Sampled-data Control and Signal Processing
– Beyond the Shannon Paradigm

Workshop in honor of Eduardo Sontag
on the occasion of his 60th birthday

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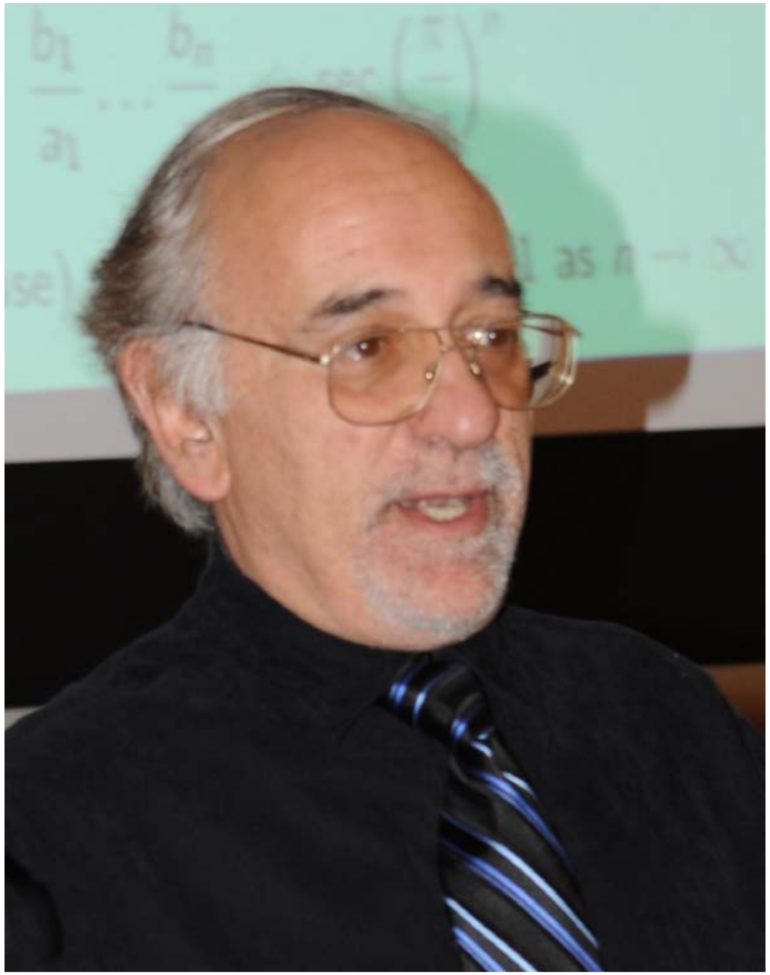
Thanks to



My schoolmate, dear friend, colleague, and even a teacher

One of the very rare pictures of Eduardo with a Tie: at my (YY) fest

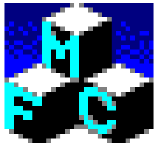
Outline

- Current signal processing paradigm
  - Via Shannon
  - $\Rightarrow$ Upper limit in high frequencies
- CAN BE SAVED via sampled-data control theory
- Some examples

Message of this talk

- We can do better in signal processing using sampled-data control theory
- ⇒ Optimal recovery of freq. components beyond the Nyquist freq. (=1/2 of sampling freq.)

Let's first listen to a demo



アプリケーション

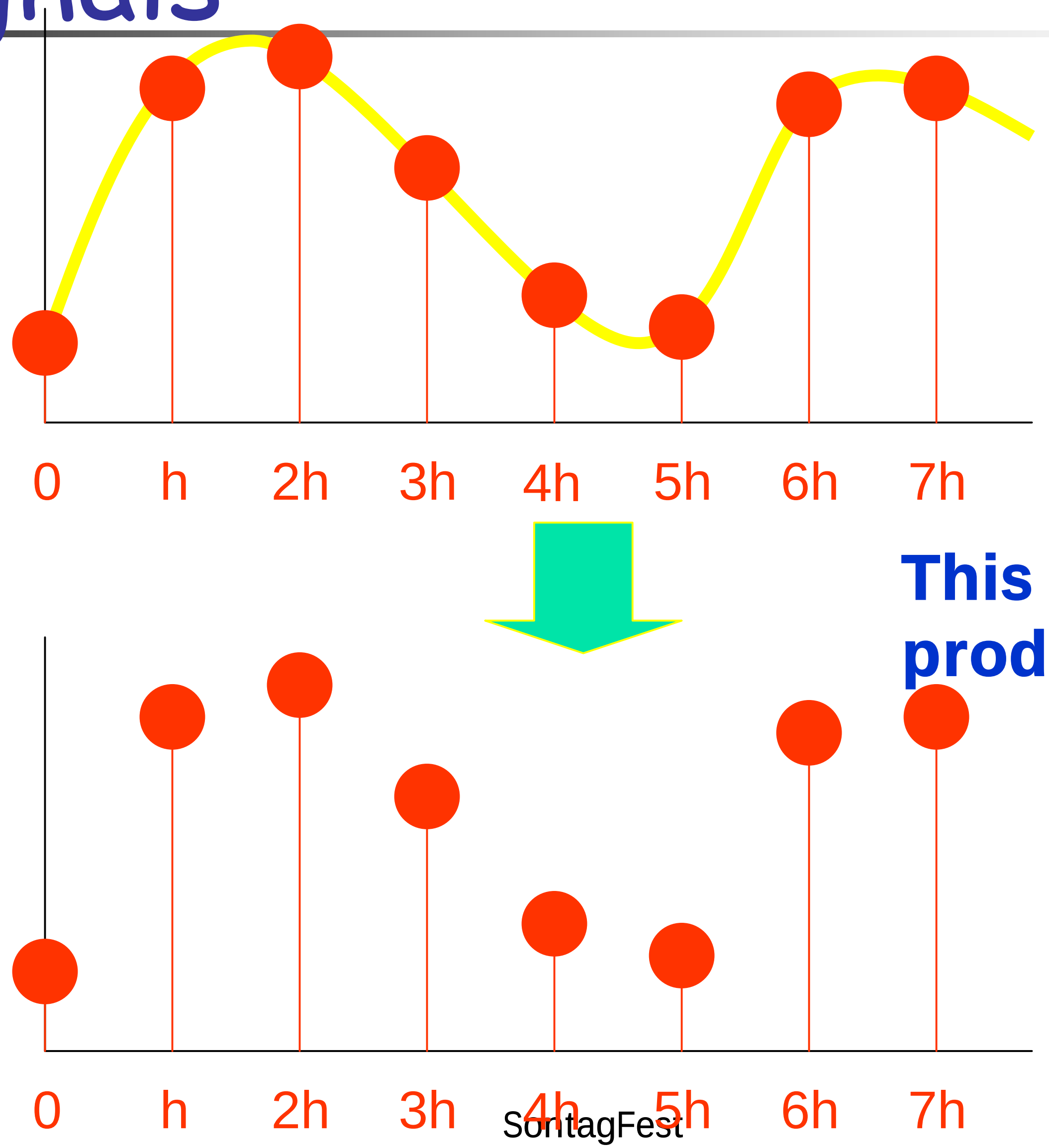
Red: Original (up to 22kHz)

Blue: downsampled to 11k, and then processed 4 times upsampled via YY filter

Did you hear the difference?

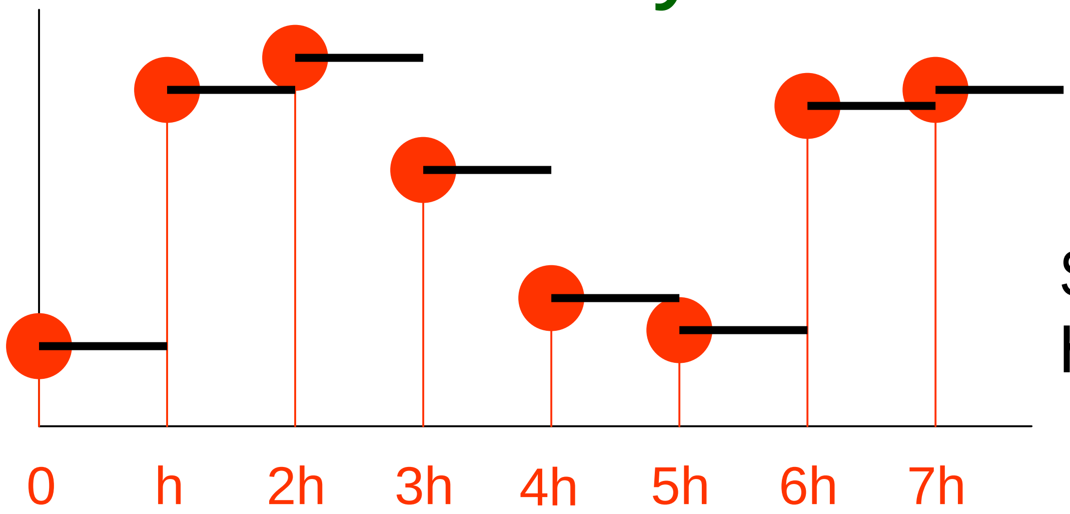
Part I: Current digital signal processing - Basics☺

Sampling continuous-time signals



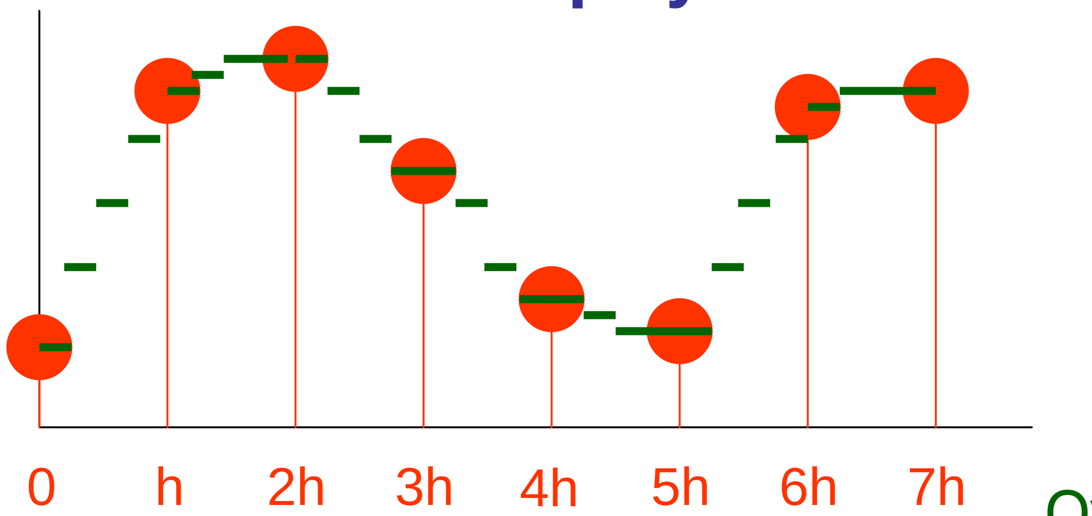
This does not produce a sound

Hold device is necessary



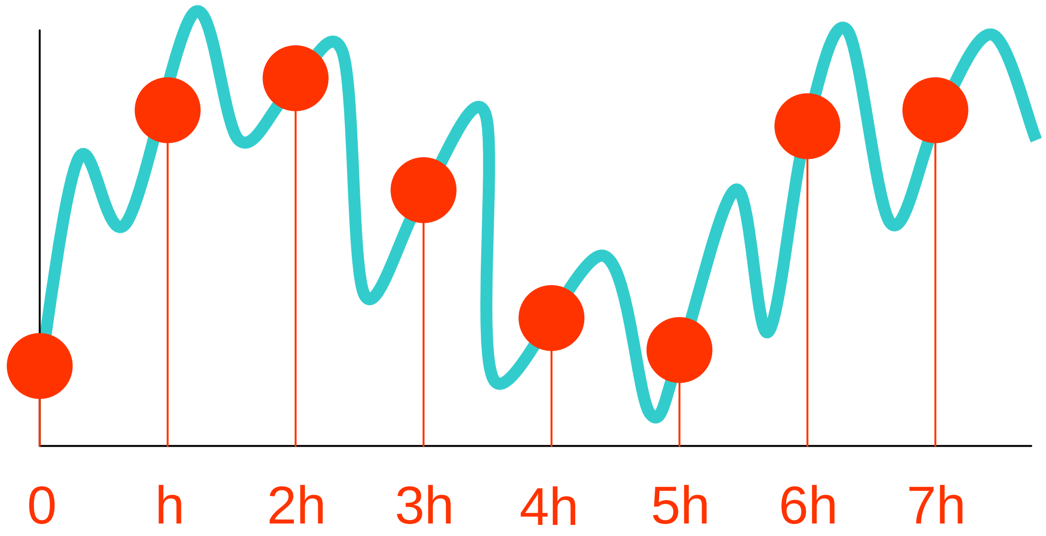
Simple 0-order hold

Old CD players

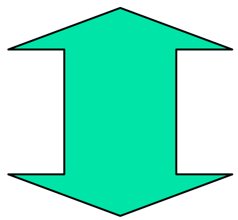


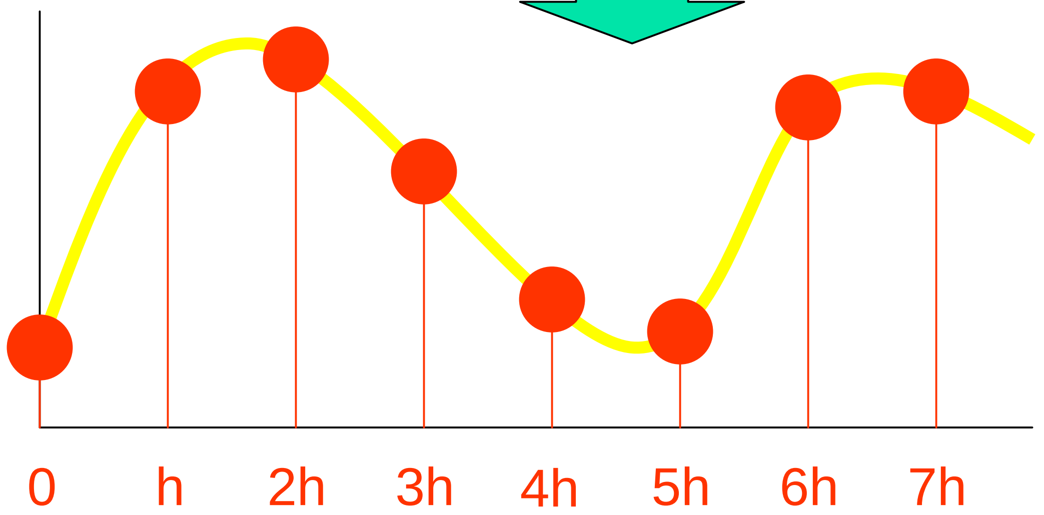
Oversampling DA converter

More recent players

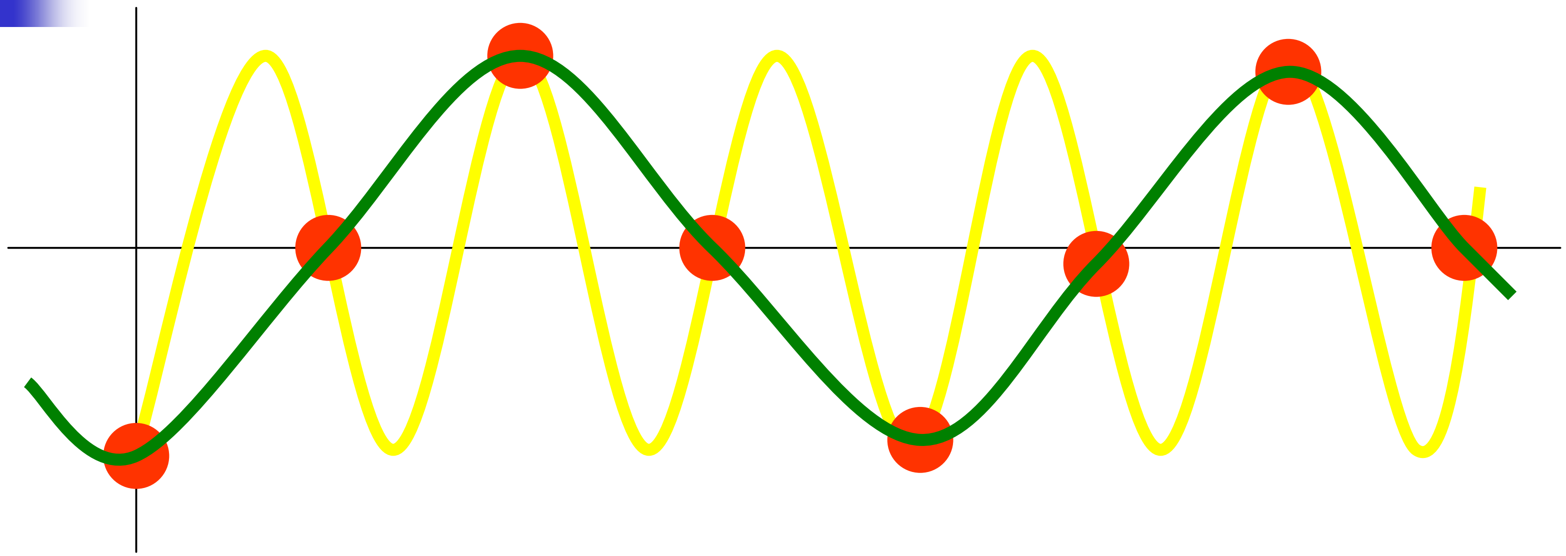


Questions





Typical problem: Sampling → Aliasing



- Intersample information can be lost
- If no high-freq. components beyond the Nyquist frequency (=1/2 of sampling freq.) → unique restoration

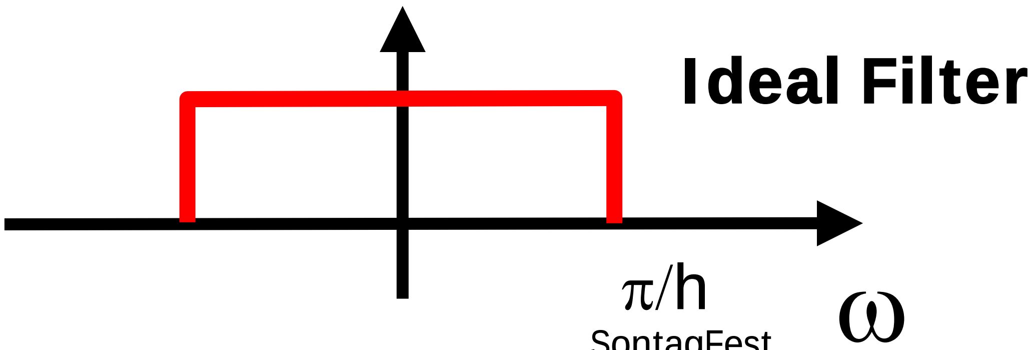
→ Whittaker-Shannon-Someya sampling theorem

Sampling Theorem

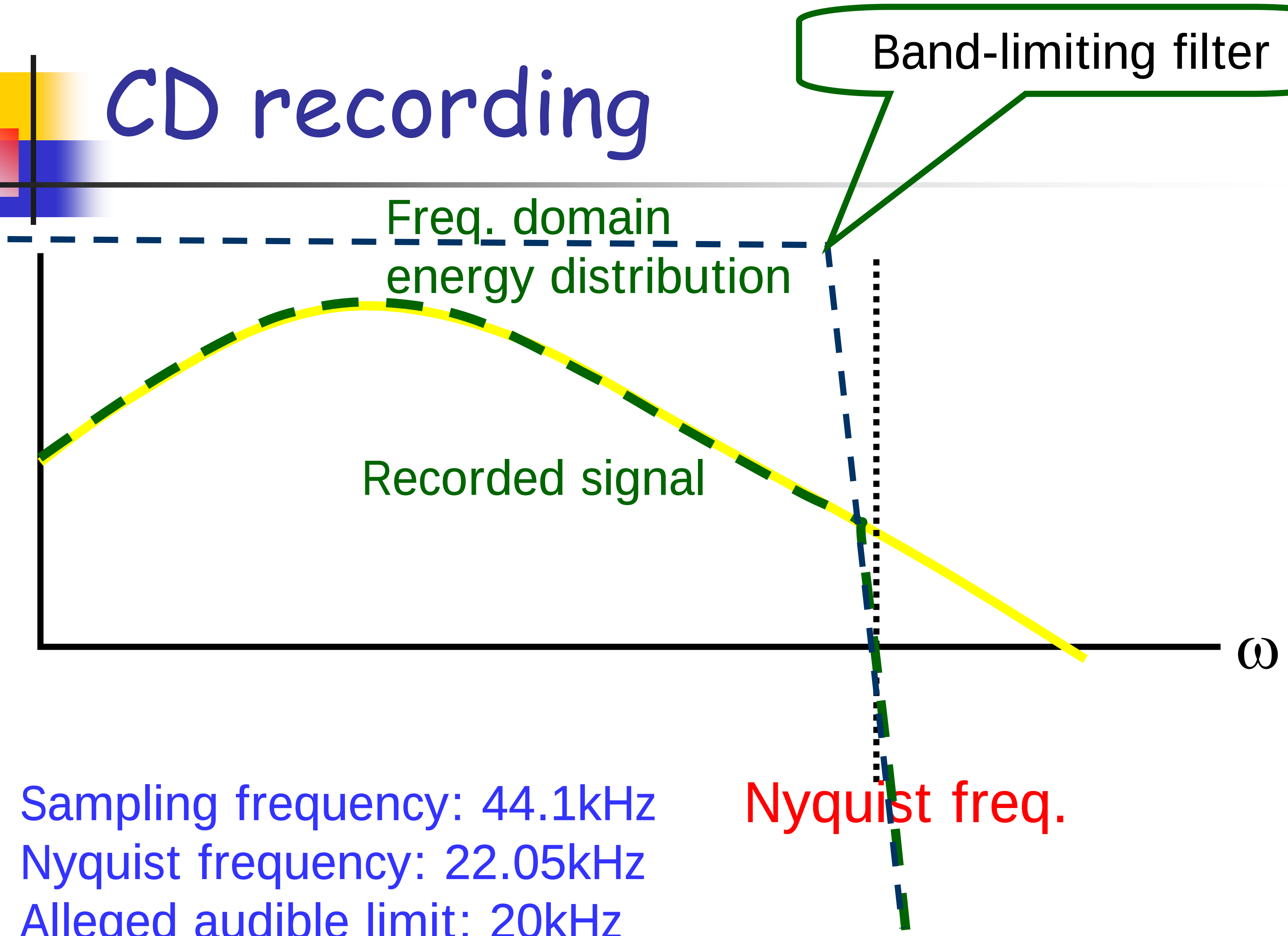
- Band limiting hypothesis $\Rightarrow$ unique recovery

$$\hat{f}(j\omega) = 0 \text{ for } |\omega| > \pi/h \Rightarrow$$

$$f(t) = \sum_{n=-\infty}^{\infty} f(nh) \frac{\sin \pi(t/h - n)}{\pi(t/h - n)}$$

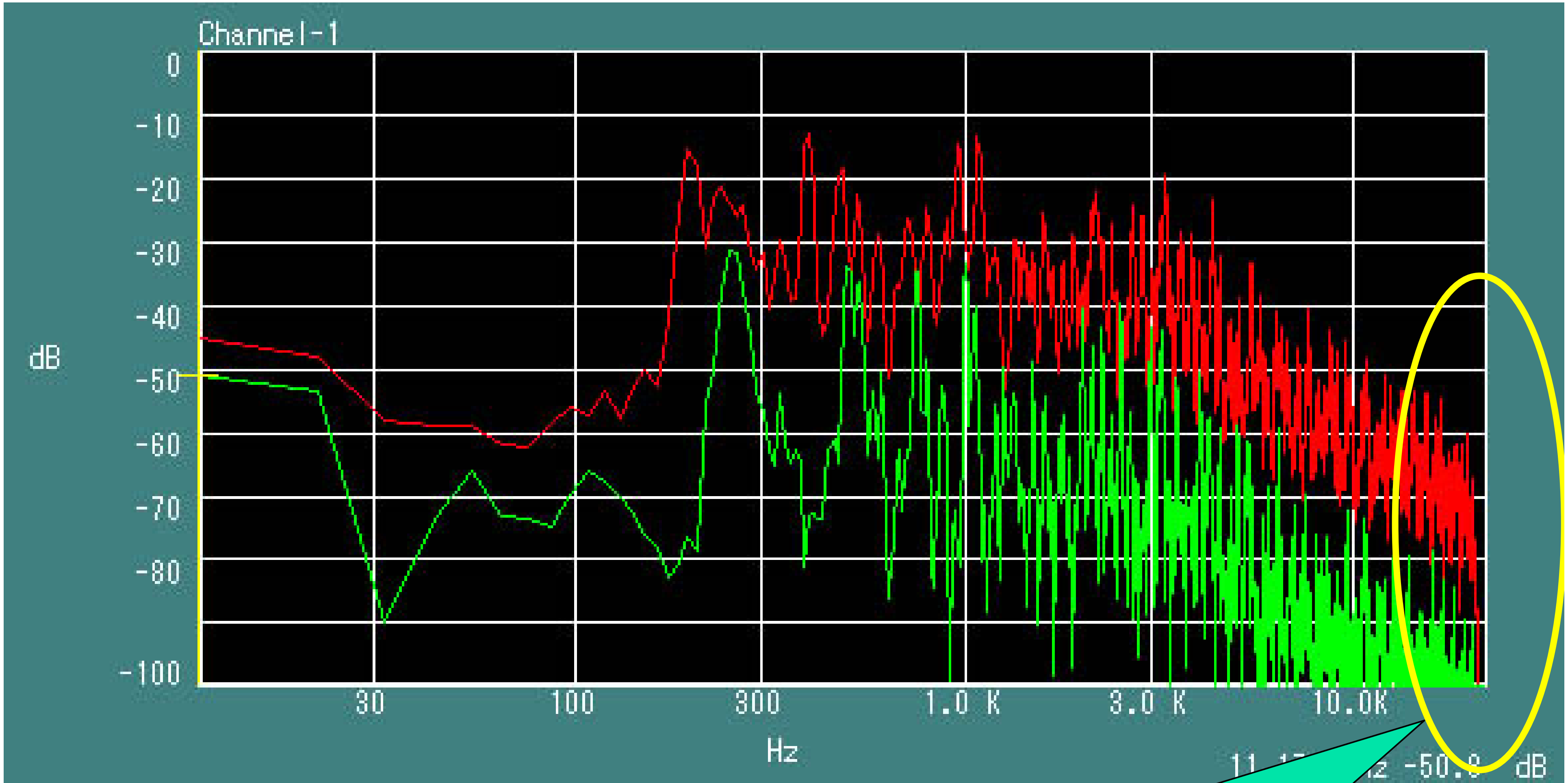


CD recording



Sampling frequency: 44.1kHz
Nyquist frequency: 22.05kHz
Alleged audible limit: 20kHz

Digital Recording (CD): sharp anti-aliasing filter
No signal components beyond 20kHz

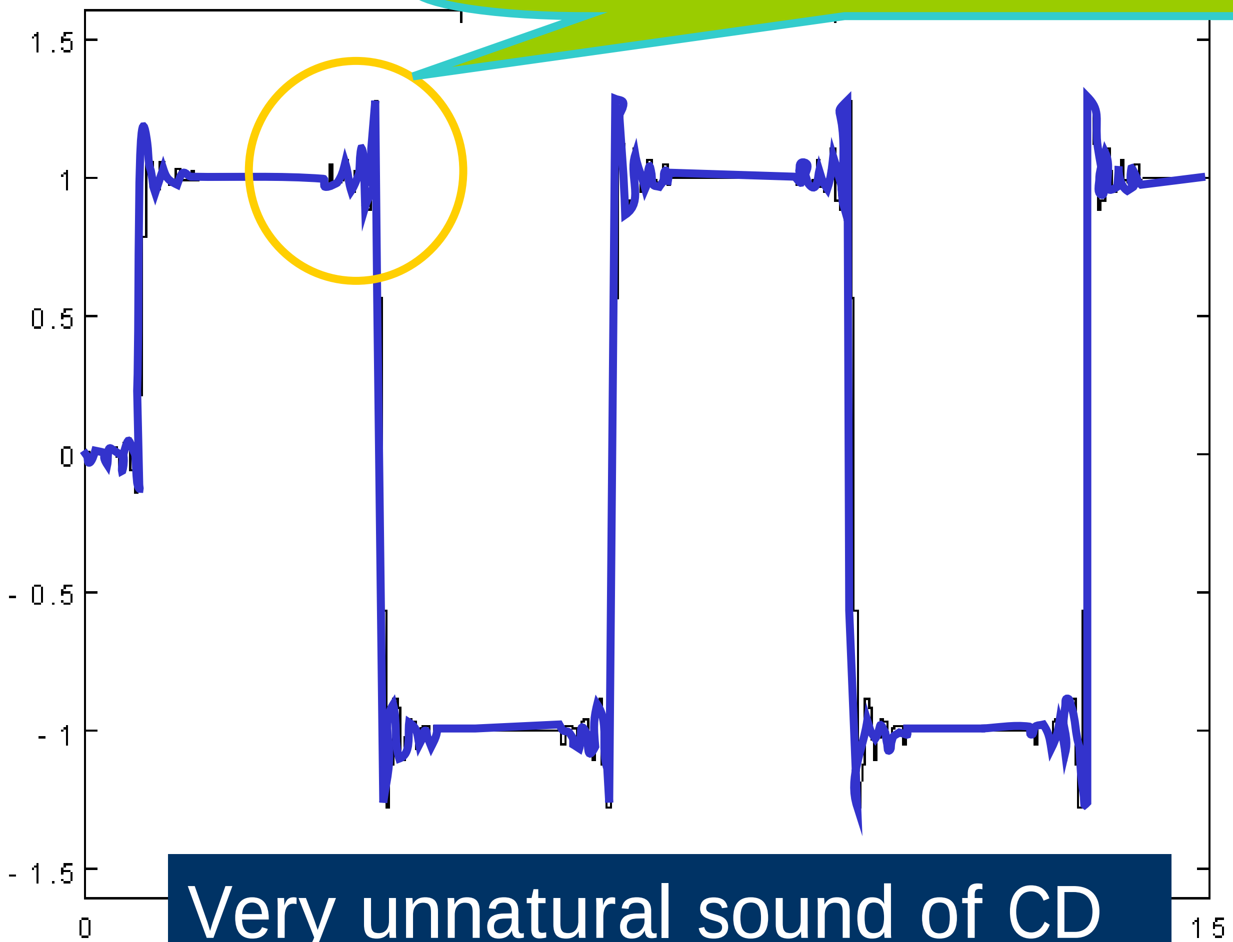


Very sharp
anti-aliasing filter

But you won’t be able to
hear them anyway??

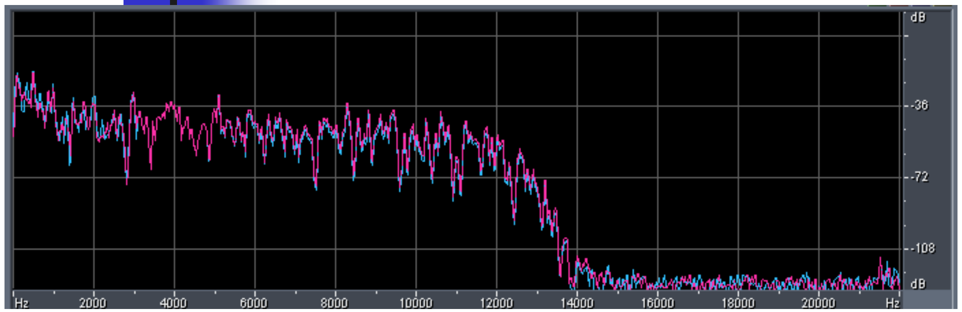
Effect of a band-limiting filter

Big amount of ringing due to the Gibbs phenomenon

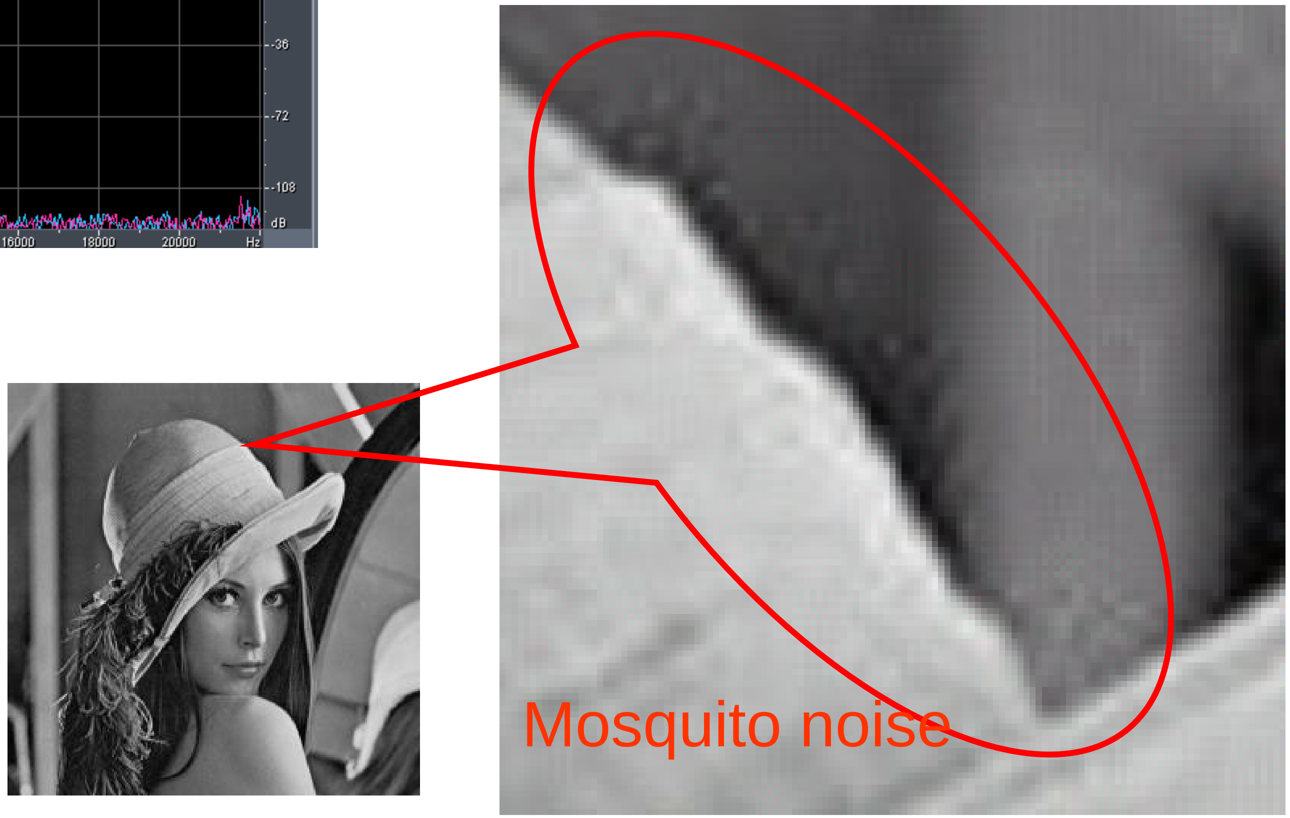


Very unnatural sound of CD

Mosquito Noise-another Gibbs phoenomenon



Truncated freq. response

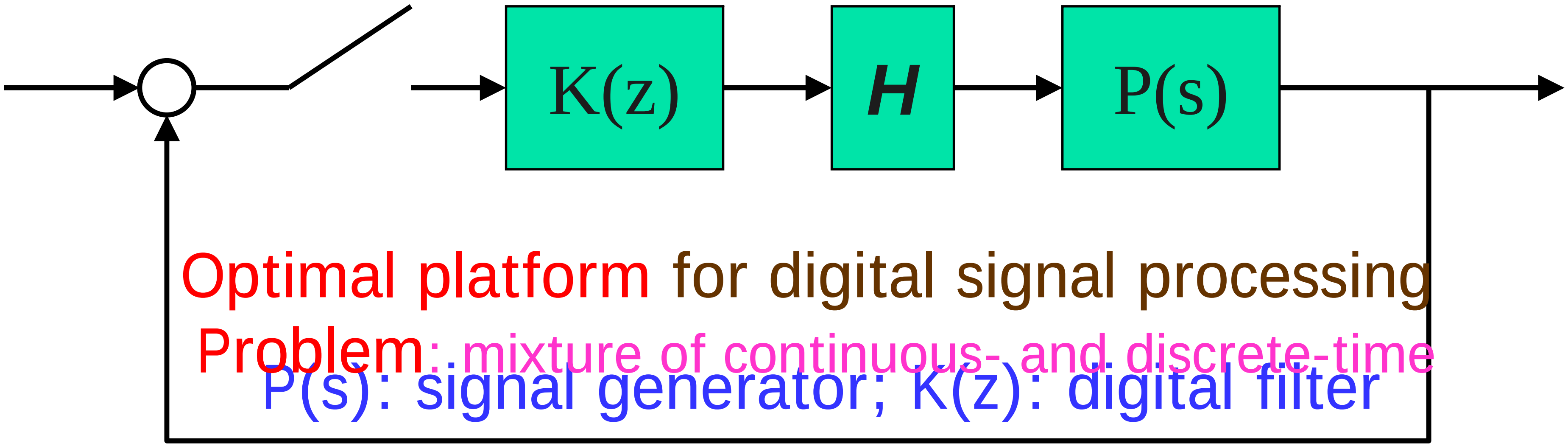


What can we do?

Part II: Review of Sampled-data Control Theory

Sampled-data Control Systems – What are they?

- Continuous-time plant
- Discrete-time controller
- sample/hold devices



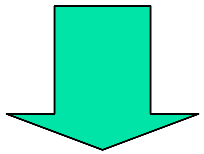
Optimal platform for digital signal processing

Problem: mixture of continuous- and discrete-time

P(s): signal generator; K(z): digital filter

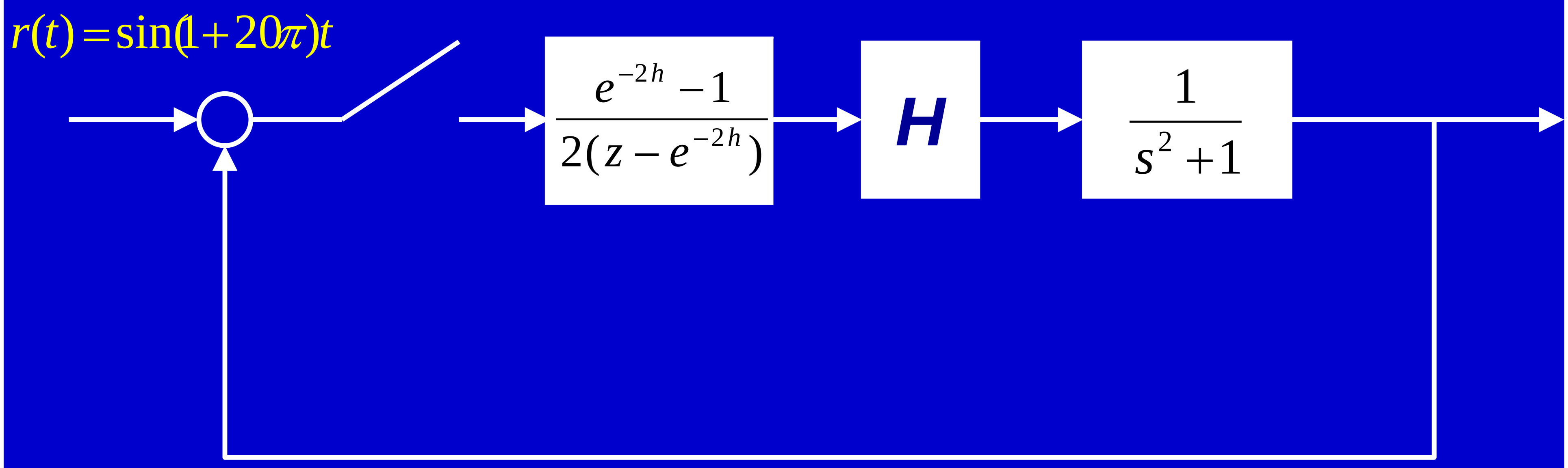
Difficulties

- Plant P(s) is continuous-time
- Controller K(z) is discrete-time
- The overall system is not time-invariant



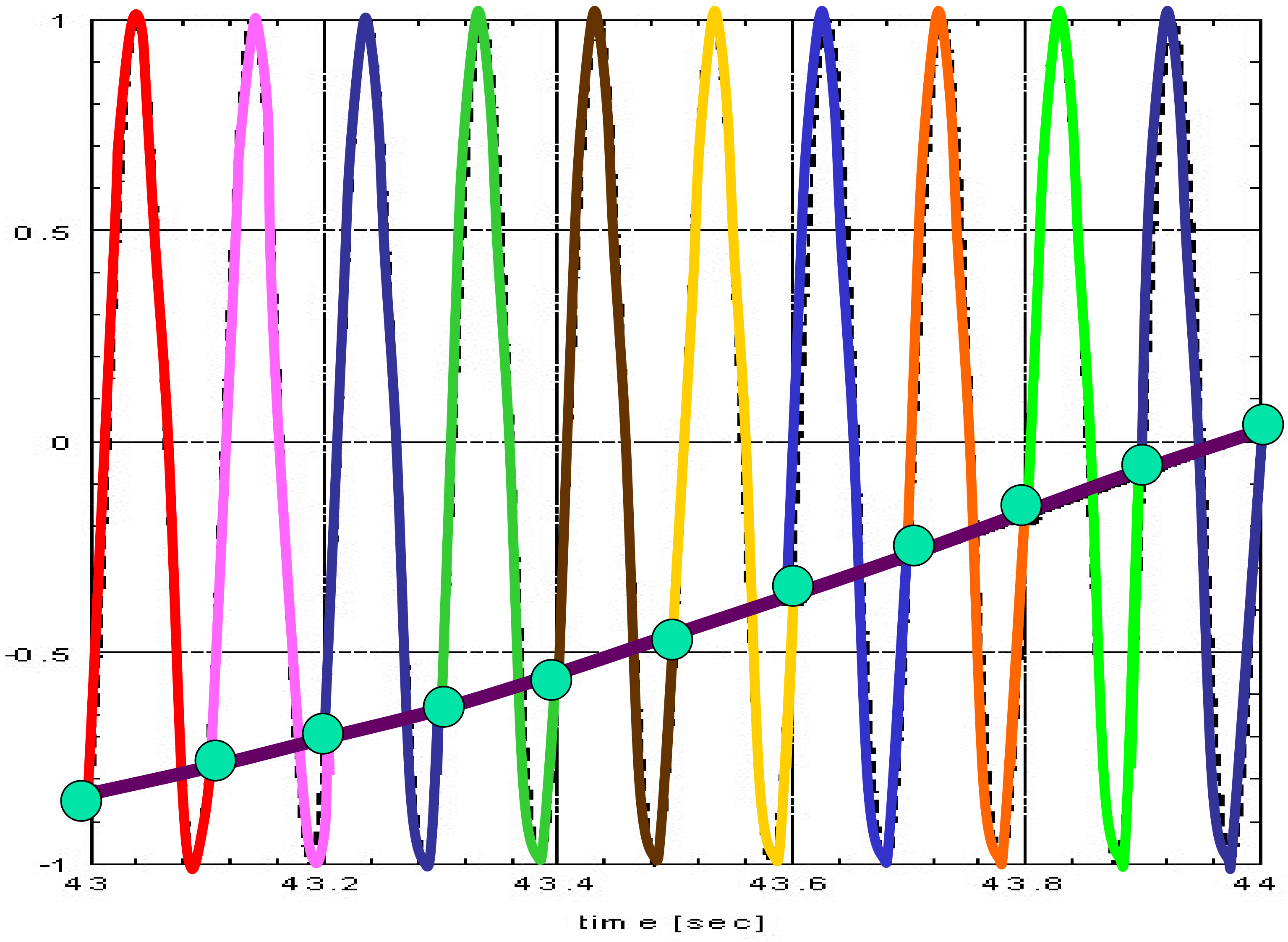
- No transfer function
- No steady-state response
- No frequency response

Response against a sinusoid



Response

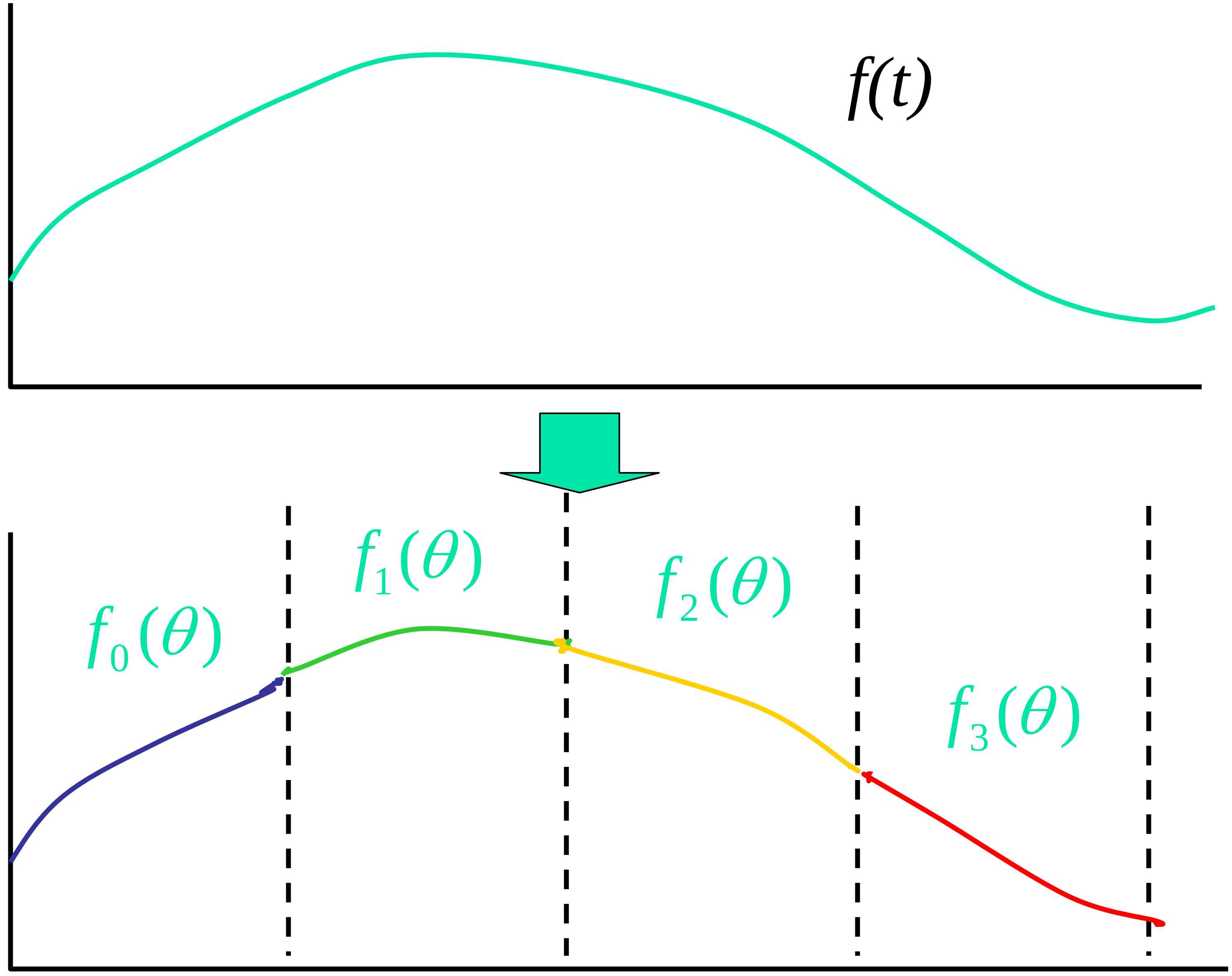
$$v(\theta) = \sin(1+20\pi)\theta$$



What to do? & solutions

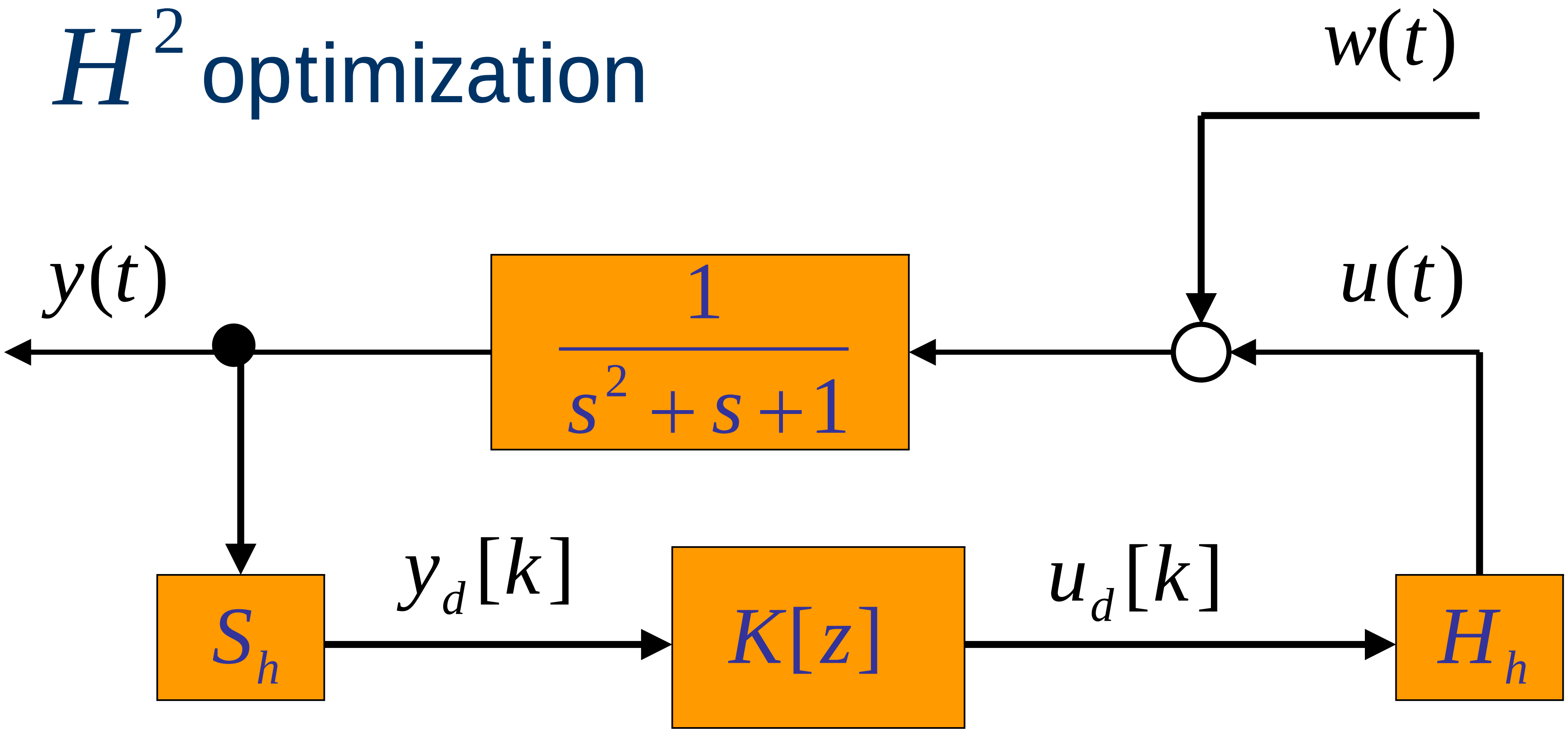
- A new technique: *lifting* (1990) that turns SD system to discrete-time LTI
- $\exists$ digital controller that makes cont.-time performance optimal

Lifting of Functions



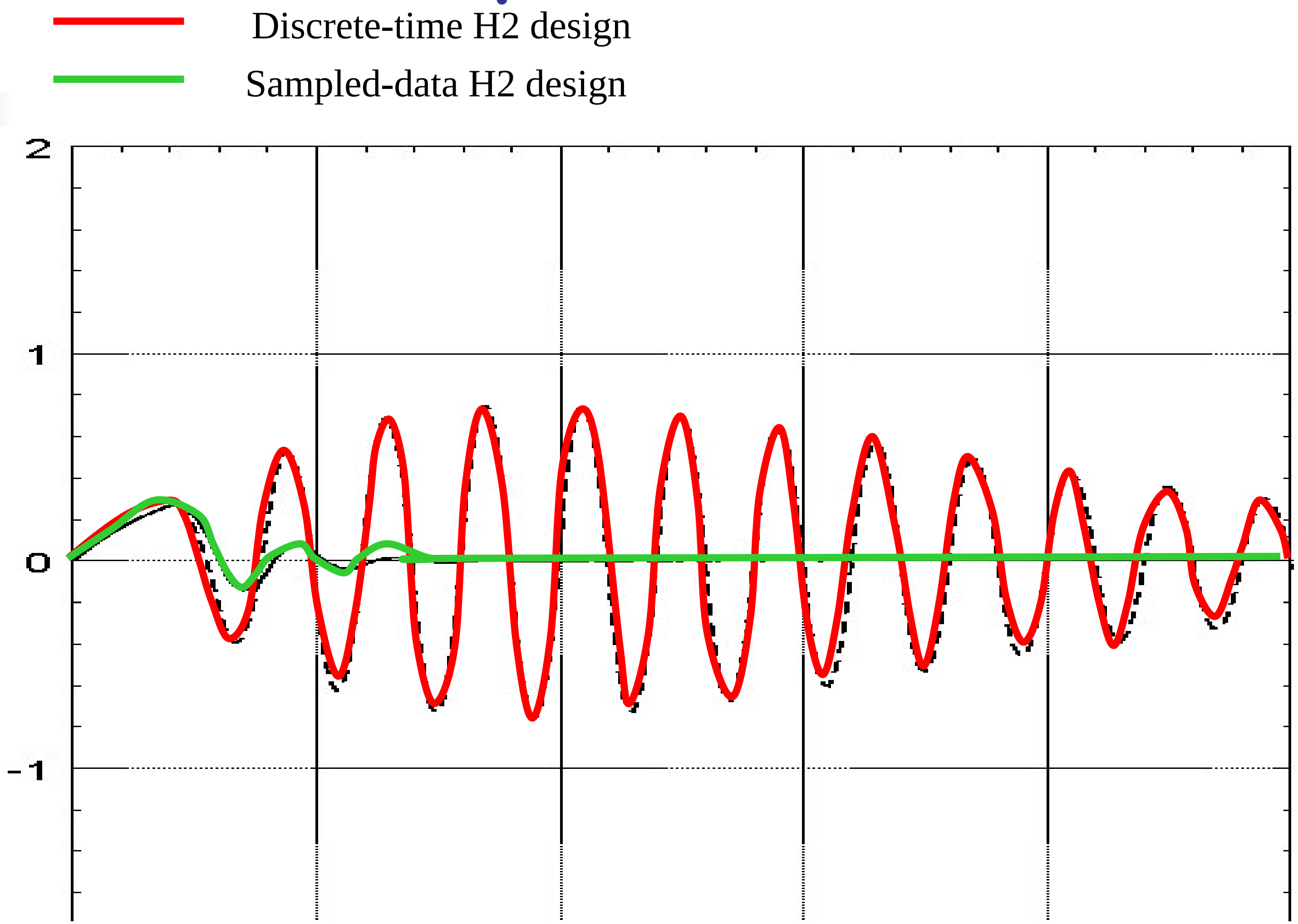
Does this make a difference? ---Yes

H^2 optimization



a) Discrete-time H^2 with no intersample consideration
b) sampled-data design

Time Response



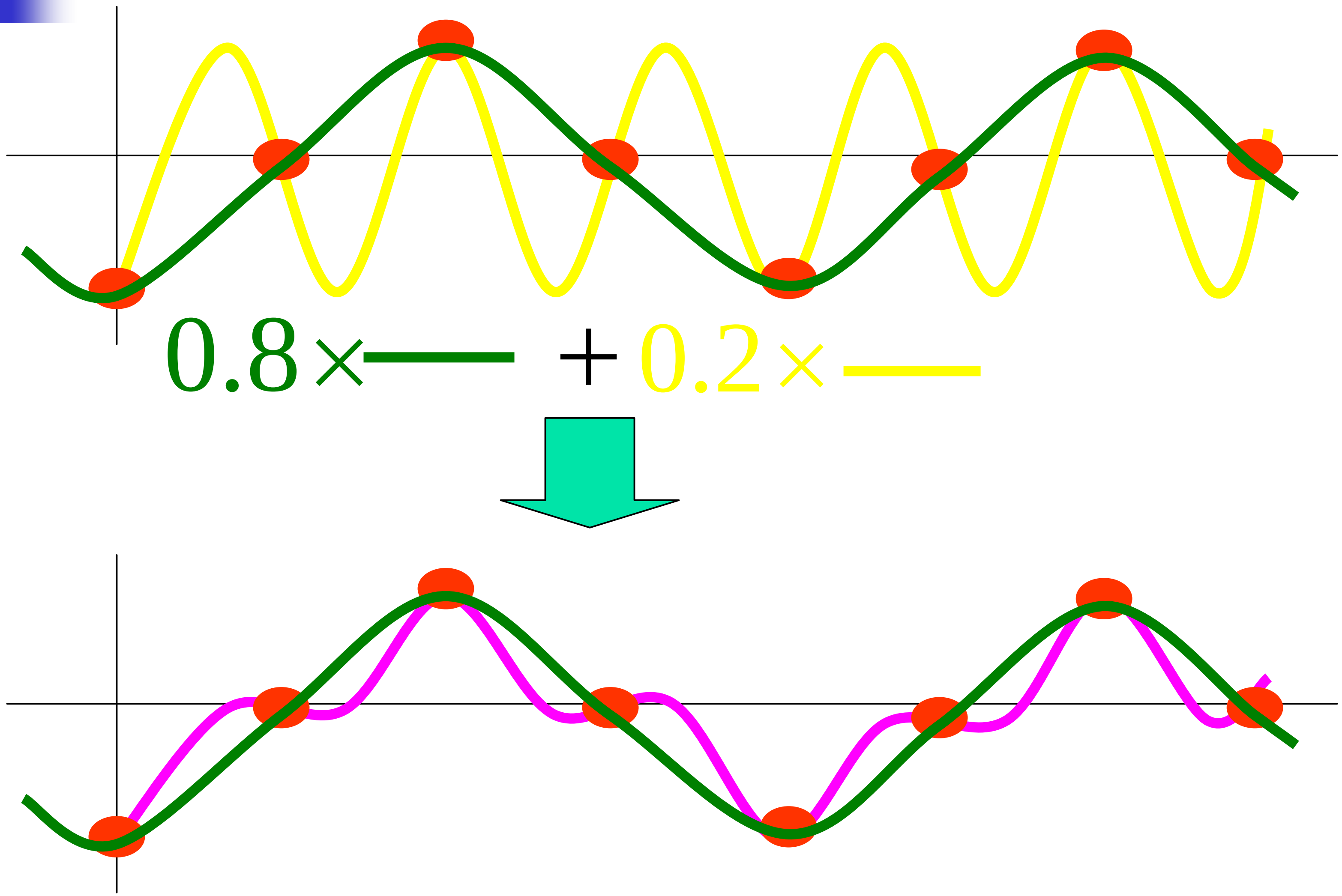
a) Discrete-time H^2 with no intersample consideration

b) sampled-data design

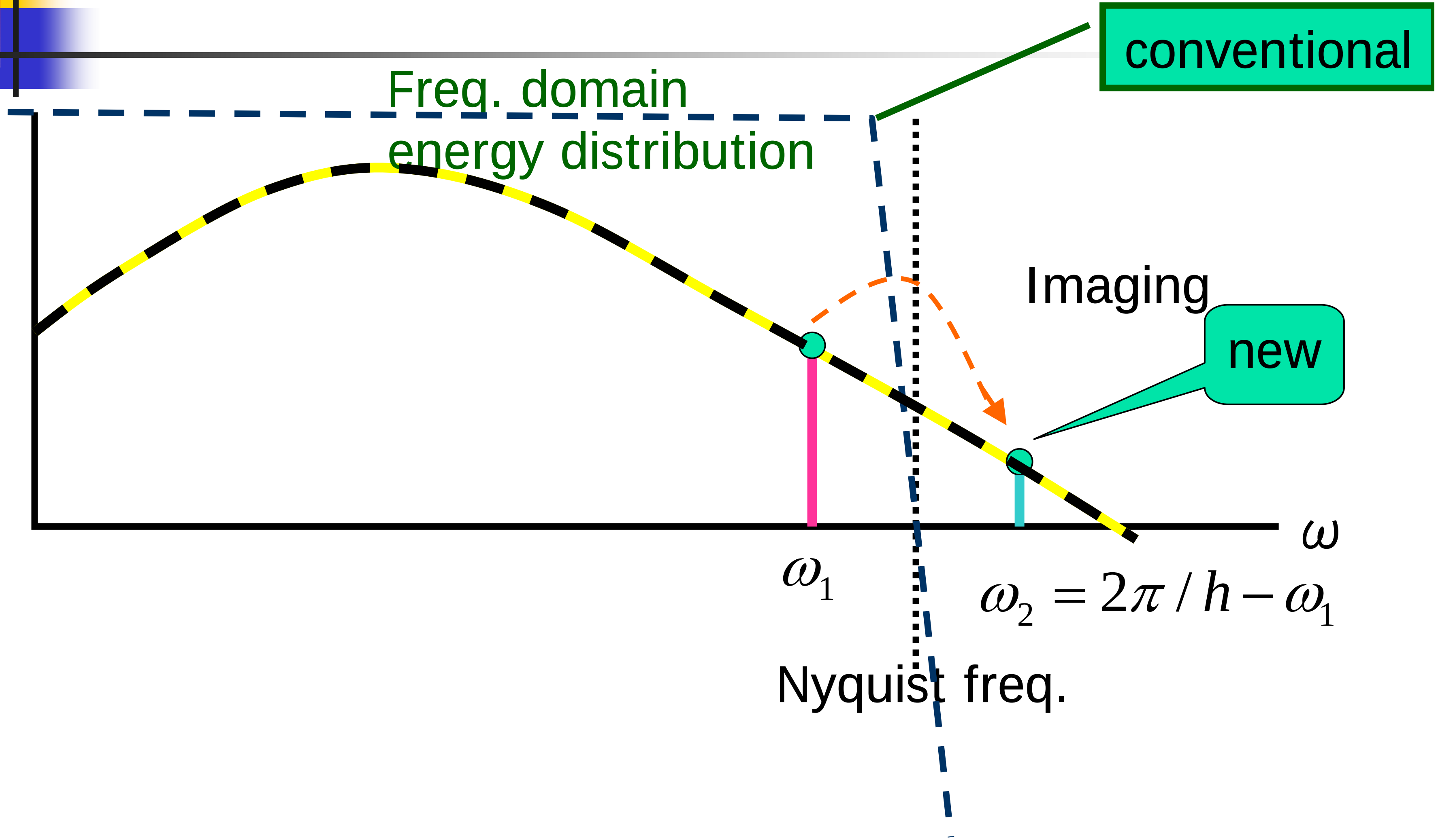
Can this be used for signal processing?

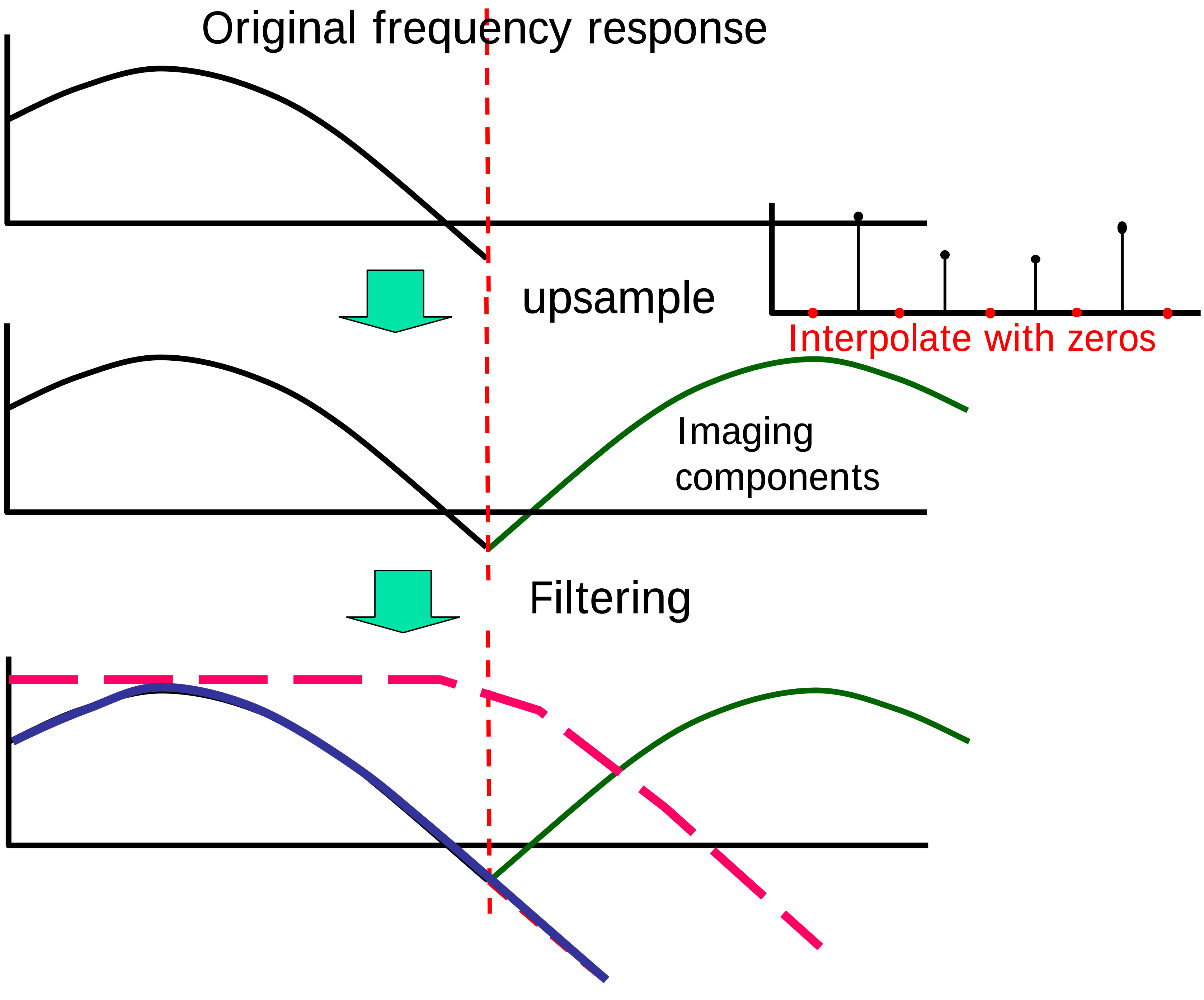
Part III: How can sampled-data theory help?

With a little bit of a priori information…



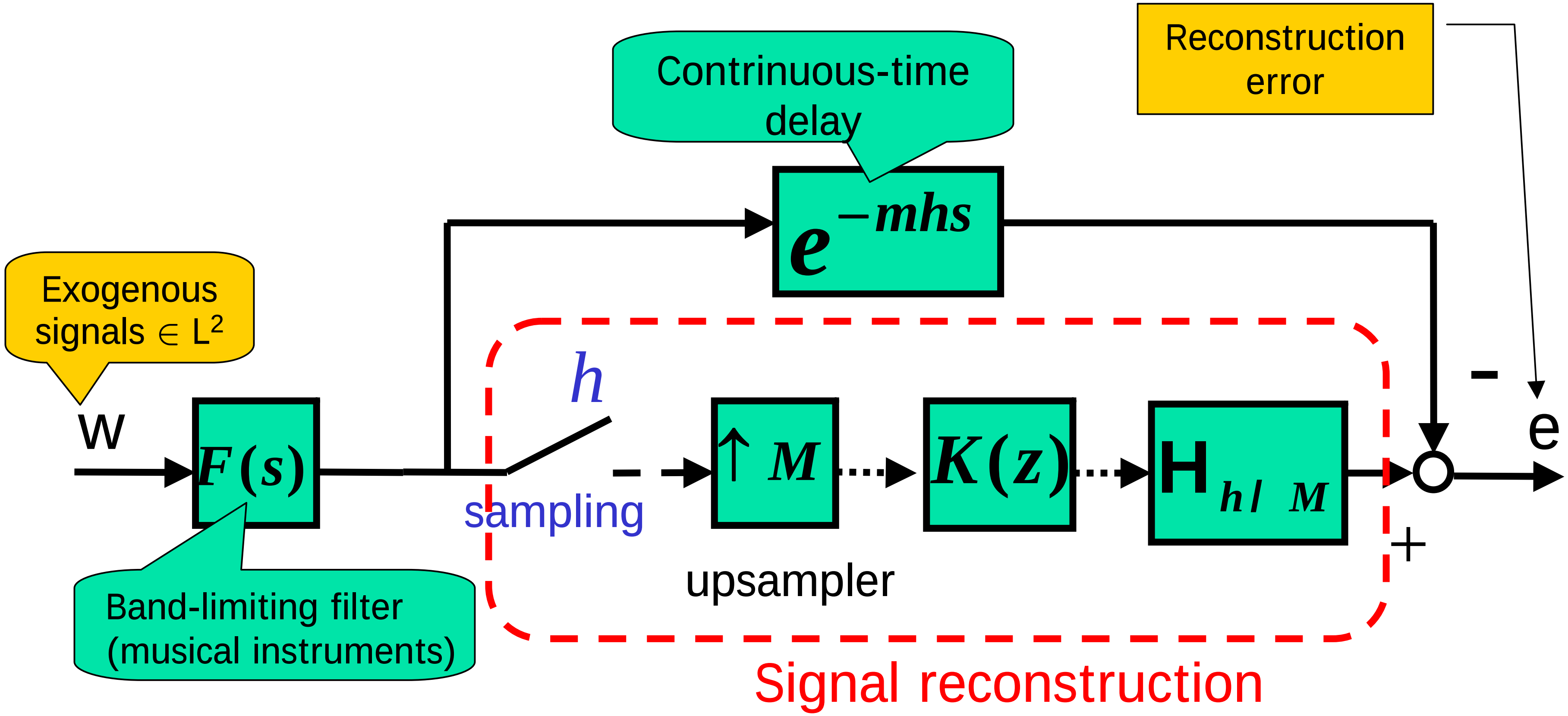
Utilizing analog characteristic





Original frequency response
upsample
Interpolate with zeros
Imaging
components
Filtering

Sampled-data Design Model

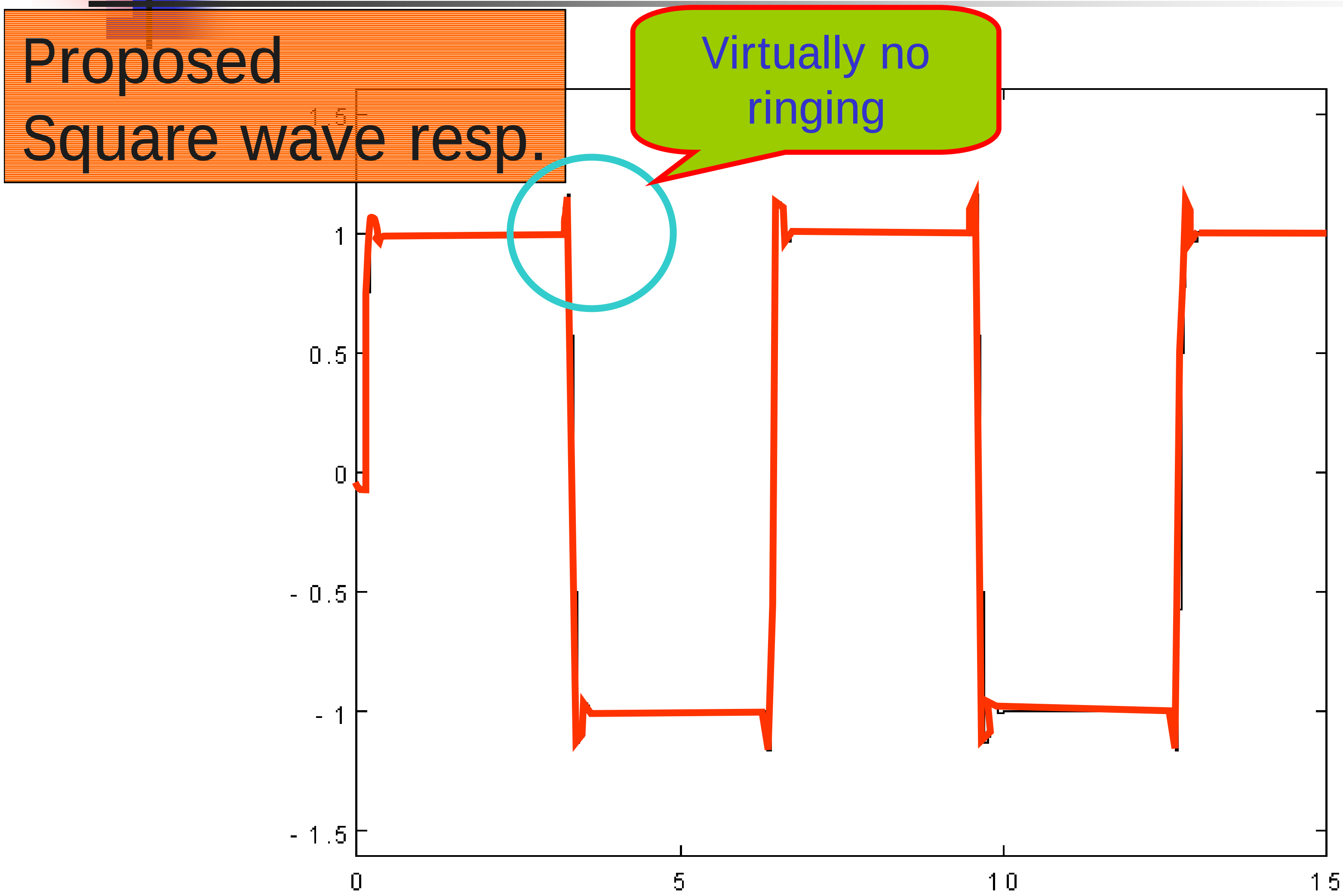


Problem: Find K[z] satisfying

$$\|T_{ew}\| < \gamma$$

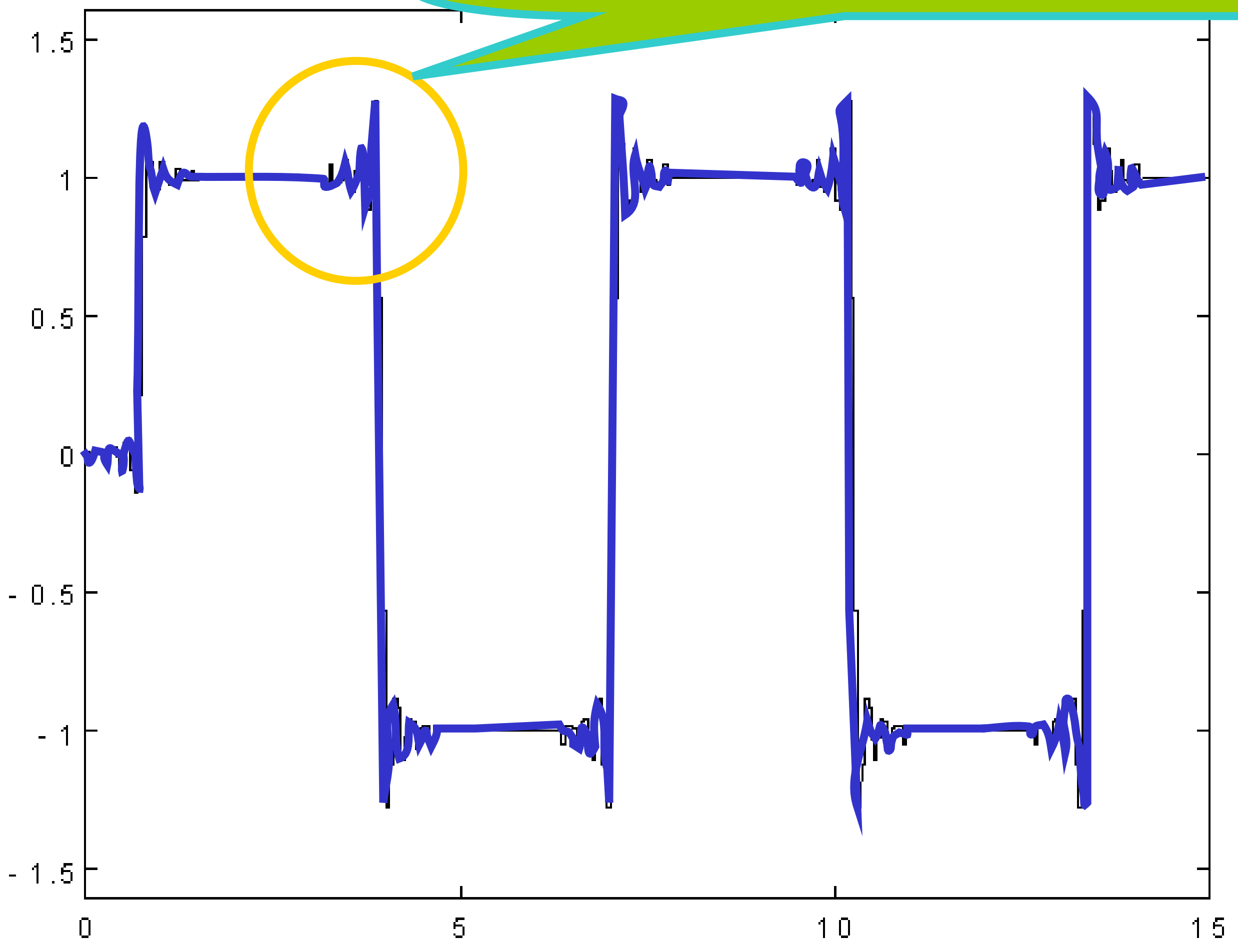
Sampled-data H$^\infty$ control problem

Interpolator via the proposed method



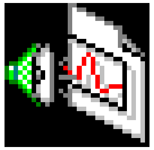
Response of the Johnston filter

Big amount of ringing due to the Gibbs phenomenon

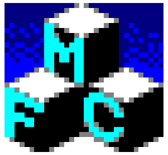


Part IV: Application to Sound Restoration

Sound restoration

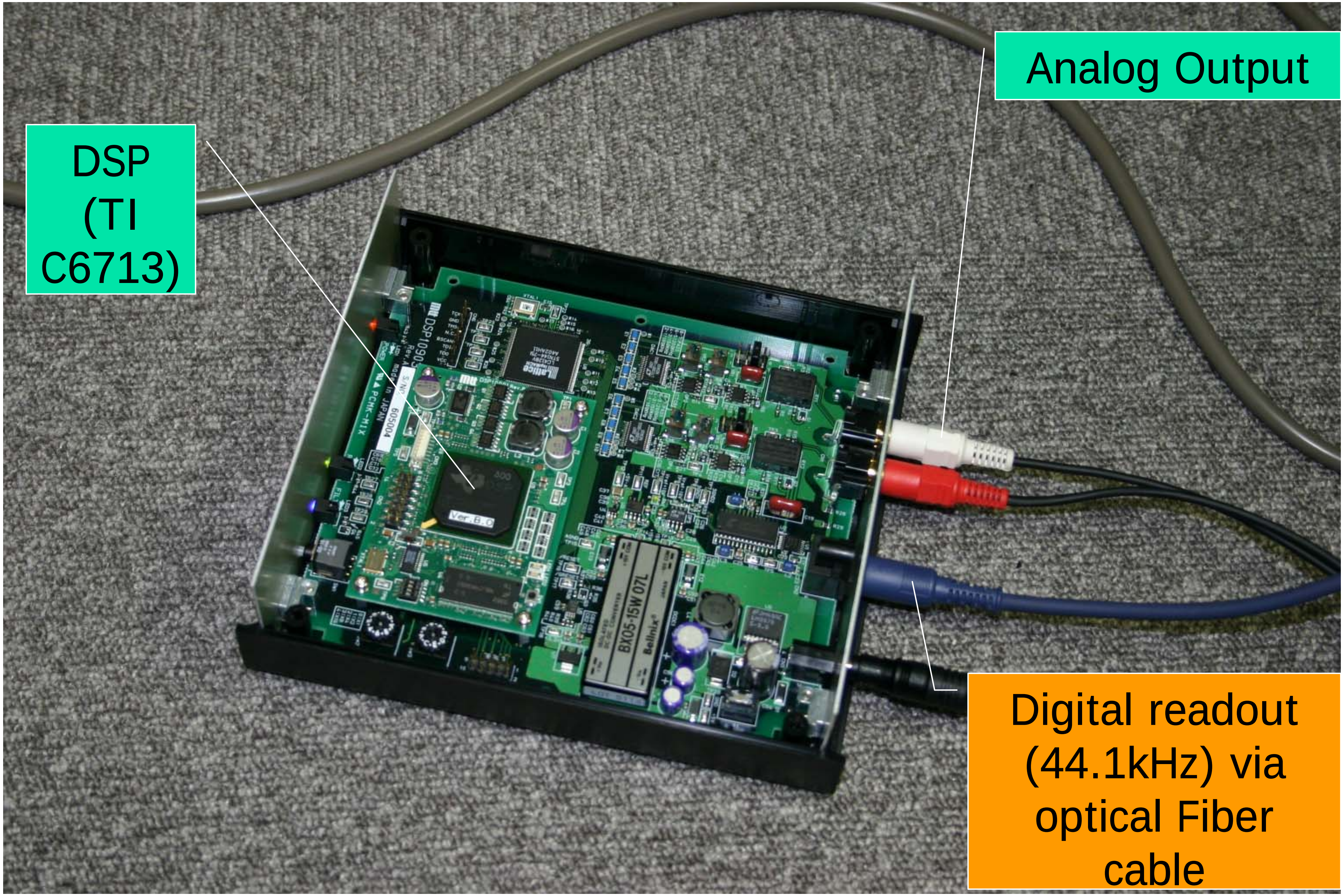


アプリケーション



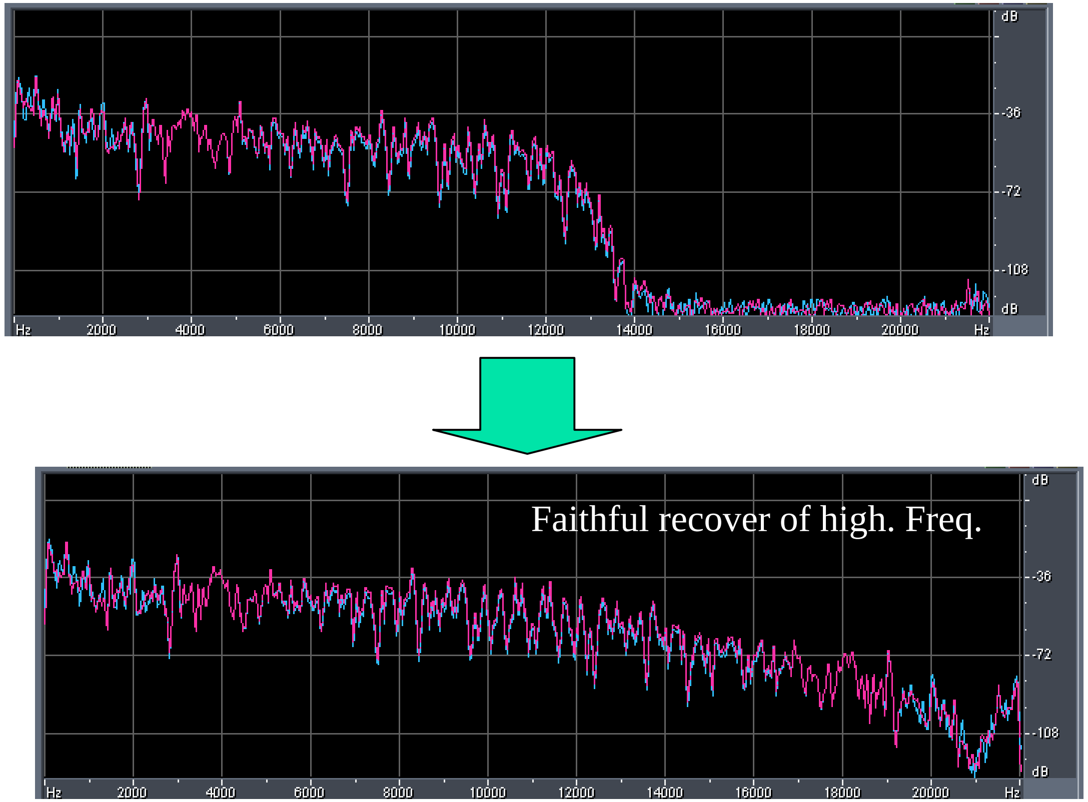
アプリケーション

YYLab


DSP (TI C6713)
Analog Output
Digital readout (44.1kHz) via optical Fiber cable

Example in MD(mini disk) players

MDLP4(66kbps)



After "YY"

More natural high freq. response

By the courtesy of SANYO Corporation

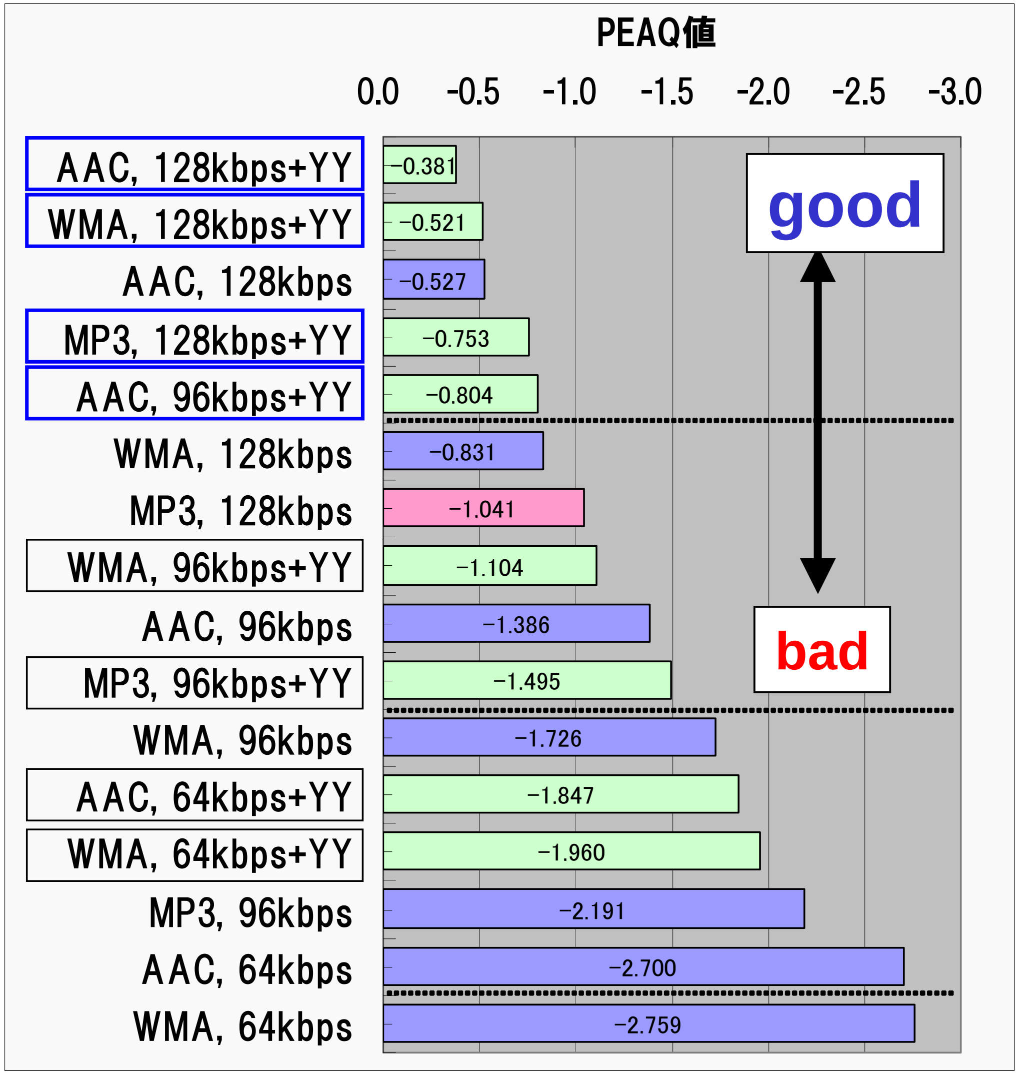
This "YY filter" is implemented in custom LSI sound chips by SANYO Coop., and being used in MP 3 players, mobile phones, voice recorders. The cumulative sale has reached over 20 million units.

Effect evaluation on compressed audio via PEAQ program

- Tested on 100 compresed music sources via PEAQ (Perceptual Evaluation of Audio Quality)
- PEAQ values:
  - 0…indistinguishable from CD
  - -1…distinguishable but does not bother the listener
  - -2…not disturbing
  - -3…disturbing
  - -4…very disturbing
- Note how YY improves the sound quality

http://en.wikipedia.org/wiki/PEAQ

By the courtesy of SANYO corporation



Compression formats: MP3, AAC, WMA
Bitrates: 64kbps, 96kbps, 128kbps
Showing average values

Part V: Application to Images

Same Problems as Sounds

- Block and Mosquito noise
- Lack of sufficient bandwidth
- Mosquito noise – Gibbs phenomenon
- Can sampled-data filter help?

Original

$\Downarrow$ 2 downsample and hold

Interpolation Via equiripple filter

YYa

4times upsample+ twice downsample via YY

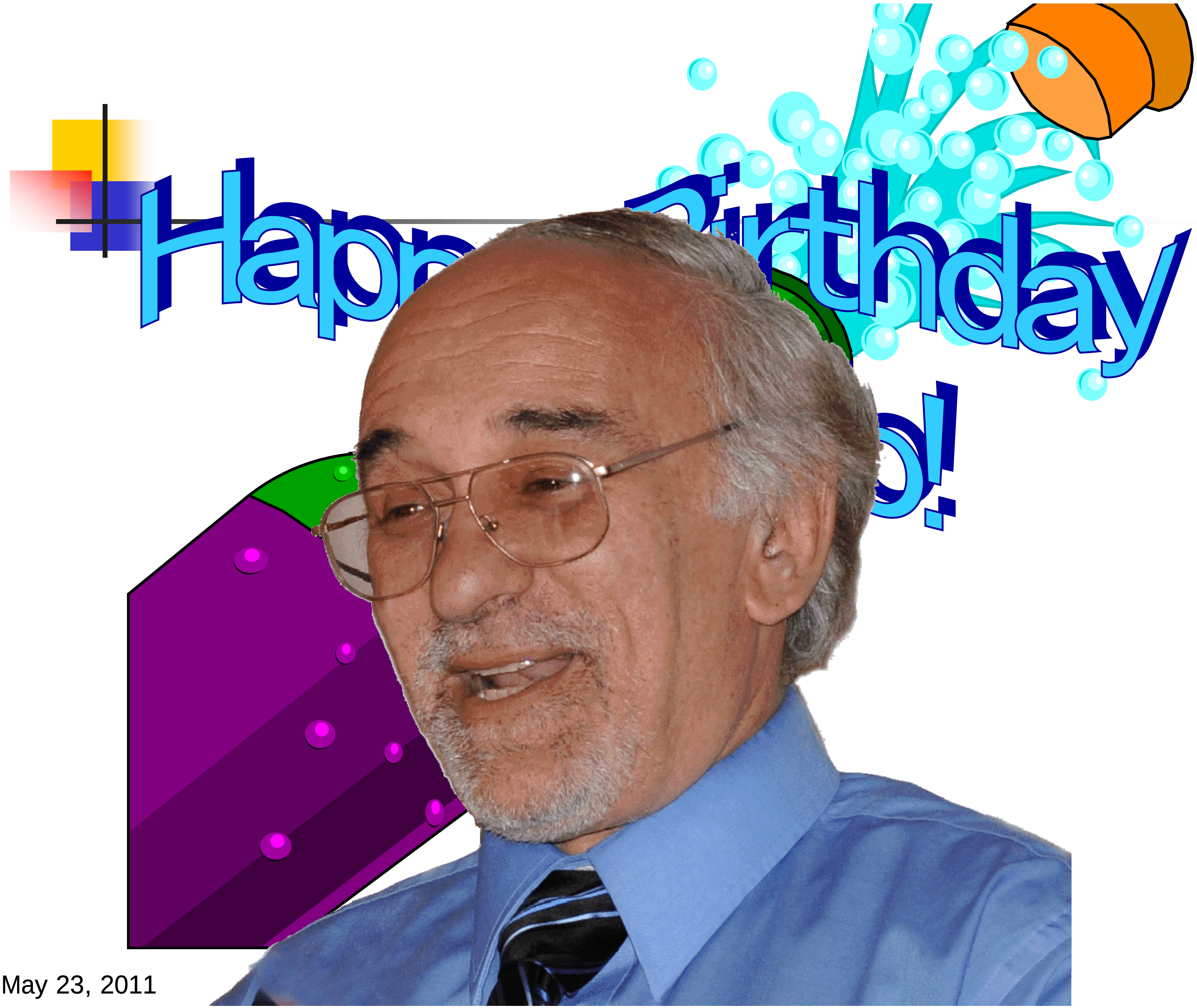
Another application: How can we zoom "digitally"?

Interpolation via bicubic filter

Interpolation via sampled-data filter

Summarizing

- Analog signal generator model
- Error frequency response to be minimized (doesn't exist in the conventional approach)
- $\Leftarrow$ sampled-data H^∞ control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