Nonlinear control: milestones, roadblocks, challenges

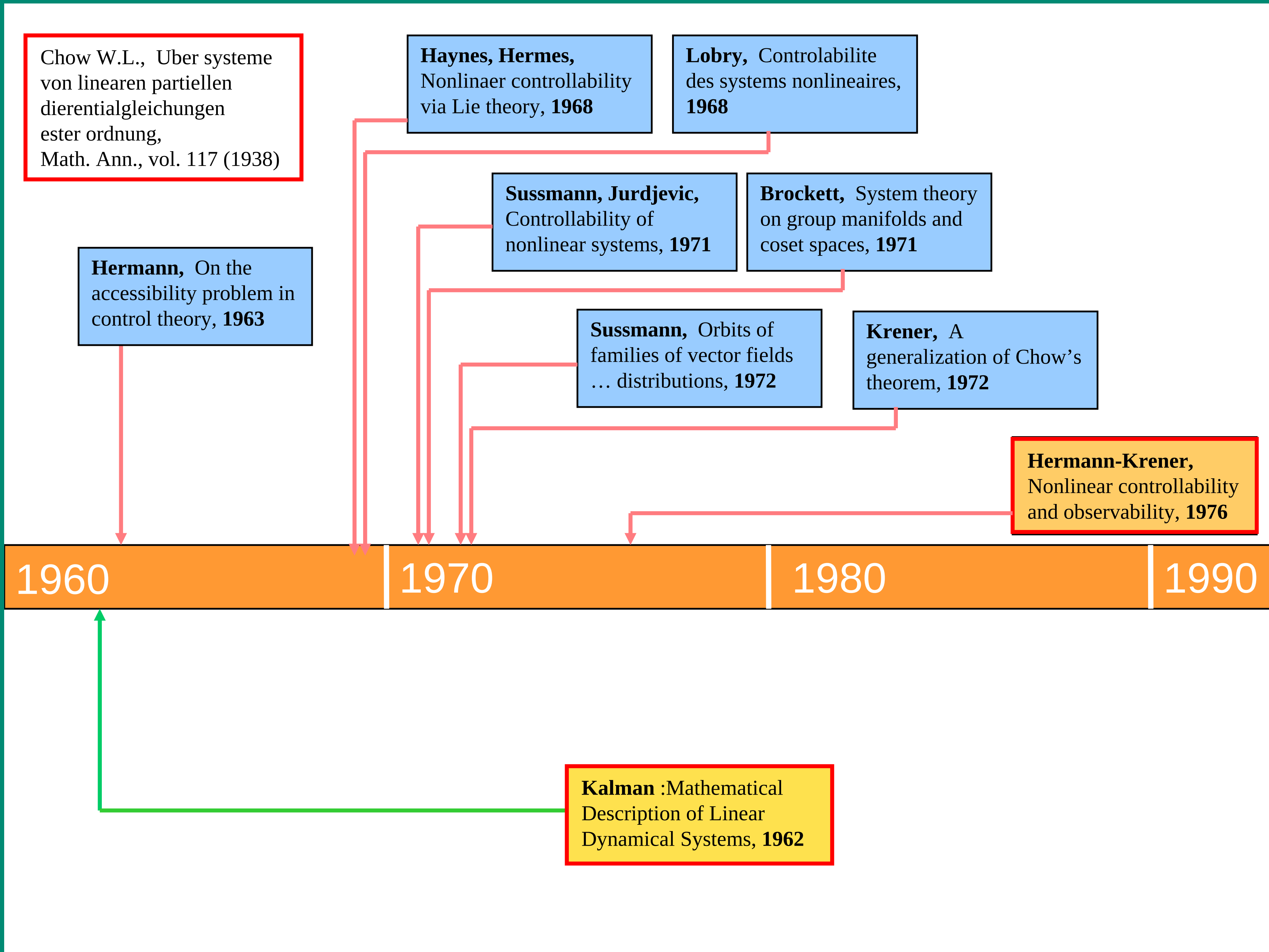
Alberto Isidori

Sontagfest - 23-25 May 2011

The milestones

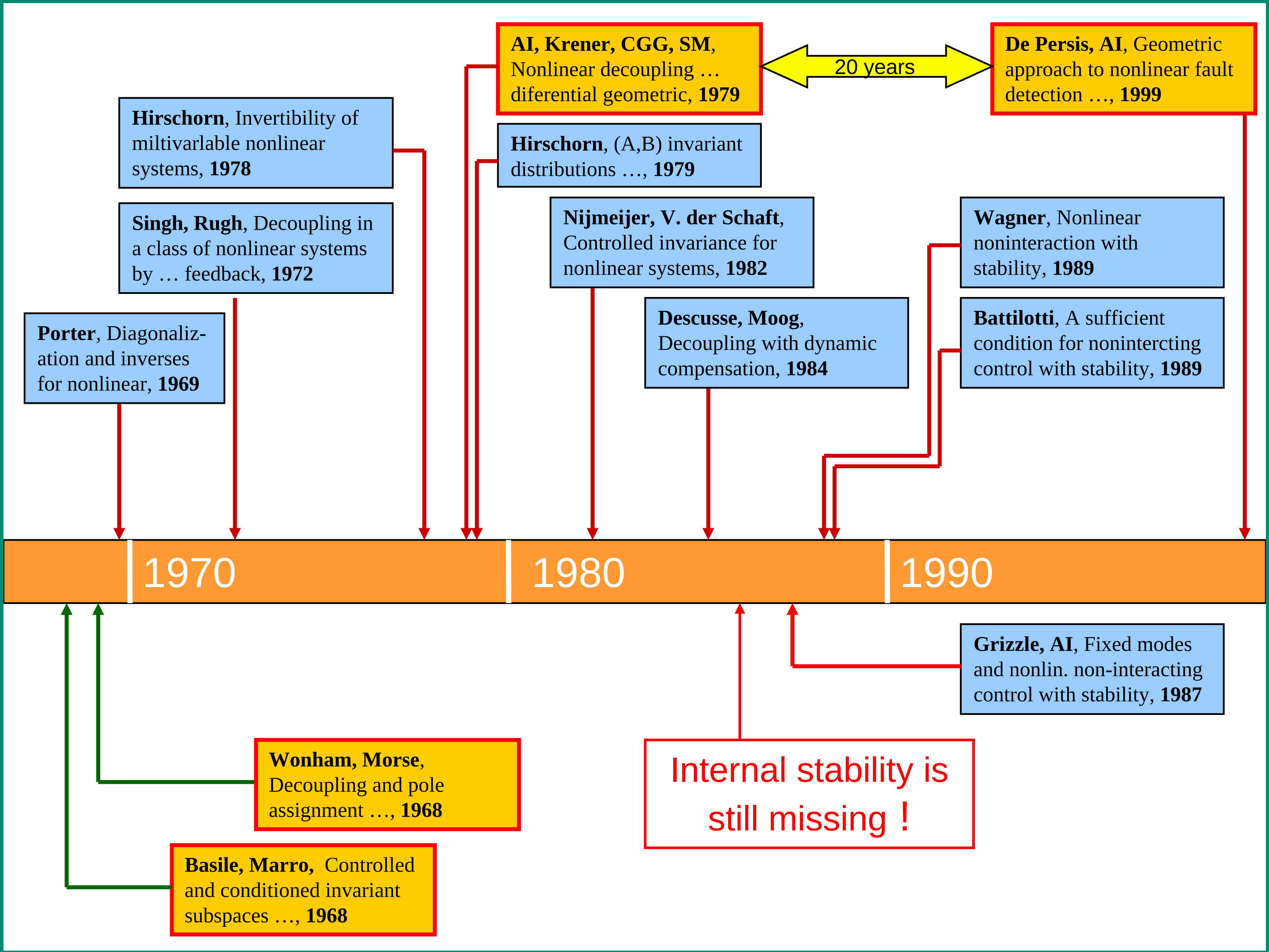
The green years: 1963-1977.

Understanding nonlinear controllability, observability and minimality.


Chow W.L., Uber systeme von linearen partiellen dierentialgleichungen ester ordnung, Math. Ann., vol. 117 (1938)
Haynes, Hermes, Nonlinaer controllability via Lie theory, 1968
Lobry, Controlabilite des systems nonlineaires, 1968
Sussmann, Jurdjevic, Controllability of nonlinear systems, 1971
Brockett, System theory on group manifolds and coset spaces, 1971
Hermann, On the accessibility problem in control theory, 1963
Sussmann, Orbits of families of vector fields … distributions, 1972
Krener, A generalization of Chow's theorem, 1972
Hermann-Krener, Nonlinear controllability and observability, 1976
1960
1970
1980
1990
Kalman :Mathematical Description of Linear Dynamical Systems, 1962

The growth of nonlinear control: 1979-1989.

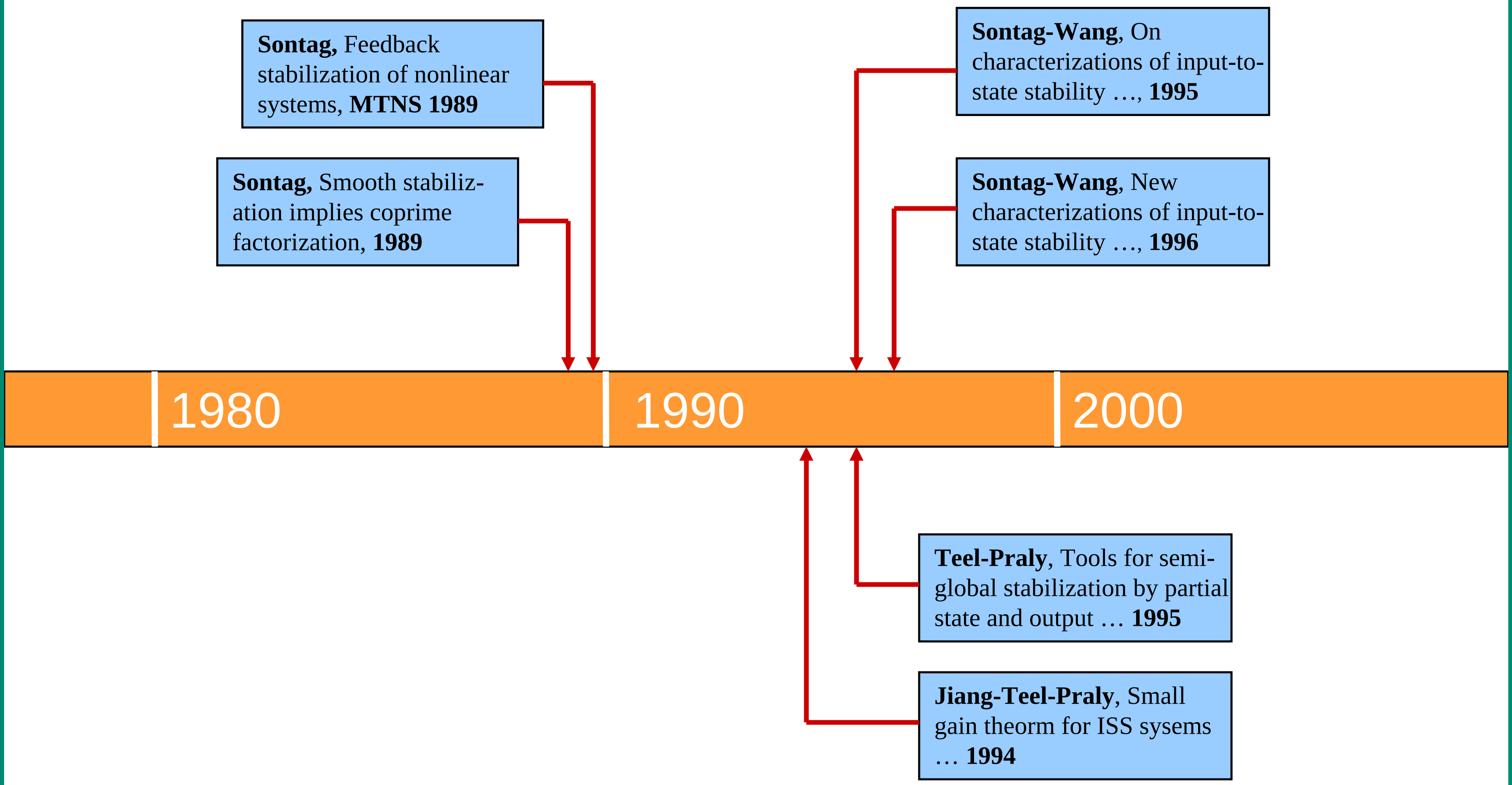
Understanding feedback design for nonlinear systems: decoupling, non-interaction, feedback linearization (only marginal emphasis on stability, though).


AI, Krener, CGG, SM, Nonlinear decoupling … diferential geometric, 1979
20 years
De Persis, AI, Geometric approach to nonlinear fault detection …, 1999
Hirschorn, Invertibility of miltivarlable nonlinear systems, 1978
Hirschorn, (A,B) invariant distributions …, 1979
Singh, Rugh, Decoupling in a class of nonlinear systems by … feedback, 1972
Nijmeijer, V. der Schaft, Controlled invariance for nonlinear systems, 1982
Wagner, Nonlinear noninteraction with stability, 1989
Porter, Diagonaliz-ation and inverses for nonlinear, 1969
Descusse, Moog, Decoupling with dynamic compensation, 1984
Battilotti, A sufficient condition for nonintercting control with stability, 1989
1970
1980
1990
Grizzle, AI, Fixed modes and nonlin. non-interacting control with stability, 1987
Wonham, Morse, Decoupling and pole assignment …, 1968
Internal stability is still missing !
Basile, Marro, Controlled and conditioned invariant subspaces …, 1968

The Copernican revolution: 1989-1995.

The introduction of the concept of Input-to-State Stability radically changes the way in which problems of feedback stabilization are handled.

The possibility of estimating the (nonlinear) gain functions via Lyapunov-like criteria makes it easy to assign such functions in the design of (globally) stabilizing feedback laws.



Sontag, Feedback stabilization of nonlinear systems, **MTNS 1989**
Sontag, Smooth stabilization implies coprime factorization, **1989**
Sontag-Wang, On characterizations of input-to-state stability …, **1995**
Sontag-Wang, New characterizations of input-to-state stability …, **1996**
1980
1990
2000
Teel-Praly, Tools for semi-global stabilization by partial state and output … **1995**
Jiang-Teel-Praly, Small gain theorm for ISS sysems … **1994**

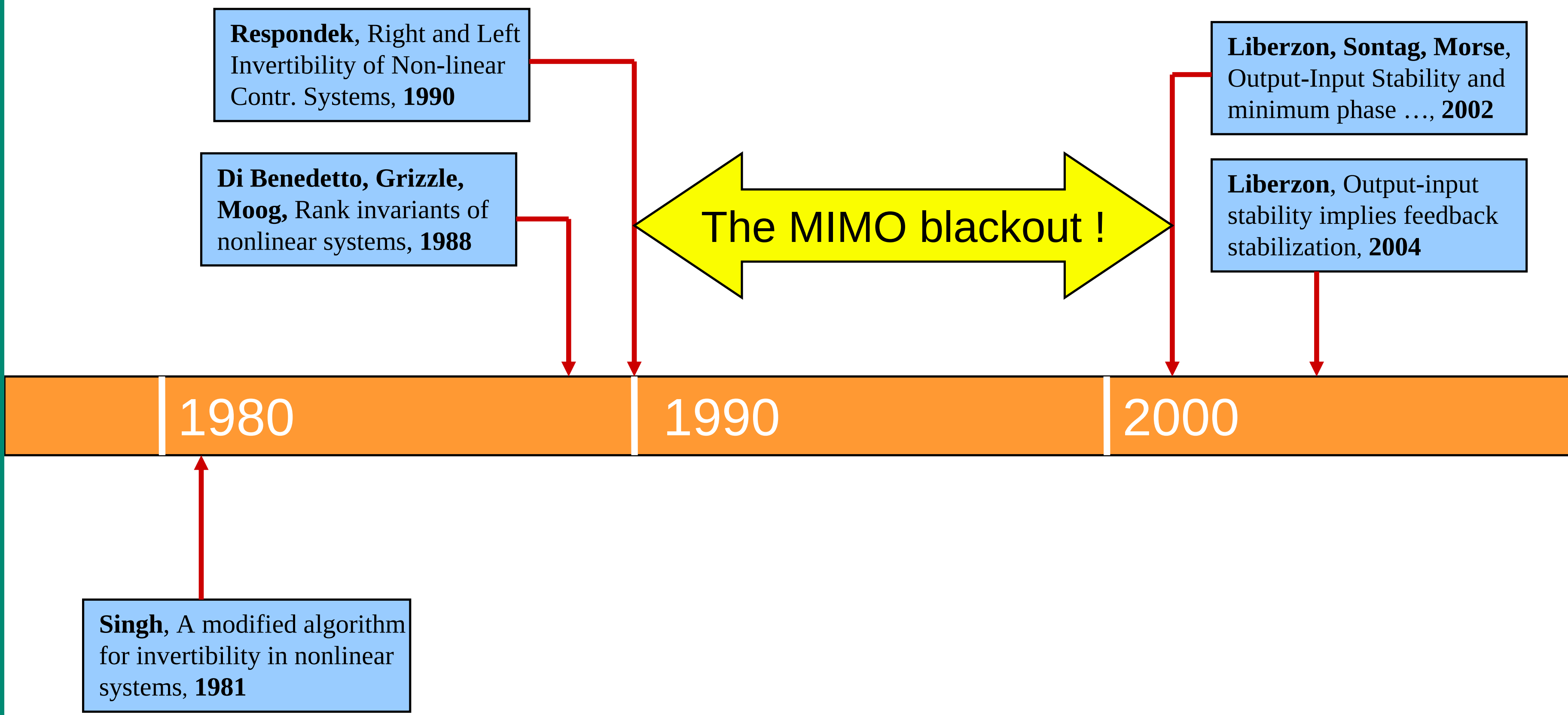
Roadblocks and Challenges

One basic question puzzles me: where did MIMO systems go ?

In the late 1960s and early 1970s, the theory of MIMO linear systems reached a high degree of sophistication (one example for all: Wonham's famous book is entitled "Linear Multivariable Control"). In the 1980s, a big a collective effort aimed at extending this theory to nonlinear systems. Sophisticated tools had been developed, yielding a rather satisfactory understanding of system inversion, zero dynamics, infinite zero structure for MIMO nonlinear system.

However, by the early 1990s, a blackout occurred.

Only in the early 2000s, interest in such ideas came back.


Respondek, Right and Left Invertibility of Non-linear Contr. Systems, **1990**
Di Benedetto, Grizzle, Moog, Rank invariants of nonlinear systems, **1988**
The MIMO blackout !
Liberzon, Sontag, Morse, Output-Input Stability and minimum phase …, **2002**
Liberzon, Output-input stability implies feedback stabilization, **2004**
1980
1990
2000
Singh, A modified algorithm for invertibility in nonlinear systems, **1981**

Stabilization of MIMO systems by output feedback

A SISO strongly minimum-phase system (having relative degree 1) can be globally stabilized by memoryless output feedback $u = \kappa(e)$.

The MIMO version of this stabilization paradigm is still a largely open domain of research.

After a "blackout" period that lasted for about a decade, interest has resumed in the problem of (globally) stabilizing MIMO nonlinear systems. Advances in this domain have been triggered by a paper of Liberzon, Morse, Sontag (2002).

A paper of Liberzon (2004), in particular, considers input-affine systems having m inputs and $p \geq m$ outputs, with the following property: for some integer N, there exist functions $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}_\infty$ such that for every initial state $x(0)$ and every admissible input $u(\cdot)$ the corresponding solution $x(t)$ satisfies

$$|x(t)| \leq \max\{\beta(|x(0)|, t), \gamma(\|\mathrm{y}^{N-1}\|_{[0,t]})\}$$

as long as it exists. The property in question is a possible extension to MIMO systems of the property of being strongly minimum phase.

Then, Liberzon (2004) assumes that the system is globally left invertible, in the sense that (the global version of) Singh's inversion algorithm terminates at a stage $k^* \leq m$ in which the input $u(t)$ can be uniquely recovered from the output $y(t)$ and a finite number of its derivatives.

Under this (and another technical) assumption it is shown that a static state feedback law $u = \alpha(x)$ exists that globally stabilizes the system. The role of this law is essentially to guarantee that – in the associated closed-loop system – the individual components of the output obey linear differential equations whose characteristic polynomials are Hurwitz.

This result is very intersting, and is the more general result available to date dealing with global stabilization of MIMO systems possessing a (strongly) stable zero dynamics. The feedback law proposed, though, is a static state feedback law. The problem of finding a UCO (in the sense of Teel-Praly) feedback law is still open.

There are classes of MIMO systems, though, in which the design paradigm based on high-gain feedback (from the output and their higher derivatives) is applicable.

The most trivial one is the class of (square) systems in which $L_g h(x)$ is nonsingular. In this case, in fact, if in addition there exist a matrix M and a number $b_0 > 0$ such that

$$[L_g h(x)]^{\mathrm{T}} M + M[L_g h(x)] \geq b_0 I$$

and if the above property holds for $N = 1$, the global stabilization paradigm described in the previous section is applicable.

So the question arises: how can a more general system be reduced to a system possessing such property ?

If the property that $L_g h(x)$ is nonsingular could be achieved via a transformation of the type

$$\tilde{y} = \phi(y, y^{(1)}, \ldots, y^{(k)})\,,$$

the paradigm in question, supplemented by the robust observer of **Teel-Praly**, can still be pursued to obtain (at least) semiglobal stability.

A special case of systems for which such transformation exists are the systems that are invertible and whose input-output behavior can be rendered linear via a transformation of the form $u = \alpha(x) + \beta(x)v$ (compare with **Liberzon (2004)**, where the autonomous behavior is rendered linear by a control law $u = \alpha(x)$). For such systems, in fact, one can find the desired $\tilde{y}$ as

$$\tilde{y} = \Lambda(s)K$$

in which K is a nonsingular matrix and $\Lambda(s)$ is a diagonal matrix of Hurwitz polynmials. If the original system was strongly minimum phase so is the modified system and the property above holds for $N = 1$.

A simple benchmark in MIMO stabilization

Consider a system with two inputs and two outputs and assume

$$L_g h_2(x) = \delta(x) L_g h_1(x)$$

for some $\delta(x)$. Define

$$\phi(x) = L_f h_2(x) - \delta(x) L_f h_1(x)$$

Then

$$\begin{aligned} \dot{y}_1 &= L_f h_1(x) + L_g h_1(x) u \\ \dot{y}_2 &= \phi(x) + \delta(x) \dot{y}_1 \\ \dot{\phi} &= L_f \phi(x) + L_g \phi(x) u \end{aligned}$$

Assume invertibility, i.e. assume

$$\begin{pmatrix} L_g h_1(x) \\ L_g \phi(x) \end{pmatrix}$$

is nonsingular for all x. How can we achieve global stability via output fedback ?

Output regulation of MIMO Systems

Let a plant

$$\begin{aligned} \dot{w} &= s(w) \\ \dot{x} &= f(w,x,u) \\ e &= h(w,x) \\ y &= k(w,x)\,, \end{aligned} \tag{1}$$

with control $u \in \mathbb{R}^m$, regulated output $e \in \mathbb{R}^p$, and supplementary measurements $y \in \mathbb{R}^q$, be controlled by

$$\begin{aligned} \dot{x}_c &= f_c(x_c, e, y) \\ u &= h_c(x_c, e, y)\,. \end{aligned} \tag{2}$$

The goal is to to obtain a closed-loop system in which all trajectories are ultimately bounded and $\lim_{t\to\infty} e(t) = 0$.

Consider, without loss of generality, the case in which the state w of the exosystem evolves on a compact invariant set W and assume that the steady-state locus of the associated closed-loop system is the graph of a C^1 map defined on W.

Then, if the problem in question is solved, there exist maps $\pi(w)$ and $\pi_c(w)$ satisfying

$$\left.\begin{array}{rcl} L_s\pi(w) & = & f(w,\pi(w),\psi(w)) \\ 0 & = & h(w,\pi(w)) \end{array}\right. \qquad \forall w \in W \tag{3}$$

and

$$\left.\begin{array}{rcl} L_s\pi_c(w) & = & f_c(\pi_c(w),0,k(w,\pi(w))) \\ \psi(w) & = & h_c(\pi_c(w),0,k(w,\pi(w)))\,. \end{array}\right. \qquad \forall w \in W \tag{4}$$

The first two are the so-called regulator equations. The last two express the property, of the controller, of generating a steady-state control that keeps $e = 0$.

In the case of linear systems, the regulator equations are robustly solvable (with respect to plant parameter uncertainties) if and only if the system is right-invertible with respect to e (which in turn implies $m \geq p$) and none of the transmission zeroes is an eigenvalue of the exosystem (non-resonance condition).

This being the case, the fulfillment of the extra two conditions (in spite of plant parameter uncertainties) is automatically guaranteed if the controller is chosen as the cascade of a "post processor" that contains of p identical controllable copies of the exosystem

$$\dot{\eta}_i = S\eta_i + G_i e_i\,, \qquad i = 1, \ldots, p \tag{5}$$

whose state $\eta = \mathrm{col}(\eta_1, \ldots, \eta_p)$ drives, along with the full measured output (e, y), a "stabilizer"

$$\begin{aligned} \dot{\xi} &= F_{11}\xi + F_{12}\eta + B_{c1}e + B_{c2}y \\ u &= H_1\xi + H_2\eta + D_{c1}e + D_{c2}y\,. \end{aligned} \tag{6}$$

In fact, appealing to the non-resonance condition, it is a simple matter to show that if the controlled plant is stabilizable and detectable so is the cascade of the controlled plant and of (5) and hence a stabilizer of the form (6) always exists.

Then, using Cayley-Hamilton's Theorem, it is not difficult to show that, regardless of what $\psi(w)$ and $\pi(w)$ are (recall that we are seeking solutions in spite of parameter variations), the equations (4) always have a solution $\pi_{\rm c}(w)$ (even if the "steady-state" supplementary measurement $k(w, \pi(w))$ is nonzero).

In the case of nonlinear systems having $m > 1$ and $p > 1$, solving the two equations (3) is not terribly difficult. This can be achieved, in fact, by means of a suitably enhanced version of the zero dynamics algorithm (such as presented in [Isidori (1995)]) and, if so desired for subsequent stabilization purposes, by bringing the system to a multivariable normal form.

However, the problem of building a controller that also solves the two equations (4) is substantially different, because the existence of $\pi_{\rm c}(w)$ is no longer automatically guaranteed by the fact that the controller is realized as an internal model driven by the error variable e which in turn drives a stabilizer.

In fact, to the current state of our knowledge, it is known how to fulfill the equations in question only if the controller is realized as a preprocessor

$$\begin{aligned} \dot{\eta} &= \varphi(\eta) + Gv \\ u &= \gamma(\eta) + v \end{aligned}$$

with $\varphi(\cdot)$ and $\gamma(\cdot)$ satisfying

$$\begin{aligned} \psi(w) &= \gamma(\tau(w)) \\ L_s\tau(w) &= \varphi(\tau(w)) \end{aligned} \qquad \forall w \in W \qquad (7)$$

for some $\tau(\cdot)$ whose input v is provided by a stabilizer only driven by the regulated variable e.

The existence of such maps was recently showed in [Marconi, Praly, Isidori (2007)]. In particular, it was shown that for a large enough d, there exists a controllable pair (F, G), in which F is a $d \times d$ Hurwitz matrix and G is a $d \times 1$ vector, a continuous map $\gamma : \mathbb{R}^d \to \mathbb{R}$ and a continuously differentiable map $\tau : W \to \mathbb{R}^d$ satisfying

$$\begin{aligned} L_s\tau(w) &= F\tau(w) + G\psi(w) \\ \psi(w) &= \gamma(\tau(w)) \end{aligned}$$

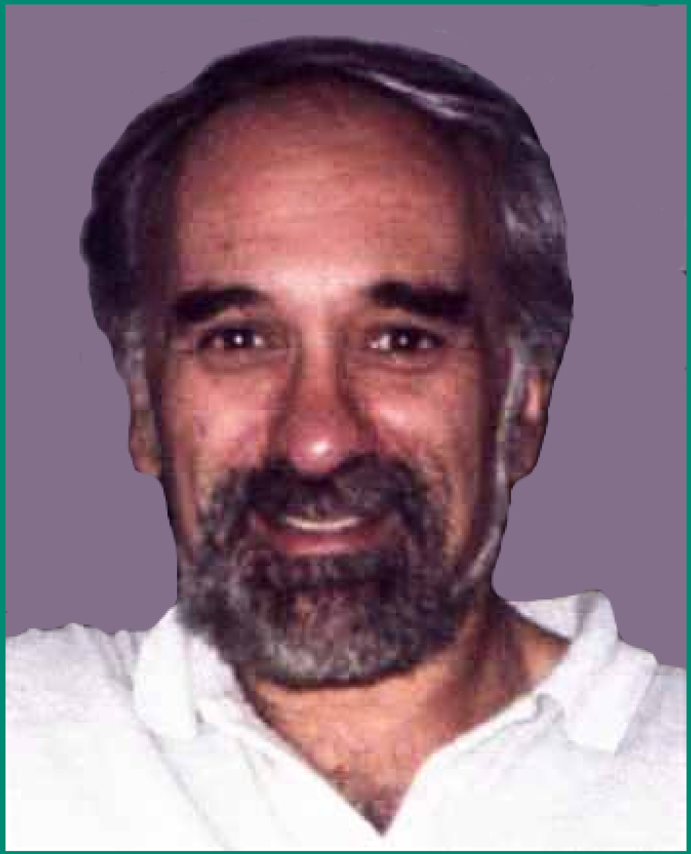
from which is is seen that (7) can be fulfilled with

$$\varphi(\eta) = F\eta + G\gamma(\eta)\,.$$

This substantially limits the generality for at least two reasons: on one side it does not allow additional measured outputs (which would be otherwise useful for stabilization purposes) because it is not immediate how their possibly nontrivial steady-state behavior could be (robustly) handled, on the other side because a scheme in which the internal model is a preprocessor requires (even in the case of linear systems) $m = p$ and this limits the availability of extra inputs (which, again, would be otherwise useful for stabilization purposes).

The theory of output regulation for MIMO nonlinear systems with $m > p$ and $q > 1$ is a completely open domain of research.

Happy Birthday EDUARDO



and Congratulations for your outstanding achievements !