Ramsey results for 3-coloring and odd cycles

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For graphs $G_1, \dots, G_k$, the Ramsey number $R(G_1, \dots, G_k)$ is the minimum integer N satisfying that for any coloring of edges of the complete graph K_N by k colors there exists a color i for which the corresponding color class contains G_i as a subgraph. Bondy and Erdős conjectured that if n is odd, $R(C_n, C_n, C_n) = 4n - 3$. This is sharp if true, as shown by some constructions.

Łuczak proved (using the Regularity Lemma) that if n is odd, then $R(C_n, C_n, C_n) = 4n + o(n),$ as $n \to \infty$. We prove that if n is odd, then

$$R(C_n, C_n, C_n) = 4n - 3.$$

We also describe the Ramsey-extremal colorings and prove some related stability theorems.