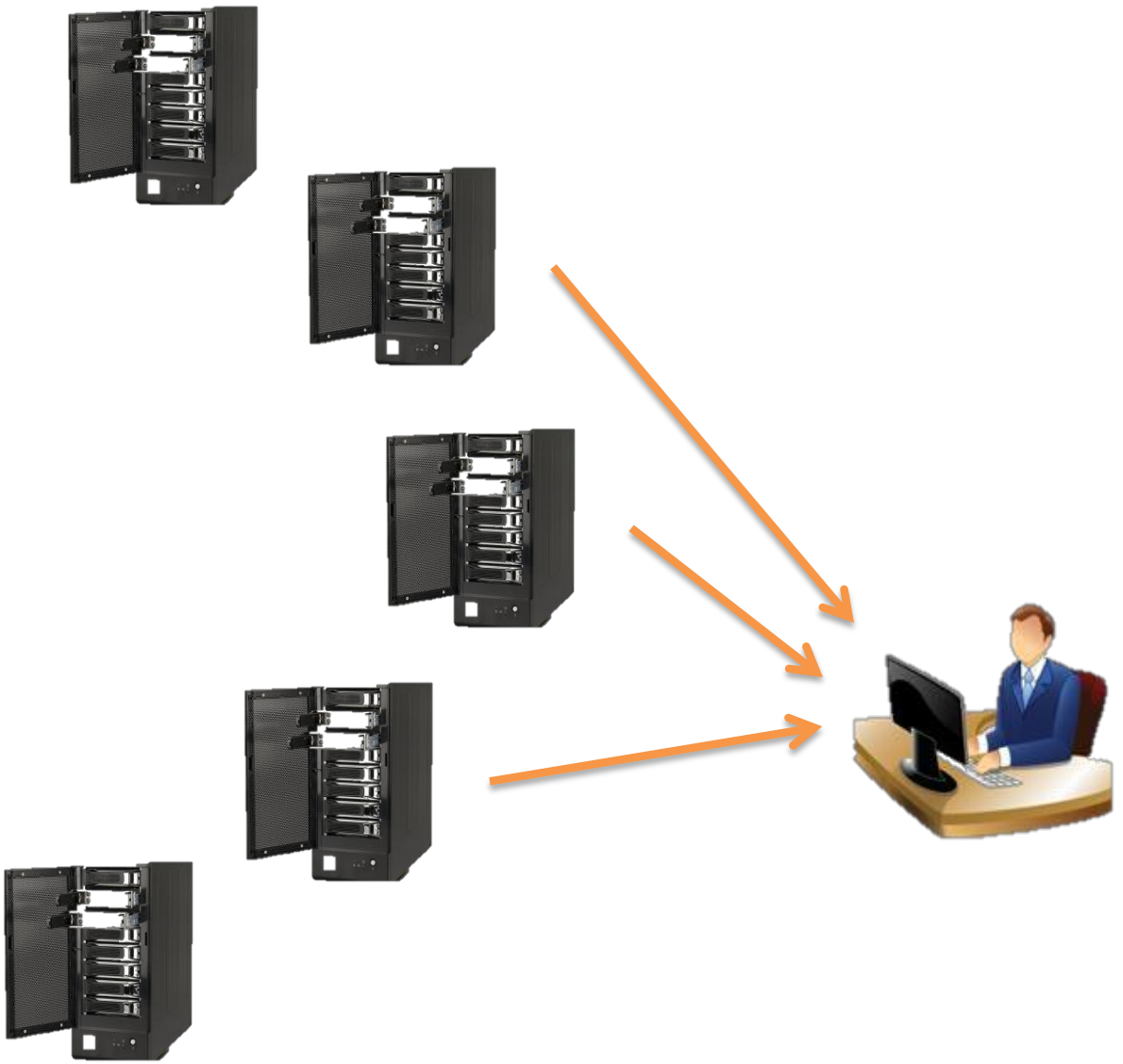
The University of Texas at Austin

December 18, 2013

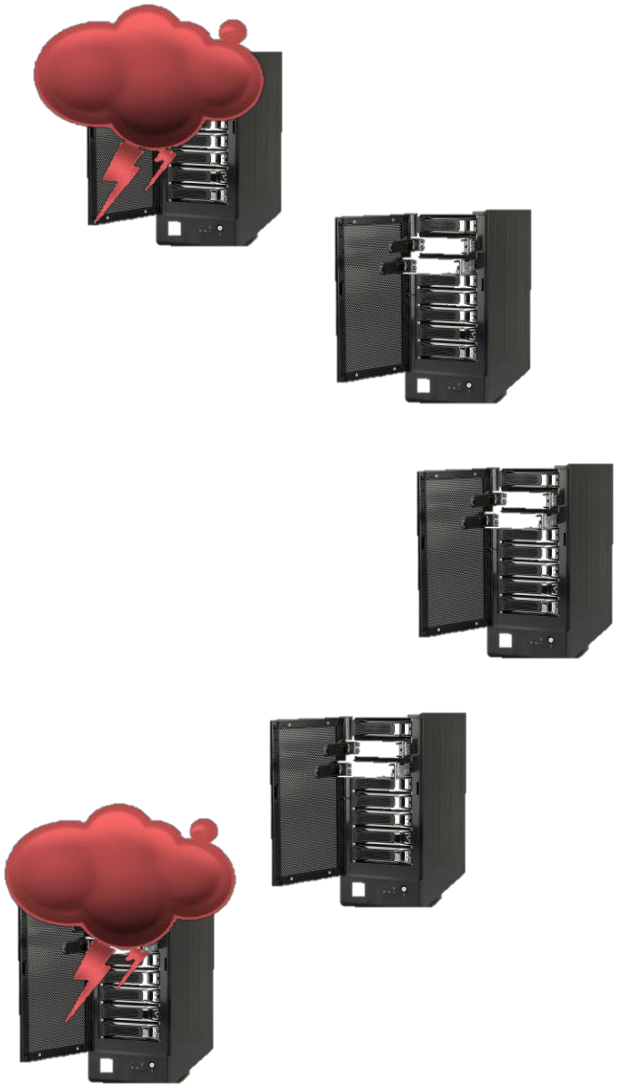
Distributed Storage System (DSS)



Data reconstruction



Node failure



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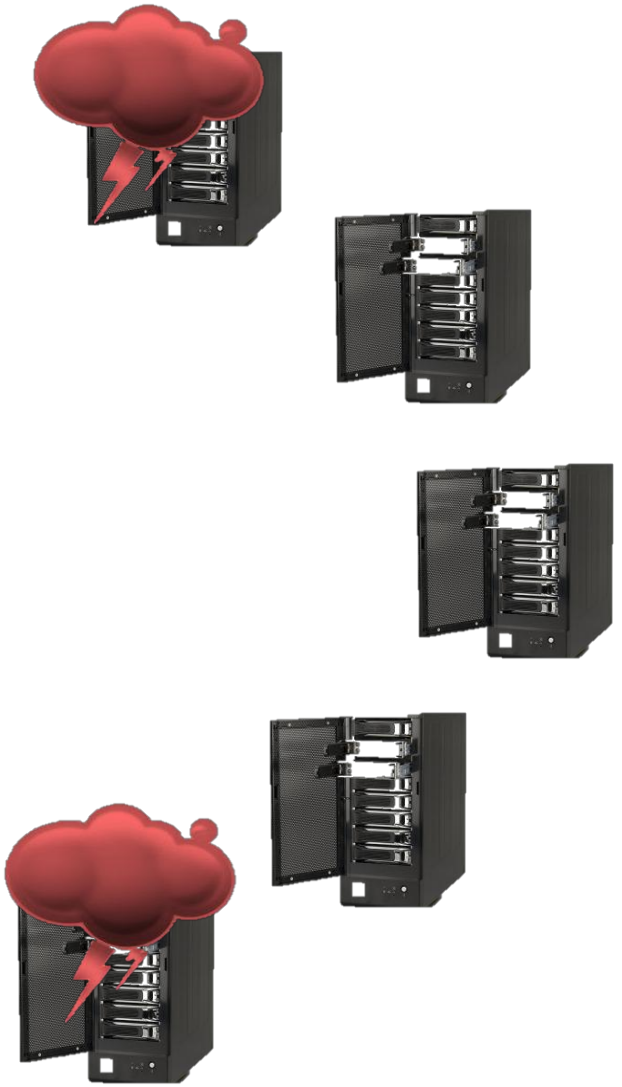
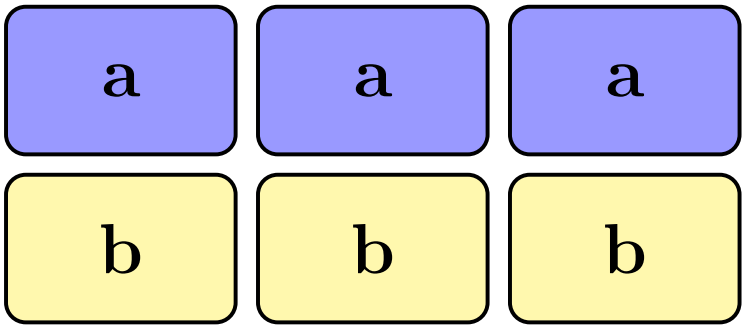
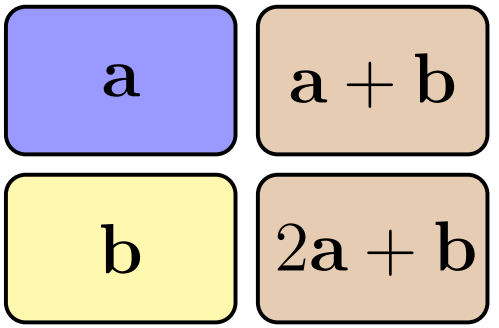


Need to introduce redundancy into the system.

Replication vs. coding

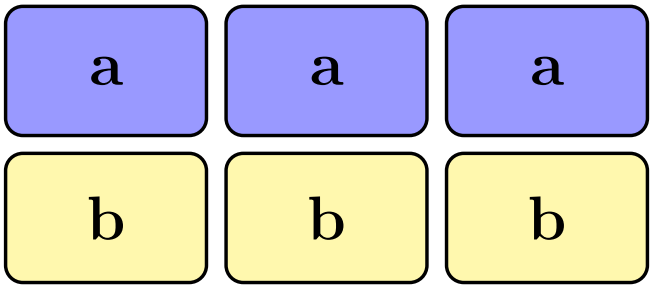


3-replication



$(4, 2)$ Erasure coding

Replication vs. coding



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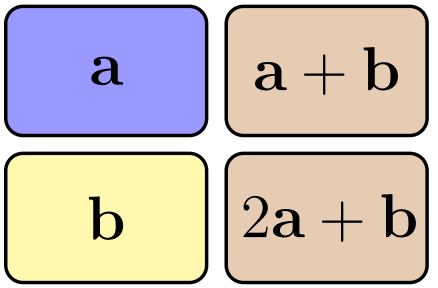

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- Traditional MDS codes are optimal for *storage vs. reliability* trade-off.

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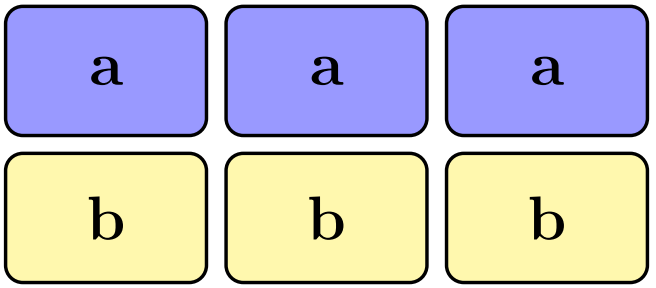

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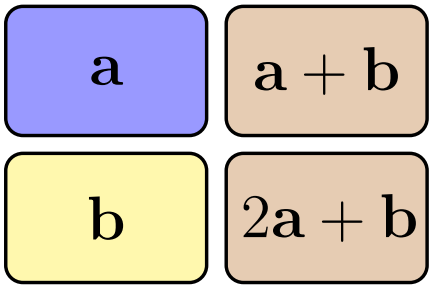

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- Traditional MDS codes are optimal for *storage vs. reliability* trade-off.
- Reliability is not the only metric of interest:
  - Overhead of updating data.
  - Node repair overhead.
  - Availability.
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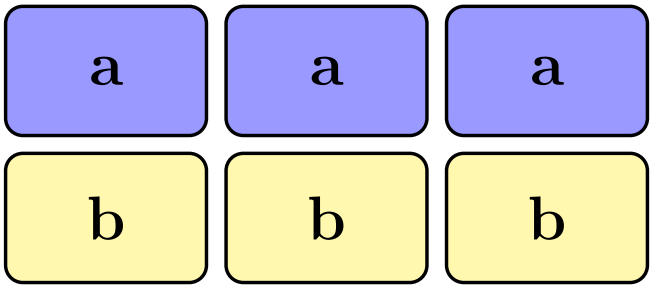

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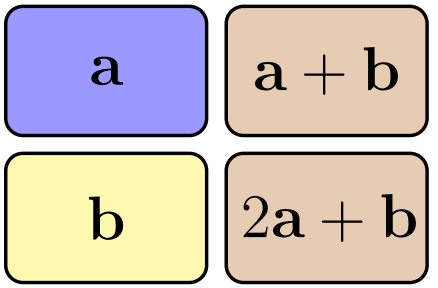

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May need to move away from MDS codes.

Overhead of updating data (Update Complexity)

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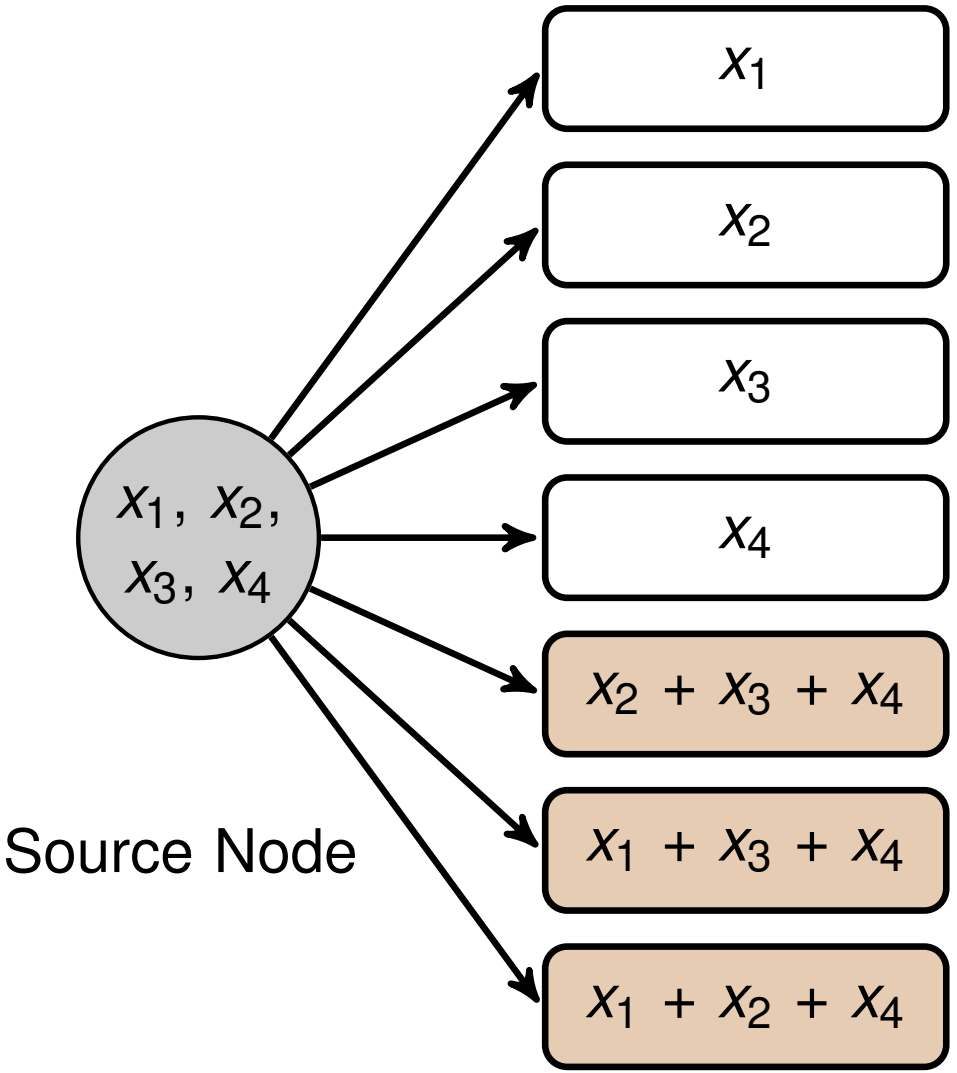
Example: $(7, 4)$ Hamming code



$$\mathbf{y} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

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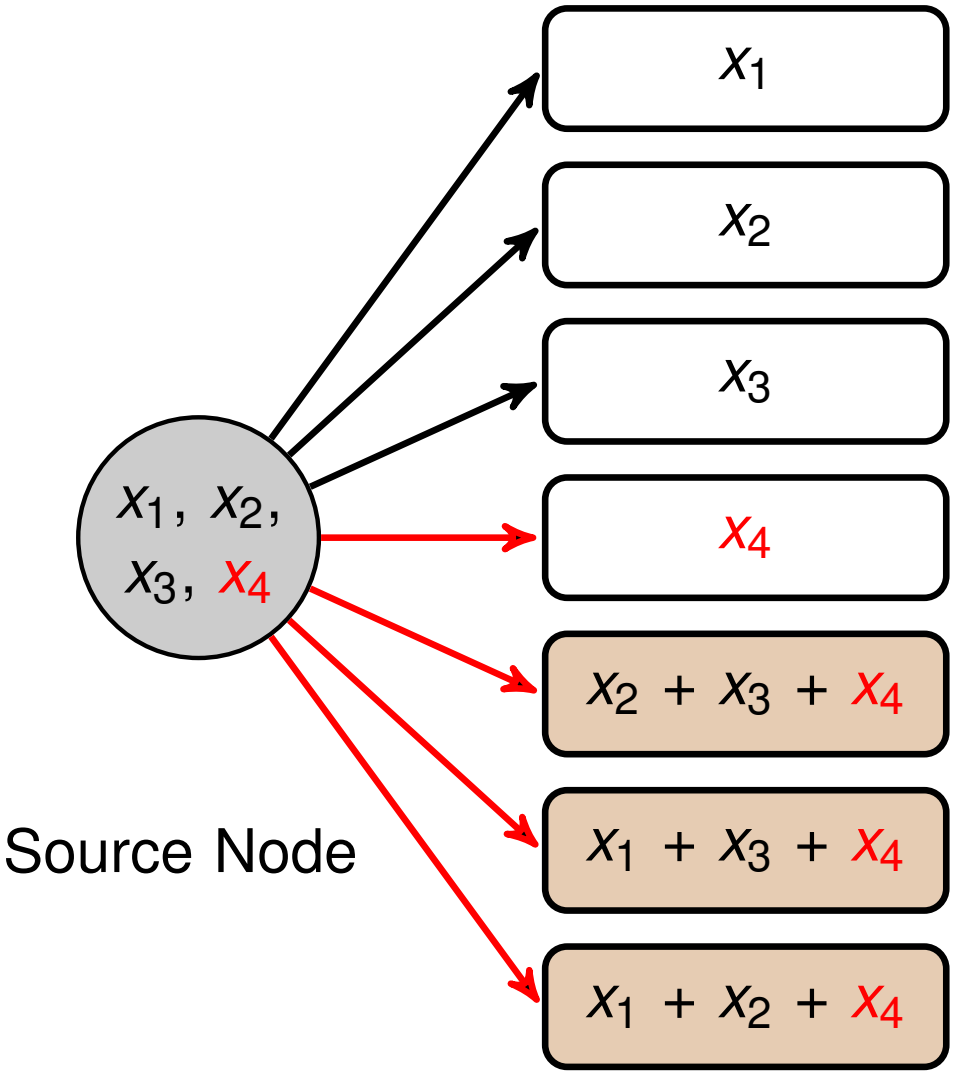

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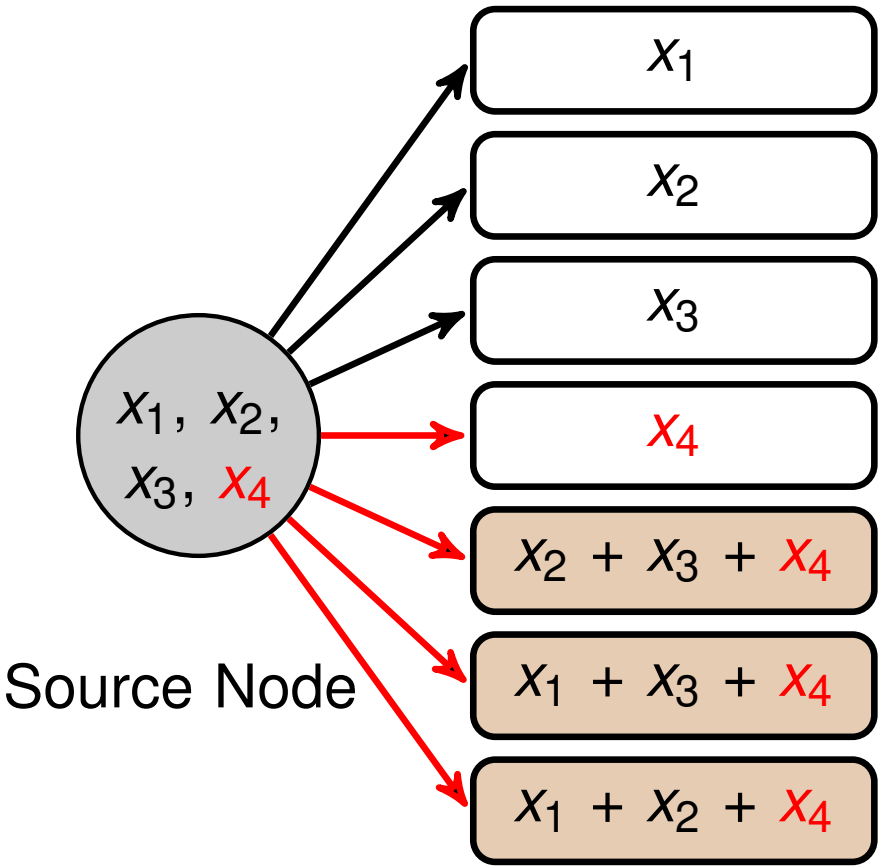

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Update complexity $\geq$ minimum distance.

Update Complexity

- Information to be stored is almost never static.
- **Update complexity:** maximum number of encoded symbols that must be updated when any single information symbol is changed.
- Updating data consumes bandwidth and energy.

Design codes with *small update complexity*.

Update Efficiency

- Constant update complexity.

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  - Low update complexity MDS codes for storage:
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  - Low update complexity MDS codes for storage:
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- Coding schemes with sub-linear update complexity.

Kolchin generator (KG) codes

- Random ensemble of codes with logarithmic update complexity.

P. Anthapadmanabhan, E. Soljanin and S. Vishwanath, in Allerton 2010.

Kolchin generator (KG) codes

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- Generator matrix:

$$G = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 1 & 0 & \cdots & 0 \end{bmatrix}_{n \times k_n}$$

$$\mathbb{P}(G_{i,j} = 1) = \mathcal{O}\left(\frac{\log n}{n}\right)$$

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$$\mathbb{P}(G_{i,j} = 1) = \mathcal{O}\left(\frac{\log n}{n}\right)$$

- **With high probability**, every column has $\mathcal{O}(\log n)$ non-zero entries.

P. Anthapadmanabhan, E. Soljanin and S. Vishwanath, in Allerton 2010.

Failure tolerance of KG codes

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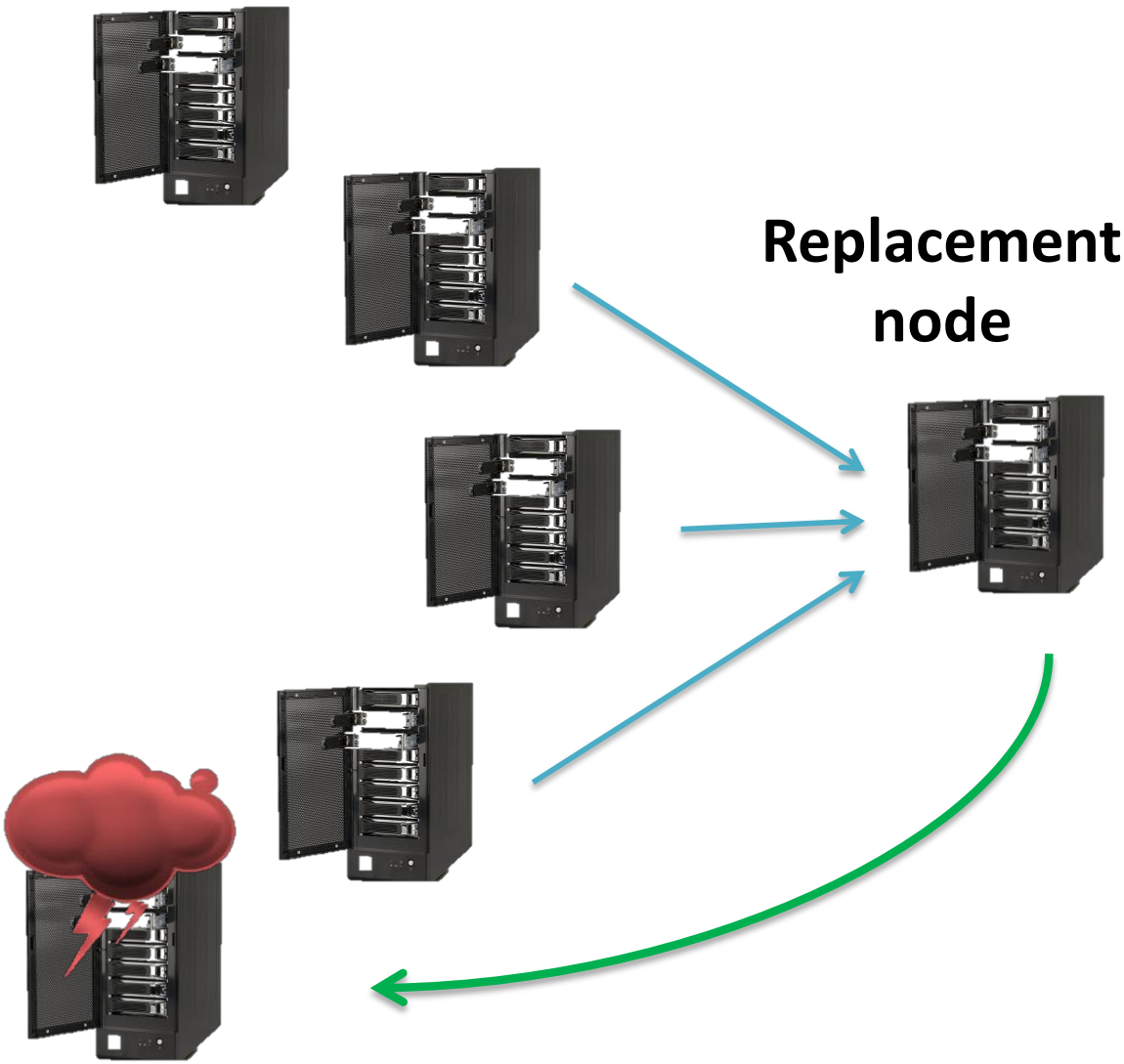
- Minimum distance of KG codes is at most $\mathcal{O}(\log n)$.
- Random erasure model: each node answer with probability p.
- **W.h.p.**, a random set of $k_n(1+\epsilon)$ encoded symbols is sufficient to reconstruct data.
  - Can tolerate erasure probability $p < \frac{n-k_n}{n}$.

Node repair



Efficient node repair

- Important to perform efficient node repairs.
- Multiple metrics to measure repair efficiency.
- **Repair bandwidth:** amount of data downloaded during a node repair.
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  - **A. S. Rawat, A. Bhowmick, S. Vishwanath and E. Soljanin**, in ISIT 2011.
- **Locality:** number of remaining nodes contacted during a node repair.
  - **M. Asteris and A. Dimakis**, in ISIT 2012.
  - Modification of KG codes appraoch.
  - **A. Mazumdar, V. Chandar and G. Wornell**, in ITA 2013.

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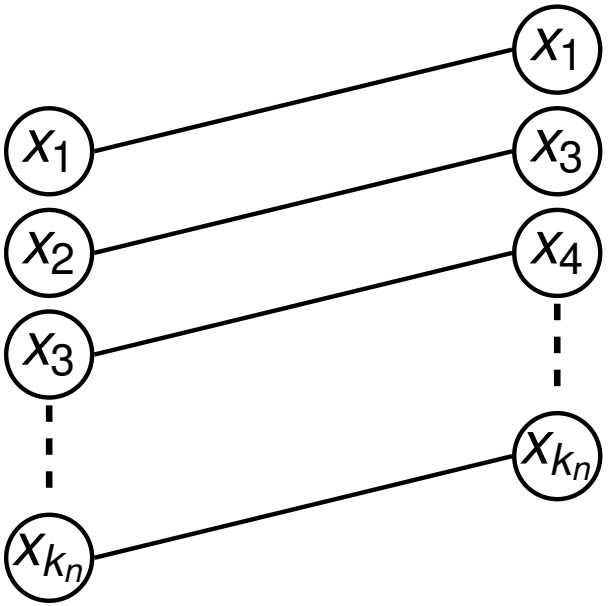

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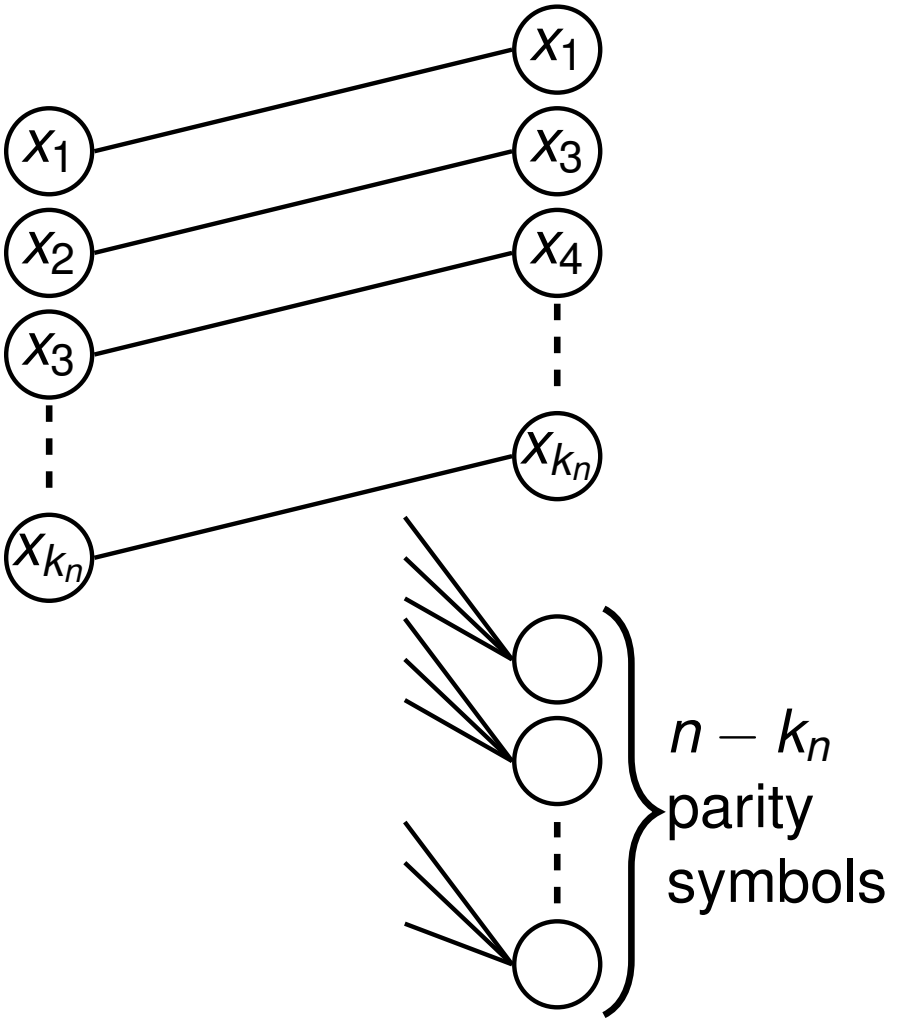

- Each parity node is obtained by randomly throwing $\Theta(\log n)$ edges.

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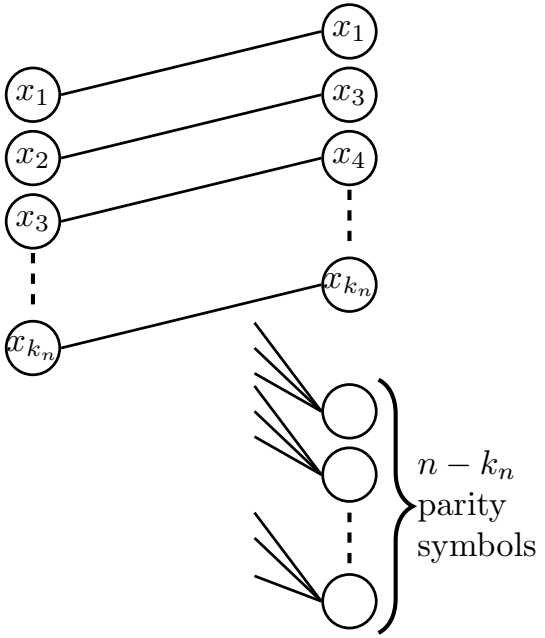
Repairable fountain codes.



- Each node has locality $\Theta(\log n)$.
- **W.h.p.**, update complexity: $\mathcal{O}(\log n)$.
  - Maximum load of throwing $\mathcal{O}(k_n \log k_n)$ balls into k_n bins.
- Failure tolerance:
  - Any random set of $k_n(1+\epsilon)$ encoded symbols are sufficient for data reconstruction.

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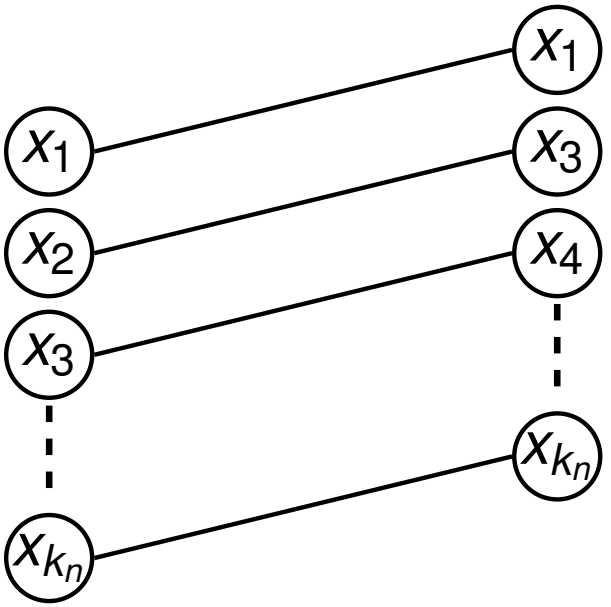

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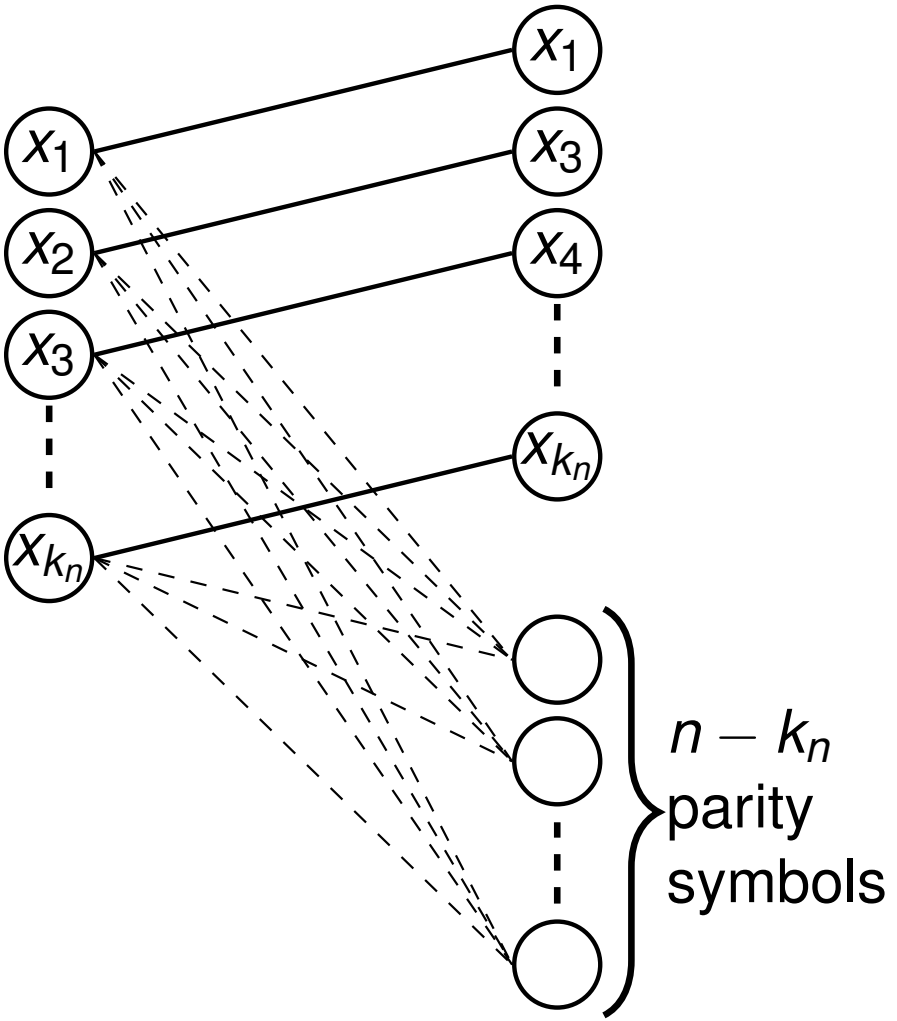
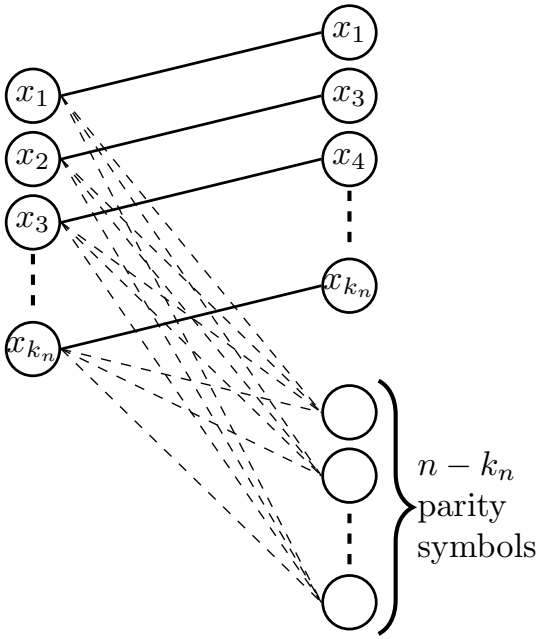


- For parity nodes, each edge is present w. p. $\Theta(\frac{\log n}{n})$.

Modification KG codes approach.



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Discussion and open questions

- Explicit constructions for update efficient codes.

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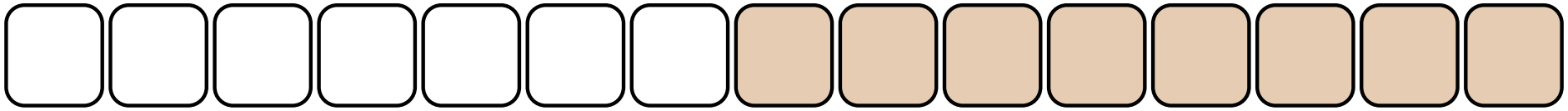
- Explicit constructions for update efficient codes.
- Study of update efficient codes under general failure model.
  - **B. Kanukurthi, N. Chandran and R. Ostrovsky**, in TCC 2014
  - Construction of update efficient *locally decodable* codes for prefix hamming metrics.

Locality and availability



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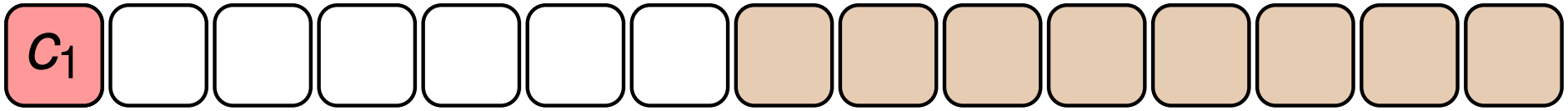

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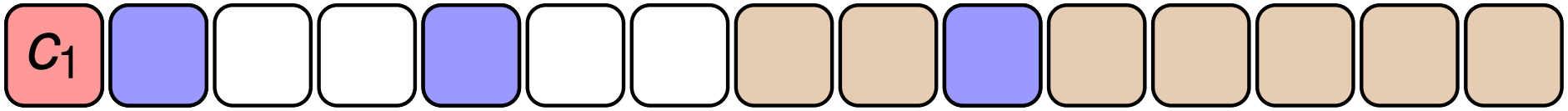

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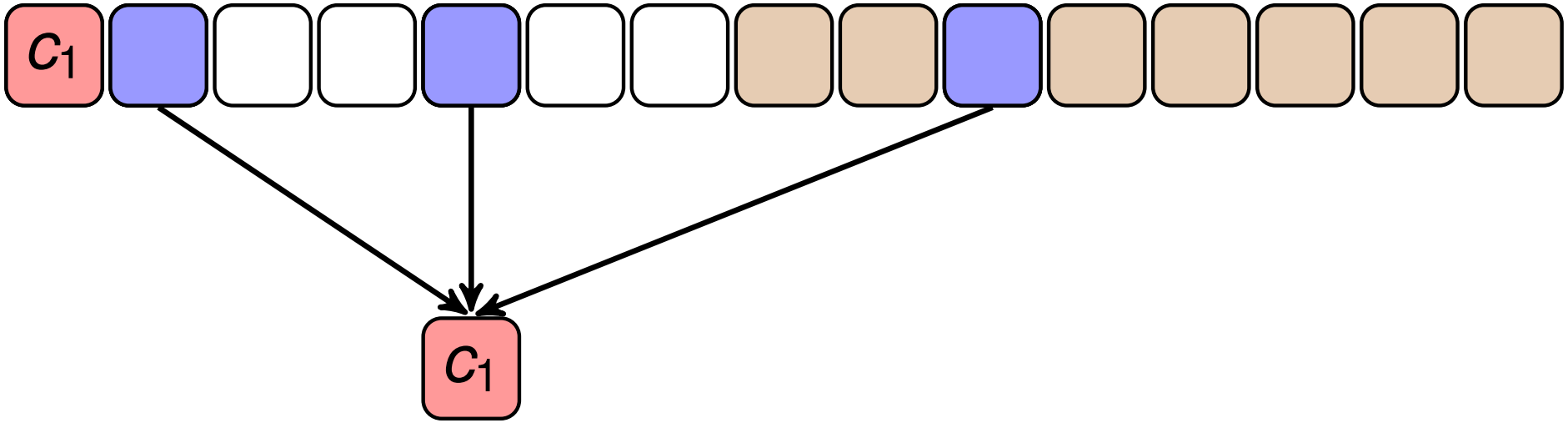

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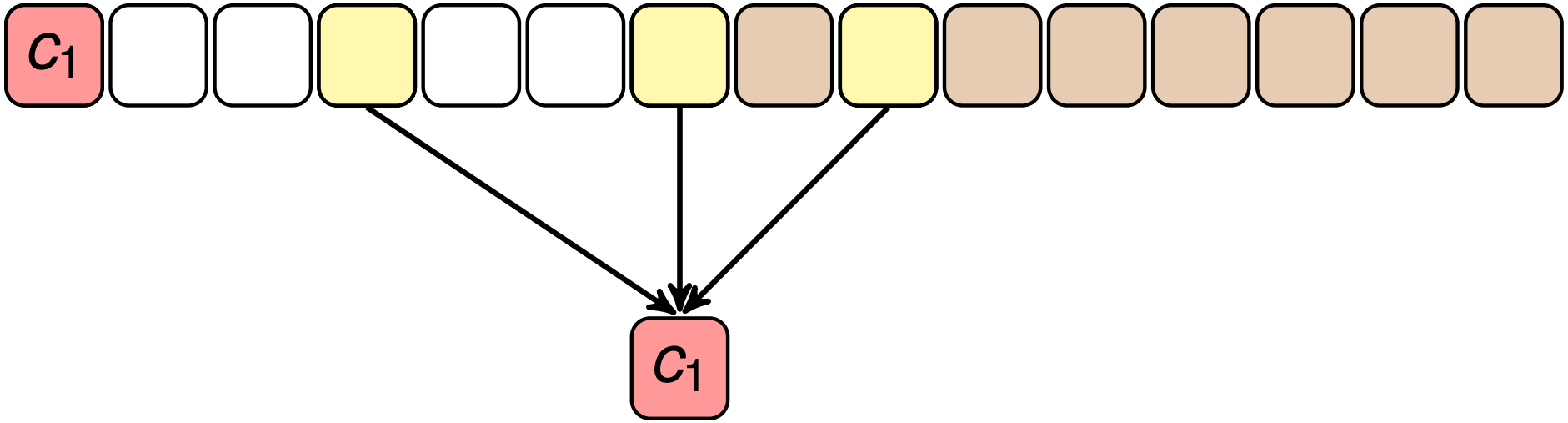

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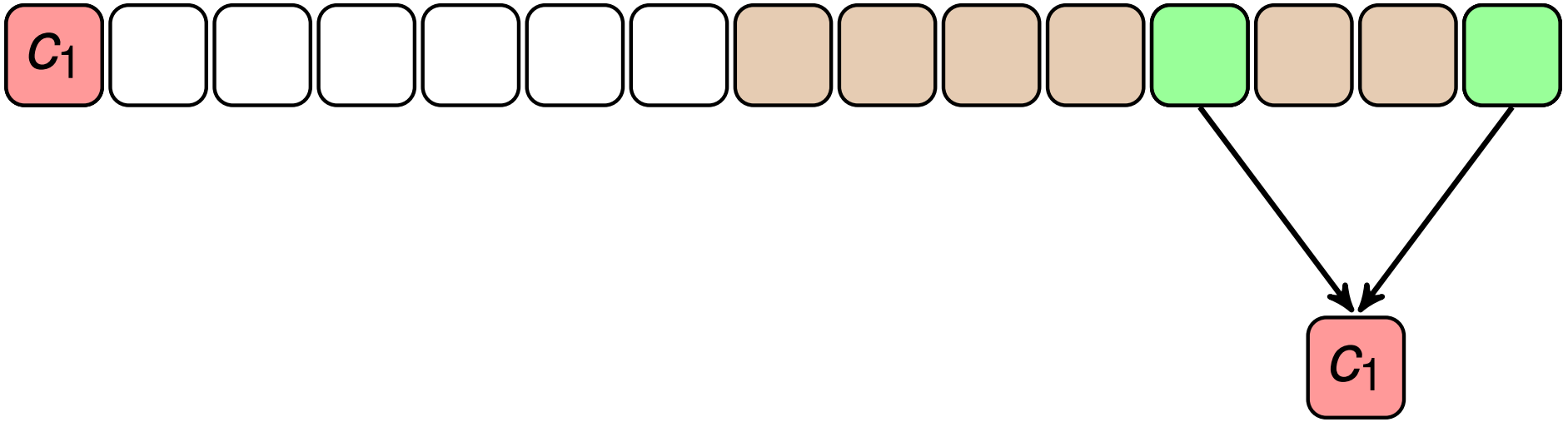

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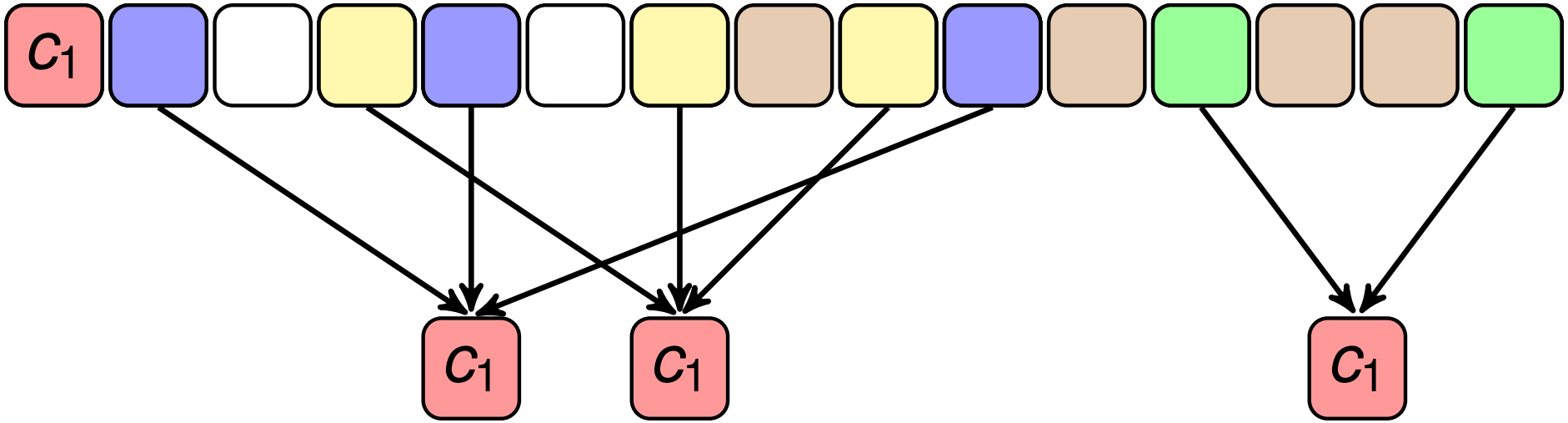

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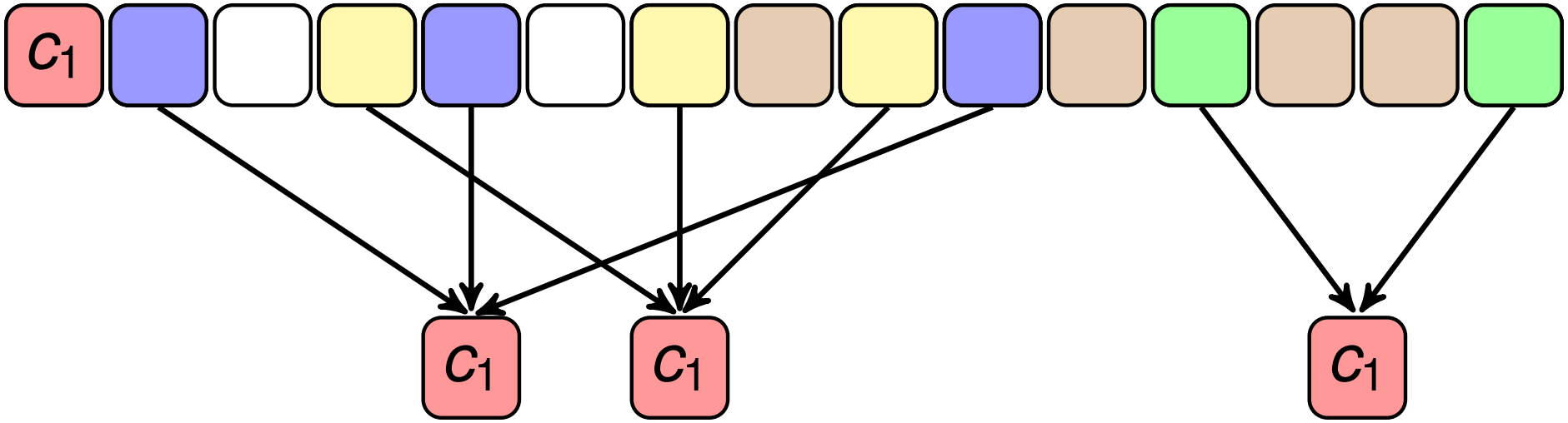

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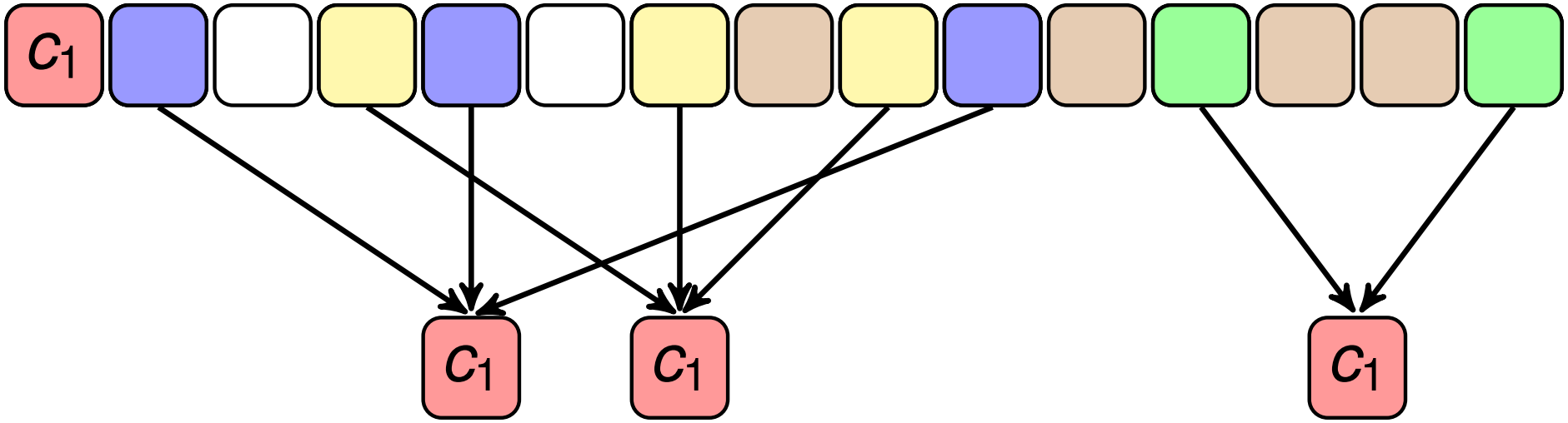

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- In general, t availability with r locality.
- **Open question:** how to address general t request patterns.
  - t requests for c_1.
  - $t/2$ requests for c_1, and $t/2$ requests for c_2.
  - $t/3$ requests for each of c_1, c_2 and c_3.

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Thank you!