# Combinatoric Auctions

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Outline

- Introduction
- Single-Minded Bidders
- Challenges

Combinatorial Auctions: Allocate $K$ items to $N$ people.

The *allocation* to i is $x^i \in \{0,1\}^K$ where $x^i_k = 1$ if and only if $i$ gets item $k$.

*Feasibility*: $x = (x^1, ..., x^N) \in F$ if and only if $x^i \in \{0,1\}^K$ and $\sum_i x^i_k \leq 1$ for all $k$.

*Utility* for i: $v^i(x^i, \theta^i) - y^i$ where $\theta^i \in \Theta^i$.
[For *reverse auctions*, use $y^i - c^i(x^i, \theta^i)$.]

Is there a combinatorial auction problem?

If agents are obedient and infinitely capable, and if the mechanism is infinitely capable, then to maximize revenue or to achieve efficiency:

Have each i report $v^i(x^i, \theta^i)$ for all $x^i \in \{0,1\}^K$.

Let $x^* =$ argmax $\sum v^i(x^i, \theta^i)$ subject to $x \in F$.

Allocate $x^{*i}$ to each i.

Charge each i, $y^i = v^i(x^{i*}, \theta^i)$.

This is efficient and revenue maximizing.

Note: If $y^i = 0$ for each i, then you get buyer efficiency.

Is there a problem?

Have each i report $v^i(x^i, \theta^i)$ for all $x^i \in \{0, 1\}^K$.
Communication: $2^K$ can be a lot of numbers.

Let $x^* =$ argmax $\sum v^i(x^i, \theta^i)$ subject to $x \in F$.
Computation: Max problem isn't polynomial.

Charge each i, $y^i = v^i(x^{i*}, \theta^i.)$
Incentives: So, why should I tell you $\theta^i$?

Subject to Communication, Computation, Voluntary Participation, and Incentive Compatibility Constraints,

What is the Best Auction Design?

## Some Design Features to Consider

Bids allowed - single items, all packages, some (which?)

Timing - synchronous, asynchronous

Pricing - pay what you bid, uniform (second price), incentive pricing

Feedback - all bids, provisional winning bids only, number of bids for each item, item prices (which?), ...

Others - minimum increments, activity rules, withdrawals, reserve prices (secret or known), retain provisional losing bids, XOR, proxies, ...

## Example Practical Questions

- Public sector - Spectrum Auctions
  Use Design #1 (single item bids, synchronous, iterative) or
  use Design #2 (package bids, synchronous, iterative) ?

- Private sector - Logisitics Acquisitions
  Use Design #1 (package bids, synchronous, iterative) or
  use Design #2 (package bids, one-shot sealed bid)?

  How Should we Decide? What about Other Designs?

## Combinatorial Auctions: The Art of Design - the 1st generation

Sealed bid, IC pricing
- Vickrey-Clarke-Groves (1963, 71, 73)

Sealed bid, pay what you bid
- Rasenti-Smith-Bulfin (1982)

Iterative, asynchronous,
- Banks, Ledyard, Porter 1989 - AUSM

Iterative, synchronous,
- Ledyard, Olson, Porter, etc. 1992 - Sears

Iterative, synchronous, no package bids, activity rules
- McMillan, Milgrom 1994 - FCC-SMR

## Combinatorial Auctions: The Art of Design - the 2nd generation

Iterative, synchronous, Proxies
- Parkes 1999 - iBEA

Iterative, synchronous, price feedback
- Kwasnica, Ledyard, Porter 2002 - RAD

Clock auction, packages, synchronous
- Porter, Rassenti, Smith 2003

CC, proxies
- Ausubel, Milgrom 2005

How should we decide

Which Design is Best for which Goals in which Situations?

## Combinatorial Auction Design: Three approaches

- Experimental: the economist's wind tunnel
- Agent-based: the computer scientist's wind tunnel
- Theoretical: the analyst's wind tunnel

| approach | behavioral model | mechanism complexity | environmental coverage |
|---|---|---|---|
| experimental | correct (naive?) | not stressed | costly |
| agent-based | open? (not str.for.) | can stress | moderate |
| theoretical | stylized | open? | complete |

## A Taste of the Experimental Approach:
(Brunner-Goeree-Holt-Ledyard)

- 12 licenses , 8 subjects (experienced - trained)
  6 regional bidders: 3 licenses each, $v \in [5, 75]$
  2 national bidders: 6 licenses each, $v \in [5, 45]$
  13,080,488 possible allocations

- 0.4 cents per point, (upto $1.25 for 3, $1.30 for 6)
  with a synergy factor $\alpha$ per license of 0.2 (national)
  and 0.125 (regional)

- Earnings averaged $50/ 2 hour session incl $10 show-up fee.
  48 sessions of 8 subjects each. 10 auctions/session.
  120 auctions /design.

## Economic Experiment Results

| | SMR | CC | RAD | FCC* |
|---|---|---|---|---|
| Average Efficiency | 90.2% | 90.8% | 93.4% | 89.7% |
| Average Revenue | 37.1% | 50.2% | 40.2% | 35.1% |
| Average Profits | 53.1% | 40.6% | 53.3% | 54.6% |

$\text{Efficiency}_{output} = (E_{actual} - E_{random})/(E_{maximum} - E_{random})$.

$\text{Revenue} = (R_{actual} - R_{random})/(R_{maximum} - R_{random})$.

Profits = Efficiency − Revenue

Is Revenue of 50% big or small?

Are these the result of Behavior, Environment, or Design?

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## A Taste of the Theoretical Approach

An auction design is $\gamma = \{N, S^1, ..., S^N, g(s)\}$.

Bidders behavior is $b^i : \{(I^i, v^i, \gamma)\} \rightarrow S^i$.

The Design Problem is:

- Choose $\gamma$ so that $g(b(I, v, \gamma)) = [x(v), y(v)]$ is desirable.

The Economist's approach:
(1) Get an upper bound on performance; ignore Computation and Communication Constraints.
(2) Use all information available; Assume the seller has a prior $\pi(\theta)d\theta = d\Pi(\theta) = d\Pi^1(\theta^1)...d\Pi^N(\theta^N)$.

Using the revelation principle, choose $(x, y) : \Theta^N \to \{(x, y)\}$ to maximize expected revenue

$$\max \int \sum_i y^i(\theta) d\Pi(\theta)$$
$$\text{subject to}$$
$$(x(\cdot), y(\cdot)) \in F^* \cap IC \cap VP.$$

Question: Interim or ex-post? Bayesian or Dominance?
Answer: Will see it doesn't matter.

Consider a special class of environments

Single-Minded Bidders

- Each bidder has a preferred package $x^{*i}$ that is common knowledge (including the auctioneer).

$v^i(x, \theta^i) = \theta^i q^i(x)$ where

$$\begin{aligned} q^i(x) &= 1 \quad \text{if} \quad x^i \geq x^{*i} \\ q^i(x) &= 0 \qquad \text{otherwise} \end{aligned}$$

Probability of winning is $Q^i(\theta^i) = \int q^i(x(\theta))d\Pi(\theta|\theta^i)$

Expected payment is $T^i(\theta^i) = \int y^i(x(\theta))d\Pi(\theta|\theta^i)$

Expected Utility is $\theta^i Q^i(\theta^i) - T^i(\theta^i)$

Incentive compatibility is $T(\theta) = T_0 + \int_{\theta_1}^{\theta} s dQ(s)$ and $dQ/d\theta \geq 0$

Voluntary participation is $\theta_1^i Q^i(\theta_1^i) - T^i(\theta_1^i) \geq 0$

Combine these with revenue maximization and
get that $T = \theta Q - \int_{\theta_1}^{\theta} Q(s)ds$

So Expected revenue from i is $\int [\theta^i - \frac{1-\Pi(\theta^i)}{\pi(\theta^i)}] q^i(\theta) d\Pi(\theta)$

The optimal *interim* mechanism for single minded-bidders (where $\Pi(\theta)$ is common-knowledge) solves

$$\begin{aligned} x(\theta) &\in \arg\max_{x \in F^*} \sum w_i(\theta^i) q^i(x) \\ y^i(\theta) &= \theta^i Q^i(\theta^i) - \int_{\theta_1}^{\theta^i} Q^i(s)\, ds \\ \text{where } w_i(\theta^i) &= \theta^i - \frac{1 - \Pi^i(\theta^i)}{\pi^i(\theta^i)} \end{aligned}$$

Requires $dw^i/d\theta^i \geq 0$, for incentive compatibility SOC.
An increasing hazard rate is sufficient.

This is a (very slight) generalization of Myerson (1981).
Only $F^*$ is different.

Using Mookherjee and Reichelstein (1992), monotonicity implies one can convert the *interim* mechanism to an *ex-post* mechanism with the same interim payoffs to everyone.

$$x^*(\theta) \in \arg\max_{x \in F} \sum w_i(\theta^i) q^i(x)$$

$$y^{*i}(\theta) = \theta^i q^i(x^*(\theta)) - \int_{\theta_1}^{\theta^i} q^i(x^*(\theta/s^i)) ds^i$$

This mechanism is the optimal *ex post* mechanism because

*ex-post* $F^* \cap IC \cap VP \subset$ *interim* $F^* \cap IC \cap VP$

Note that $q^i(x^*(\theta)) = 1$ if

$$\max_{x \in F} \sum_{j=1}^{N} w^j(\theta^j) q^j(x) > \max_{x \in F} \sum_{j \neq i} w^j(\theta^j) q^j(x)$$

Let

$$\theta^{*i}(\theta_{-i}) = \inf\{\theta^i | q^i(x^*(\theta)) = 1\}$$

The optimal *ex-post* mechanism is:

$$\begin{aligned} q^i(x^*(\theta)) &= 1 \text{ iff } \theta^i \geq \theta^{*i}(\theta_{-i}) \\ \text{and } y^{*i}(\theta) &= \theta^{*i}(\theta_{-i}) q^i(x^*(\theta)) \end{aligned}$$

The optimal *ex-post* mechanism is not VGC.

It is closely related. They both look like

$$\begin{aligned} q^i(x(\theta)) &= \text{ iff } \theta^i \geq \theta^i(\theta_{-i}) \\ \text{and } y^i(\theta) &= \theta^i(\theta_{-i}) q^i(x(\theta)) \end{aligned}$$

but the Optimal $\theta^{*i}(\theta_{-i}) \neq$VCG $\widehat{\theta}^i(\theta_{-i})$

$$\begin{aligned} x^*(\theta) &\in \arg\max_{x \in F} \sum_i \left( \theta^i - \frac{1 - \Pi^i(\theta)}{\pi^i(\theta)} \right) q^i(x) \\ \widehat{x}(\theta) &\in \arg\max_{x \in F} \sum_i \theta^i q^i(x) \end{aligned}$$

The optimal *ex post* mechanism is not output-efficient.

Even if conditioned on participation (as in Myerson).

The optimal *ex post* optimal mechanism is VCG with preferences.

- Request sealed bids for packages: $b^i$
- Subtract an *individual* "preference": $p^i = \frac{1-\Pi^i(b^i)}{\pi^i(b^i)}$
- Maximize *adjusted* bid revenue: max $\sum_i (b^i - p^i)\nu^i$ subject to $\nu^i \in \{0,1\}$ and $(\nu^1, ..., \nu^N)$ feasible
- Charge *pivot* prices: $y^i = \inf\{b^i | \nu^i = 1\}$

## Interesting Special Case

If values are uniformly distributed, then

$\theta^i \sim U[m^i, M^i]$, then $p^i(b^i) = M^i - b^i$ and $b^i - p^i(b^i) = 2b^i - M^i$.

In this case, the optimal auction is equivalent to:

- Charge a reserve price of: $r^i = M^i/2$
- Maximize the reserve-adjusted surplus: $\sum(b^i - r^i)\nu^i$.

Example: $K = 2, N = 3$

$x^{*1} = (1, 0), x^{*2} = (0, 1, ), x^{*3} = (1, 1)$
$\theta^1, \theta^2$ are uniformly distributed on $[0, 1]$
$\theta^3$ is uniformly distributed on $[0, a]$

Revenue as a % of maximum extractable

| | if a=1 | if a=2 | if a=3 |
|---|---|---|---|
| OA | 0.585 | 0.625 | 0.613 |
| VGC | 0.240 | 0.452 | 0.426 |
| Random | 0.480 | 0.465 | 0.413 |

OA & VCG highest for $a = 2$, the most competitive situation.

Random (5 allocations possible) looks as good as VCG.

New Experiments

* 2 items, 3 subjects
* Tested SMR, RAD, and SB
* 1 session for each auction
* 9 subjects per session
* Randomly matched into groups of 3 at beginning
* 10 rounds for each group (the first 2 were practice rounds).
* Before round, bidders randomly assigned to role .
* Values for 1 and 2 are in [0,100], values for 1,2 are in [0,200]
* No withdrawals, no activity rules

Experiment Results (24 auctions of each type)

Mean (Std. Dev.)

| | Revenue | Efficiency | Rev/Max Possible |
|---|---|---|---|
| OA | 77.31 (38.52) | 0.86 (.29) | 0.59 (.23) |
| SMR | 58.13 (43.16) | 0.90 (0.20) | 0.46 (0.33) |
| RAD | 66.71 (46.99) | 0.97 (0.09) | 0.53 (0.30) |

RAD $>$ SMR in revenue.

# rounds for RAD (5.65) $<$ SMR (7.46).

But OA $>$ RAD

Experiment Results (24 auctions of each type)

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| RAD | 66.71 (46.99) | 0.97 (0.09) | 0.53 (0.30) |
| SB | 89.79 (36.99) | 0.96 (0.19) | 0.74 (0.19) |

SB > OA > RAD > SMR.

No reserve price used in SB.

Summary to here

For combinatorial auctions with single minded bidders

We find the DSIC design that maximizes expected revenue.
- It is neither VGC nor output efficient.
- It is VCG with individualized bid preferences.

In a small experiment, SB $>$ OA $>$ RAD $>$ SMR,
- RAD gets 85% of the revenue of the theoretical upper bound.
- SB gets 116% of the revenue of the theoretical upper bound.

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Combinatorial Auctions:

- The auction design: $\gamma = \{N, S^1, ..., S^N, g(\cdot)\}$.
- Bidders behavior: $b^i : \{(I^i, \theta^i, \gamma)\} \to S^i$
- Choose a feasible $\gamma$ so that $g(b(I, \theta, \gamma))$ is desirable.

The tension is between theory and practice.

## Choose a feasible $\gamma$ so that $g(b(I,\theta,\gamma))$ is desirable.

- Which $\gamma$ are feasible?

  Need pliable communication and computation constraints
  - A finer grid than NP-hard, polynomial, etc.
  - An analytic version that can be used as constraints in a maximization problem.

  Need a revelation principle for feasible mechanisms, $G^F \subset G$.
  - Usual: $\forall\gamma \in G^F$, $\exists\ \gamma^* \in G^D$ with $\gamma^* = \{N, \Theta, h(\cdot)\}$ such that $h(\theta) = g(b(\theta,\gamma))$ and $b(\theta,\gamma^*) = \theta$.
  - But inverse is now a problem. Need to characterize $G^{D*}$ such that if $\gamma^* \in G^{D*}$ then $\exists\gamma \in G^F \ni h(b(\theta,\gamma^*)) = g(b(\theta,\gamma))$.

## Choose a feasible $\gamma$ so that $g(b(I, \theta, \gamma))$ is desirable.

- What is the "right" theory of behavior?

  Need better theory of behavior in iterative auctions
  - Game theoretic equilibria such as Dominance & Bayes make sense for simple, direct revelation auctions but are "wrong."
  - With iteration, straight-forward bidding tempting, but "wrong."
  - Incorporate behavioral learning models (agents) into optimal auction methodology?

  Need behavior model to be more sensitive to details
  - Designing to prevent collusion often involves information issues finessed by direct mechanisms.
  - Reveal bids and bidders? Reveal only winning bids? Endogenous sunshine?

Choose a feasible $\gamma$ so that $g(b(I, \theta, \gamma))$ is desirable.

- What does desirable mean?

  Need to consider all costs and benefits

  - Tradeoff between mechanism and bidder computations

  - Iteration may reduce costs of determining values but increase costs of bidding?

- How do we choose?

  Can we always reduce to an optimization problem?

  - Need to deal with multi-dimensional incentive constraints

  - Need to find a simple characterization for feasible $\gamma$.

  - Or do we just need to generate a lot of experiments?