

## Efficient Online mechanisms for persistent, periodically inaccessible self-interested agents

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(Harvard)

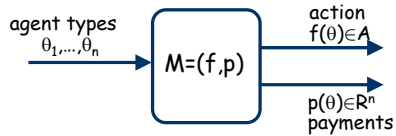
Satinder Singh  
(Michigan)

David C. Parkes  
Harvard University  
<http://www.eecs.harvard.edu/econcs>

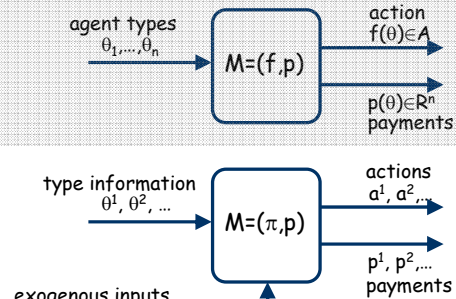
## Motivating question

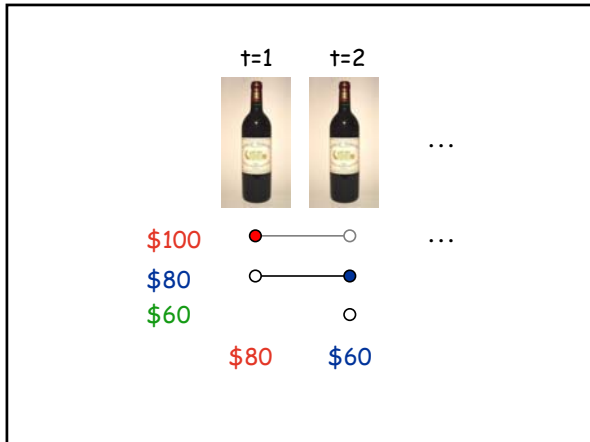
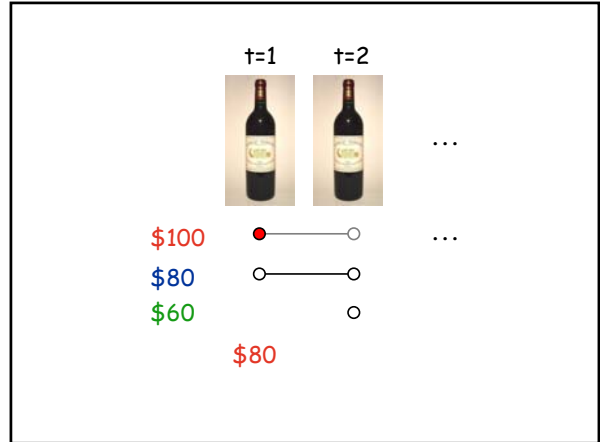
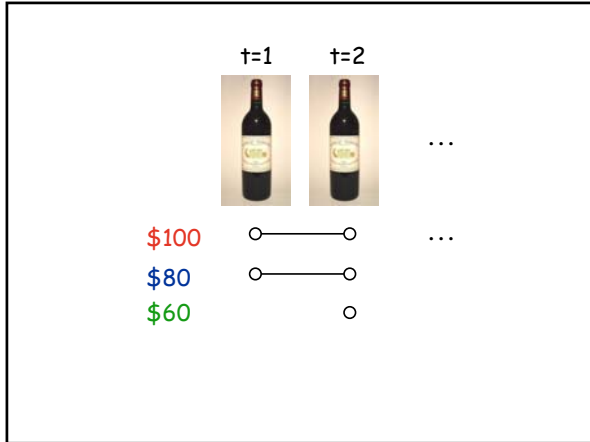
- MD in dynamic environments!

	dynamics	non-episodic
internet ads	demand & supply	contracts
social learning	information	persistent
last minute tix	demand	expressiveness
peer production	demand & supply	long tasks
pref. elicitation	information	bounded supply
task allocation	tasks	long tasks
...	...	...



## Dynamic Incentive Mechanisms





**One view from CS**

- Prior free:
 
$$\min_{z \in \mathcal{Z}} \mathbb{E} \left\{ \frac{\text{Val}(\pi(\theta_z))}{V^*(\theta_z)} \right\} \geq \frac{1}{c},$$
- Typical setting:  $(e_i, d_i, v_i, q_i)$

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- Typical setting:  $(e_i, d_i, v_i, q_i)$
- Body of work
  - Lavi & Nisan '00
  - Awerbuch et al. '03
  - Porter '04
  - Hajiaghayi, Kleinberg, Parkes'04
  - Blum & Hartline '05
  - Hajiaghayi, Kleinberg, Mahdian, Parkes'05
  - Lavi & Nisan '05
  - ...
- DSIC, monotonicity-based characterizations.
- Limited misreports: no-early arrival, no-late dep., etc.

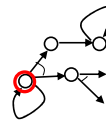
## A second view

- Center (and/or agents) have a probabilistic model of dynamics of environment
- $\pi^*(s) \in \arg \max_a [r(s,a) + \gamma \sum_{s' \in \mathcal{S}^{t+1}} \Pr(s',a,s) V^*(s')]$
- Agents can misreport local model, local state

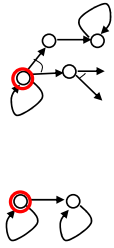
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- Agents can misreport local model, local state
- Body of work:
  - Parkes & Singh '03, '04
  - Cavallo, Parkes & Singh '06
  - Bergemann and Valimaki '06
  - Cavallo, Parkes & Singh '07
  - Athey & Segal '07
- typically interim IC, sometimes DSIC

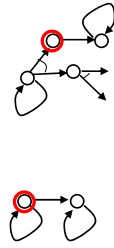
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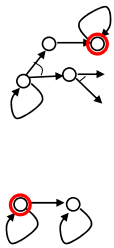
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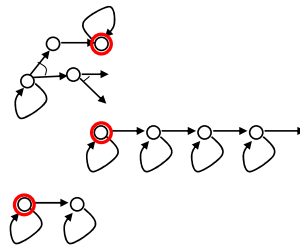
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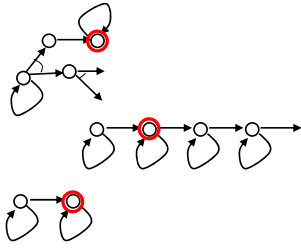
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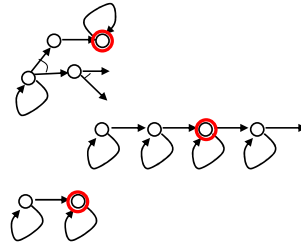
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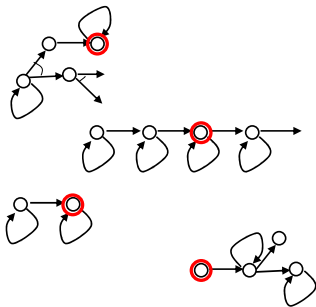
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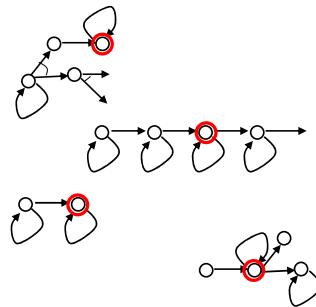
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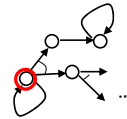
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- Dynamic types, persistent agents
- Dynamic types, arrival + departures
- Dynamic types, accessible/inaccessible

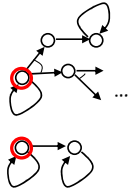
MDP:  $(S_i, A, r_i, \tau_i)$   
 $\tau_i(s_i, a)$   $r_i(s_i, a)$   
 initial state  $s_i^0$

policy  $\pi_i: S_i \rightarrow A$   
 $V_i^\pi(s) = E[\sum_{k \geq t} \gamma^{k-t} r^k(s^k, \pi(s^k))]$   
 $V_i^*(s): \pi_i^* \in \arg \max_{\pi} V_i^\pi(s)$



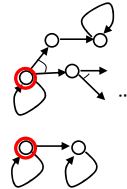
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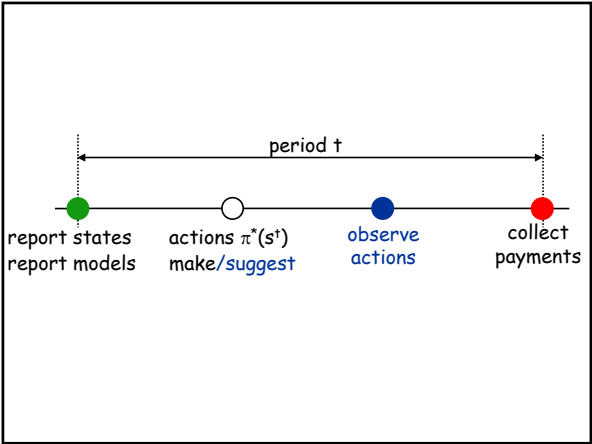
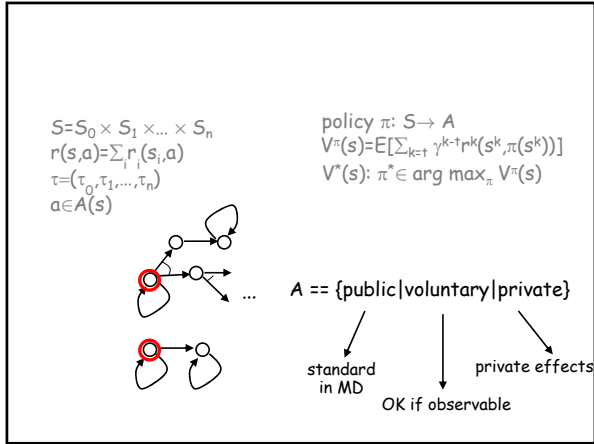
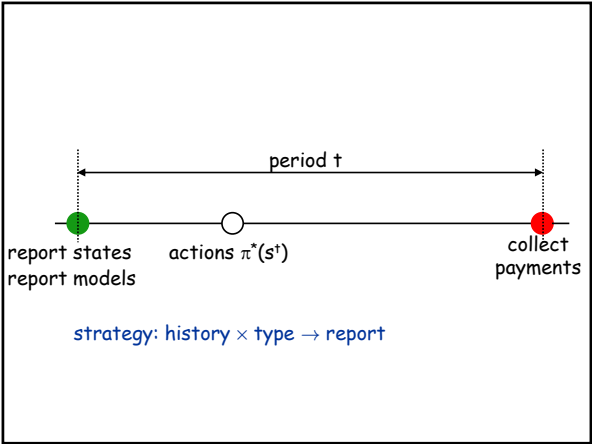
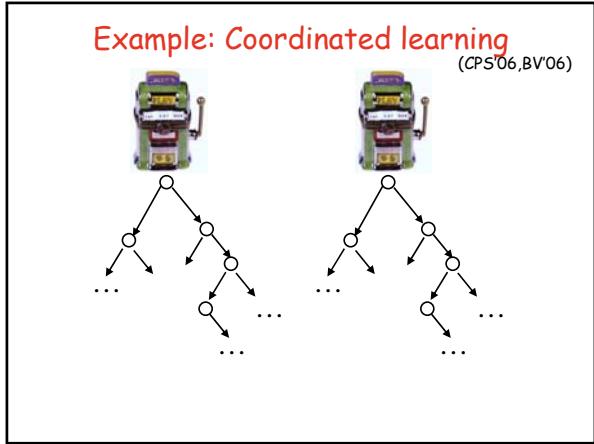


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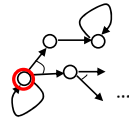


**Assumption: CIA**  
 (Conditional Independence given Actions)  
 $r_i((s_i, s_{-i}), a) = r_i((s_i, s_{-i}'), a)$   
 $\tau_i((s_i, s_{-i}), a) = \tau_i((s_i, s_{-i}'), a)$

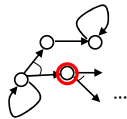


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- **Dynamic types, arrival + departures**
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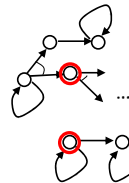
arrival/departure process



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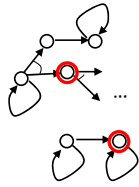


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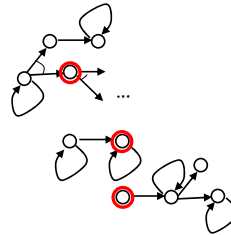




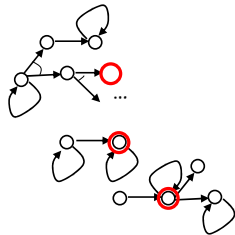
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static type == (local model, initial state)

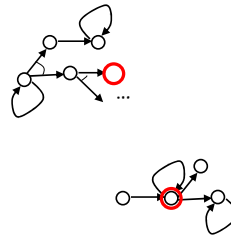
$H(s_0)$ : set of static types present

$s = (s_0, \{s_i\}_{i \in H(s_0)}) \in S$

$\tau_0: S_0 \times A \rightarrow S_0$  ← arrival process

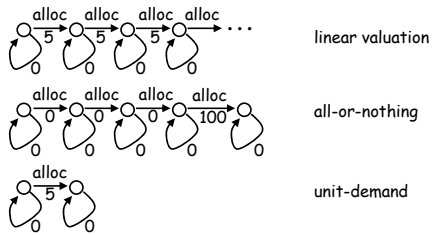
$r(s, a) = \sum_{i \in H(s_0)} r_i(s_i, a)$

departure: local absorbing state



### Special case: Deterministic Local Models

- Only dynamics are arrival/departure of static types
- Agents arrive and declare reward for all future sequences of actions via deterministic local model



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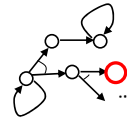
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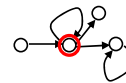
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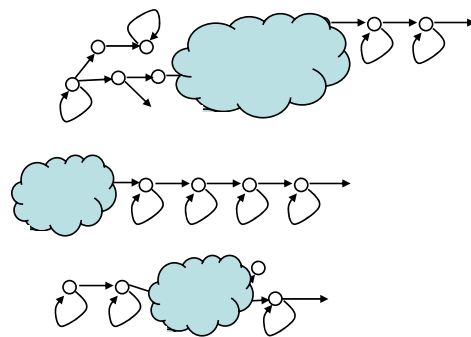
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arrival/departure = inaccess → access → inaccess

- Augment local state with accessible/inaccessible
- **Inaccessible == no messages, no payments**
- Can pretend to be inaccessible when not accessible (but not vice versa)
- Actions can make an agent become inaccessible
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• Assumption: run but can't hide // pay the piper

### A Belief-state MDP model

- **Partially Observable Markov Decision Process**
- **Model as a belief-state MDP** (Kaelbling et al. 96)
  - $BS = S_0 \times BS_1 \times \dots \times BS_n$ ,  $BS_i = \Delta(S_i)$
  - when accessible,  $bs_i \in BS_i$ , reduces to point mass
  - $r_i(bs_i^t a)$ : in expectation on underlying states
  - policy  $\pi^*$ :  $BS \rightarrow A$

#### Persistent & Dynamic type

CIA:  $r(s,a)$   $\tau(s,a)$   
 BV'06  
 $V^*(s) - V^*(s|\pi^*(s))$   
 charge each period

#### Arr/dep & Static type

$r(s,a)$   $\tau(s,a)$   
 $\Pr(\theta^t | \theta^{1:t-1}, a^{1:t-1}) = \Pr(\theta^t | a^{1:t-1})$   
 PS'03  
 $v_t - (V^*(s^e) - V^*(s^e))$   
 charge @ departure

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$r(s,a)$   $\tau(s,a)$   
 $\Pr(\theta^t | \theta^{1:t-1}, a^{1:t-1}) = \Pr(\theta^t | a^{1:t-1})$   
 CPS'07  
 $V^*(s) - V^*(s|\pi^*(s))$   
 charge each period

#### Persistent, Access/inaccess, dynamic type

$r(s,a)$   $\tau(s,a)$  RBCH  
 CPS'07  
 charge  $\hat{T}_t^*(bs^t) = \sum_{k=t-\delta^*(t)}^t \frac{T_k^*(bs^k)}{\gamma^{t-k}}$   
 where  $\delta^*$  is # periods been inaccessible

• Expected equilibrium payoff to agent  $i$  is  $V^*(s^t | f_i) - C_i(s^t)$ , where  $C_i(s^t)$  is independent of strategy  $f_i$

- persistent: in every state
- arrival/departure: in arrival period
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- Subtle:  $V^*(s^t) - V^*(s^t_{-i})$  does not work!

### Relating to "Offline VCG"

#### offline VCG

- optimal  $x^*$  // DSIC
  - ex post IR
  - ex post ND
- need private values

#### dynamic VCG

- optimal  $\pi^*$  // interim IC
- interim IR ( $V^*(s^t) \geq V^*_{-i}(s^t_{-i})$ )
- interim ND ( $V^*_{-i}(s^t_{-i}) \geq V^*_{-i}(s^t)$ )

$\epsilon$ -correctness of model  
 $\epsilon$ -optimality of policy  
 critical for incentive properties

### Computational issues

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- Tree-sampling (Kearns, Mansour & Ng'99, Ng & Jordan'00)
  - applied to online MD by Parkes & Singh'04
  - scale exponentially in number of actions and look-ahead, but independent of state space
  - $\epsilon$ -optimality and  $\epsilon$ -equilibrium (poly in  $1/\epsilon$ )



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  - $\epsilon$ -optimality and  $\epsilon$ -equilibrium (poly in  $1/\epsilon$ )
- Trajectory-sampling (van Hentzenryck et al.)
  - suitable in deterministic local model // independent arrival process setting
  - avoids exponential scaling, but hard to prove bounds (but see Mercier, Upfal & van Hentzenryck '07)



### Parallel literature: DSIC

- set-valued setting  $(e,d,r,L)$
- limited misreports and/or delayed allocation and late departures
- makes monotonicity in sense of earlier arrival, later departure, smaller  $L$  sufficient

competitive analysis

online stochastic combinatorial optimization + "output ironing"  
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### Open problems

### Indirect methods

- Direct revelation of type information costly, and counter to privacy concerns
- Prefer to allow agents to retain private model, perform local planning, report enough information to allow optimal joint actions
- Look for price-based methods

### Cashless design

- What about dynamic coordination in systems without money?
- Dynamic voting protocols
- c.f. Jackson "linking mechanism"

### Richer models, Economic Qs

- Two-sided models, e.g. dynamic (combinatorial) exchanges. Only have results for simple settings. E.g., Blum et al.'06, Parkes & Bredin'07
- Redistribution methods. Can the methods of Bailey, Cavallo, Guo & Conitzer, Moulin, Hartline etc. be leveraged?
- What about revenue?
- Learning by center?
- etc...

### Approximate incentive compatibility

- Likely will need to relax IC requirements
- One approach
  - have local agents help with computation
  - suggest improved policies
  - fold new computational methods into the mechanism
  - c.f. Nisan & Ronen (second-chance)
  - c.f. Cavallo (belief-coordination mechanism)
- Another approach
  - local stability
  - first price vs. generalized second price vs. GSP, Threshold rule, etc.
  - tolerable manipulability

### Applications/Comput. Directions

- Meta-deliberation auctions
  - use these techniques to coordinate the deliberation process of agents
  - extend MD to embrace more of the process of decision making
- Online CAs
  - obvious application area
  - lots of algorithmic challenges
- AI architectures
  - can this be used to coordinate computational processes?

## Summary

- Rich agenda in embracing dynamics within mechanism design
- Relevant to many kinds of coordination problems, including computational processes
- Fun new challenges ☺

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