An algorithmic solution for an optimal decision making process within emission trading markets

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Abstract

We present a new algorithmic approach which leads towards a procedure for an international emissions trading system within the so-called Kyoto game. The economic background is described in detail. The Kyoto-game is introduced. An algorithmic solution which results from the player's cost-game is presented. We describe several allocation principles and an suitable auction procedure. The results can be used to support an optimal decision making process.

1 Introduction

The conferences of Rio de Janeiro 1992 and Kyoto 1997 demand for new economic instruments which have a focus on environmental protection in the macro and micro economy. An important economic tool being part of the treaty of Kyoto in that area is *Joint-Implementation.* It is a program which intends to strenghten international cooperations between enterprises in order to reduce CO_2-emissions. A sustainable development can only be guaranteed if the instrument is embedded into an optimal energy management. For that reason, the Technology-Emissions-Means (TEM) model was developed, giving the possibility to simulate such an extraordinary market situation. The realization of Joint-Implementation (JI) is restricted by technical and financial constraints. In a JI Program, the reduced emissions resulting from technical cooperations are recorded at the *Clearing House*. The TEM model integrates the simulation of both the technical and the financial parameters. In Pickl (1999) the TEM model is treated as a time-discrete control problem. Furthermore, the analysis of the feasible set is examined in Pickl (2001). In the following, a short introduction into the TEM model is given. Furthermore, we want to present a new algorithmic approach which leads to a procedure for an international emissions trading procedure within the so-called Kyoto auctioning game.

2 The Basic Model

The presented TEM model describes the economic interaction between several players (sometimes we say equivalently actors) which intends to maximize their reduction of emissions (E_i) caused by technologies (T_i), by expenditures of money or by financial means (M_i). The index stands for the i-th player, $i \in \{1, \ldots, n\}$. The players are linked by technical cooperations and by the market.
The effectivity measure parameter em_{ij} describes the effect on the emissions of the i-th player if the j-th actor invests money for his technologies. We can say that it expresses how effective technology cooperations are (like an innovation factor), which is the central element of a JI Program. The variable φ can be regarded as a memory parameter of the financial investigations, whereas the value λ_i acts as a growth parameter. For a deeper insight see Pickl (1999). The TEM model is represented by the following two equations:

$$E_i(t+1) = E_i(t) + \sum_{j=1}^{n} em_{ij}(t) M_j(t), \tag{2.1}$$

$$M_i(t+1) = M_i(t) - \lambda_i M_i(t)[M_i^* - M_i(t)]\{E_i(t) + \varphi_i \Delta E_i(t).\} \tag{2.2}$$

It is a great advantage of the TEM model, that we are able to determine the em_{ij}-parameter empirically. In the first equation, the level of the reduced emissions at the $t+1$-th time-step depends on the previous value plus a *market effect*. This effect is represented by the additive terms which might be negative or positive.

In general, $E_i > 0$ implies that the actors have yet reached the demanded value $E_i = 0$ (normalized *Kyoto-level*). A value $E_i < 0$ expresses that the emissions are less than the requirements of the treaty. In the second equation we see that for such a situation the financial means will increase, whereas $E_i > 0$ leads to a reduction of $M_i(t+1)$:

$$M_i(t+1) = M_i(t) - \lambda_i M_i(t)[M_i^* - M_i(t)]\{E_i(t) + \varphi_i \Delta E_i(t)\,.\}$$

The second equation contains the logistic functional dependence and the memory parameter φ_i which describes the effect of the preceding investment of financial means. The dynamics does not guarantee, that the parameter $M_i(t)$ lies in the interval, which can be regarded as a budget for the i-th actor. For that reason we have additionally to impose the following restrictions to the dynamical representation:

$$0 \leq M_i(t) \leq M_i^*, \quad i = 1, \ldots, n \quad \text{and} \quad t = 0, \ldots, N.$$

These restrictions ensure that the financial investigations can neither be negative nor exceed the budget of each actor. Now, it is easy to show that

$$-\lambda_i M_i(t)[M_i^* - M_i(t)] \leq 0 \quad \text{for} \quad i = 1, \ldots, n \quad \text{and} \quad t = 0, \ldots, N.$$

We have guaranteed that $M_i(t+1)$ increases if $E_i(t) + \varphi_i \Delta E_i(t) \leq 0$ and it decreases if $E_i(t) + \varphi_i \Delta E_i(t) \geq 0$. Applying the memory parameter φ_i, we have developed a reasonable model for the *money expenditure* - *emission* - interaction, where the influence of the technologies is integrated in the *em*-matrix of the system.
We can use the TEM model as a time-discrete model where we start with a special parameter set and observe the resulting trajectories. Usually, the actors start with a negative value, i.e., they lie under the baseline mentioned in Kyoto Protocol, see Kyoto (1997). They try to reach a positive value of E_i . By adding control parameters, we enforce this development by an additive financial term. For that reason the control parameters are added only to the second equation of our model:

$$M_i(t+1) \;=\; M_i(t) - \lambda_i M_i(t)[M_i^* - M_i(t)]\{E_i(t) + \varphi_i \Delta E_i(t)\} + u_i(t)\,.$$

The introduction of the control parameter $u_i(t)$ implies that each actor makes an additional investigation at each time-step. In the sense of environmental protection, the aim is to reach a state, mentioned in the treaty of *Kyoto*, by choosing the control parameters such that the emissions of each player become minimized. The focus is the realization of the necessary optimal control parameters via a played cost game, which is determined by the way of actors cooperation.

3 The Cost-Game in the TEM Model

Let us regard the nonlinear time-discrete dynamics of the TEM-model

$$\begin{aligned} E_i(t+1) &= E_i(t) + \sum_{j=1}^{n} em_{ij}(t) M_j(t)\,, && (3.1)\\ M_i(t+1) &= M_i(t) - \lambda_i M_i(t)[M_i^* - M_i(t)]\{E_i(t) + \varphi_i \Delta E_i(t)\,\}. && (3.2) \end{aligned}$$

We can replace (3.2) by

$$\begin{aligned} M_i(t+1) &= M_i(t) - \lambda_i M_i(t)[M_i^* - M_i(t)]\{E_i(t) + \varphi_i \sum_{j=1}^{n} em_{ij}(t) M_j(t)\}\\ &\qquad \text{taking into account (3.1)} \mathit{and} \quad \Delta E_i(t) = E_i(t+1) - E_i(t) \end{aligned}$$

In order to reach steady states, which are determined in Pickl (1999), an independent institution may coordinate the *trade relations* between the actors (clearing house mechanism). The trade relations are expressed by the *em*-matrix. In practice, the imposing of *taxes* or the giving of *incentives* means that in the

TEM-model the em-parameters will change.
Now, the principle of JI implies that technical cooperation will be benefitted. If there is a cooperation between player 1 and player 2, we introduce an additional parameter $\epsilon, \epsilon > 0$, which implies that the measure of effectivity increases. The cooperation of the grand coalition is expressed by the parameter ω:

$$\begin{pmatrix} em_{11} & em_{12}+\epsilon & em_{13} \\ em_{21}+\epsilon & em_{22} & em_{23} \\ em_{31} & em_{32} & em_{33} \end{pmatrix}$$

Example: Actor 1 and Actor 2 do cooperate

$$\begin{pmatrix} em_{11} & em_{12}+\omega & em_{13}+\omega \\ em_{21}+\omega & em_{22} & em_{23}+\omega \\ em_{31}+\omega & em_{32}+\omega & em_{33} \end{pmatrix}$$

Example: All players do cooperate

This extension of the TEM model results in a cost-saving effect at each time-step, which can be expressed by an cooperative cost-game. According to (3.1) and (3.2) let us begin with the construction of the cost-game:

$$\begin{aligned} v_t(K) &:= \underbrace{\sum_{j\in K} M_j(t)}_{without\,cooperation} - \underbrace{M(K)}_{cooperation} \\ &= \begin{pmatrix} K_1^*(t) & K_2^*(t) & K_3^*(t) \end{pmatrix} \begin{pmatrix} 0 & \epsilon & \delta \\ \epsilon & 0 & \gamma \\ \delta & \gamma & 0 \end{pmatrix}_{Ind(K)} \begin{pmatrix} M_1(t) \\ M_2(t) \\ M_3(t) \end{pmatrix} \end{aligned} \tag{3.3}$$

where $K_i^*(t) := \varphi_i \tilde{M}_i(t)$, $\tilde{M}_i(t) := [M_i^* - M_i(t)]\,(i = 1, \ldots, n)$ and $K \in Pot(\mathcal{N})$, $\mathcal{N}$ indicates the great coalition, i.e. the cooperation of all players.

Above, we also we used the notation $(B)_{Ind(K)}$ denoting the submatrix of $B = (b_{ij})$ corresponding to the indices $j, k \in K$, filled up by zero entries:

$$(B)_{Ind(K)} = A, \quad \text{with} \begin{cases} a_{ij} = b_{ij} & \text{, if } \; i \in K \; \text{ and } \; j \in K\,, \\ a_{ij} = 0 & \text{, otherwise}\,. \end{cases}$$

For the time-dependent (i.e. at each time-step there might exists another coalition) grand coalition we get:

$$\begin{aligned} v_t(\mathcal{N}) &:= \underbrace{\sum_{j\in \mathcal{N}} M_j(t)}_{without\,cooperation} - \underbrace{M(\mathcal{N})}_{cooperation} \\ &= \begin{pmatrix} K_1^*(t) & K_2^*(t) & K_3^*(t) \end{pmatrix} \begin{pmatrix} 0 & \omega & \omega \\ \omega & 0 & \omega \\ \omega & \omega & 0 \end{pmatrix} \begin{pmatrix} M_1(t) \\ M_2(t) \\ M_3(t) \end{pmatrix} \end{aligned}$$

For $\tilde{M}_i(t)\varphi_i M_i(t) \geq 0$ $(i \in \{1, \ldots, n\})$ the difference between the cooperative and the non-cooperative case is always positive.

So we have constructed a reasonable cost-game. This might be the bases for a decision analysis within emission trading markets where tolerances (according to Janowitz 1994)of underlying empirical data play an important role.

4 Conclusion

The method is that at each time step, this amount is put into a central fund (according to the clearing house mechanism). The results can be used to optimze and support a decision making process within the establishment of emission trading markets. They can be regarded as an actual example of *Computer Science and Decision Theory*.

5 References

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