Computing Projective Clusters via Certificates

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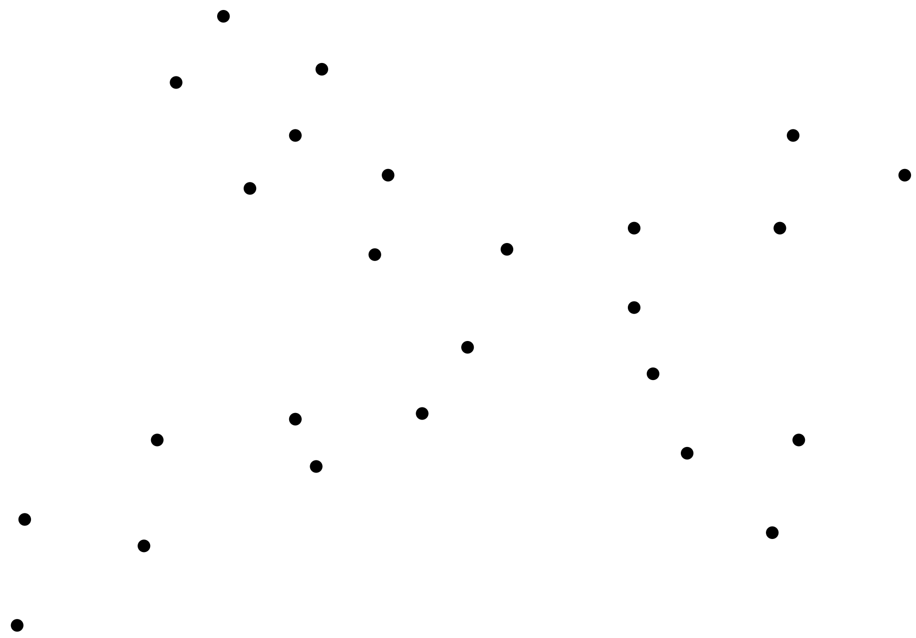
(joint work with Pankaj Agarwal and Kasturi Varadarajan)

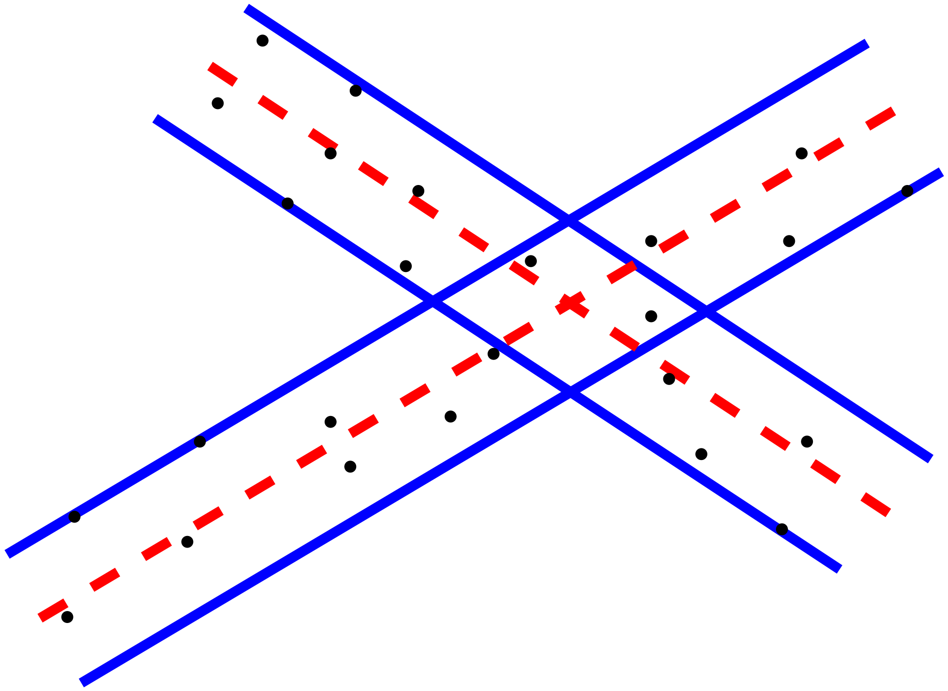
Applications

- Shape Fitting
- Database Indexing
- Information Retrieval
- Data Compression
- Image Processing

Example



Example



Definition

- S: set of n points
- k: integer

k-Line-Center: Find k lines $\ell_1, \ldots, \ell_k$ that minimize

$$\max_{p \in S} \min_{1 \le j \le k} d(p, \ell_i).$$

$$w^* = \text{minimum value so that } S \text{ can be covered by } k \text{ hyper-cylinders of diameter } w^*.$$

Projective Clustering: Find q-dimensional flats $h_1, \ldots, h_k$, for some integer q.

Results

1. Most variants of projective clustering problems are NP-Hard: Meggido and Tamir '82.

2. $d = 2, 3, k = 1, 2$: Houle & Toussaint '98, Agarwal & Sharir '96, Jaromczyk & Kowaluk '95.

3. $k = 1$, general d, $(1+\varepsilon)$-approx.:
   - $q = d - 1$ (width): Duncan et al. '97, Chan '00.
   - $q = 1$ (enclosing cyl.):Har-Peled & Varadarajan '01, Bădoiu et al. '02.
   - general q: Har-Peled & Varadarajan '03.

4. General k and d:
   - $O(dk \log k)$ hyper-cylinders of diameter $8w^*$ in $\tilde{O}(dnk^3)$ time: Agarwal & Procopiuc '00
   - k hyper-cylinders of diameter $(1+\varepsilon)w^*$ in $\tilde{O}(nf(k, d, \varepsilon))$ time: Agarwal, Procopiuc & Varadarajan '02.
   - k q-flats of diameter $(1+\varepsilon)w^*$ in $dn^{O(g(k,q,\varepsilon))}$ time: Har-Peled & Varadarajan '02.

Core-Sets (Har-Peled & Varadarajan)

For each flat h in optimal cover, there exists small subset Q_h s.t. $subspace(Q_h)$ contains ε-approx. flat.

Q_h: core-set of h.

$|\bigcup_h Q_h| = f(k, q, \varepsilon)$: *independent of* n *and* d*!*

1. Find core-sets Q_h (brute force enumeration).
2. Compute ε-approx. solution (brute force).

Certificates (Agarwal, Procopiuc & Varadarajan)

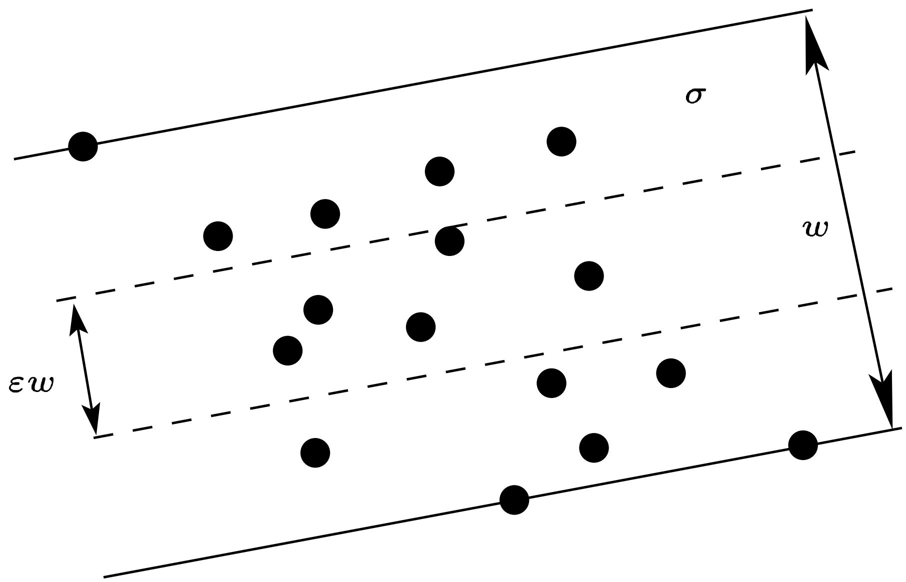
There exists small subset Q s.t. Q covered by k congruent hyper-cylinders $\Rightarrow$ S covered by the ε-expanded hyper-cylinders.

Q: certificate of S.

$|Q| = f(k, \varepsilon, d)$: *independent of* n*!*

1. Find certificate Q (iterative sampling).
2. Compute optimal solution on Q (brute force).
3. Expand to solution on S.

1-Strip Certificate



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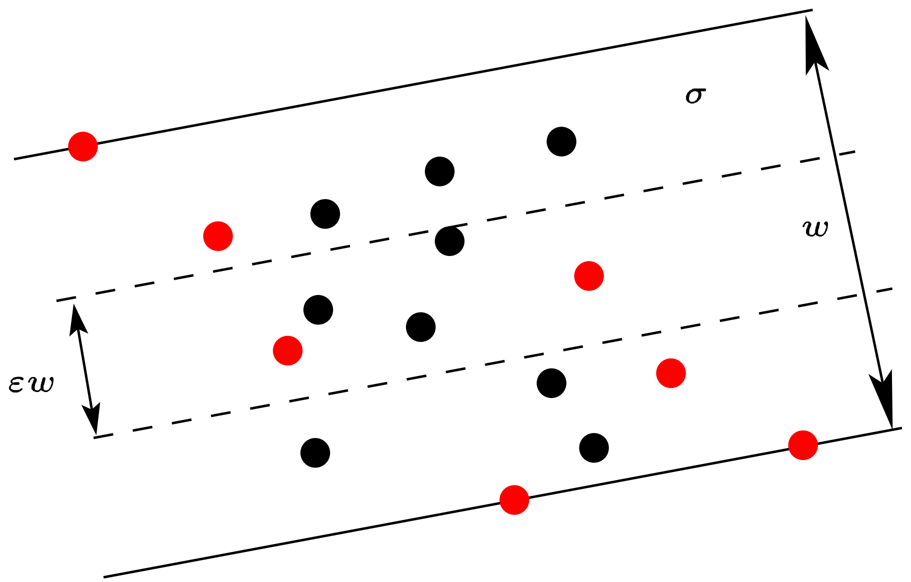

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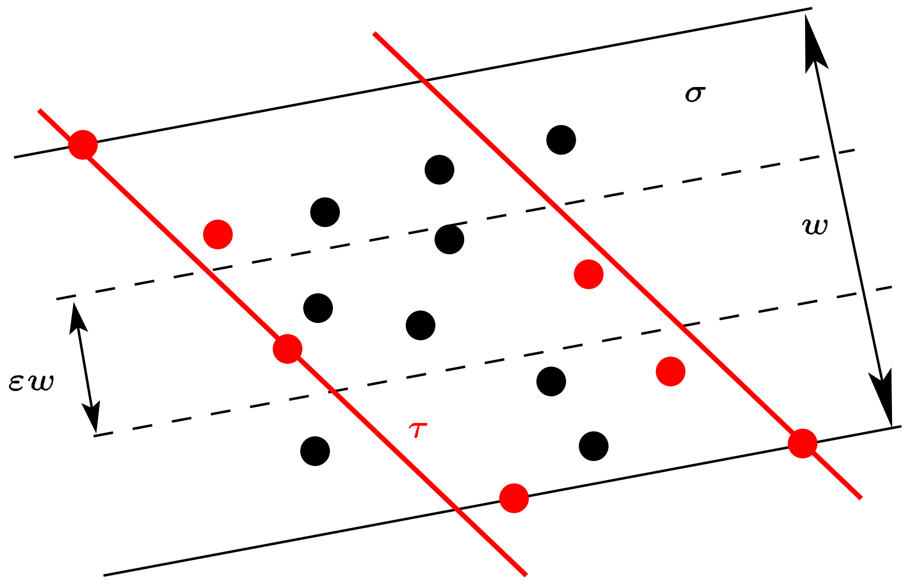

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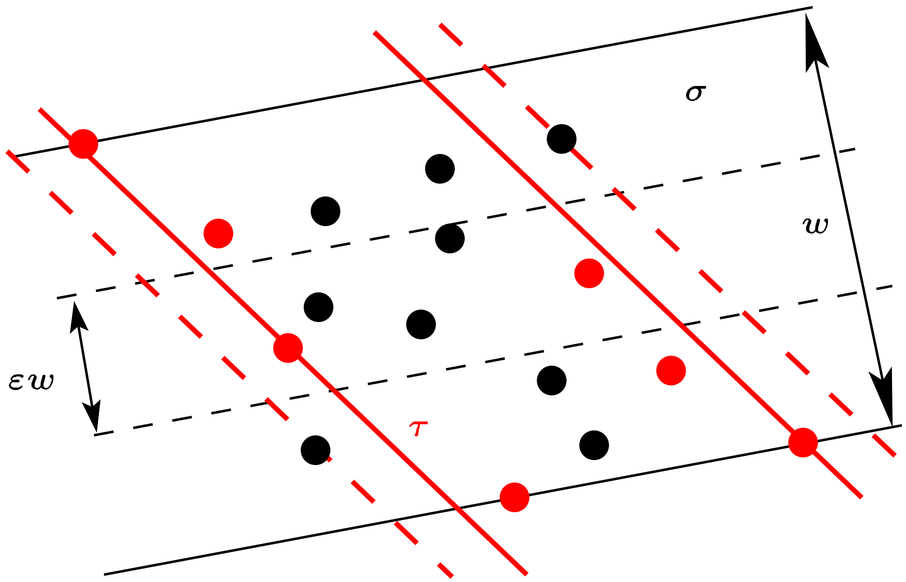

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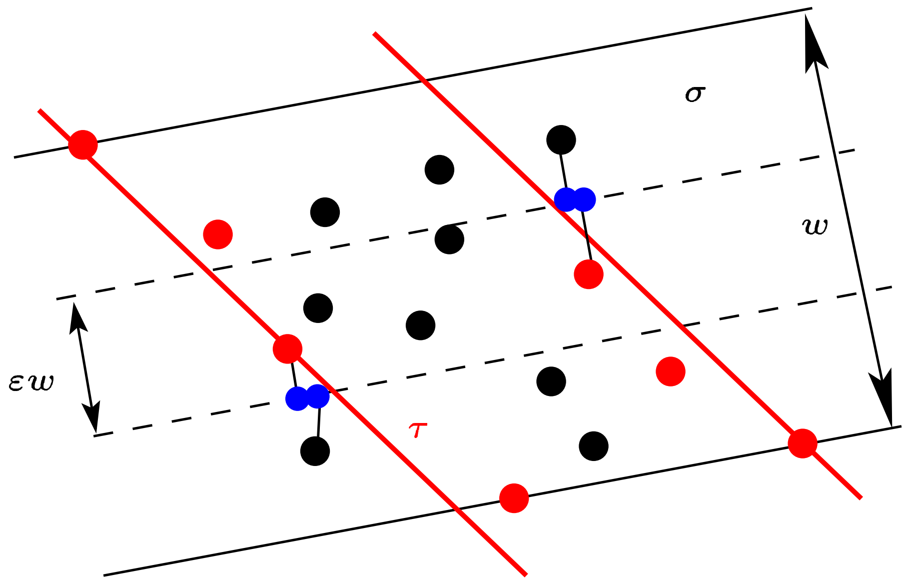

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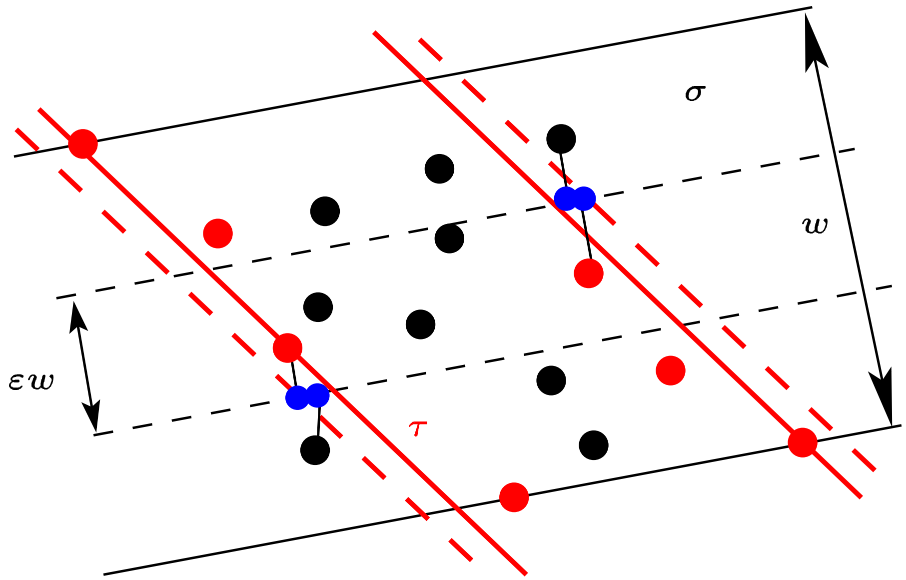

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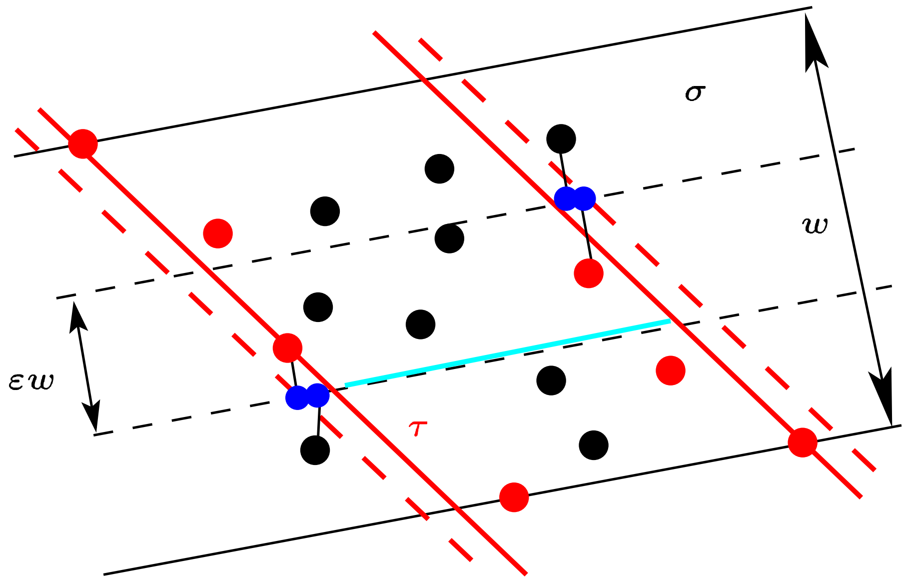

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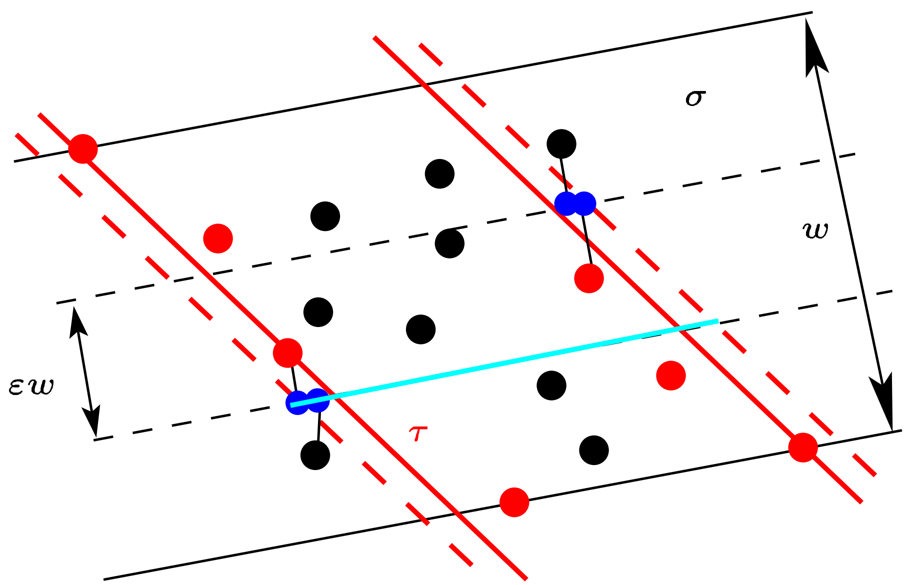

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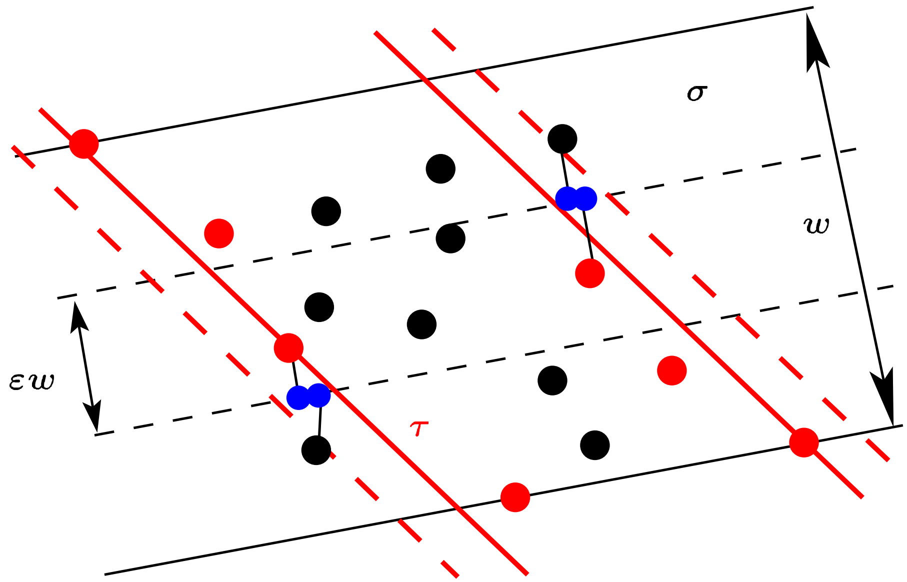
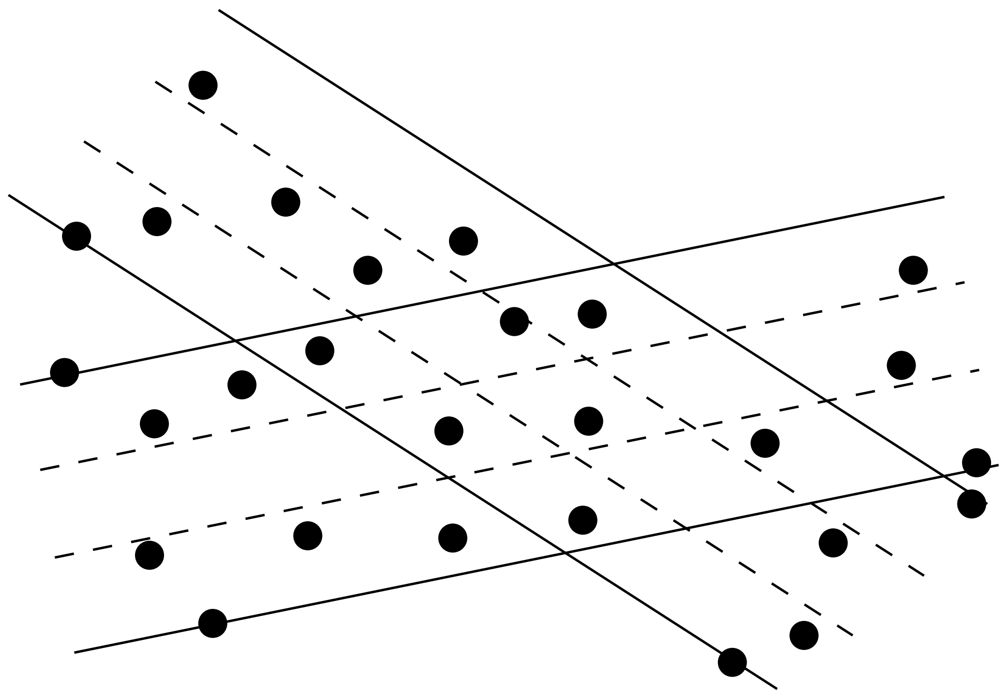


2-Strip Certificate



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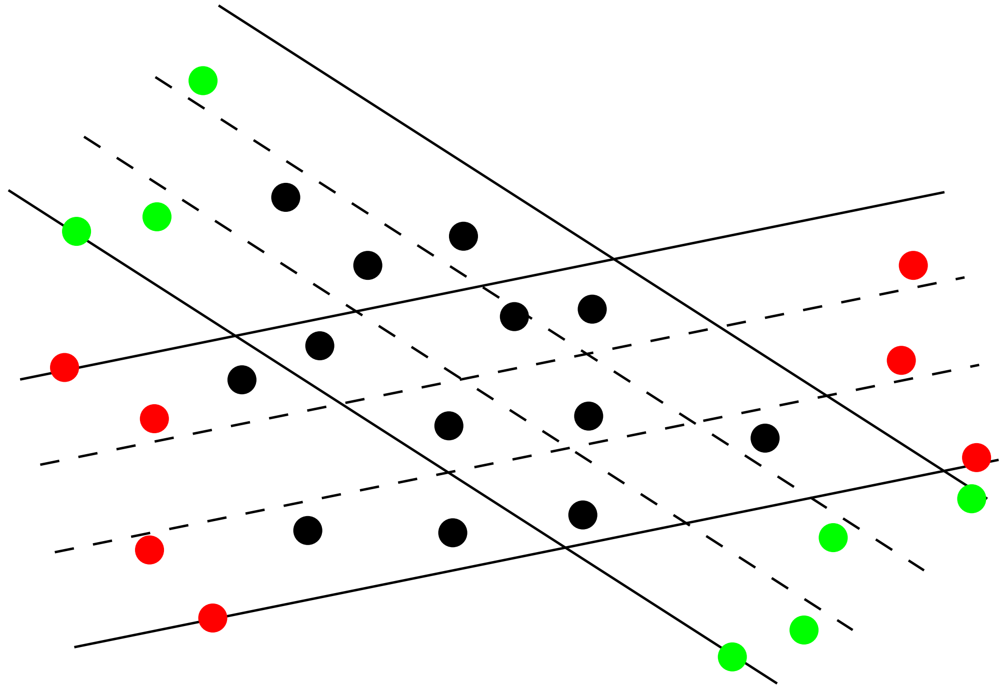

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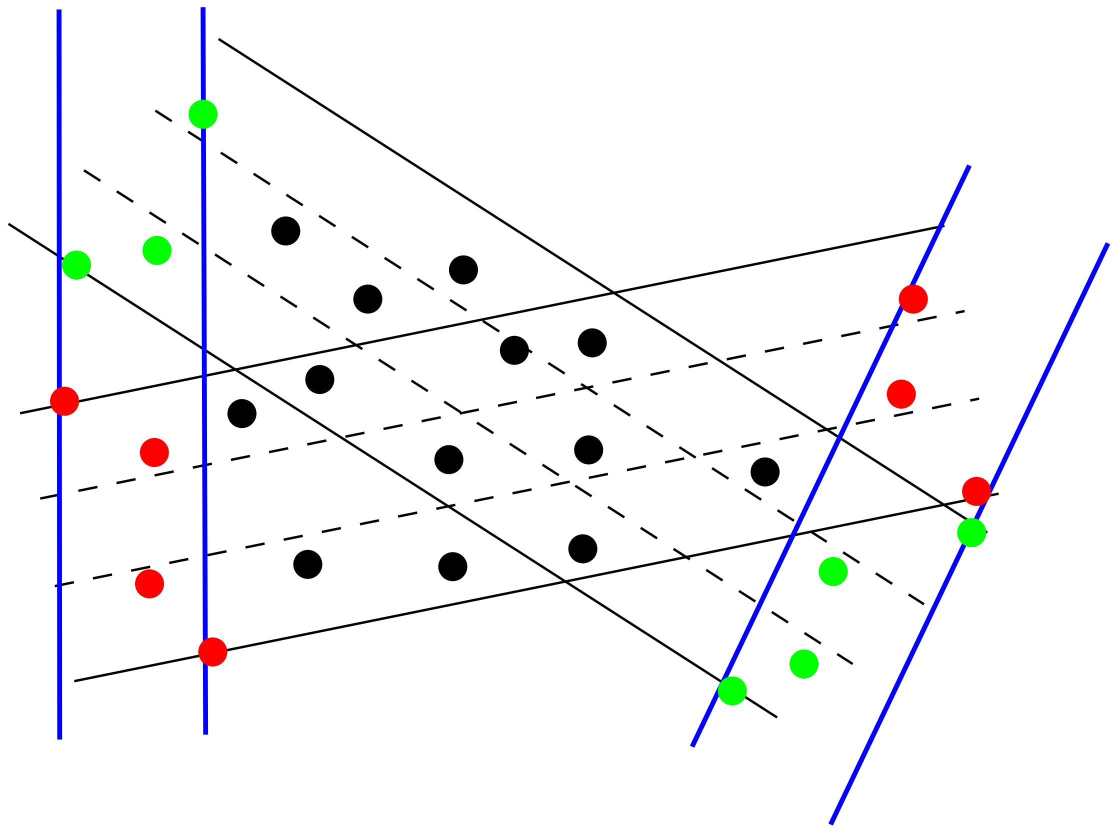

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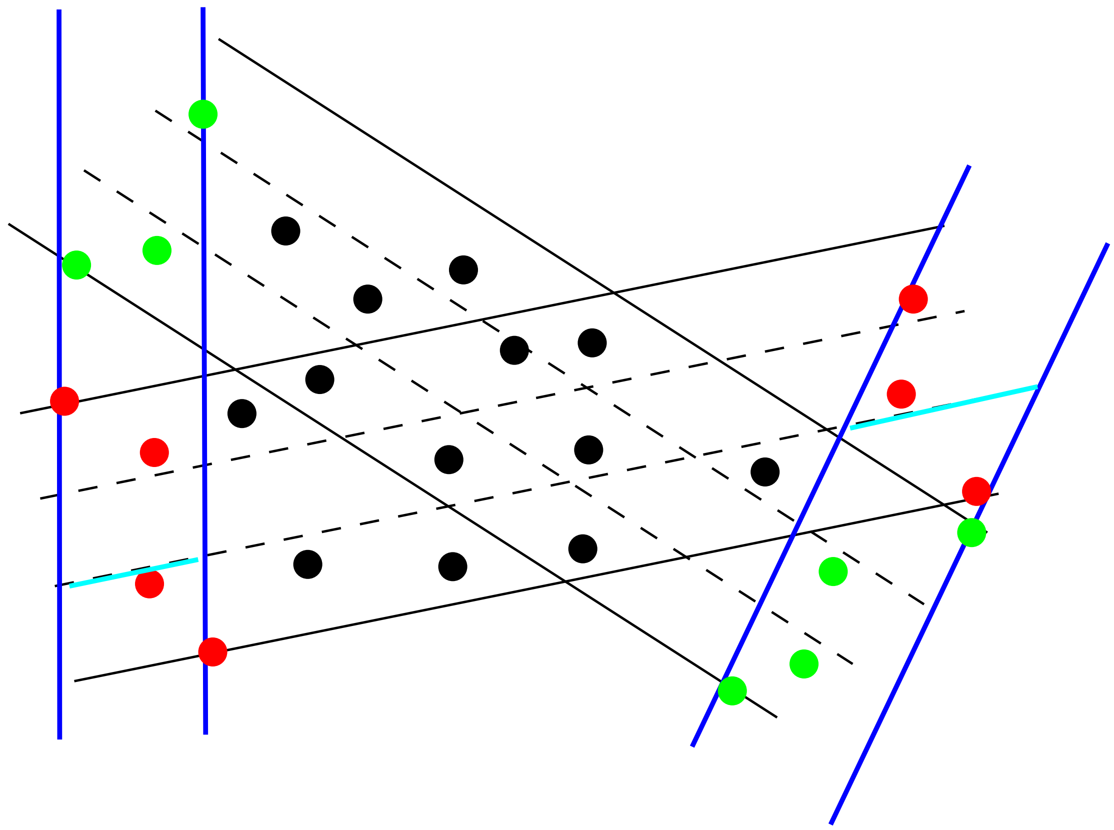
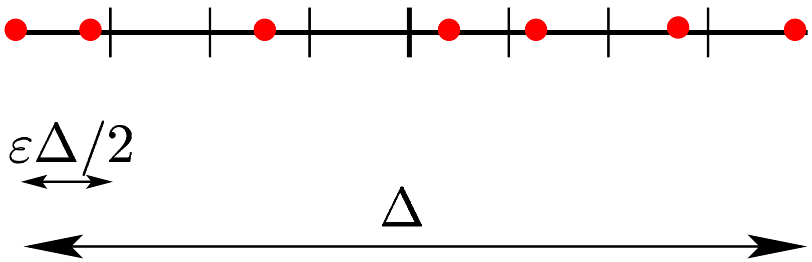


Line Certificate

- P: set of points on *real line*.
- $Q \subseteq P$: k-certificate if any k intervals that cover Q can be ε-expanded to cover P.

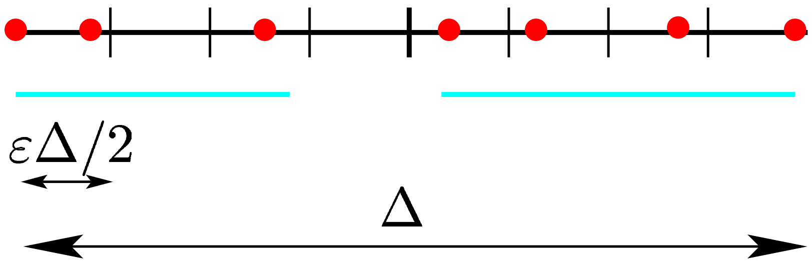
Claim: A k-strip certificate can be obtained from the union of k-certificates of all grid lines.

Line Certificate



$k = 2$

Line Certificate



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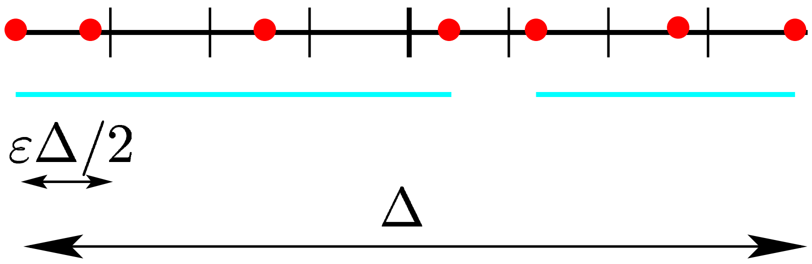
Line Certificate



$k = 2$

Line Certificate

Lemma 1: For any set of points in $\mathbb{R}$, there exists a line certificate of size $(k/\varepsilon)^{O(k)}$.

Lemma 2: For any set of points in $\mathbb{R}^d$, there exists a certificate of size $k^{O(k)}/\varepsilon^{O(d+k)}$.

- Iterative random sampling

Line Certificate

Open Problems

1. Certificates of smaller size?
2. Constructive proof for certificates.
3. Extensions to q-flats.