Applications of the sparse regularity lemma

Y. Kohayakawa

Department of Mathematics and Computer Science
Emory University, Atlanta, GA 30322
`yoshi@mathcs.emory.edu`**

Many applications of the regularity lemma are based on the following fact, often referred to as the *counting lemma*: *Let G be an s-partite graph with vertex partition $V(G) = \bigcup_{i=1}^{s} V_i$, where $|V_i| = m$ for all i and all pairs (V_i, V_j) are ε-regular of density d. Then G contains $(1 + f(\varepsilon))d^{\binom{s}{2}}m^s$ cliques K_s, where $f(\varepsilon) \to 0$ as $\varepsilon \to 0$.* The combined application of the regularity lemma followed by the counting lemma is now often called the *regularity method.*

In recent years, considerable advances have occurred in the applications of the regularity method, of which we mention two: (i) the regularity lemma and the counting lemma have been generalized to the hypergraph setting and (ii) the case of sparse graphs is now better understood.

In the sparse setting, that is, when n-vertex graphs with $o(n^2)$ edges are involved, most applications have so far dealt with random graphs. In this talk, we shall discuss a new approach that allows one to apply the regularity method in the sparse setting in purely *deterministic* contexts. Roughly speaking, this approach allows one to prove an appropriate counting lemma.

This is joint work with V. Rödl, M. Schacht, and J. Skokan.

** On leave from Instituto de Matemática e Estatística, Universidade de São Paulo, Rua do Matão 1010, 05508–090 São Paulo, Brazil