# Double Auction Markets vs. Matching & Bargaining Markets: Comparing the Rates at which They Converge to Efficiency

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# Overview

- Bargaining, private information, inefficiency, and market size
- Rates of convergence for the static double auction: convergence is in $m$, the number of buyers and the number of sellers.
- Rates of convergence for matching and bargaining games: convergence is in $\tau$, the severity of search frictions, and $\kappa$, the magnitude of participation costs
- A nested model of dynamic trading with either
  - the double auction as the bargaining protocol
  - The random offer protocol within bilateral matches as the bargaining protocol
- Demonstration that both protocols result in equivalent allocations and have identical convergence rates in $\tau$ and $\kappa$.

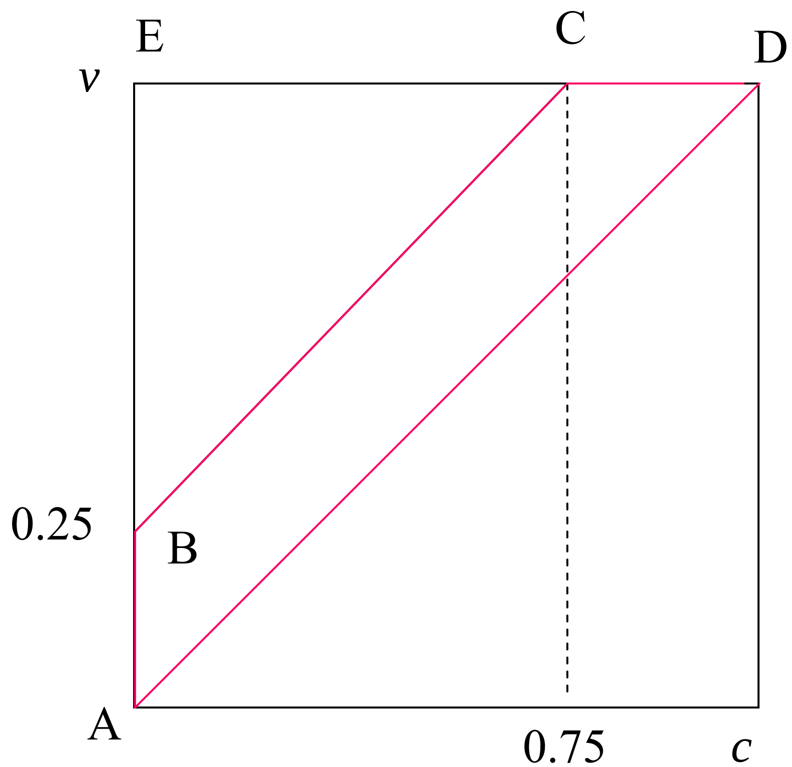
# Inefficiency of Bargaining with Private Information

- Private information about costs/values is ubiquitous. To understand the consequences of this consider split-the-difference bilateral trade.
- Seller has an indivisible good that he might be willing to sell to the single buyer.
- Buyer has value v and seller has cost c, each drawn independently and uniformly from [0,1].
- Both simultaneously write down a bid/offer, $v'$ and $c'$ respectively.
- If the bid is at least as greater as the offer, then they trade at

$$p = (v' + c')/2$$

  and otherwise do not trade. If trade occurs, seller's utility is $p - c$ and buyers' utility is $v - p$. If no trade occurs, utility is 0 for each.
- Inefficiency occurs because buyers understate values and sellers overstate costs.
- Chatterjee and Samuelson (1983) discovered that a linear equilibrium exists for this game.

## The trading region




How fast does ABCD shrink as $N$ increases?

- Trade should occur whenever $(c,v)$ lies in AEC.
- Trade only occurs when $(c,v)$ lies in BEC.
- The trapezoid ABCD are possible trades that are inefficiently unrealized. Notice, however, all high value trades are realized.
- Myerson and Satterthwaite (1983) showed that for this environment this is the most efficient bargaining mechanism among those that are incentive compatible, interim individually rational, and budget balanced.
- We know that when $m$ (the number of traders on each side of the market) goes to infinity that no trader has any ability to affect price and the inefficiency vanishes. But how fast does this happen?

# Double Auction (DA)

- Consider a centralized market with *m* sellers and *m* buyers where each seller wants to sell one unit and each buyer wants to buy one unit. Each seller's cost $c_i$ and each buyer's value $v_j$ is private knowledge. The costs/valuations of entering traders are drawn from common knowledge distribution *F* whose densities on [0, 1] is bounded away from zero
- A double auction is a one-shot game to allocate the *m* units of supply to the *m* traders who most highly value them.
- Simultaneously each buyer submits a bid and each seller submits an offer. The auctioneer accepts the bids and offers as truthful statements of value and cost and allocates the *m* units to the *m* traders who submitted the *m* highest offers/bids. The price is set within the interval of possible market clearing prices.
- Formally, sort the $2m$ bids/offer from lowest to highest:

$$s_{(1)}, s_{(2)}, \cdots, s_{(m)}, s_{(m+1)}, \cdots, s_{(2m)}.$$

- Set price at the midpoint of the interval $[s_{(m)}, s_{(m+1)}]$. This is the ½ DA.
- Assign the *m* units of supply to the traders who bid/offered at least that price. Buyers who trade gain $v_i - p$ and sellers who trade gain $p - c_i$. Traders who do not trade have zero gain.
- **There are no participation costs.**

# Theorems for Independent Private Values

- Equilibrium notion is Bayesian Nash equilibrium. Sellers' strategy is a function from cost to offer. Buyers' strategy is a function from value to bid

- The potential gains from trade (GFT) are the expected gains from trade if all traders reported their true values/costs. The inefficiency of an equilibrium is the potential GFT that the equilibrium fails to realize in expectation. The relative inefficiency of an equilibrium is the ratio of its inefficiency to the potential GFT.

- *I*, the relative inefficiency of an equilibria, decreases at quadratic rate: for all equilibria: a $\xi > 0$ exists such that

$$I < \frac{\xi}{m^2}.$$

- These results were established by Satterthwaite and Williams (1989) and Rustichini, Satterthwaite, and Williams (1994).

# Theorems for Correlated Private Values

- For large markets an equilibrium with trade exists and the deviation from of traders' strategies from truth-telling is order $1/m$ (Fudenberg, Mobius, and Szeidel, 2003).
- For large markets the relative inefficiency of all equilibria is order $1/m^{2-\varepsilon}$ where $\varepsilon$ is positive and arbitrarily small (Cripps and Swinkels, 2003). Their model permits multiple unit supply for sellers, multiple unit demand for buyers, and very general notions of correlation.

# Theorem for Interdependent Values and Affiliated Private Signals

- Consider a large double auction market with $2m$ traders in which bids and offers are constrained to be on a discrete grid in the interval [0,1].
- The state $\omega \in [0,1]$ of the market is an unobservable random variable with density $g(\omega)$ drawn from [0,1]. Each trader *i* observes an independently drawn signal $x_i$ with density $f(x_i \mid \omega)$. The $2m+1$ random variables $x_1, \dots, x_{2m}, \omega$ are strictly affiliated. The value of the good for trader *i* is $v(x_i, \omega)$ where $v_x > 0$ and $v_\omega \geq 0$.
- Benchmark: In an exchange economy with a continuum of agents the unique fully revealing REE has price $P(\omega) = v(x(\omega), \omega)$ where $F(x(\omega) \mid \omega) = 0.5$.
- Theorem (Reny and Perry, 2003). If the market is large enough, then
  - An equilibrium to the double auction exists.
  - The equilibrium is arbitrarily close to efficient and converges to the REE.
- Parametric examples (Satterthwaite, Williams, and Zachariadis, 2006) exist in which the equilibrium converges the REE and the rate of convergence to efficiency is $1/m^2$.

# Static Double Auctions: Summary

- The purpose of the double auction—a centralized market mechanism—is to simultaneously elicit traders' costs/values and assign the *m* units of supply to those *m* traders who have the highest costs/values..
- Buyers have an incentive to misrepresent their valuations, which causes inefficiency.
  - These incentives linearly decline with market size because the probability of being the marginal buyer decreases with market size.
  - As the market increases in size, the gains from trade that an inefficiently excluded trader would have realized if the market were efficient approaches zero.

  These two effects imply quadratic convergence of relative inefficiency to zero.
- This is true for private values, correlated values, and (with caveats) interdependent values.
- Suggests that centralized markets can be wonderfully efficient.

# Relaxing the One-shot Assumption

- Most markets are not centralized and traders who fail to trade at a particular point in time can and do attempt to trade again a short while later. They are anything but one-shot games. *How can this be if the DA is so good?*

- We want to know if competition across time works as well as competition at a point in time in a market with incomplete information. Economists believe this is so.

- In recent years there has been a series of matching and bargaining models in which there is incomplete information on both sides of the market and which, as the length of time between trading opportunities shrinks to zero, converge to the competitive allocation. See, for example, Atakan (2007a, 2007b), Lauermann (2006), Satterthwaite and Shneyerov (2007a, 2007b), and Shneyerov and Wong (2007).

- In matching and bargaining models the rate of convergence is in terms the expected length of time $\tau$ an individual has to wait to be matched and his participation cost $\kappa$ per unit time. These rates are $1/\tau$ and $1/\kappa$. It is not clear how they relate to the $1/m^2$ rate of static double auctions.

- To resolve this puzzle the DA and a matching and bargaining protocol must be nested within the same dynamic framework.

# A Dynamic Market: Framework

- I adopt a stripped down version of the setup that Shneyerov and Wong (2006) use for their matching and bargaining model.
- The market is infinite horizon with continuous time.
  - Potential buyers and sellers enter continuously, each at the rate 1 unit mass per unit time. Only potential traders who have non-negative expected utility actually enter and become active traders.
  - Each trader incurs a participation cost of $\kappa$ per unit time. Traders do not time discount their utility.
- The costs/valuations of potential traders are independently drawn from $F$, the uniform distribution on [0, 1]. Its density, $f$, is bounded away from zero and infinity. Sellers' costs and buyers' values are private.
- To make my point, I only need to consider symmetric equilibria in which buyers and sellers follow strategies $B$ and $S$ such that $B(x) = 1 - S(1- x)$.
- $G_B$ and $G_S$ are the endogenous distributions of the costs/valuation of traders who are active in the market in a given period, are endogenous. Let $A_B$, and $A_S$ denote the steady state masses of these buyers and sellers respectively.

# Equilibrium

- A steady state equilibrium has the following properties:
  - Traders' strategies $S$ and $B$ are time invariant.
  - The strategies generate the (i) steady state distributions $G_S$ and $G_B$ of active traders' types, (ii) the measure $A_S$ of active sellers at the beginning of each period, and (iii) the measure $A_B$ of active buyers at the beginning of each period.
  - No type of trader can increase his expected utility by a unilateral deviation from the strategies $S$ and $B$ taking full account of their continuation values
  - The equilibrium strategies $S$ and $B$, the masses $A_S$ and $A_B$, and the distributions $G_S$ and $G_B$ are common knowledge among all active and potential traders.
- Beliefs are simple because matching is anonymous and there are continua of traders.

# The DA Market with Participation Cost

- Let $\tau$ be a small length of time and $m$ be a positive, possibly integer.
- A trader enters at time $t$. At time $t + \tau$ he is matched in a large DA that, in total, has $m$ buyers and $m$ sellers. Note that each has incurred $\tau\kappa$ participation cost and would not have entered the market if he did not expect recover at least that cost.
- The $2m$ traders play the ½ double auction.
- Traders who succeed in buying or selling leave the market with their gains from trade.
- Traders who fail to buy or sell stay in the market and are rematched into a new DA after a time interval of $\tau$. This rematching continues for each trader until he trades (or decides to give up).

# Theorem for DA Market with Participation Costs

- Theorem (Wu, 2003). If $m$ is sufficiently large, then an equilibrium exists and the number of buyers and sellers are equal, then
  - Only buyers for whom $v > 0.5 + \tau\kappa$ enter.
  - Only sellers for whom $c < 0.5 - \tau\kappa$ enter.
  - Every trader bids/offers 0.5.
  - Every trader trades immediately and at least recovers his participation cost.
  - The supports of active buyers and sellers' values do not overlap.
  - The only inefficiencies in the market are the participation costs $\tau\kappa$.for each trader and the exclusion of buyers with $v$ in $[0.5, 0.5 + \tau\kappa]$ and sellers with $c$ in $[0.5 - \tau\kappa, 0.5]$.

# ½-DA Equilibrium with Participation Costs When Supports of Active Buyers and Sellers' Values/Costs Overlap

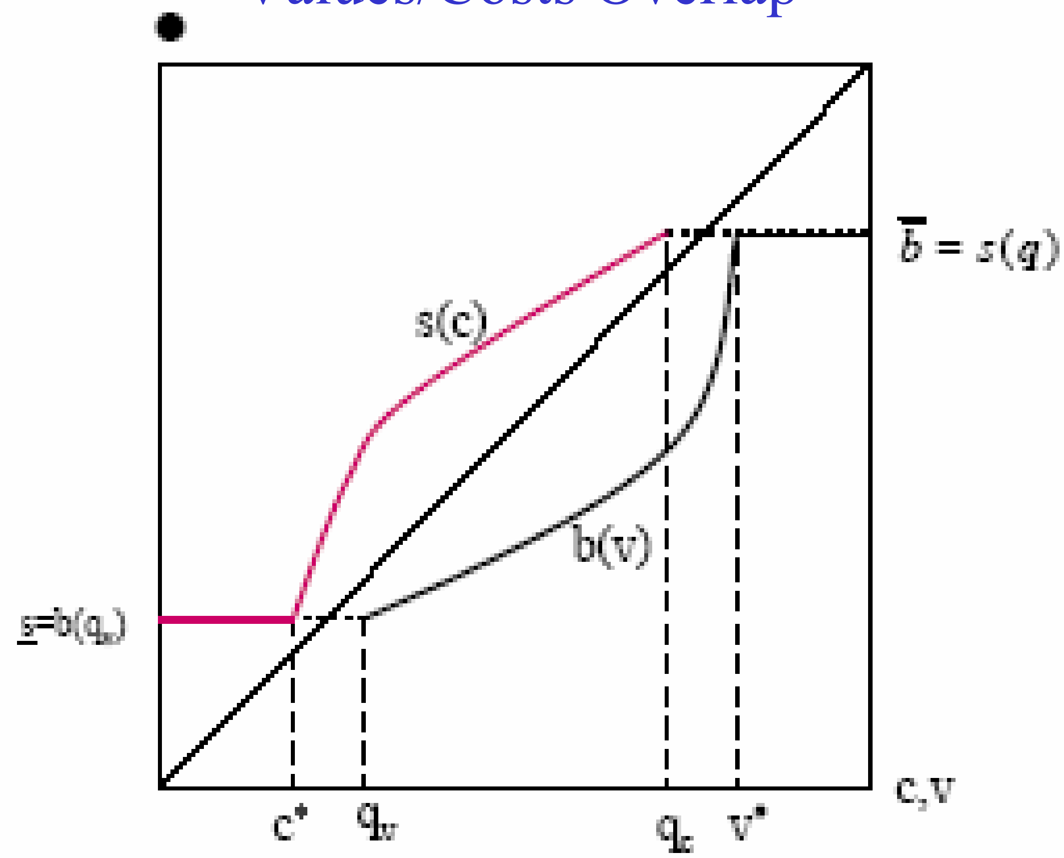


Figure 5: Kinked Strategy when $m = n$

In this equilibrium the supports of active sellers costs and active buyers values overlap. Wu (2003) shows that if *m* sufficiently large, then the supports separate and the strategies described on the previous slide become an equilibrium.

# The Matching and Bargaining Market (as in Shneyerov and Wong, 2006)

- The market is decentralized and matching into bilateral pairs occurs continuously.
- Each time interval *dt* each buyer's has probability of being matched bilaterally with a seller is $(M(A_B, A_S)/\tau A_B)dt$. The analogous probability for a seller is $(M(A_B, A_S)/\tau A_S)dt$.
  - The matching function *M* is homogenous of degree 1, continuous, symmetric in its arguments. Let $M(A, A) = 1$ for any positive *A*.
  - This implies that if the steady state masses are equal, then $\tau$ is the expected time until a trader's next match.
- Once matched, a fair coin flip selects either the buyer or seller to make a take-it-or-leave-it offer to the other. This is the random offer bargaining protocol.
- If the offer is accepted the pair leaves with their gains from trade. If it is not accepted, then they stay in the market and are randomly and anonymously rematched.

# Theorem for Matching and Bargaining Market with Participation Costs and Search Frictions

- Theorem (Shneyerov and Wong, 2006). An equilibrium exists in which:
  - Only buyers for whom $v > 0.5 + \tau\kappa$ enter.
  - Only sellers for whom $c < 0.5 - \tau\kappa$ enter.
  - The buyer, if he is the proposer, offers $0.5 - \tau\kappa$.
  - The seller, if he is the proposer, offers $0.5 + \tau\kappa$.
  - Every match results in a trade.
  - The only inefficiencies in the market are the participation costs $\tau\kappa$.for each trader and the exclusion of buyers with v in $[0.5, 0.5 + \tau\kappa]$ and sellers with c in $[0.5 - \tau\kappa, 0.5]$.
- This is the same allocation as under the DA.

# Conclusions

- The $1/m^2$ convergence to efficiency of the static DA mechanism is not relevant when it is embedded into a dynamic environment with participation costs.
- The DA mechanism and the random offer bargaining protocol work equally well in the dynamic framework. They produce equivalent allocations and converge to the competitive limit at the same rate as the search friction $\tau$ and participation cost $\kappa$ approach zero.
- Independent private values is a very restrictive assumption. What happens when costs and values are correlated or interdependent are open, important questions.

# The End