Model predictive control without terminal constraints: stability and performance

Lars Grüne

Mathematisches Institut, Universität Bayreuth

in collaboration with

Anders Rantzer (Lund), Nils Altmüller (Bayreuth), Thomas Jahn (Bayreuth), Jürgen Pannek (Perth), Karl Worthmann (Bayreuth)

supported by DFG priority research program 1305 and Marie-Curie ITN SADCO

SontagFest'11, DIMACS, Rutgers, May 23–25, 2011

Model predictive control without terminal constraints: stability and performance

Lars Grüne

Mathematisches Institut, Universität Bayreuth

in collaboration with

Anders Rantzer (Lund), Nils Altmüller (Bayreuth), Thomas Jahn (Bayreuth), Jürgen Pannek (Perth), Karl Worthmann (Bayreuth)

supported by DFG priority research program 1305 and Marie-Curie ITN SADCO

SontagFest'11, DIMACS, Rutgers, May 23–25, 2011

Happy 60th Birthday Eduardo!

Setup

We consider nonlinear discrete time control systems

$$x(n+1) = f(x(n), u(n))$$

with $x(n) \in X$, $u(n) \in U$, $\quad X$, U arbitrary metric spaces

Setup

We consider nonlinear discrete time control systems

$$x(n+1) = f(x(n), u(n))$$

with $x(n) \in X$, $u(n) \in U$, X, U arbitrary metric spaces

Problem: feedback stabilization

Setup

We consider nonlinear discrete time control systems

$$x(n+1) = f(x(n), u(n))$$

with $x(n) \in X$, $u(n) \in U$, $\quad X$, U arbitrary metric spaces

Problem: Optimal feedback stabilization via infinite horizon optimal control:

For a running cost $\ell : X \times U \to \mathbb{R}_0^+$ penalizing the distance to the desired equilibrium solve

$$\text{minimize } J_\infty(x,u) = \sum_{n=0}^{\infty} \ell(x(n), u(n)) \quad \text{with } u(n) = F(x(n))$$

Setup

We consider nonlinear discrete time control systems

$$x(n+1) = f(x(n), u(n))$$

with $x(n) \in X$, $u(n) \in U$, $\quad X$, U arbitrary metric spaces

Problem: Optimal feedback stabilization via infinite horizon optimal control:

For a running cost $\ell : X \times U \to \mathbb{R}_0^+$ penalizing the distance to the desired equilibrium solve

$$\text{minimize } J_\infty(x,u) = \sum_{n=0}^{\infty} \ell(x(n), u(n)) \quad \text{with } u(n) = F(x(n)),$$

subject to state/control constraints $x \in \mathbb{X}$, $u \in \mathbb{U}$

Model predictive control

Direct solution of the problem is numerically hard

Alternative method: model predictive control (MPC)

Model predictive control

Direct solution of the problem is numerically hard

Alternative method: model predictive control (MPC)

Idea: replace the original problem

$$\text{minimize } J_\infty(x,u) = \sum_{n=0}^{\infty} \ell(x(n), u(n))$$

by the iterative (online) solution of finite horizon problems

$$\text{minimize } J_N(x,u) = \sum_{k=0}^{N-1} \ell(x_u(k), u(k))$$

with $x_u(k) \in \mathbb{X}$, $u(k) \in \mathbb{U}$

Model predictive control

Direct solution of the problem is numerically hard

Alternative method: model predictive control (MPC)

Idea: replace the original problem

$$\text{minimize } J_\infty(x,u) = \sum_{n=0}^{\infty} \ell(x(n), u(n))$$

by the iterative (online) solution of finite horizon problems

$$\text{minimize } J_N(x,u) = \sum_{k=0}^{N-1} \ell(x_u(k), u(k))$$

with $x_u(k) \in \mathbb{X}$, $u(k) \in \mathbb{U}$

We obtain a feedback law F_N by a moving horizon technique

Model predictive control

Basic moving horizon MPC concept:

Model predictive control

Basic moving horizon MPC concept:

At each time instant n solve for the current state $x(n)$

$$\text{minimize } J_N(x,u) = \sum_{k=0}^{N-1} \ell(x_u(k), u(k)), \quad x_u(0) = x(n)$$

Model predictive control

Basic moving horizon MPC concept:

At each time instant n solve for the current state $x(n)$

$$\text{minimize } J_N(x,u) = \sum_{k=0}^{N-1} \ell(x_u(k), u(k)), \quad x_u(0) = x(n)$$

$\rightsquigarrow$ optimal trajectory $\quad x^{opt}(0), \ldots, x^{opt}(N-1)$

Model predictive control

Basic moving horizon MPC concept:

At each time instant n solve for the current state $x(n)$

$$\text{minimize } J_N(x,u) = \sum_{k=0}^{N-1} \ell(x_u(k), u(k)), \quad x_u(0) = x(n)$$

$\rightsquigarrow$ optimal trajectory $x^{opt}(0), \ldots, x^{opt}(N-1)$

with optimal control $u^{opt}(0), \ldots, u^{opt}(N-1)$

Model predictive control

Basic moving horizon MPC concept:

At each time instant n solve for the current state $x(n)$

$$\text{minimize } J_N(x,u) = \sum_{k=0}^{N-1} \ell(x_u(k), u(k)), \quad x_u(0) = x(n)$$

$\rightsquigarrow$ optimal trajectory $x^{opt}(0), \ldots, x^{opt}(N-1)$

with optimal control $u^{opt}(0), \ldots, u^{opt}(N-1)$

$\rightsquigarrow$ MPC feedback law $F_N(x(n)) := u^{opt}(0)$

Model predictive control

Basic moving horizon MPC concept:

At each time instant n solve for the current state $x(n)$

$$\text{minimize } J_N(x,u) = \sum_{k=0}^{N-1} \ell(x_u(k), u(k)), \quad x_u(0) = x(n)$$

$\rightsquigarrow$ optimal trajectory $x^{opt}(0), \ldots, x^{opt}(N-1)$

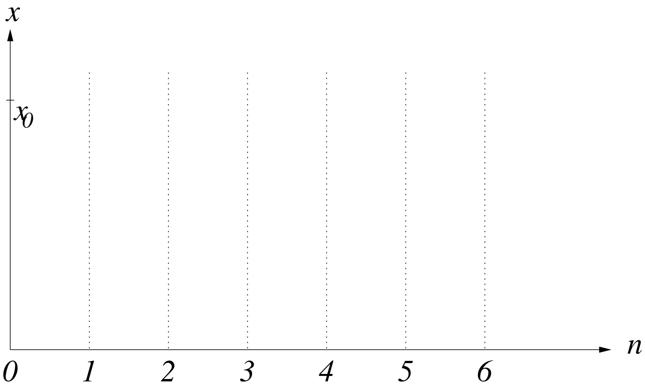
with optimal control $u^{opt}(0), \ldots, u^{opt}(N-1)$

$\rightsquigarrow$ MPC feedback law $F_N(x(n)) := u^{opt}(0)$

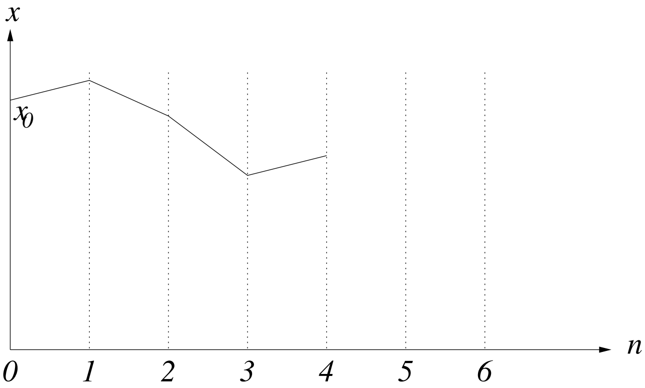
$\rightsquigarrow$ closed loop system

$$x(n+1) = f(x(n), F_N(x(n))) = f(x^{opt}(0), u^{opt}(0)) = x^{opt}(1)$$

MPC from the trajectory point of view

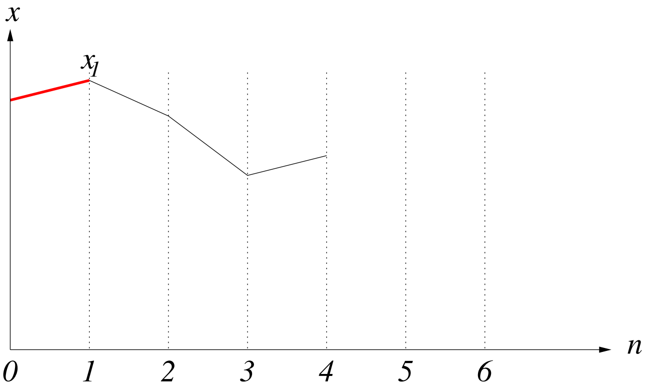


MPC from the trajectory point of view



black = predictions (open loop optimization)

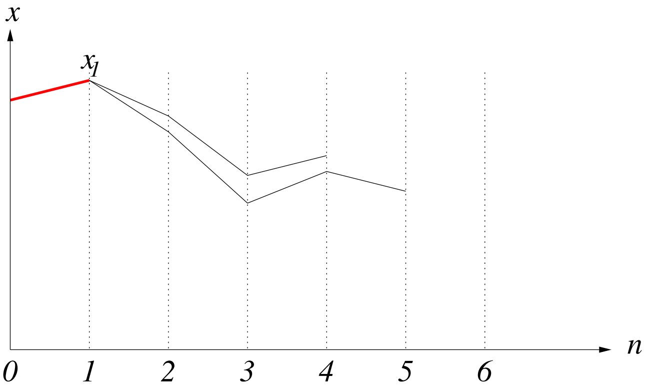
MPC from the trajectory point of view



black = predictions (open loop optimization)

red = MPC closed loop $\quad x(n+1) = f(x(n), F_N(x(n)))$

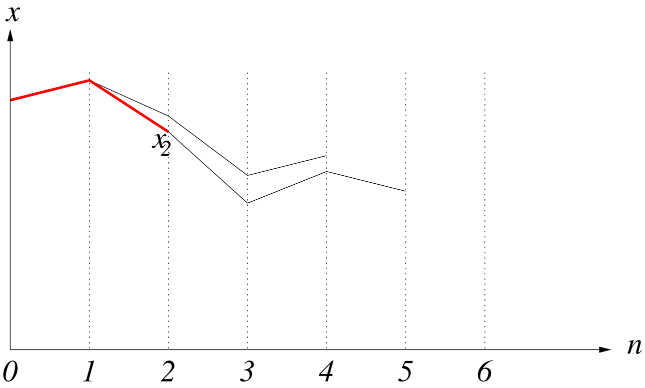
MPC from the trajectory point of view



black = predictions (open loop optimization)

red = MPC closed loop $\quad x(n+1) = f(x(n), F_N(x(n)))$

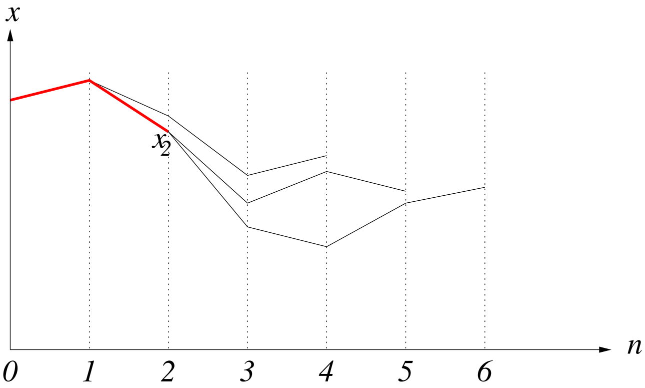
MPC from the trajectory point of view



black = predictions (open loop optimization)

red = MPC closed loop $\quad x(n+1) = f(x(n), F_N(x(n)))$

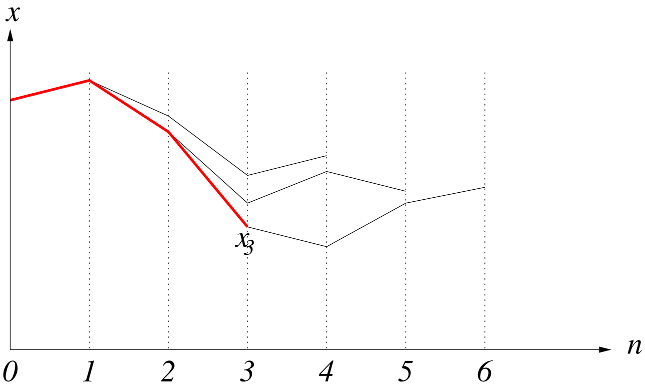
MPC from the trajectory point of view



black = predictions (open loop optimization)

red = MPC closed loop $\quad x(n+1) = f(x(n), F_N(x(n)))$

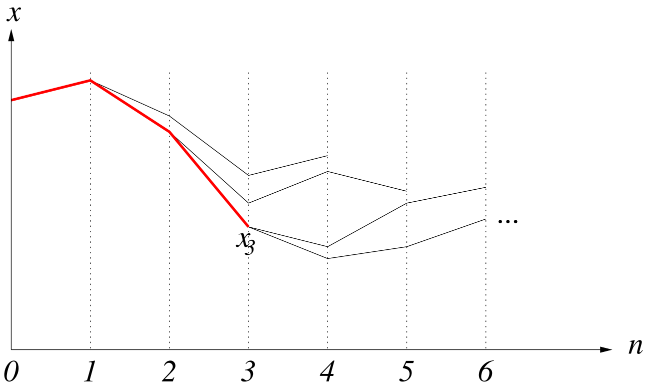
MPC from the trajectory point of view



black = predictions (open loop optimization)

red = MPC closed loop $\quad x(n+1) = f(x(n), F_N(x(n)))$

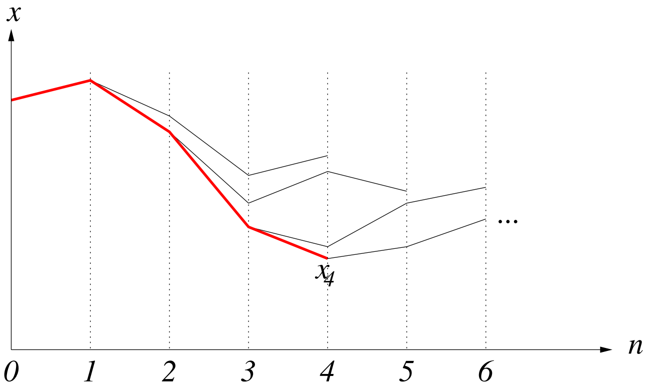
MPC from the trajectory point of view



black = predictions (open loop optimization)

red = MPC closed loop $\quad x(n+1) = f(x(n), F_N(x(n)))$

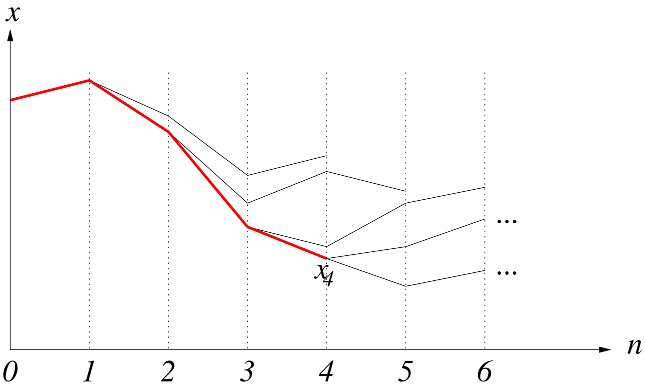
MPC from the trajectory point of view



black = predictions (open loop optimization)

red = MPC closed loop $\quad x(n+1) = f(x(n), F_N(x(n)))$

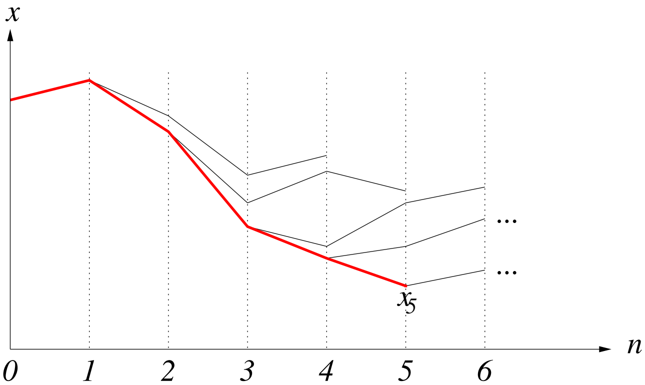
MPC from the trajectory point of view



black = predictions (open loop optimization)

red = MPC closed loop $\quad x(n+1) = f(x(n), F_N(x(n)))$

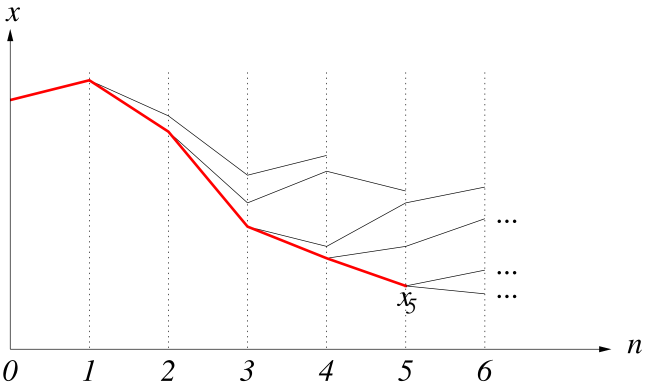
MPC from the trajectory point of view



black = predictions (open loop optimization)

red = MPC closed loop $\quad x(n+1) = f(x(n), F_N(x(n)))$

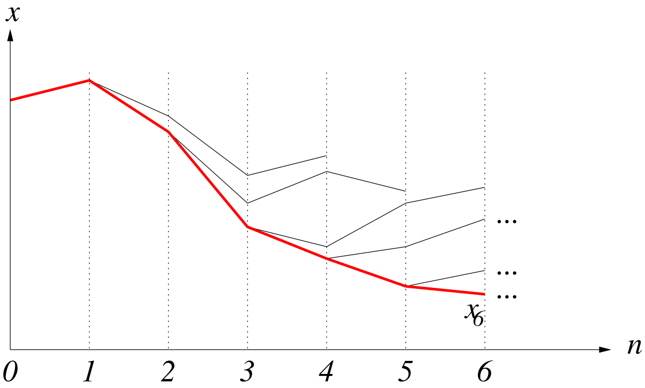
MPC from the trajectory point of view



black = predictions (open loop optimization)

red = MPC closed loop $\quad x(n+1) = f(x(n), F_N(x(n)))$

MPC from the trajectory point of view



black = predictions (open loop optimization)

red = MPC closed loop $\quad x(n+1) = f(x(n), F_N(x(n)))$

UNIVERSITÄT BAYREUTH

MPC: Questions

Questions in this talk:

- When does MPC stabilize the system?

MPC: Questions

Questions in this talk:

- When does MPC stabilize the system?
- How good is the MPC feedback law compared to the infinite horizon optimal solution?

MPC: Questions

Questions in this talk:

- When does MPC stabilize the system?
- How good is the MPC feedback law compared to the infinite horizon optimal solution?

Part 1: stabilizing MPC

MPC: Questions

Questions in this talk:

- When does MPC stabilize the system?
- How good is the MPC feedback law compared to the infinite horizon optimal solution?

Part 1: stabilizing MPC

Part 2: economic MPC

MPC: Questions

Questions in this talk:

- When does MPC stabilize the system?
- How good is the MPC feedback law compared to the infinite horizon optimal solution?

Part 1: stabilizing MPC — survey on recent results

Part 2: economic MPC

MPC: Questions

Questions in this talk:

- When does MPC stabilize the system?
- How good is the MPC feedback law compared to the infinite horizon optimal solution?

Part 1: stabilizing MPC — survey on recent results

Part 2: economic MPC — some very recent results

MPC: Questions

Questions in this talk:

- When does MPC stabilize the system?
- How good is the MPC feedback law compared to the infinite horizon optimal solution?

Part 1: stabilizing MPC — survey on recent results

Part 2: economic MPC — some very recent results

In stabilizing MPC, stability can be ensured by including additional "stabilizing" terminal constraints in the finite horizon problem. Here we consider problems without such stabilizing constraints.

MPC: Questions

Questions in this talk:

- When does MPC stabilize the system?
- How good is the MPC feedback law compared to the infinite horizon optimal solution?

Part 1: stabilizing MPC — survey on recent results

Part 2: economic MPC — some very recent results

In stabilizing MPC, stability can be ensured by including additional "stabilizing" terminal constraints in the finite horizon problem. Here we consider problems without such stabilizing constraints.

Main motivation: even for small optimization horizons N we can — in principle — obtain large feasible sets, i.e., sets of initial values for which the finite horizon problem is well defined

Stability without stabilizing terminal constraints

Without stabilizing constraints, stability is known to hold for "sufficiently large optimization horizon N" [Alamir/Bornard '95, Jadbabaie/Hauser '05, Grimm/Messina/Tuna/Teel '05]

Stability without stabilizing terminal constraints

Without stabilizing constraints, stability is known to hold for "sufficiently large optimization horizon N" [Alamir/Bornard '95, Jadbabaie/Hauser '05, Grimm/Messina/Tuna/Teel '05]

How large is "sufficiently large"?

Stability without stabilizing terminal constraints

Without stabilizing constraints, stability is known to hold for "sufficiently large optimization horizon N" [Alamir/Bornard '95, Jadbabaie/Hauser '05, Grimm/Messina/Tuna/Teel '05]

How large is "sufficiently large"?

For obtaining a quantitative estimate we need quantitative information.

Stability without stabilizing terminal constraints

Without stabilizing constraints, stability is known to hold for "sufficiently large optimization horizon N" [Alamir/Bornard '95, Jadbabaie/Hauser '05, Grimm/Messina/Tuna/Teel '05]

How large is "sufficiently large"?

For obtaining a quantitative estimate we need quantitative information.

A suitable condition is "exponential controllability through ℓ":

there exist constants $C > 0$, $\sigma \in (0,1)$ such that for each $x_u(0) \in \mathbb{X}$ there is $u(\cdot)$ with $x_u(k) \in \mathbb{X}$, $u(k) \in \mathbb{U}$ and

$$\ell(x_u(k), u(k)) \leq C\sigma^k \ell^*(x_u(0))$$

with $\ell^*(x) = \min_{u \in \mathbb{U}} \ell(x, u)$

Stability and performance conditions

C, σ-exp. controllability: $\ell(x(k), u(k)) \leq C\sigma^k \ell^*(x_u(0))$

Stability and performance conditions

C, σ-exp. controllability: $\qquad \ell(x(k),u(k)) \le C\sigma^k \ell^*(x_u(0))$

Define $\alpha := 1 - \dfrac{(\gamma_N - 1)\prod_{i=2}^{N}(\gamma_i - 1)}{\prod_{i=2}^{N}\gamma_i - \prod_{i=2}^{N}(\gamma_i - 1)}$ with $\gamma_i = \sum_{k=0}^{i-1} C\sigma^k$

Stability and performance conditions

C, σ-exp. controllability: $\ell(x(k), u(k)) \le C\sigma^k \ell^*(x_u(0))$

Define $\alpha := 1 - \dfrac{(\gamma_N - 1)\prod_{i=2}^{N}(\gamma_i - 1)}{\prod_{i=2}^{N}\gamma_i - \prod_{i=2}^{N}(\gamma_i - 1)}$ with $\gamma_i = \sum_{k=0}^{i-1} C\sigma^k$

Theorem: If $\alpha > 0$, then the MPC feedback F_N stabilizes all C, σ-exponentially controllable systems and we get

$$J_\infty(x, F_N) \le \inf_{u \in \mathbb{U}^\infty} J_\infty(x, u)/\alpha$$

Stability and performance conditions

C, σ-exp. controllability: $\ell(x(k), u(k)) \leq C\sigma^k \ell^*(x_u(0))$

Define $\alpha := 1 - \dfrac{(\gamma_N - 1)\prod_{i=2}^{N}(\gamma_i - 1)}{\prod_{i=2}^{N}\gamma_i - \prod_{i=2}^{N}(\gamma_i - 1)}$ with $\gamma_i = \sum_{k=0}^{i-1} C\sigma^k$

Theorem: If $\alpha > 0$, then the MPC feedback F_N stabilizes all C, σ-exponentially controllable systems and we get

$$J_\infty(x, F_N) \leq \inf_{u \in \mathbb{U}^\infty} J_\infty(x, u)/\alpha$$

If $\alpha < 0$ then there exists a C, σ-exponentially controllable system, which is not stabilized by F_N

Stability and performance conditions

C, σ-exp. controllability: $\ell(x(k), u(k)) \leq C\sigma^k \ell^*(x_u(0))$

Define $\alpha := 1 - \dfrac{(\gamma_N - 1)\prod_{i=2}^{N}(\gamma_i - 1)}{\prod_{i=2}^{N}\gamma_i - \prod_{i=2}^{N}(\gamma_i - 1)}$ with $\gamma_i = \sum_{k=0}^{i-1} C\sigma^k$

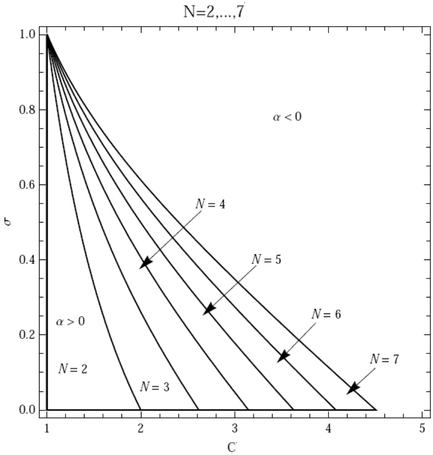
Theorem: If $\alpha > 0$, then the MPC feedback F_N stabilizes all C, σ-exponentially controllable systems and we get

$$J_\infty(x, F_N) \leq \inf_{u \in \mathbb{U}^\infty} J_\infty(x, u)/\alpha$$

If $\alpha < 0$ then there exists a C, σ-exponentially controllable system, which is not stabilized by F_N

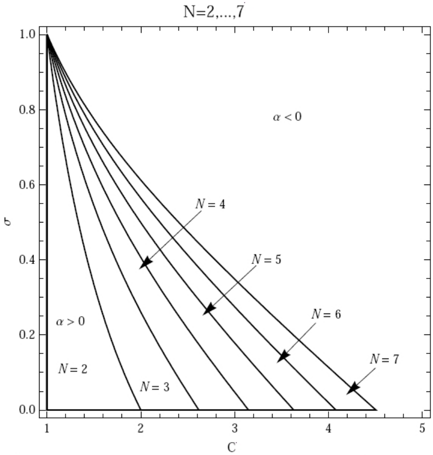
Moreover, $\alpha \to 1$ as $N \to \infty$

Stability chart for C and σ



(Figure: Harald Voit)

Stability chart for C and σ



(Figure: Harald Voit)

Conclusion: try to reduce C, e.g., by choosing ℓ appropriately

A PDE example

We illustrate this with the 1d controlled PDE

$$y_t = y_x + \nu y_{xx} + \mu y(y+1)(1-y) + u$$

with

domain $\Omega = [0,1]$

solution $y = y(t,x)$

boundary conditions $y(t,0) = y(t,1) = 0$

parameters $\nu = 0.1$ and $\mu = 10$

and distributed control $u : \mathbb{R} \times \Omega \to \mathbb{R}$

A PDE example

We illustrate this with the 1d controlled PDE

$$y_t = y_x + \nu y_{xx} + \mu y(y+1)(1-y) + u$$

with

domain $\Omega = [0,1]$

solution $y = y(t,x)$

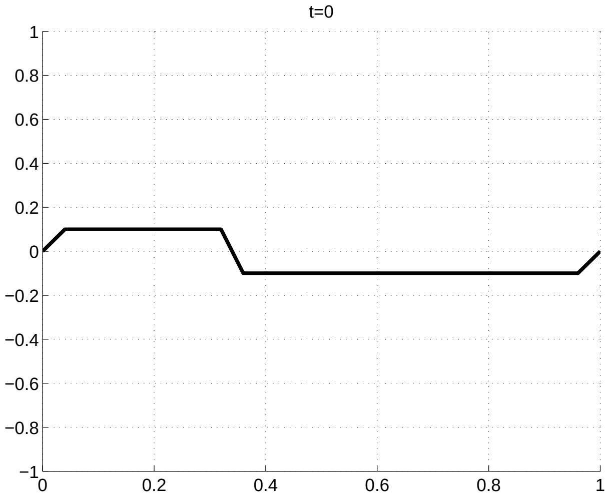
boundary conditions $y(t,0) = y(t,1) = 0$

parameters $\nu = 0.1$ and $\mu = 10$

and distributed control $u : \mathbb{R} \times \Omega \to \mathbb{R}$

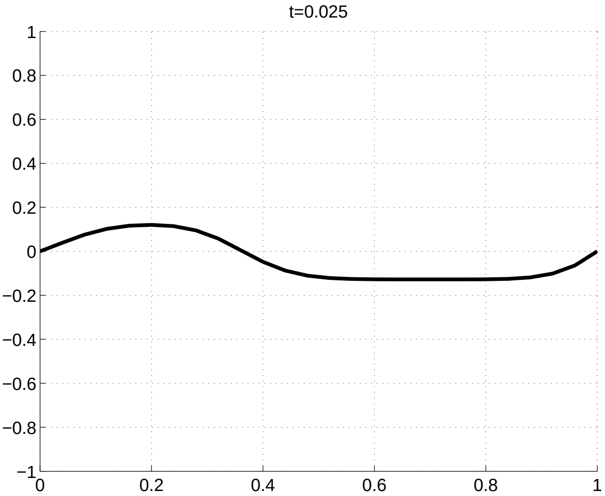
Discrete time system: $y(n) = y(nT, \cdot)$ for some $T > 0$
("sampled data system with sampling time T")

The uncontrolled PDE



uncontrolled ($u \equiv 0$)

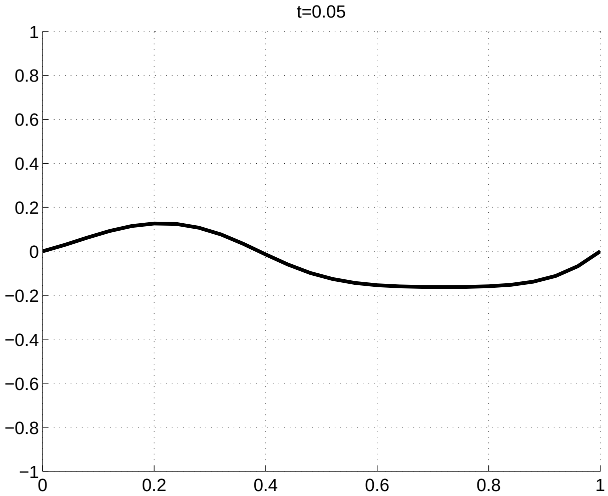
The uncontrolled PDE



uncontrolled ($u \equiv 0$)

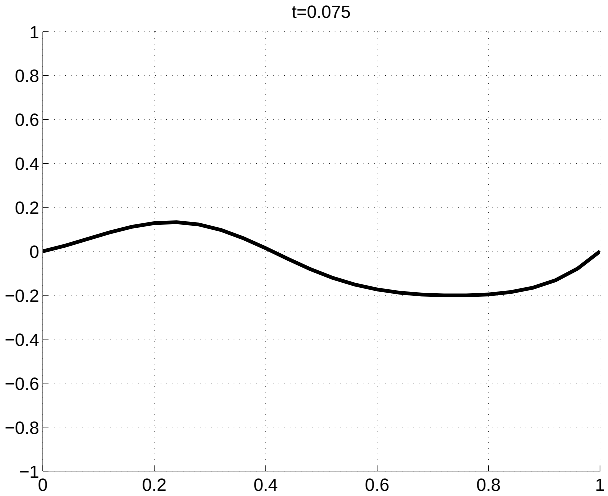
UNIVERSITÄT BAYREUTH

The uncontrolled PDE



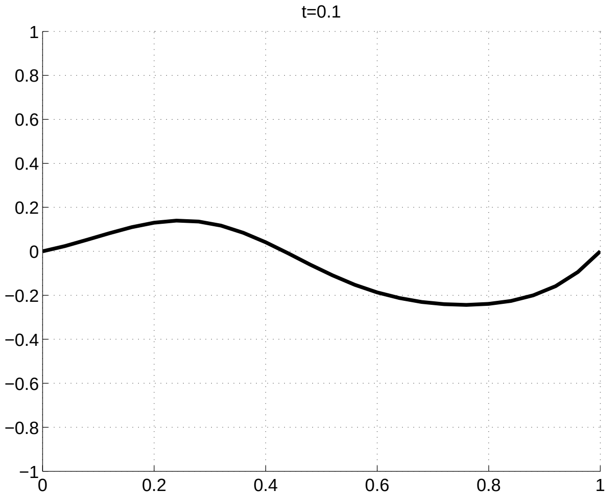
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



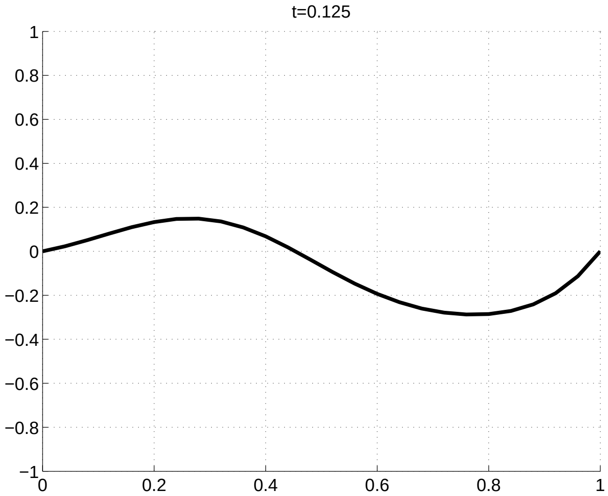
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



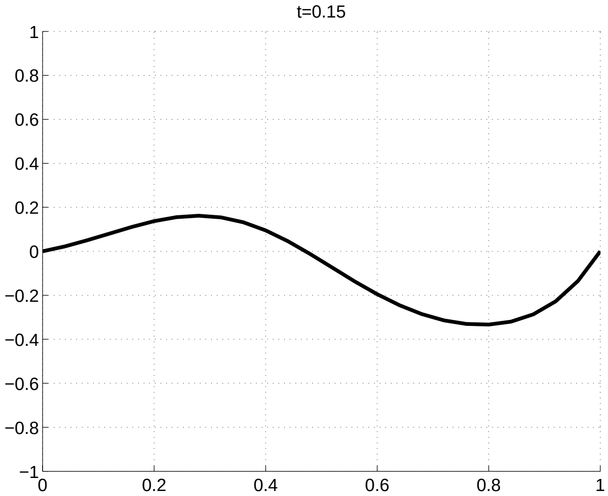
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



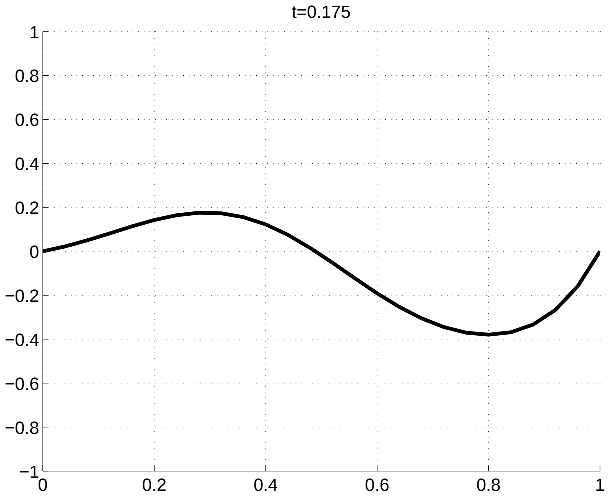
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



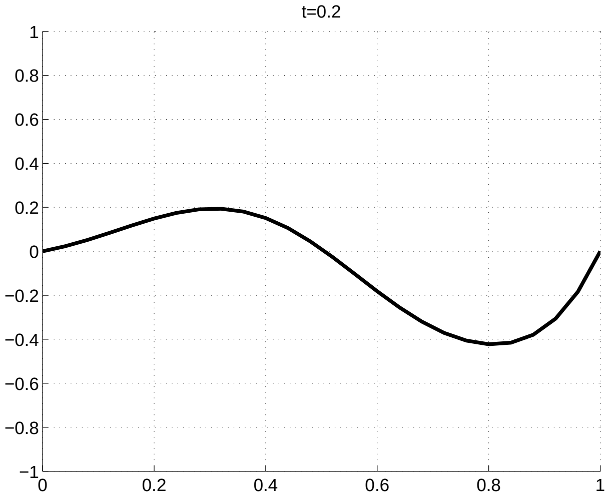
uncontrolled $(u \equiv 0)$

The uncontrolled PDE



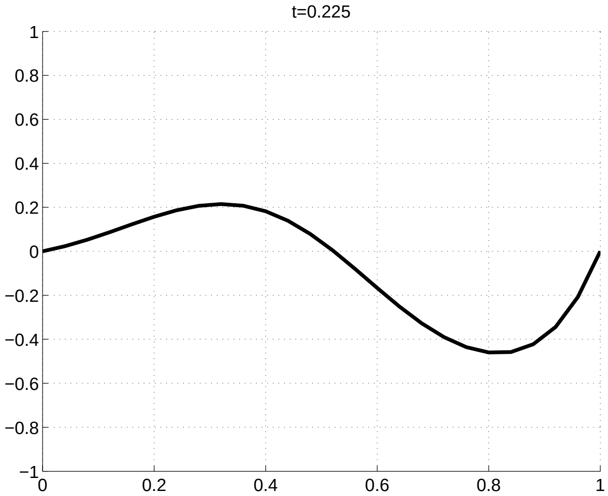
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



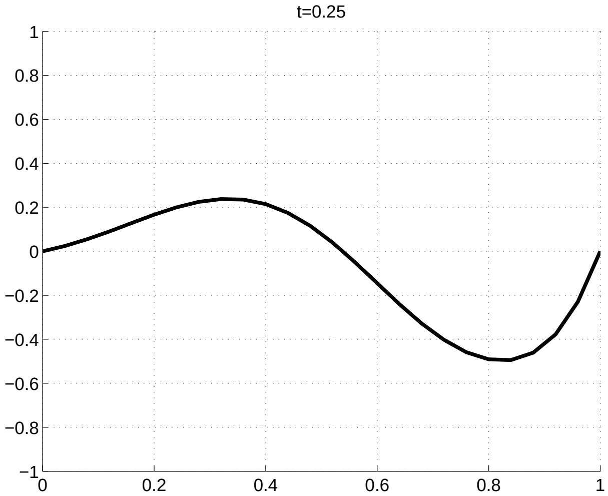
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



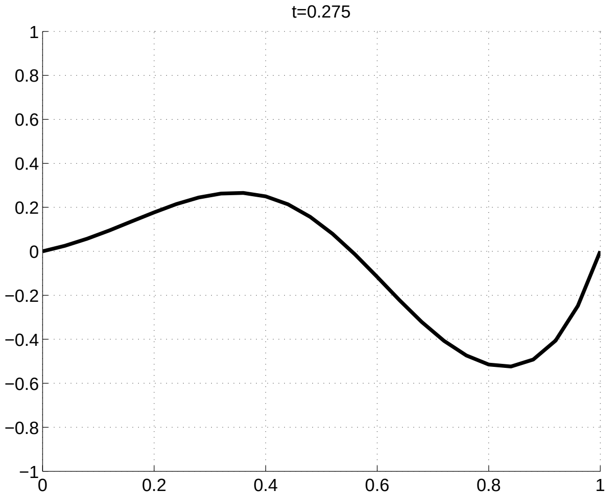
uncontrolled $(u \equiv 0)$

The uncontrolled PDE



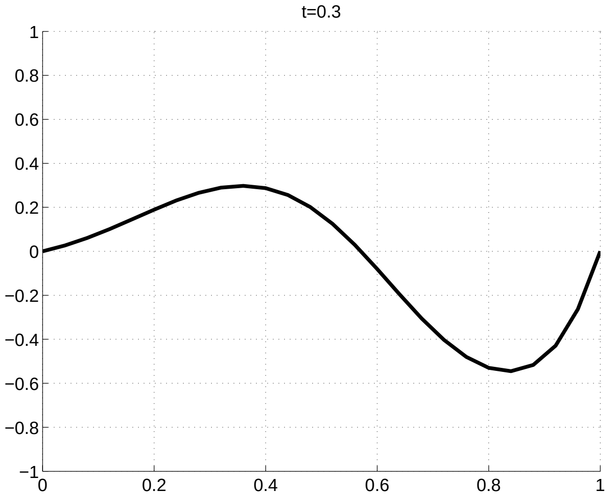
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



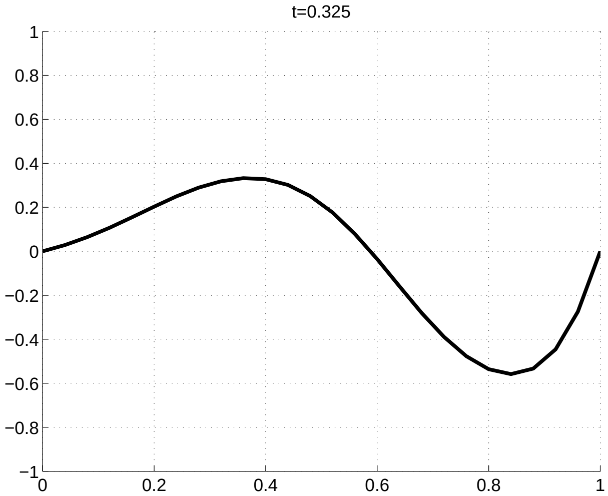
uncontrolled $(u \equiv 0)$

The uncontrolled PDE



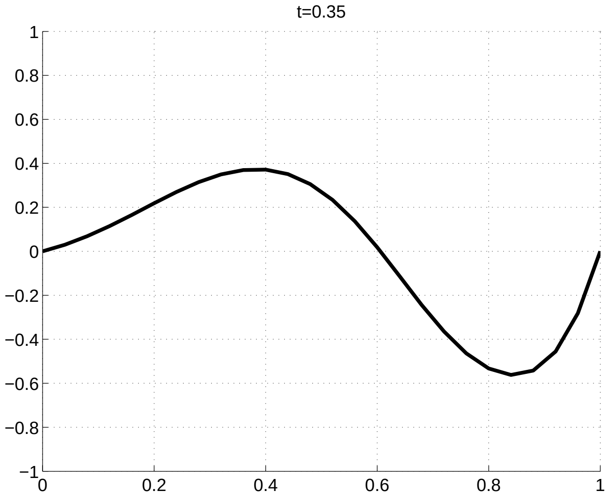
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



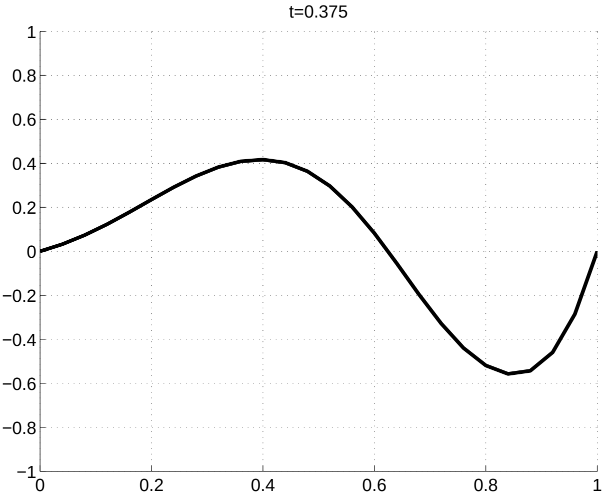
uncontrolled $(u \equiv 0)$

The uncontrolled PDE



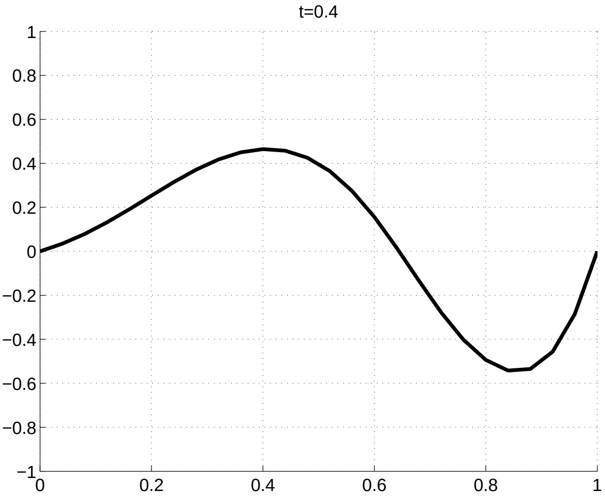
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



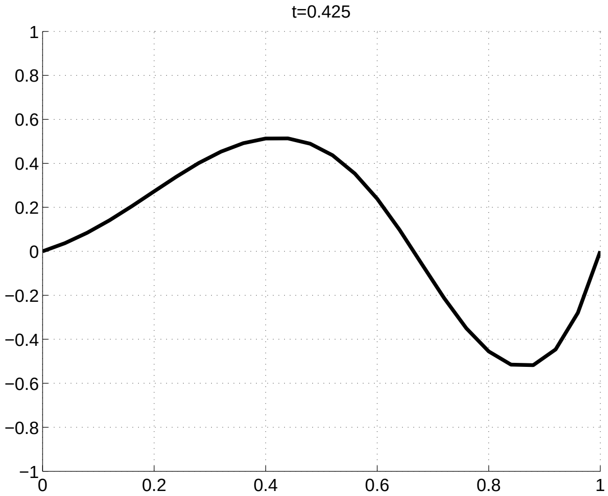
uncontrolled $(u \equiv 0)$

The uncontrolled PDE



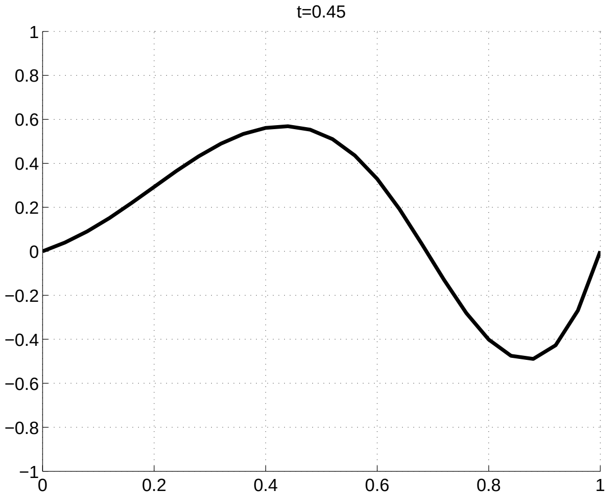
uncontrolled $(u \equiv 0)$

The uncontrolled PDE



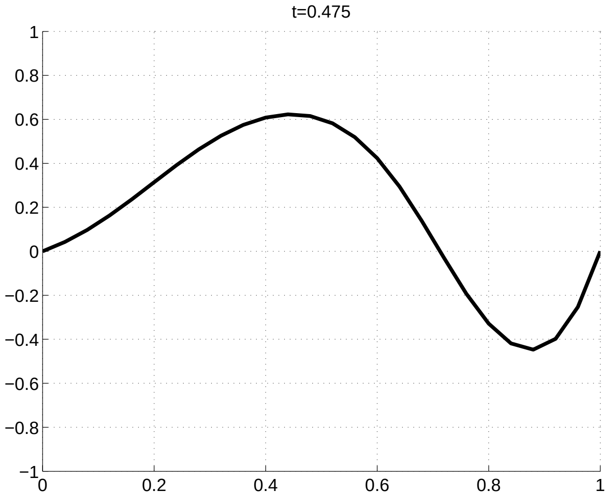
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



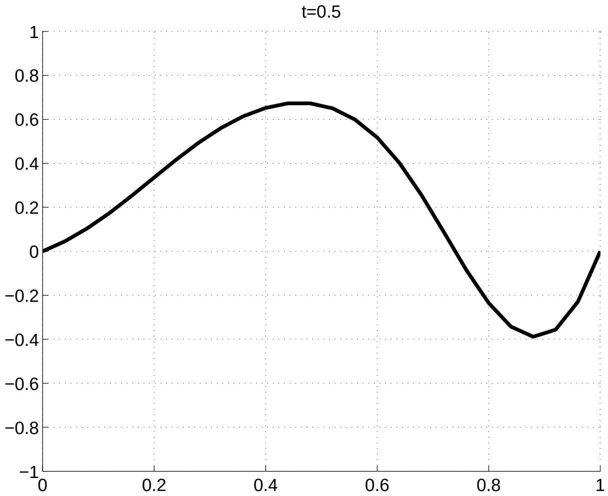
uncontrolled $(u \equiv 0)$

The uncontrolled PDE



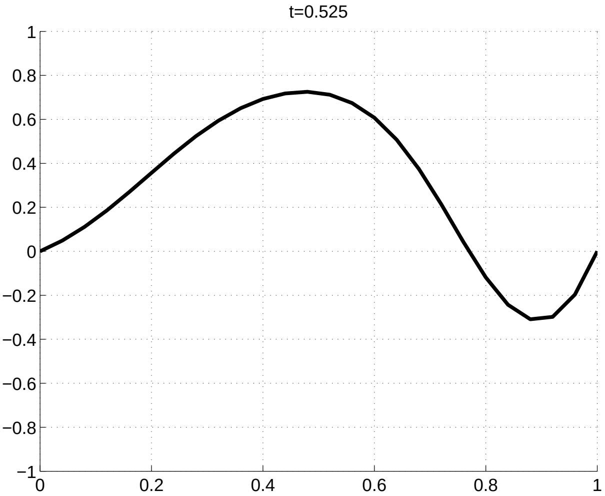
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



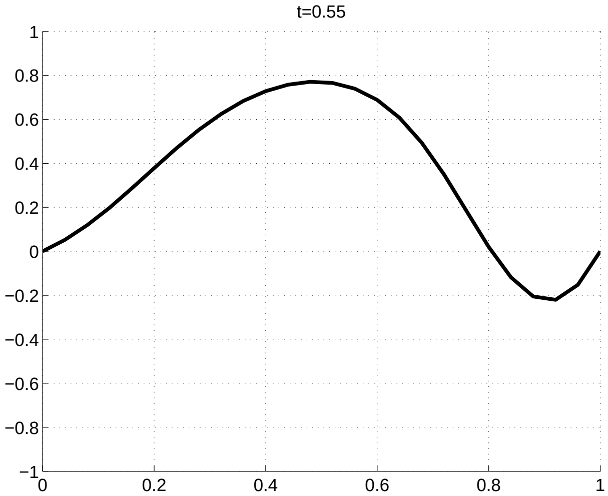
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



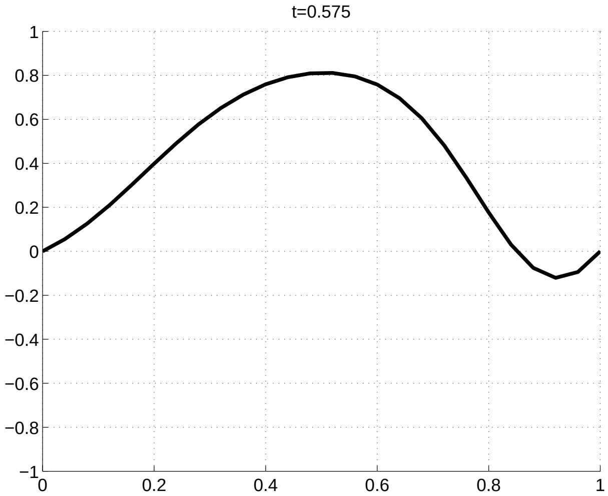
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



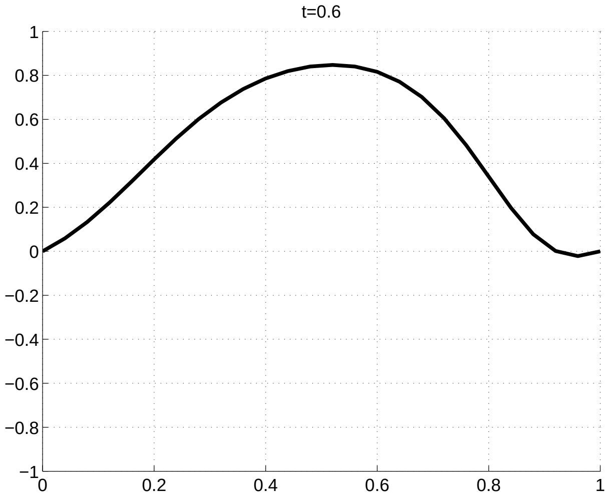
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



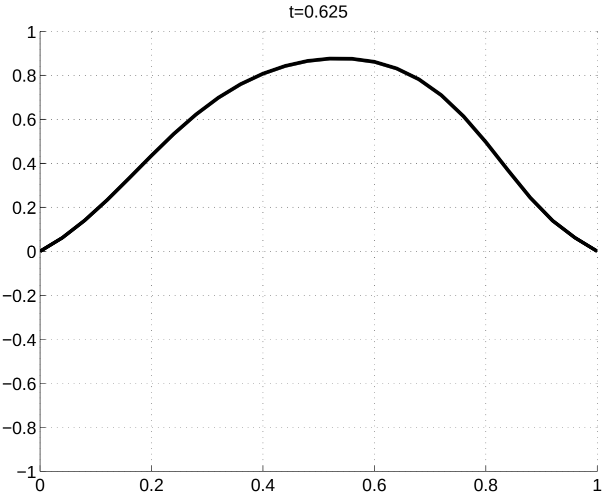
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



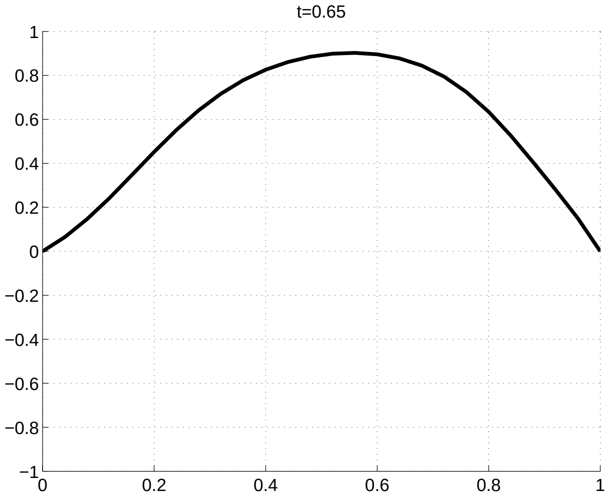
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



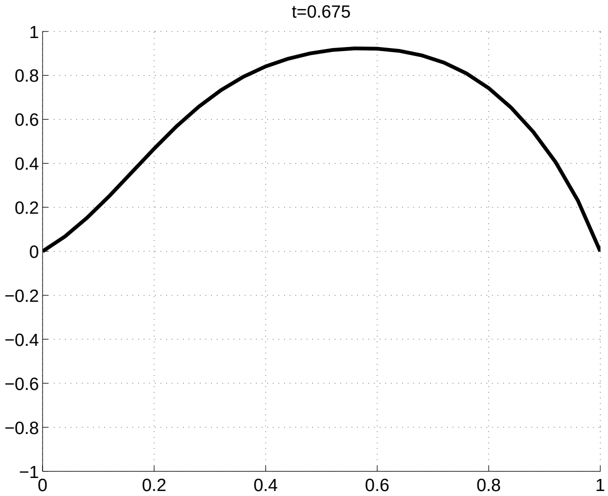
uncontrolled $(u \equiv 0)$

The uncontrolled PDE



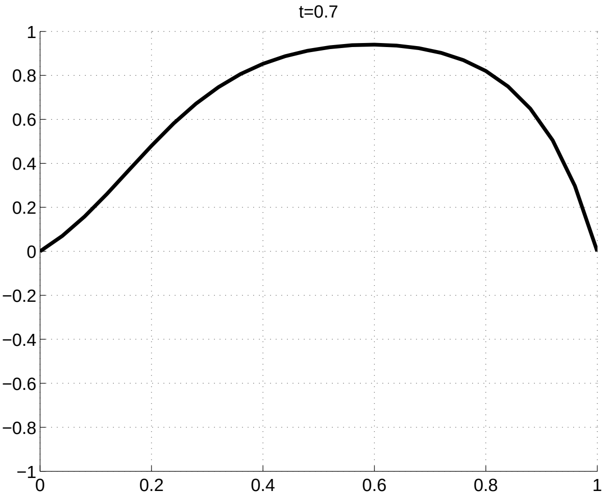
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



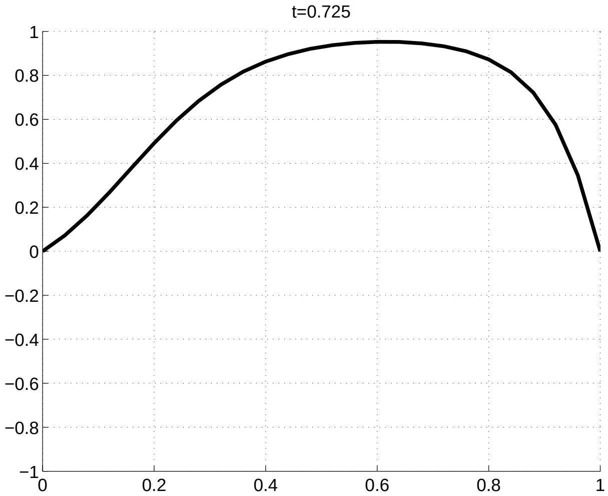
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



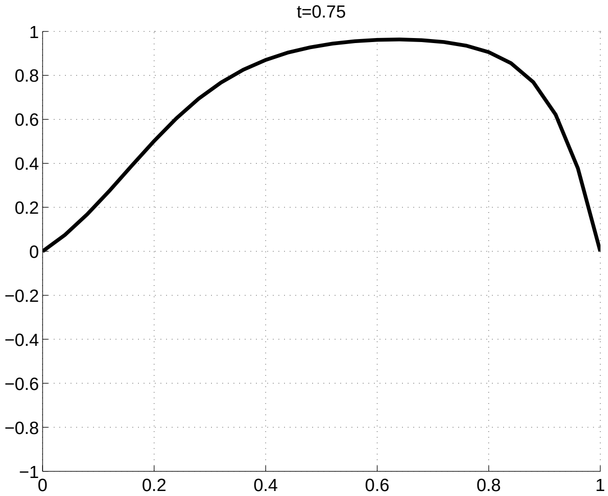
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



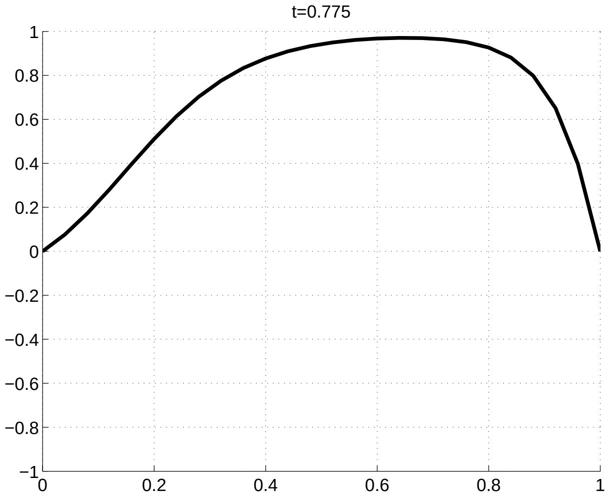
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



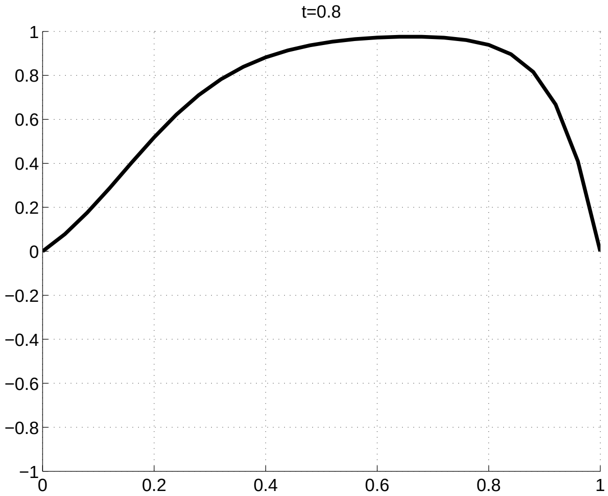
uncontrolled $(u \equiv 0)$

The uncontrolled PDE



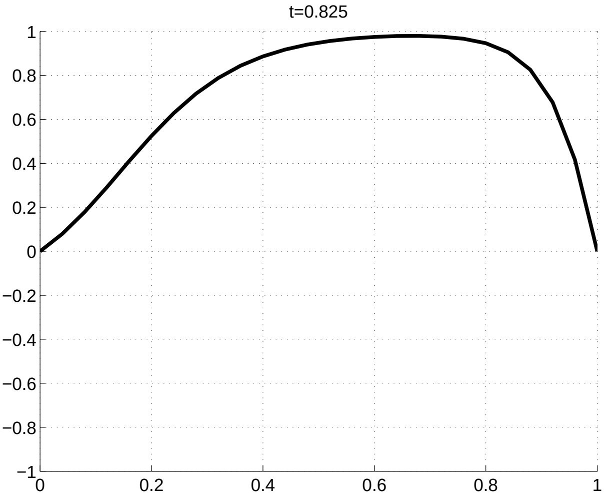
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



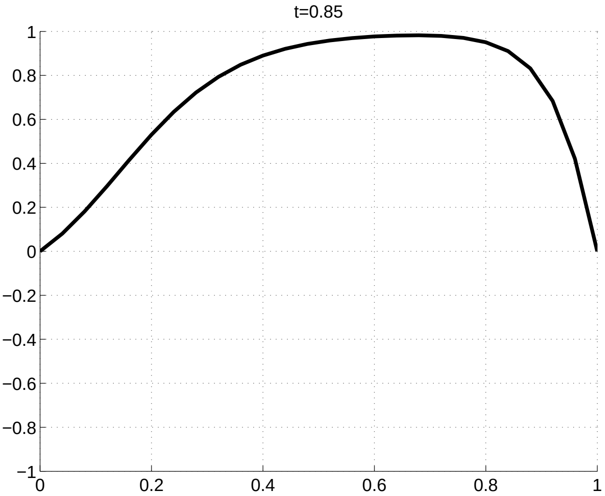
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



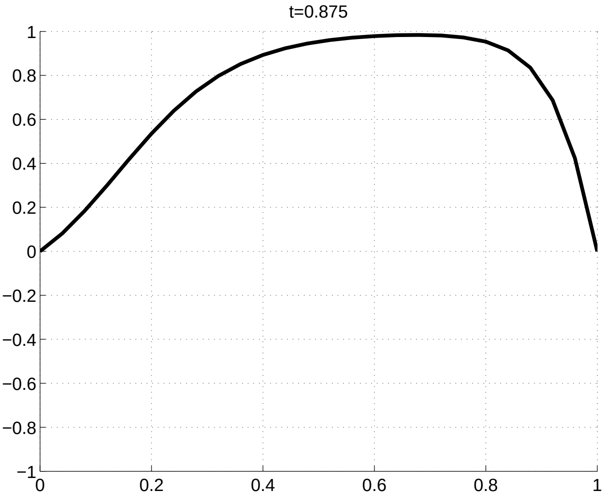
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



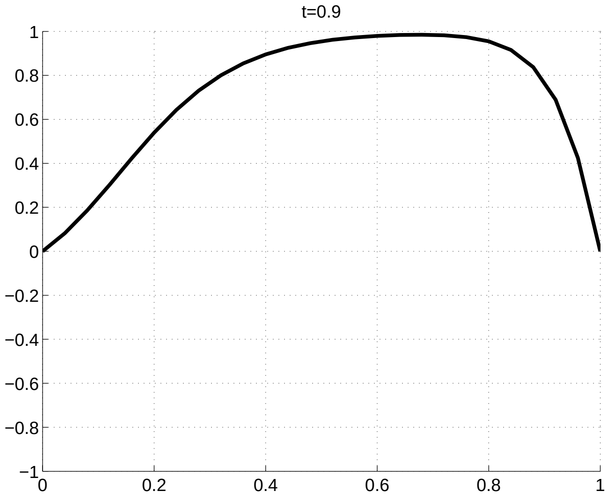
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



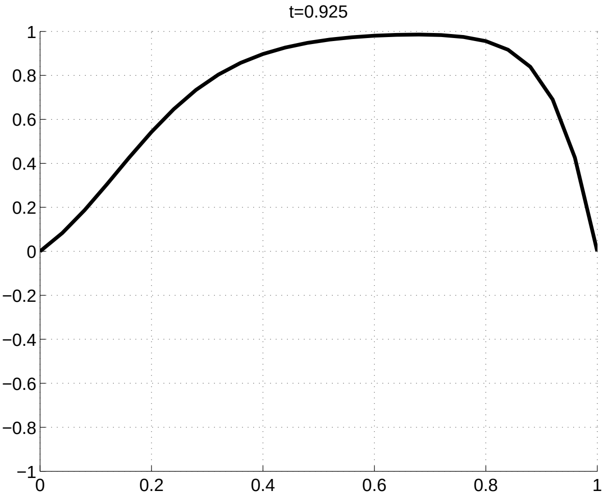
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



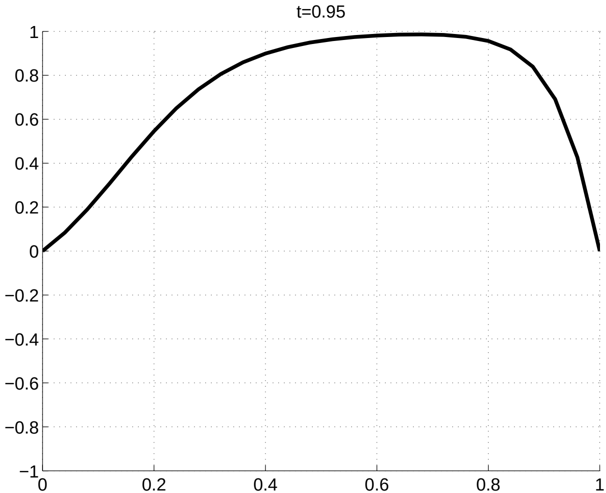
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



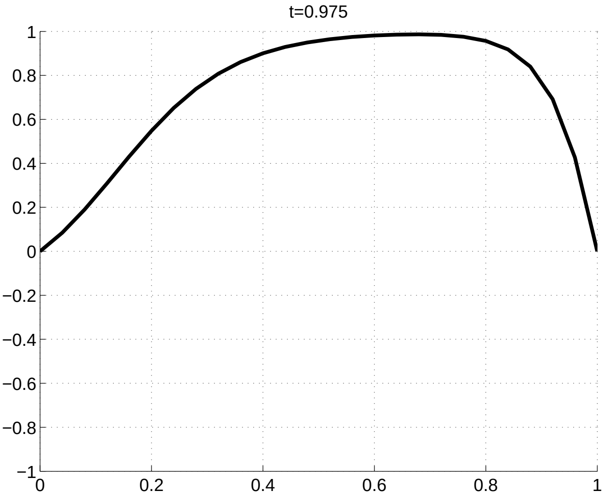
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



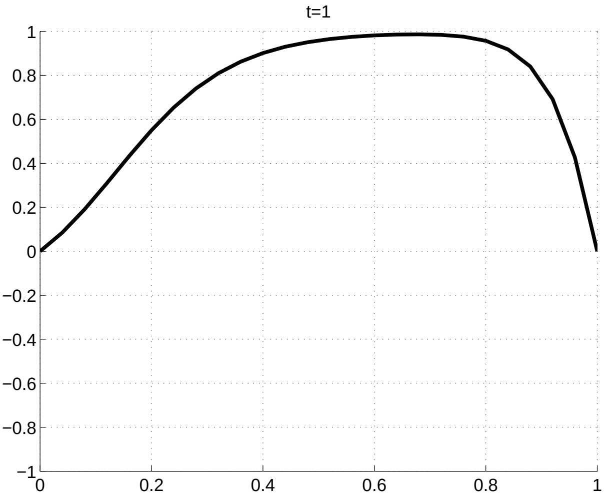
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



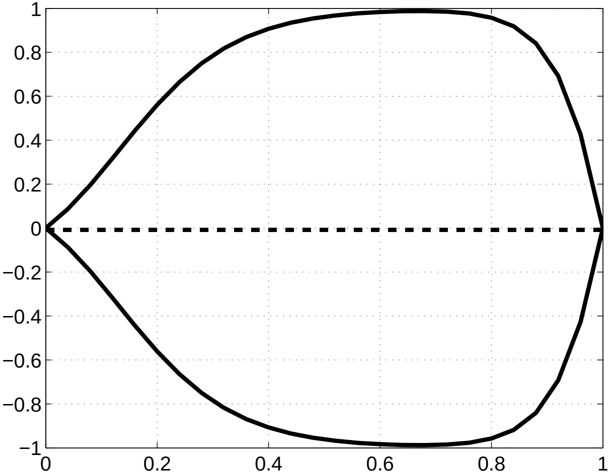
uncontrolled ($u \equiv 0$)

The uncontrolled PDE



uncontrolled ($u \equiv 0$)

The uncontrolled PDE



all equilibrium solutions

MPC for the PDE example

$$y_t = y_x + \nu y_{xx} + \mu y(y+1)(1-y) + u$$

MPC for the PDE example

$$y_t = y_x + \nu y_{xx} + \mu y(y+1)(1-y) + u$$

Goal: stabilize the sampled data system $y(n)$ at $y \equiv 0$

MPC for the PDE example

$$y_t = y_x + \nu y_{xx} + \mu y(y+1)(1-y) + u$$

Goal: stabilize the sampled data system $y(n)$ at $y \equiv 0$

For $y \approx 0$ the control u must compensate for $y_x \rightsquigarrow u \approx -y_x$

MPC for the PDE example

$$y_t = y_x + \nu y_{xx} + \mu y(y+1)(1-y) + u$$

Goal: stabilize the sampled data system $y(n)$ at $y \equiv 0$

For $y \approx 0$ the control u must compensate for $y_x \rightsquigarrow u \approx -y_x$

This observation and a little computation reveals:

For the (usual) quadratic L^2 cost

$$\ell(y(n), u(n)) = \|y(n)\|_{L^2}^2 + \lambda \|u(n)\|_{L^2}^2$$

the constant C is much larger than for the quadratic H^1 cost

$$\ell(y(n), u(n)) = \underbrace{\|y(n)\|_{L^2}^2 + \|y_x(n)\|_{L^2}^2}_{=\|y(n)\|_{H^1}^2} + \lambda \|u(n)\|_{L^2}^2.$$

MPC for the PDE example

$$y_t = y_x + \nu y_{xx} + \mu y(y+1)(1-y) + u$$

Goal: stabilize the sampled data system $y(n)$ at $y \equiv 0$

For $y \approx 0$ the control u must compensate for $y_x \rightsquigarrow u \approx -y_x$

This observation and a little computation reveals:

For the (usual) quadratic L^2 cost

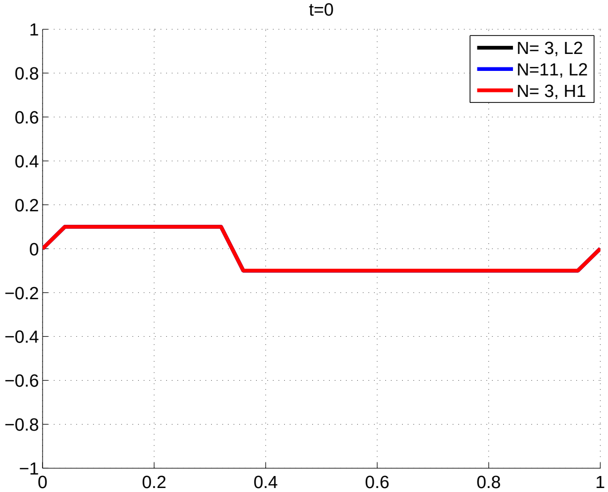
$$\ell(y(n), u(n)) = \|y(n)\|_{L^2}^2 + \lambda \|u(n)\|_{L^2}^2$$

the constant C is much larger than for the quadratic H^1 cost

$$\ell(y(n), u(n)) = \underbrace{\|y(n)\|_{L^2}^2 + \|y_x(n)\|_{L^2}^2}_{=\|y(n)\|_{H^1}^2} + \lambda \|u(n)\|_{L^2}^2.$$

$\rightsquigarrow$ H^1 should perform better that L^2

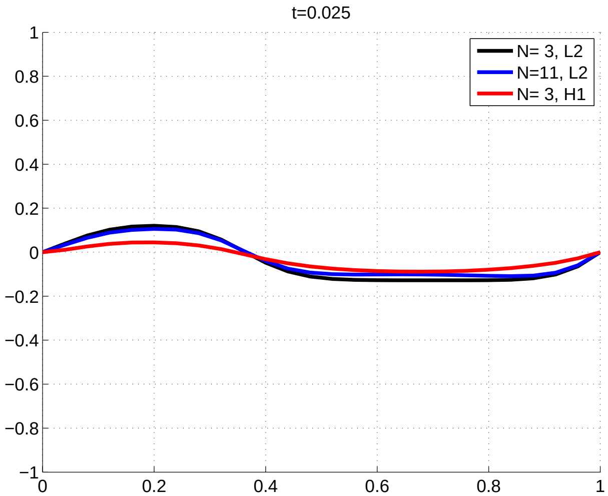
MPC with L_2 vs. H_1 cost



MPC with L_2 and H_1 cost, $\lambda = 0.1$, sampling time $T = 0.025$

UNIVERSITÄT BAYREUTH

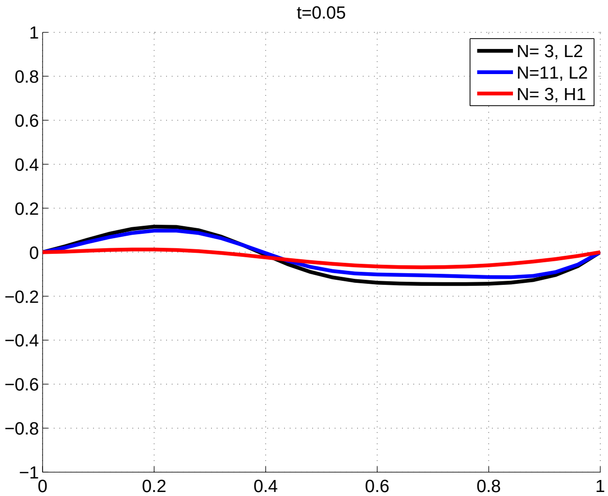
MPC with L_2 vs. H_1 cost



MPC with L_2 and H_1 cost, $\lambda = 0.1$, sampling time $T = 0.025$

UNIVERSITÄT BAYREUTH

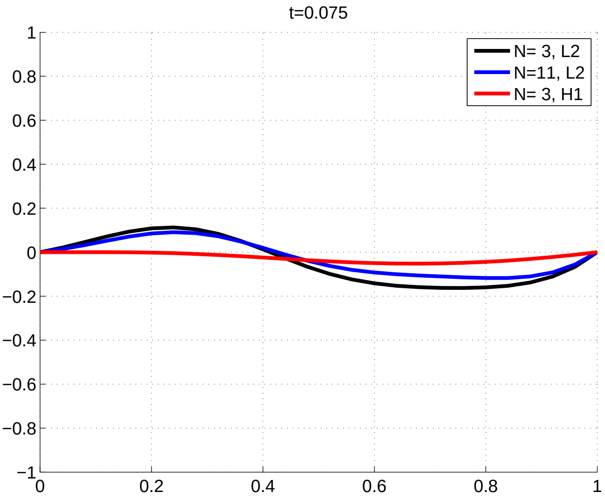
MPC with L_2 vs. H_1 cost



MPC with L_2 and H_1 cost, $\lambda = 0.1$, sampling time $T = 0.025$

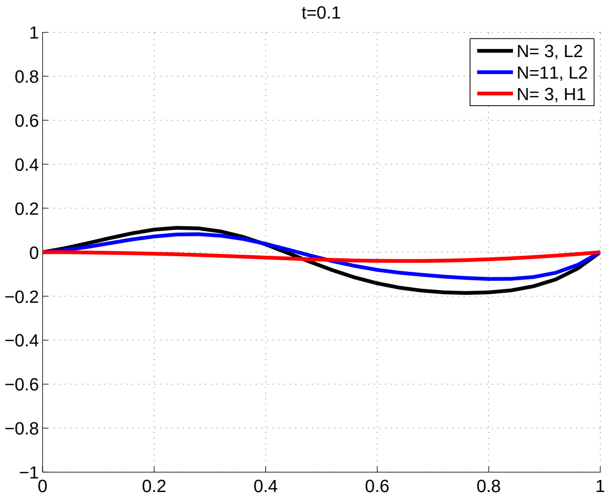
UNIVERSITÄT BAYREUTH

MPC with L_2 vs. H_1 cost



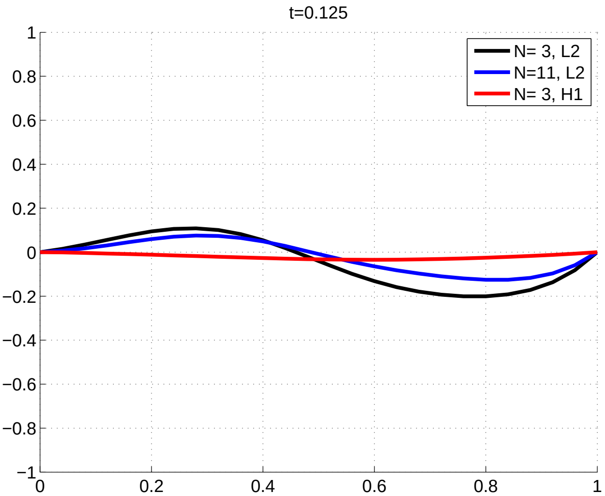
MPC with L_2 and H_1 cost, $\lambda = 0.1$, sampling time $T = 0.025$

MPC with L_2 vs. H_1 cost



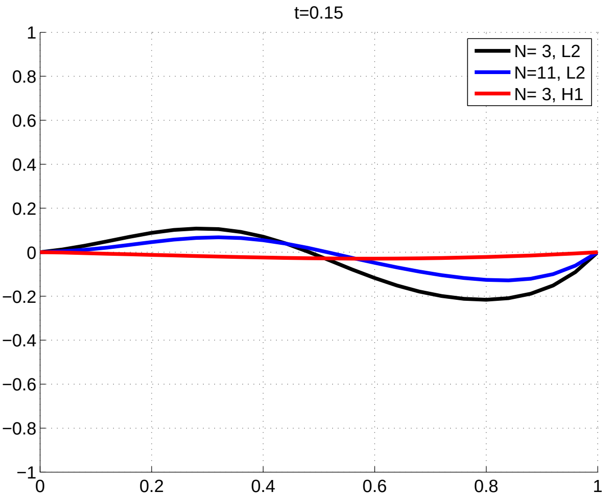
MPC with L_2 and H_1 cost, $\lambda = 0.1$, sampling time $T = 0.025$

MPC with L_2 vs. H_1 cost



MPC with L_2 and H_1 cost, $\lambda = 0.1$, sampling time $T = 0.025$

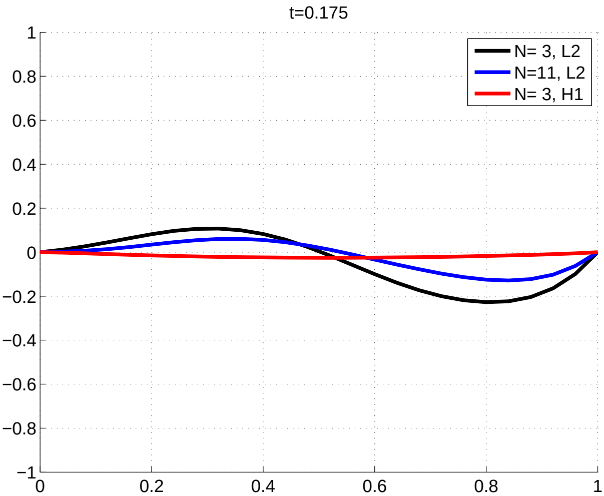
MPC with L_2 vs. H_1 cost



MPC with L_2 and H_1 cost, $\lambda = 0.1$, sampling time $T = 0.025$

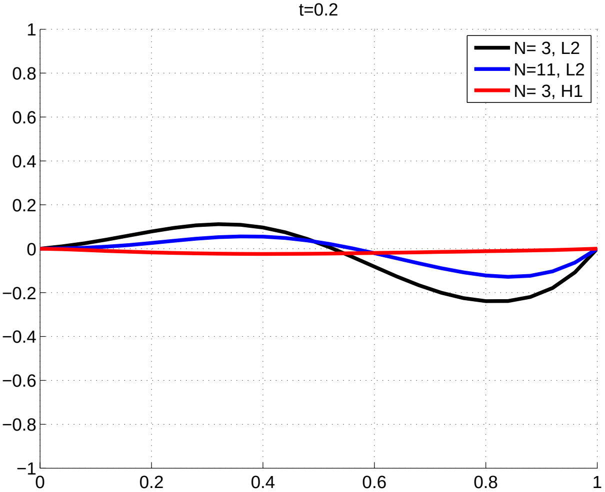
UNIVERSITÄT BAYREUTH

MPC with L_2 vs. H_1 cost



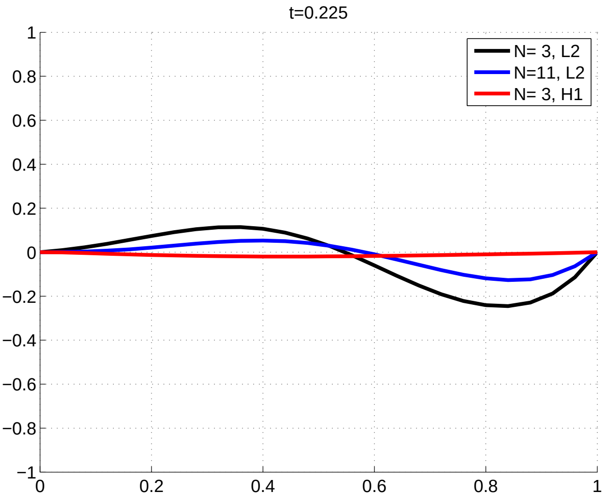
MPC with L_2 and H_1 cost, $\lambda = 0.1$, sampling time $T = 0.025$

MPC with L_2 vs. H_1 cost



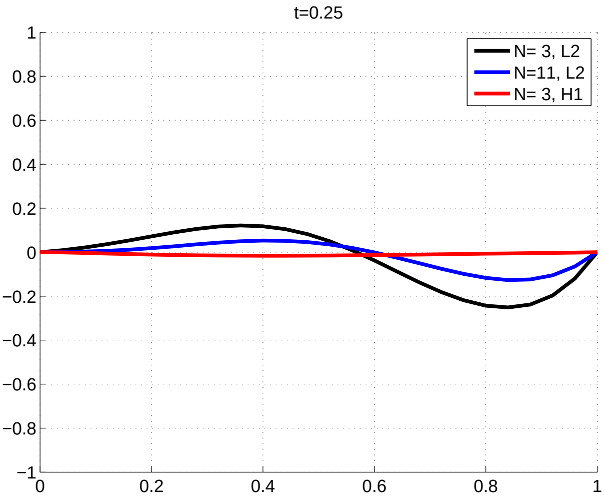
MPC with L_2 and H_1 cost, $\lambda = 0.1$, sampling time $T = 0.025$

MPC with L_2 vs. H_1 cost



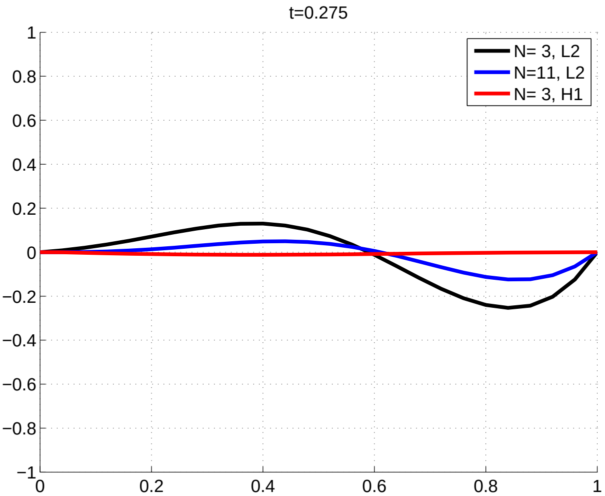
MPC with L_2 and H_1 cost, $\lambda = 0.1$, sampling time $T = 0.025$

MPC with L_2 vs. H_1 cost



MPC with L_2 and H_1 cost, $\lambda = 0.1$, sampling time $T = 0.025$

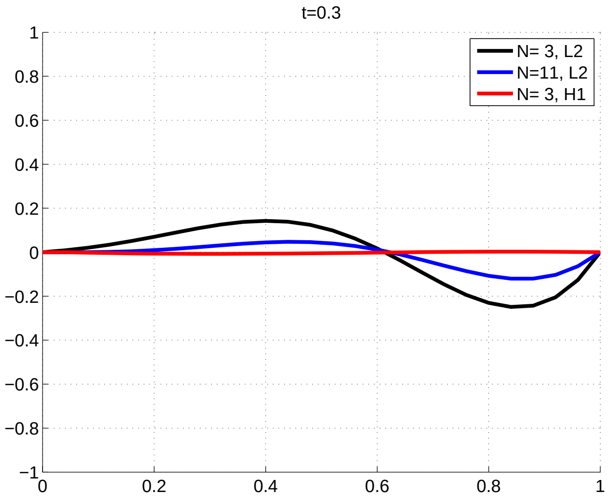
MPC with L_2 vs. H_1 cost



MPC with L_2 and H_1 cost, $\lambda = 0.1$, sampling time $T = 0.025$

UNIVERSITÄT BAYREUTH

MPC with L_2 vs. H_1 cost



MPC with L_2 and H_1 cost, $\lambda = 0.1$, sampling time $T = 0.025$

Boundary Control

Now we change our PDE from distributed to (Dirichlet-) boundary control, i.e.

$$y_t = y_x + \nu y_{xx} + \mu y(y+1)(1-y)$$

with

domain $\Omega = [0,1]$

solution $y = y(t,x)$

boundary conditions $y(t,0) = u_0(t)$, $y(t,1) = u_1(t)$

parameters $\nu = 0.1$ and $\mu = 10$

Boundary Control

Now we change our PDE from distributed to (Dirichlet-) boundary control, i.e.

$$y_t = y_x + \nu y_{xx} + \mu y(y+1)(1-y)$$

with

domain $\Omega = [0,1]$

solution $y = y(t,x)$

boundary conditions $y(t,0) = u_0(t)$, $y(t,1) = u_1(t)$

parameters $\nu = 0.1$ and $\mu = 10$

with boundary control, stability can only be achieved via large gradients in the transient phase

Boundary Control

Now we change our PDE from distributed to (Dirichlet-) boundary control, i.e.

$$y_t = y_x + \nu y_{xx} + \mu y(y+1)(1-y)$$

with

domain $\Omega = [0, 1]$

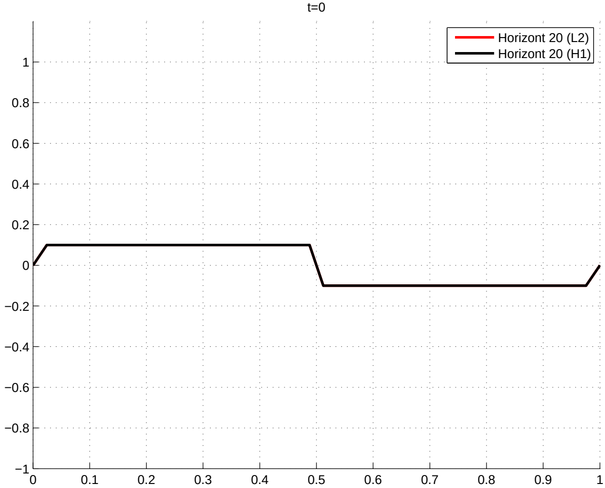
solution $y = y(t, x)$

boundary conditions $y(t, 0) = u_0(t)$, $y(t, 1) = u_1(t)$

parameters $\nu = 0.1$ and $\mu = 10$

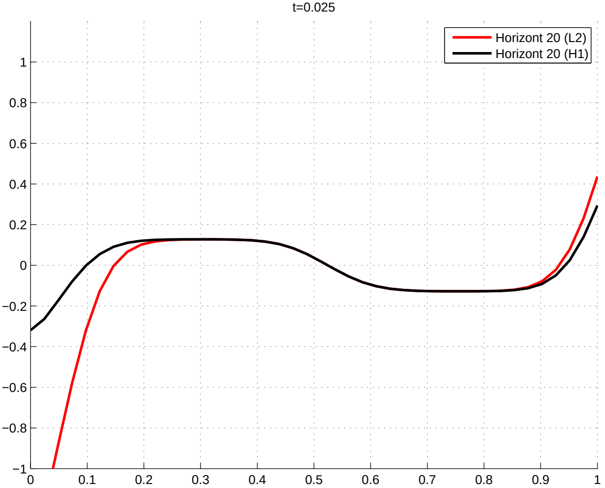
with boundary control, stability can only be achieved via large gradients in the transient phase
$\rightsquigarrow$ L^2 should perform better that H^1

Boundary control, L_2 vs. H_1, $N = 20$



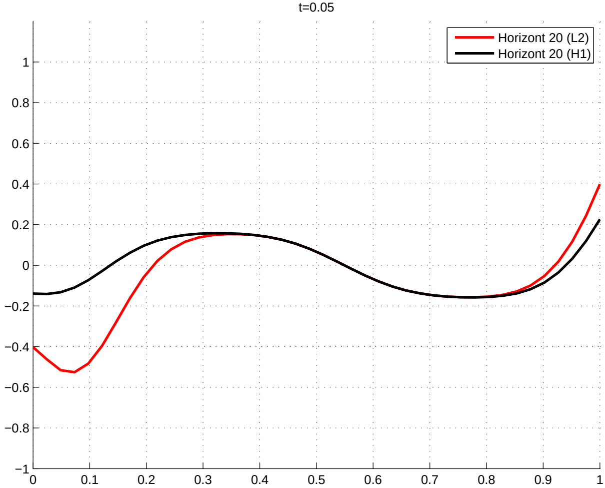
Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

Boundary control, L_2 vs. H_1, $N = 20$



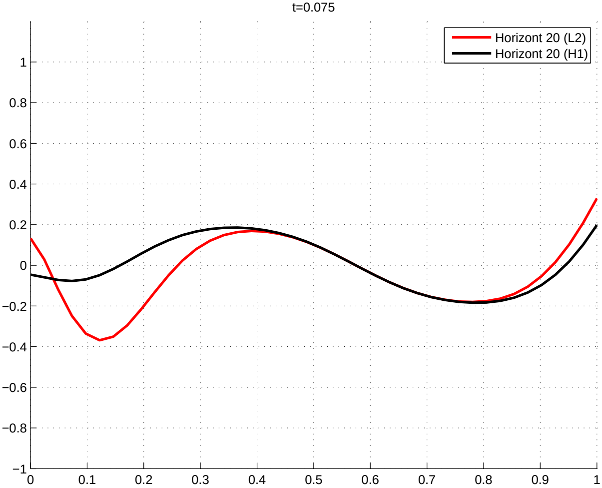
Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

Boundary control, L_2 vs. H_1, $N = 20$



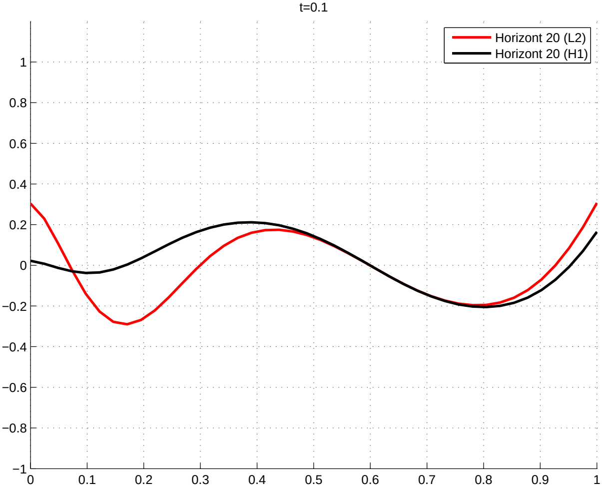
Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

Boundary control, L_2 vs. H_1, $N = 20$



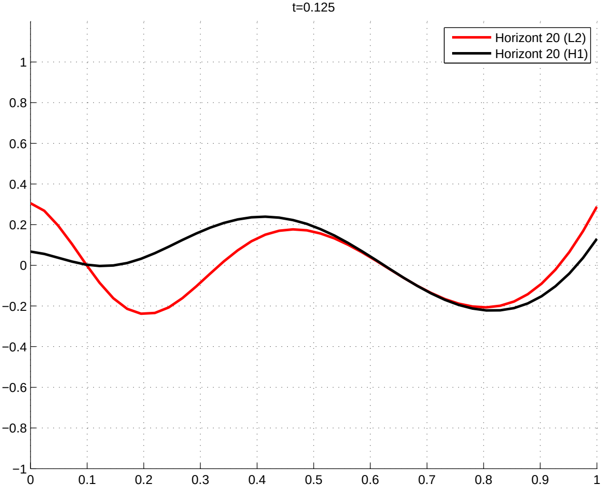
Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

Boundary control, L_2 vs. H_1, $N = 20$



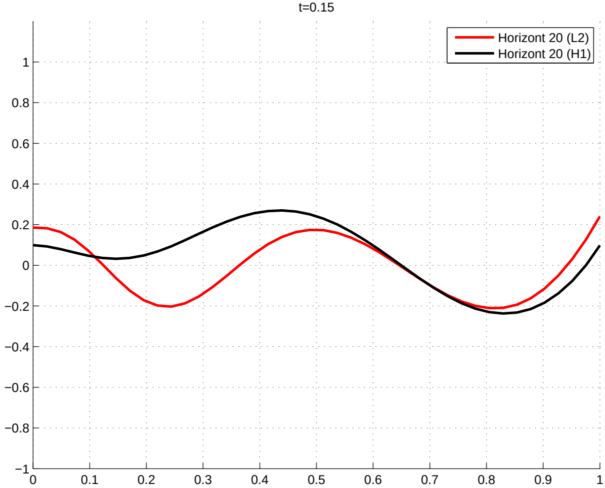
Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

Boundary control, L_2 vs. H_1, $N = 20$



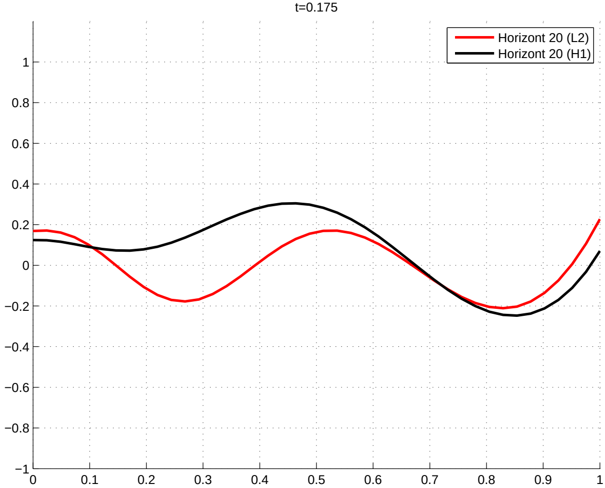
Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

Boundary control, L_2 vs. H_1, $N = 20$



Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

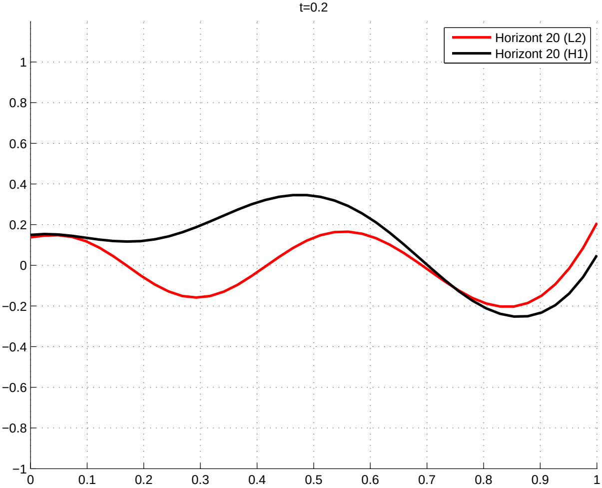
Boundary control, L_2 vs. H_1, $N = 20$



Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

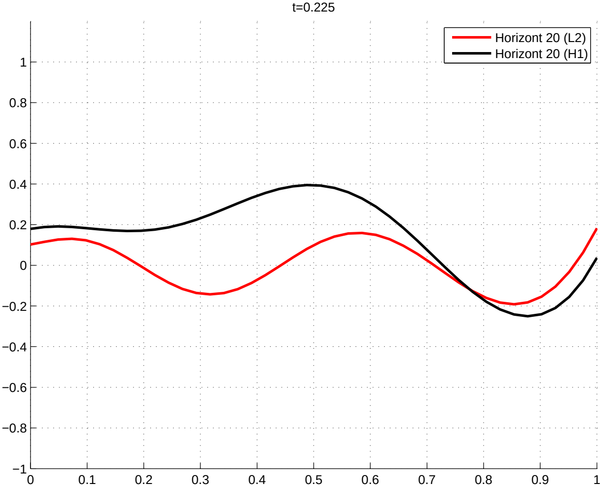
UNIVERSITÄT BAYREUTH

Boundary control, L_2 vs. H_1, $N = 20$



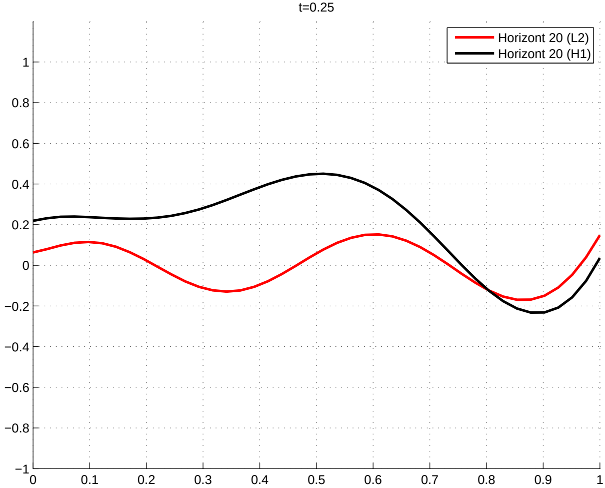
Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

Boundary control, L_2 vs. H_1, $N = 20$



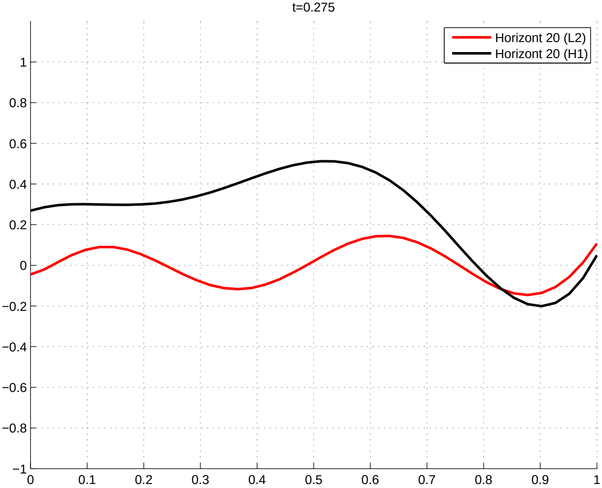
Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

Boundary control, L_2 vs. H_1, $N = 20$



Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

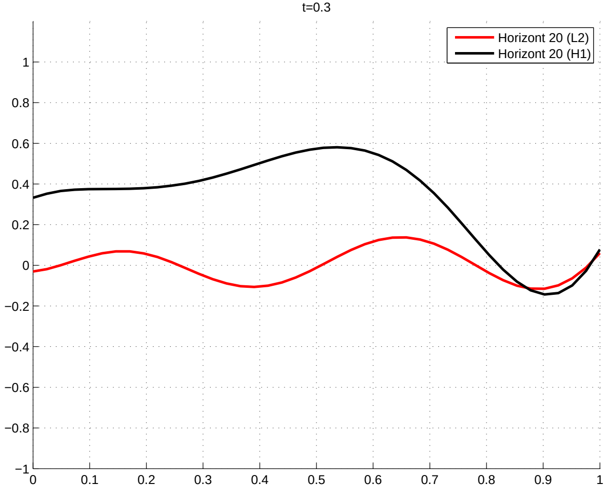
Boundary control, L_2 vs. H_1, $N = 20$



Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

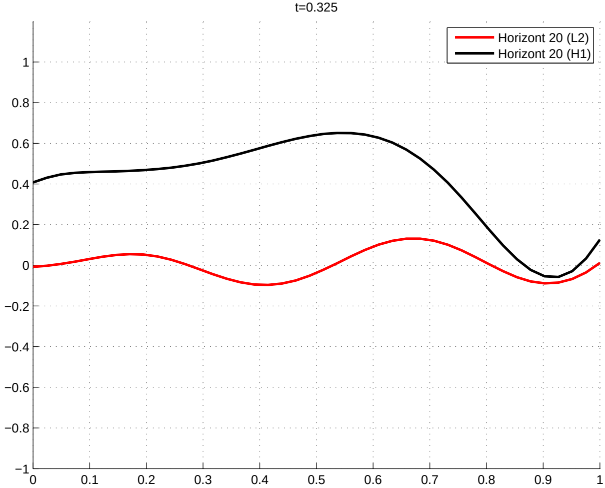
UNIVERSITÄT BAYREUTH

Boundary control, L_2 vs. H_1, $N = 20$



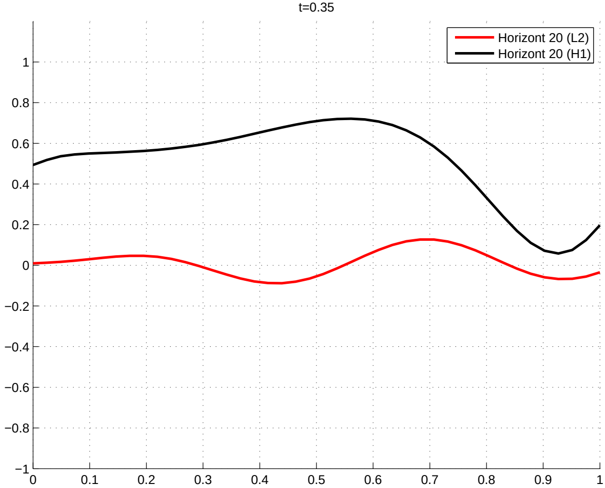
Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

Boundary control, L_2 vs. H_1, $N = 20$



Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

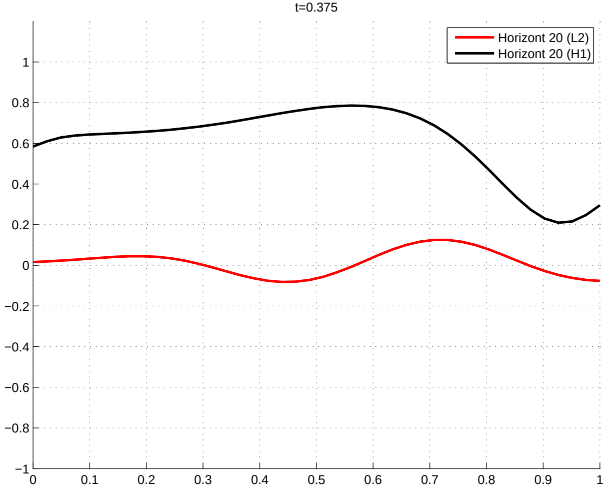
Boundary control, L_2 vs. H_1, $N = 20$



Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

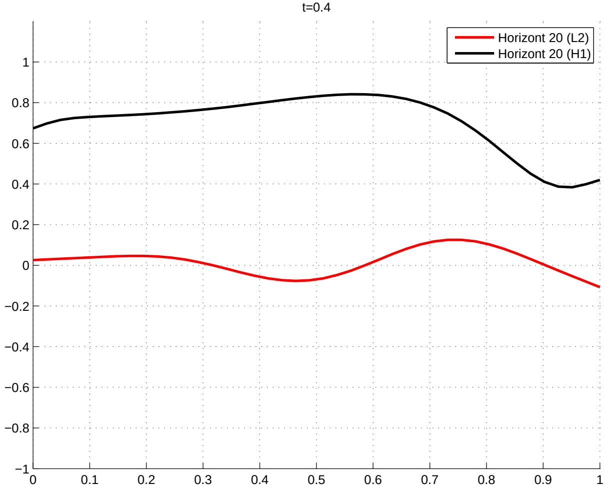
UNIVERSITÄT BAYREUTH

Boundary control, L_2 vs. H_1, $N = 20$



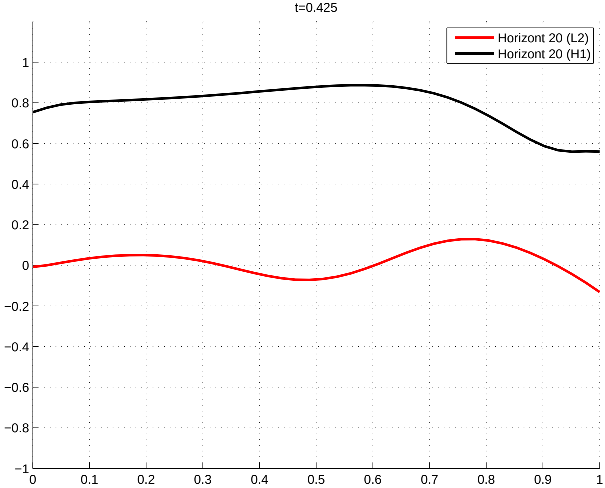
Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

Boundary control, L_2 vs. H_1, $N = 20$



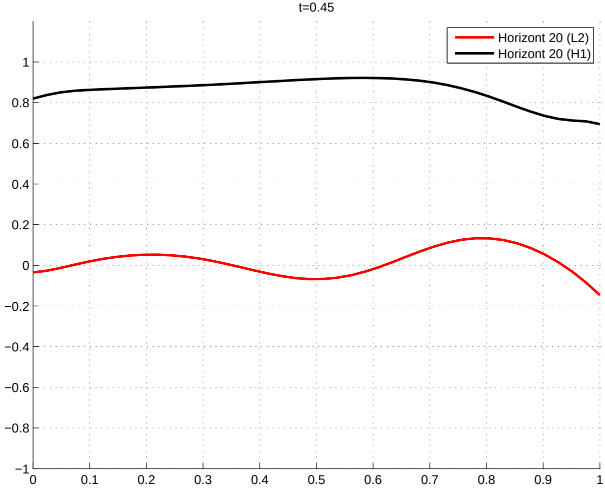
Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

Boundary control, L_2 vs. H_1, $N = 20$



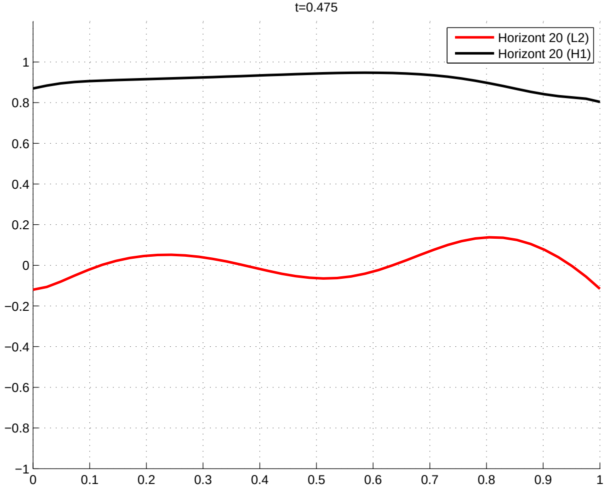
Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

Boundary control, L_2 vs. H_1, $N = 20$



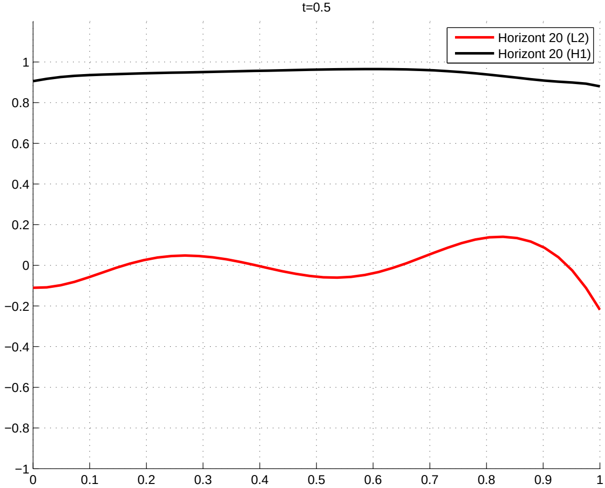
Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

Boundary control, L_2 vs. H_1, $N = 20$



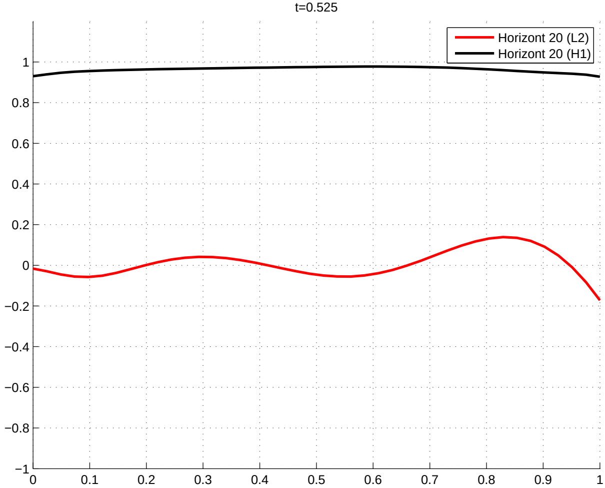
Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

Boundary control, L_2 vs. H_1, $N = 20$



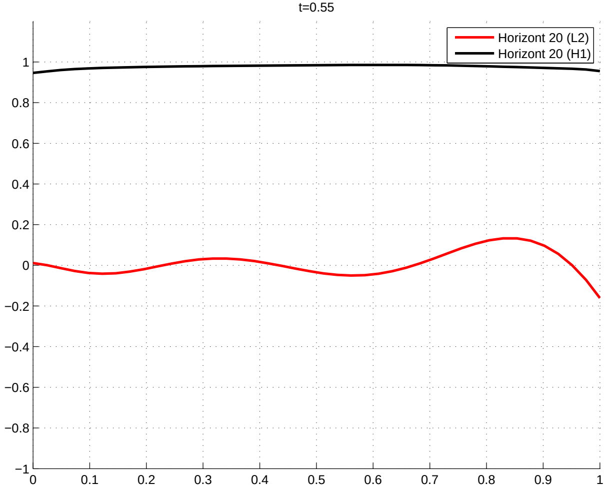
Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

Boundary control, L_2 vs. H_1, $N = 20$



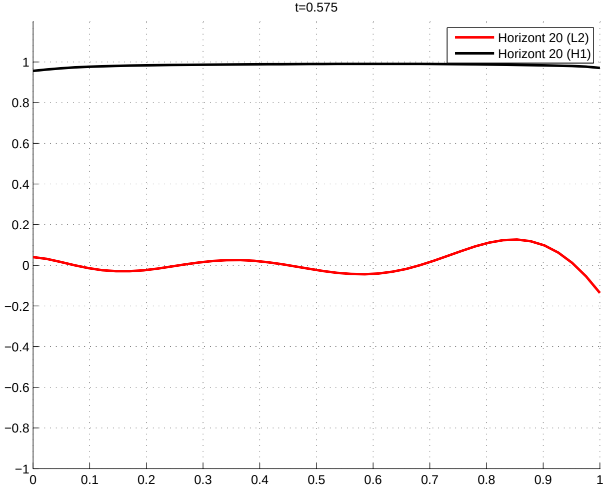
Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

Boundary control, L_2 vs. H_1, $N = 20$



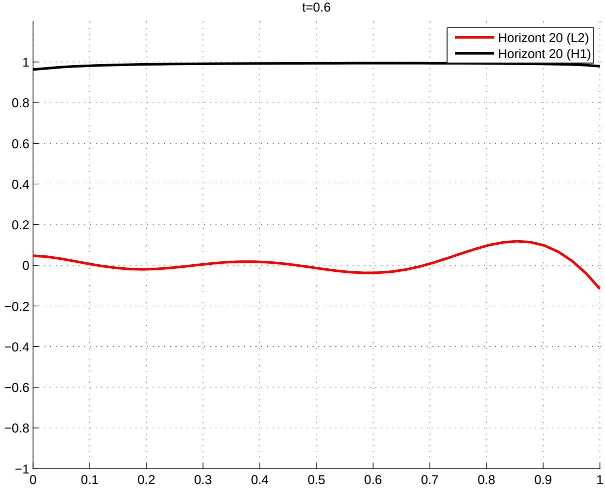
Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

Boundary control, L_2 vs. H_1, $N = 20$



Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

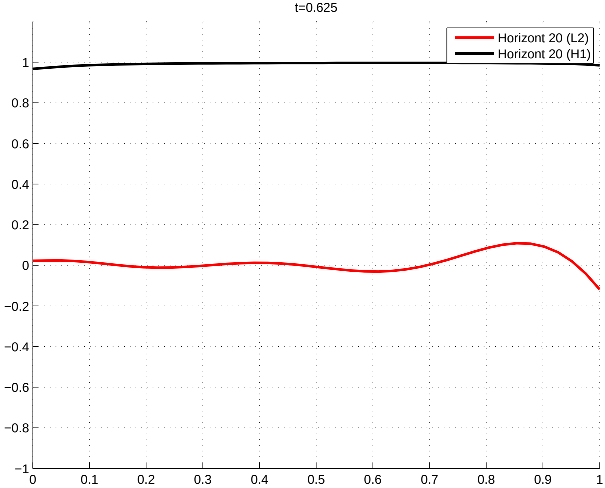
Boundary control, L_2 vs. H_1, $N = 20$



Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

UNIVERSITÄT BAYREUTH

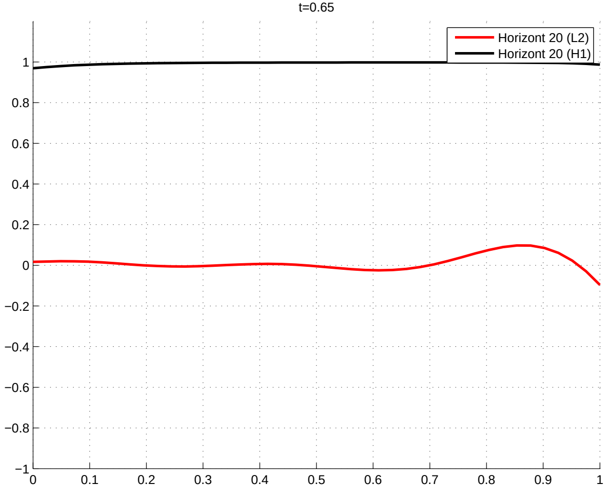
Boundary control, L_2 vs. H_1, $N = 20$



Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

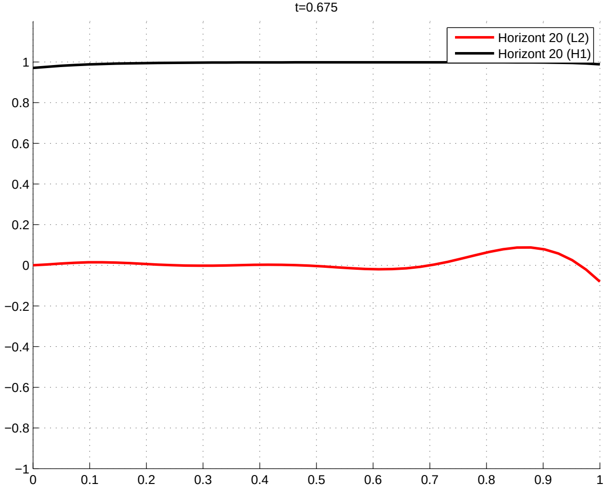
UNIVERSITÄT BAYREUTH

Boundary control, L_2 vs. H_1, $N = 20$



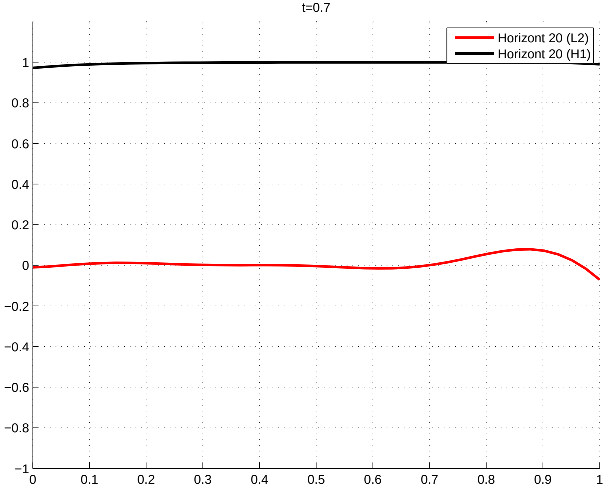
Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

Boundary control, L_2 vs. H_1, $N = 20$



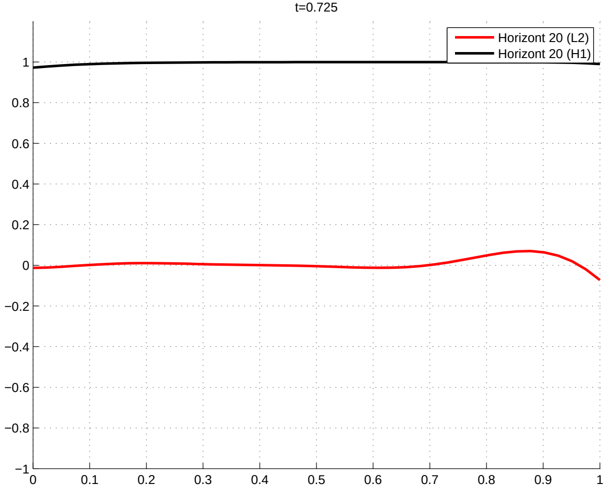
Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

Boundary control, L_2 vs. H_1, $N = 20$



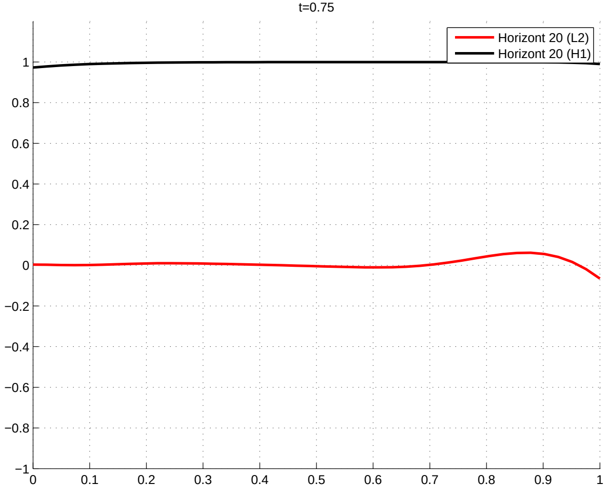
Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

Boundary control, L_2 vs. H_1, $N = 20$



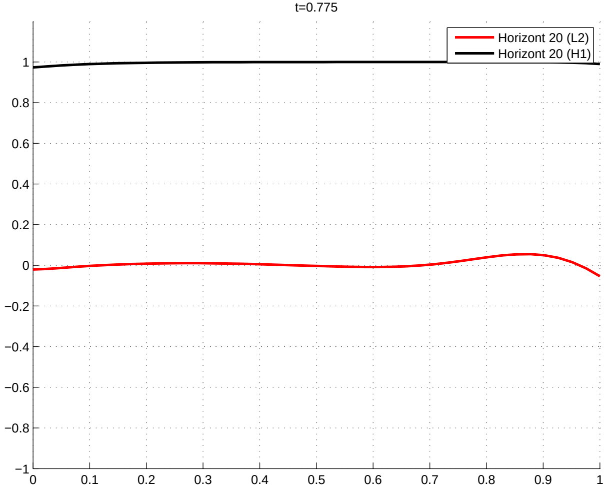
Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

Boundary control, L_2 vs. H_1, $N = 20$



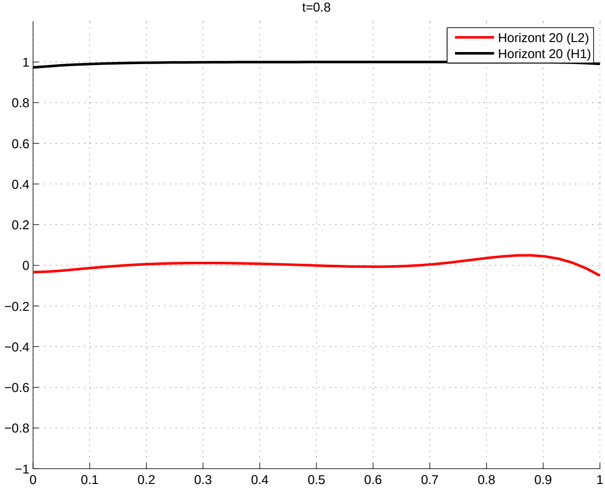
Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

Boundary control, L_2 vs. H_1, $N = 20$



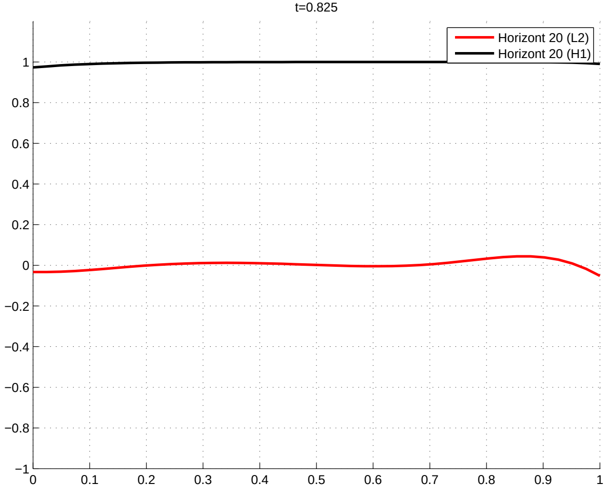
Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

Boundary control, L_2 vs. H_1, $N = 20$



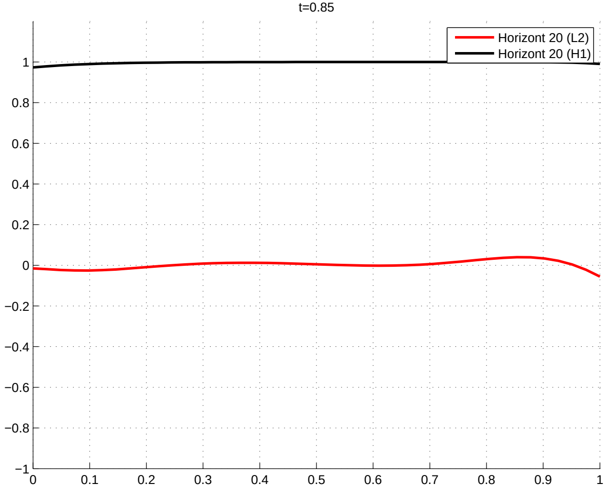
Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

Boundary control, L_2 vs. H_1, $N = 20$



Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

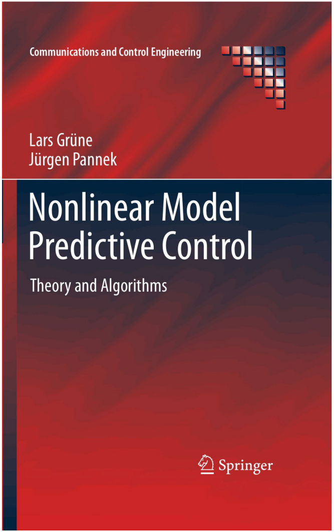
Boundary control, L_2 vs. H_1, $N = 20$



Boundary control, $\lambda = 0.001$, sampling time $T = 0.025$

Proofs, references etc.

For proofs, references, historical notes etc. please see:



www.nmpc-book.com

Economic MPC

In principle, the receding horizon MPC paradigm can also be applied for stage cost ℓ not related to any stabilization problem

Economic MPC

In principle, the receding horizon MPC paradigm can also be applied for stage cost ℓ not related to any stabilization problem

[Angeli/Rawlings '09, Angeli/Amrit/Rawlings '10, Diehl/Amrit/Rawlings '11] consider MPC for the infinite horizon averaged performance criterion

$$\overline{J}_\infty(x,u) = \limsup_{K\to\infty} \frac{1}{K} \sum_{k=0}^{K-1} \ell(x_u(k,x), u(k))$$

Here ℓ reflects an "economic" cost (like, e.g., energy consumption) rather than penalizing the distance to some desired equilibrium

Economic MPC with terminal constraints

Typical result: Let $x^* \in \mathbb{X}$ be an equilibrium for some $u^* \in \mathbb{U}$, i.e., $f(x^*, u^*) = x^*$. Consider an MPC scheme where in each step we minimize

$$\overline{J}_N(x, u) = \frac{1}{N} \sum_{k=0}^{N-1} \ell(x_u(k), u(k))$$

subject to the terminal constraint $x_u(N) = x^*$.

Economic MPC with terminal constraints

Typical result: Let $x^* \in \mathbb{X}$ be an equilibrium for some $u^* \in \mathbb{U}$, i.e., $f(x^*, u^*) = x^*$. Consider an MPC scheme where in each step we minimize

$$\overline{J}_N(x, u) = \frac{1}{N} \sum_{k=0}^{N-1} \ell(x_u(k), u(k))$$

subject to the terminal constraint $x_u(N) = x^*$. Then for any feasible initial condition $x \in \mathbb{X}$ we get the inequality

$$\overline{J}_\infty(x, F_N) \leq \ell(x^*, u^*)$$

Economic MPC with terminal constraints

Typical result: Let $x^* \in \mathbb{X}$ be an equilibrium for some $u^* \in \mathbb{U}$, i.e., $f(x^*, u^*) = x^*$. Consider an MPC scheme where in each step we minimize

$$\overline{J}_N(x,u) = \frac{1}{N}\sum_{k=0}^{N-1} \ell(x_u(k), u(k))$$

subject to the terminal constraint $x_u(N) = x^*$. Then for any feasible initial condition $x \in \mathbb{X}$ we get the inequality

$$\overline{J}_\infty(x, F_N) \leq \ell(x^*, u^*)$$

Question: Does this also work without the terminal constraint $x_u(N) = x^*$, i.e., is MPC able to find a good equilibrium x^* "automatically"?

Economic MPC without terminal constraints

We investigate this question for the following optimal invariance problem:

Keep the state of the system inside an admissible set $\mathbb{X}$ with minimal infinite horizon averaged cost

$$\overline{J}_\infty(x,u) = \limsup_{K\to\infty} \frac{1}{K} \sum_{k=0}^{K-1} \ell(x_u(k,x),u(k))$$

Economic MPC without terminal constraints

We investigate this question for the following optimal invariance problem:

Keep the state of the system inside an admissible set $\mathbb{X}$ with minimal infinite horizon averaged cost

$$\overline{J}_\infty(x,u) = \limsup_{K\to\infty} \frac{1}{K}\sum_{k=0}^{K-1} \ell(x_u(k,x),u(k))$$

Example: $$x(k+1) = 2x(k) + u(k)$$

with $\mathbb{X} = [-2,2]$, $\mathbb{U} = [-2,2]$ and $\ell(x,u) = u^2$

Economic MPC without terminal constraints

We investigate this question for the following optimal invariance problem:

Keep the state of the system inside an admissible set $\mathbb{X}$ with minimal infinite horizon averaged cost

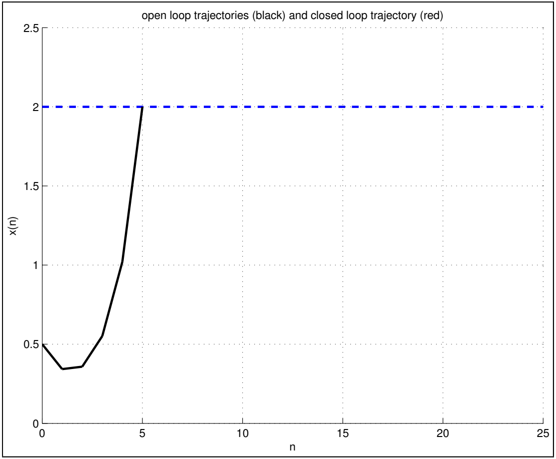
$$\overline{J}_\infty(x,u) = \limsup_{K\to\infty} \frac{1}{K} \sum_{k=0}^{K-1} \ell(x_u(k,x), u(k))$$

Example: $$x(k+1) = 2x(k) + u(k)$$

with $\mathbb{X} = [-2,2]$, $\mathbb{U} = [-2,2]$ and $\ell(x,u) = u^2$

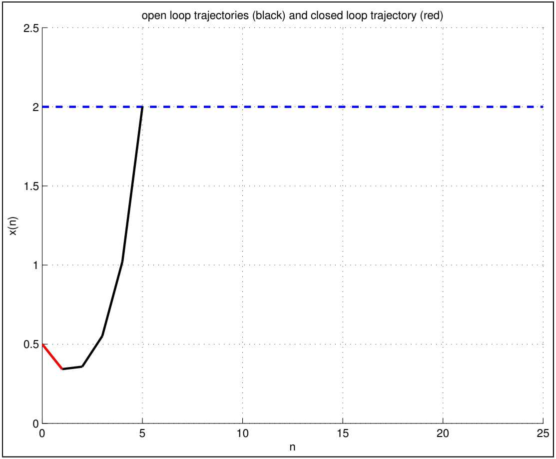
For this example, it is optimal to control the system to $x^* = 0$ and keep it there with $u^* = 0$ $\rightsquigarrow$ $\inf_{u\in\mathbb{U}^\infty} \overline{J}_\infty(x,u) = 0$

Optimal invariance example



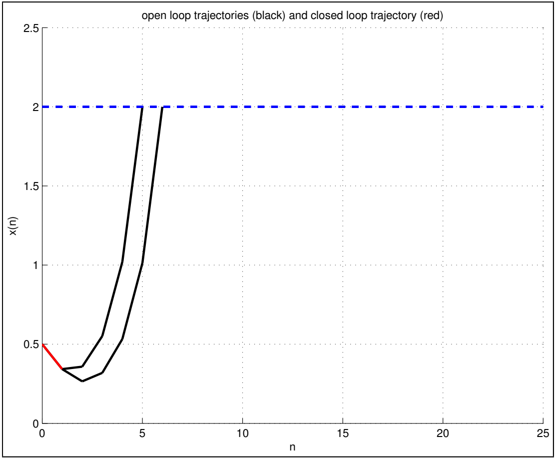
$N = 5$

Optimal invariance example



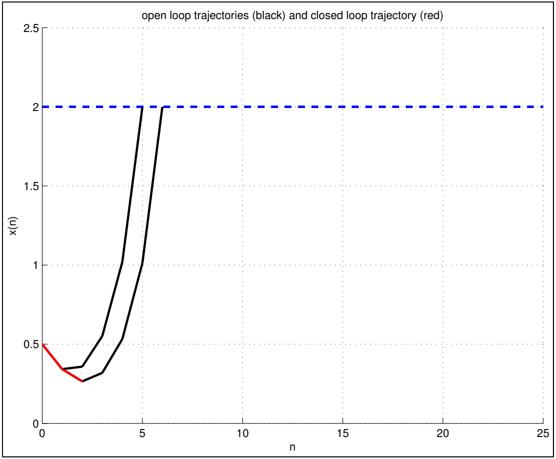
$N = 5$

Optimal invariance example



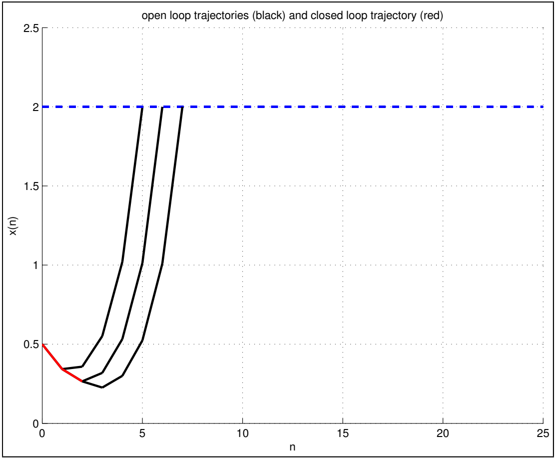
$N = 5$

Optimal invariance example



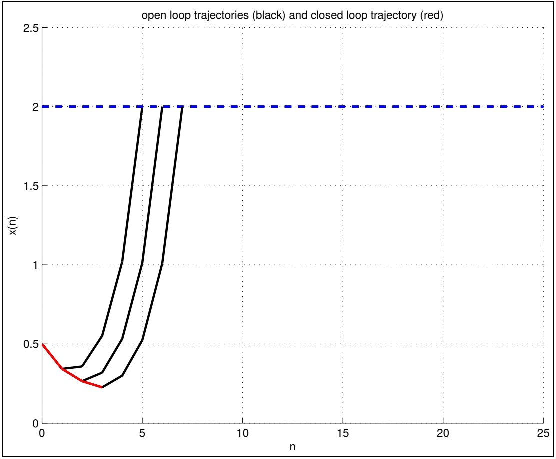
$N = 5$

Optimal invariance example



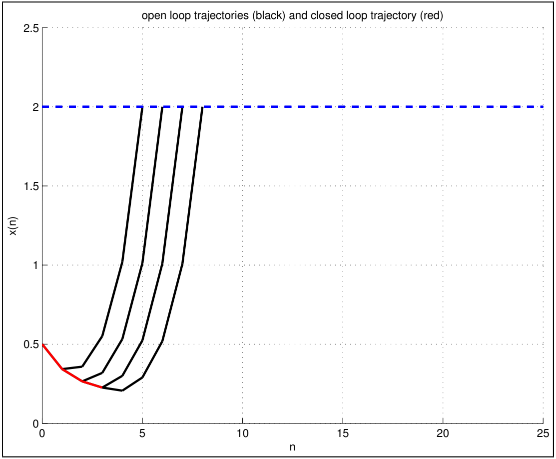
$N = 5$

Optimal invariance example



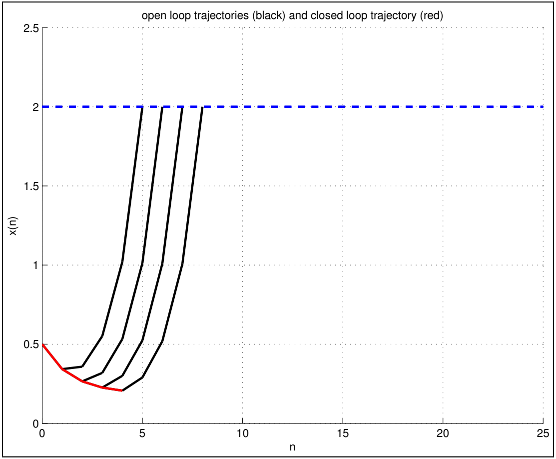
$N = 5$

Optimal invariance example



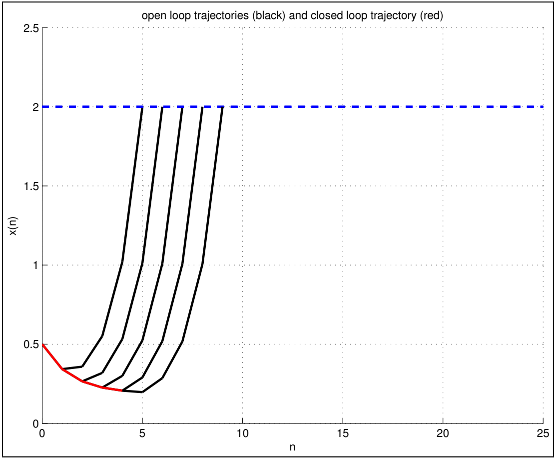
$N = 5$

Optimal invariance example



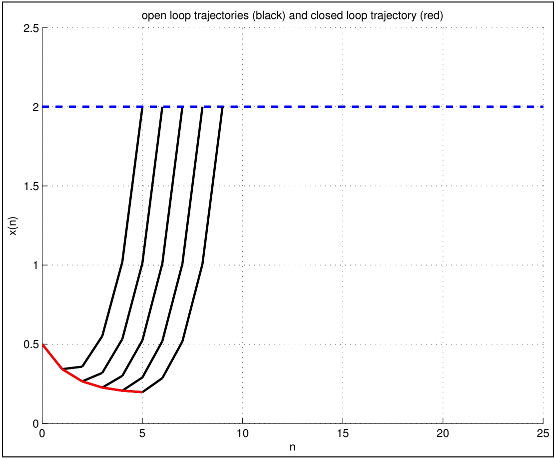
$N = 5$

Optimal invariance example



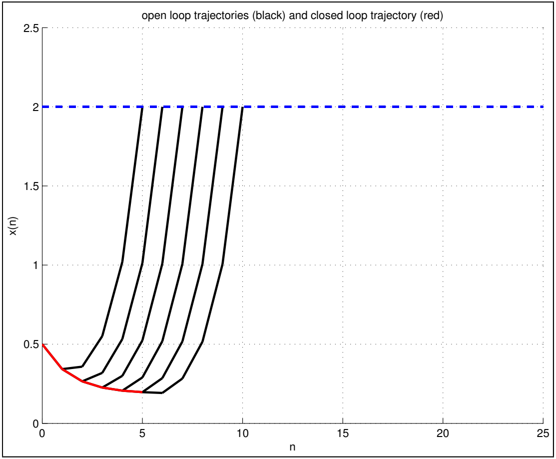
$N = 5$

Optimal invariance example



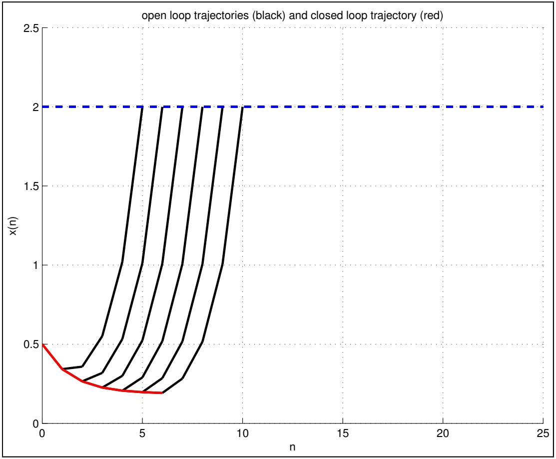
$N = 5$

Optimal invariance example



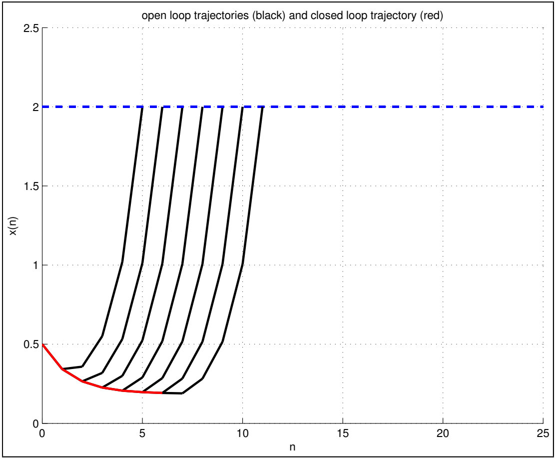
$N = 5$

Optimal invariance example



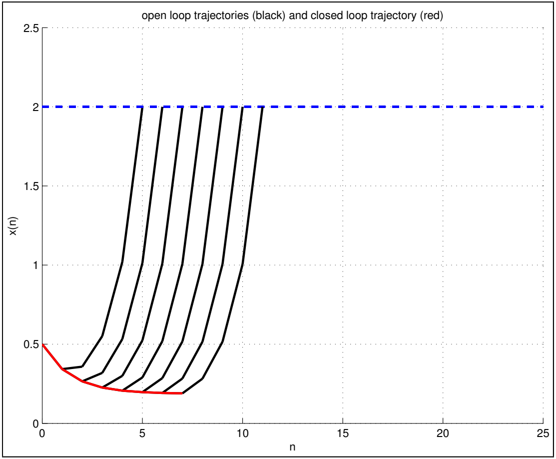
$N = 5$

Optimal invariance example



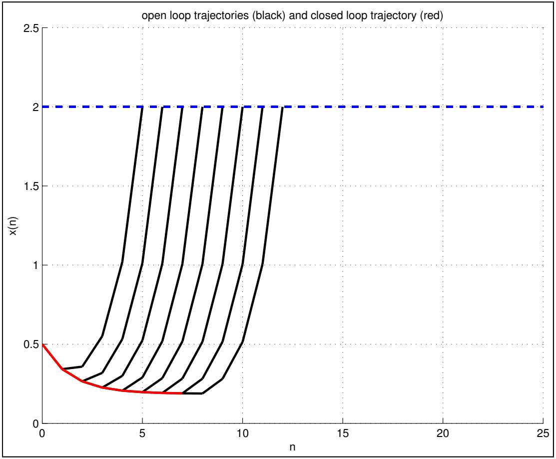
$N = 5$

Optimal invariance example



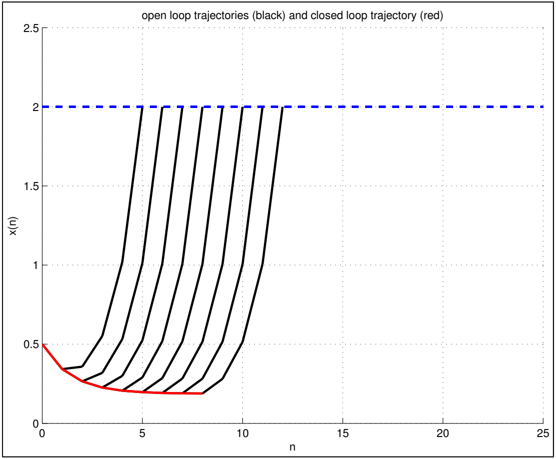
$N = 5$

Optimal invariance example



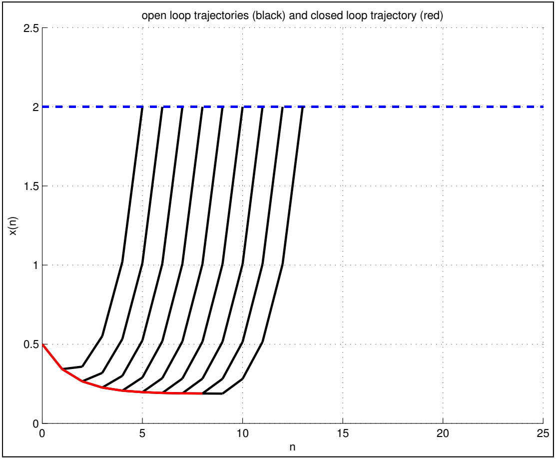
$N = 5$

Optimal invariance example



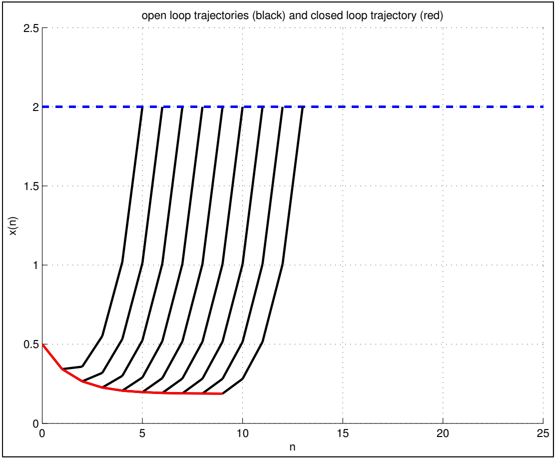
$N = 5$

Optimal invariance example



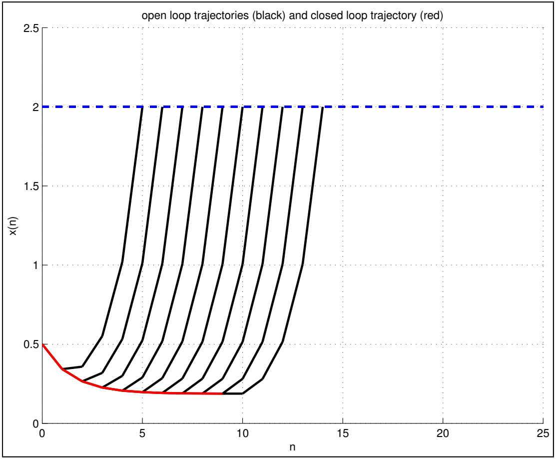
$N = 5$

Optimal invariance example



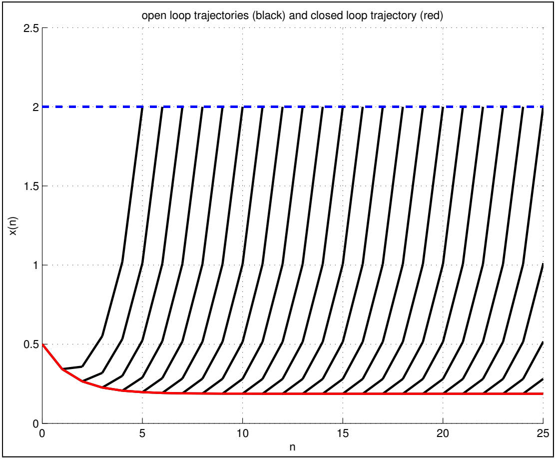
$N = 5$

Optimal invariance example



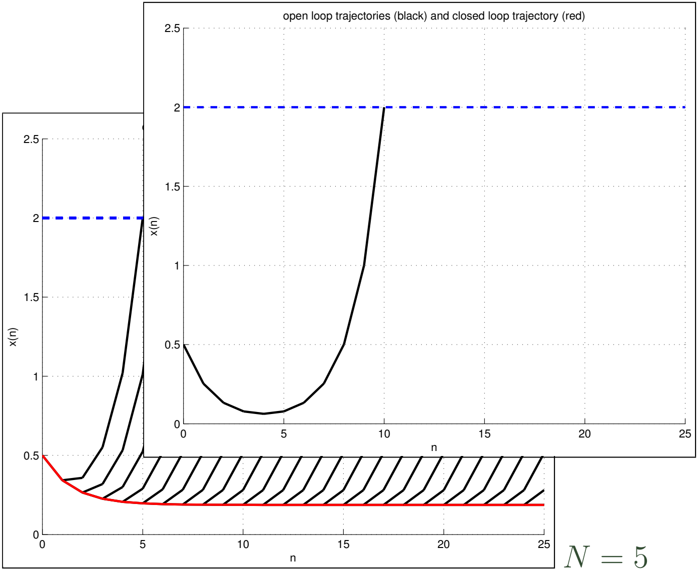
$N = 5$

Optimal invariance example



$N = 5$

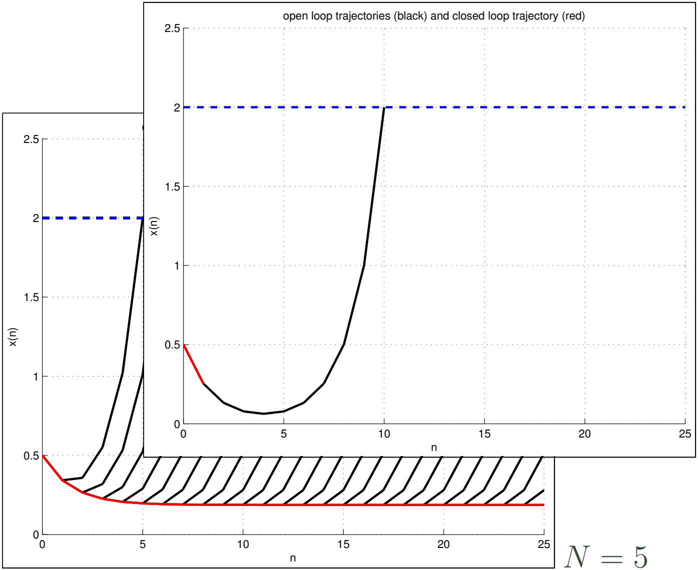
Optimal invariance example



$N = 10$

$N = 5$

Optimal invariance example

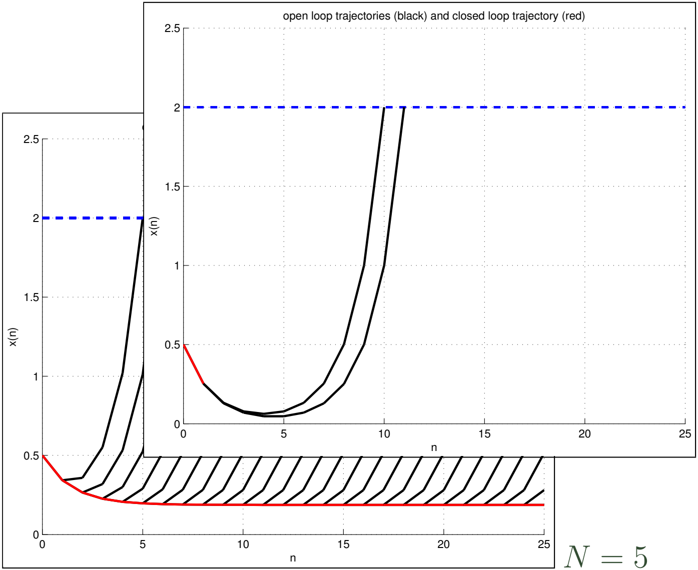


$N = 10$

$N = 5$

UNIVERSITÄT BAYREUTH

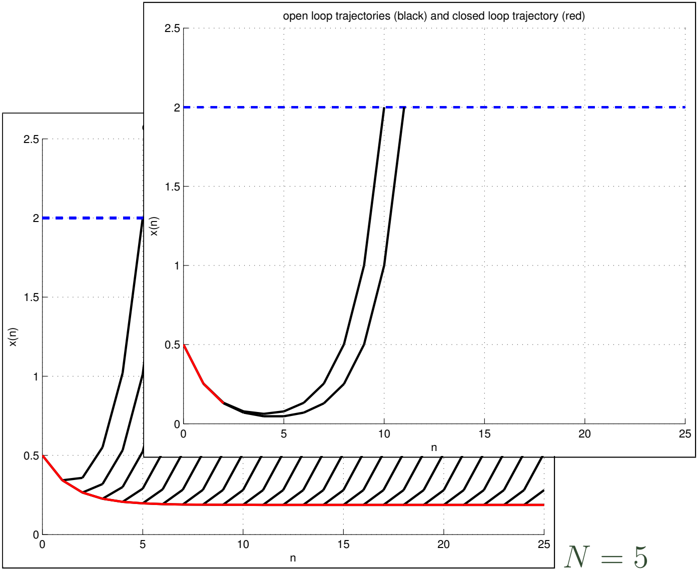
Optimal invariance example



$N = 10$

$N = 5$

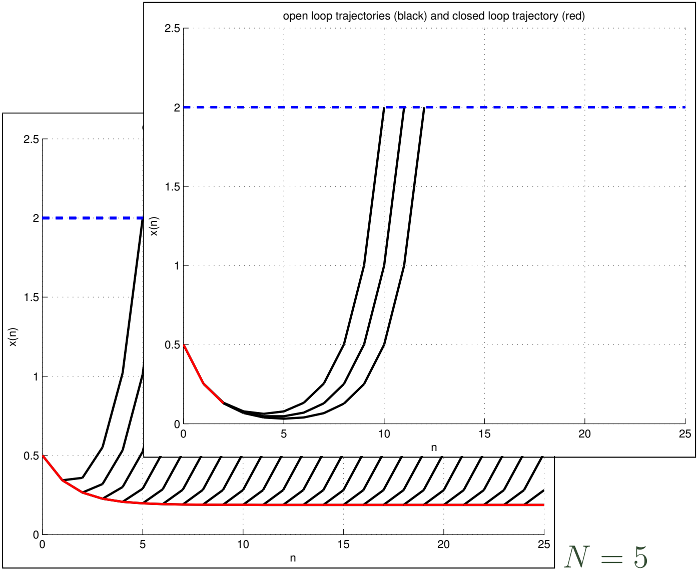
Optimal invariance example



$N = 10$

$N = 5$

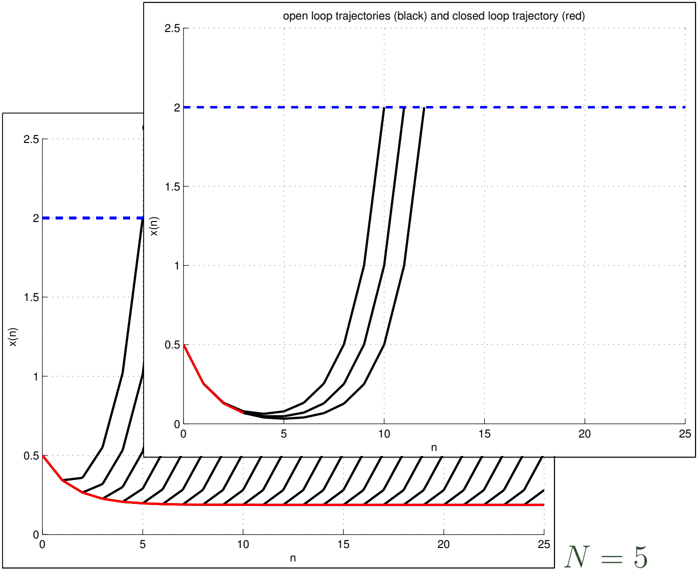
Optimal invariance example



$N = 10$

$N = 5$

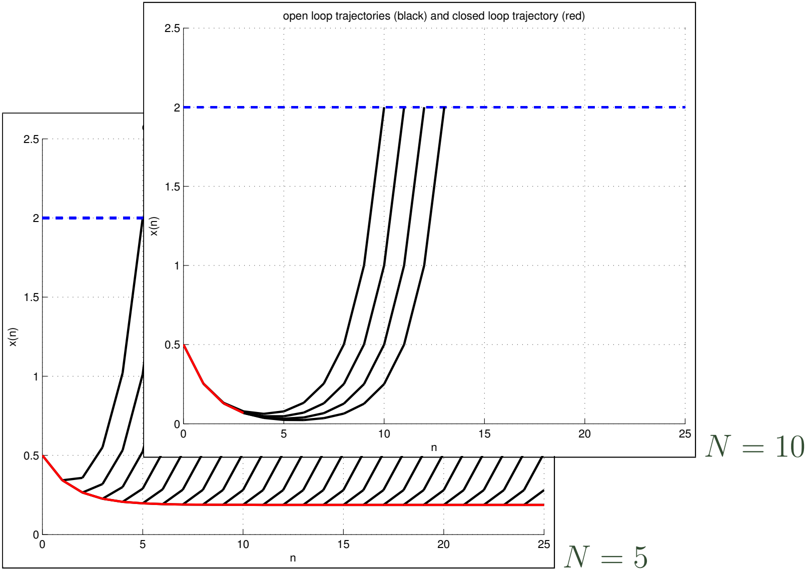
Optimal invariance example



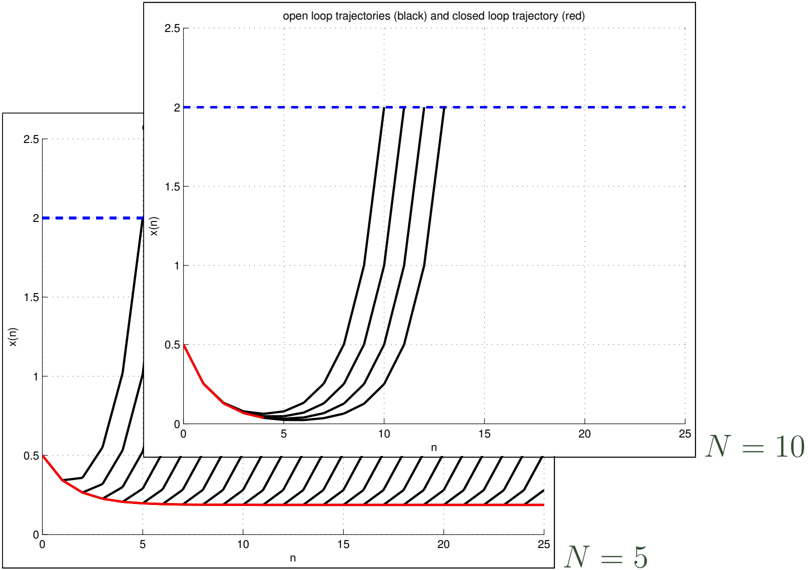
$N = 10$

$N = 5$

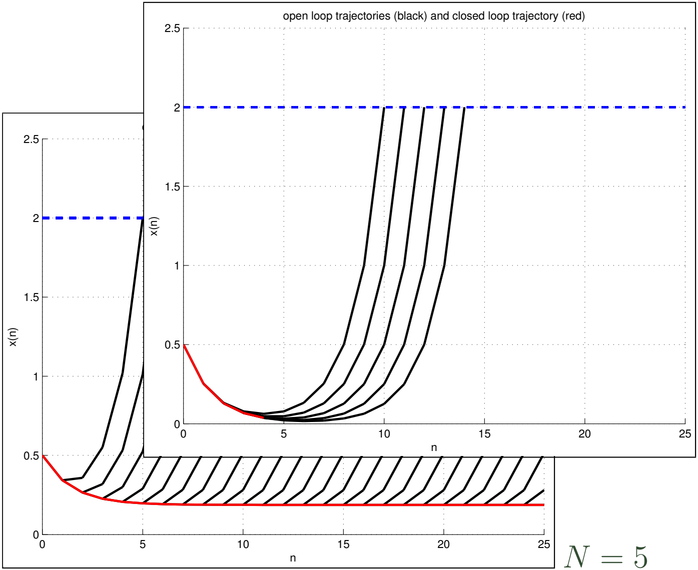
Optimal invariance example



Optimal invariance example



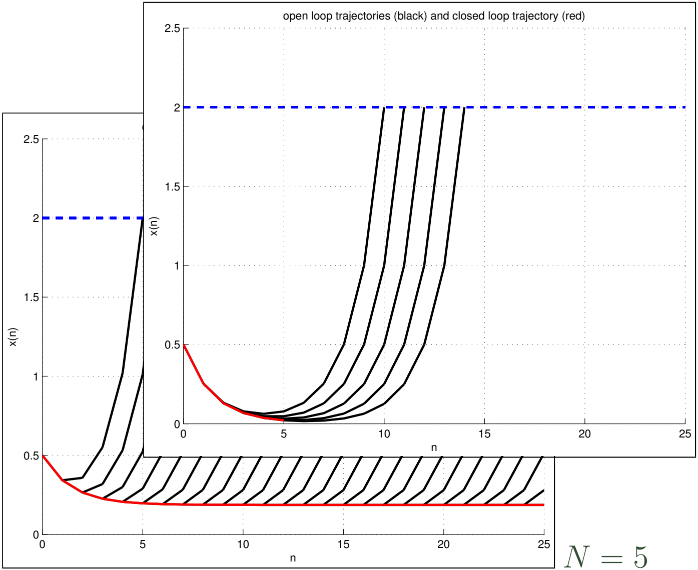
Optimal invariance example



$N = 10$

$N = 5$

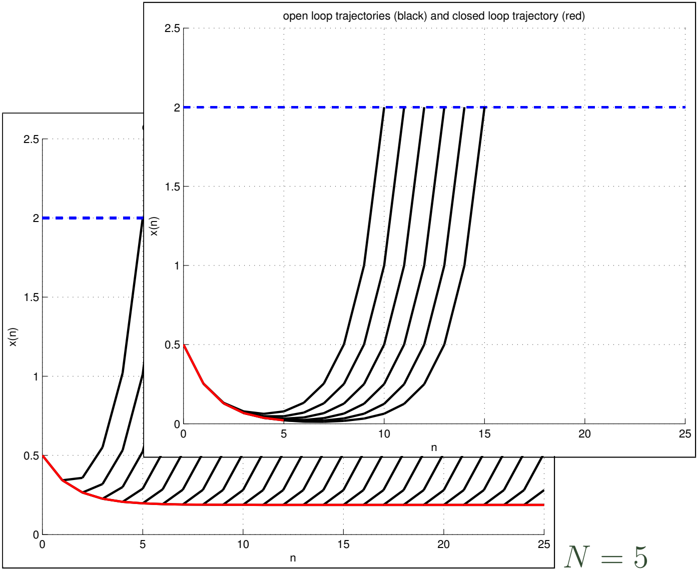
Optimal invariance example



$N = 10$

$N = 5$

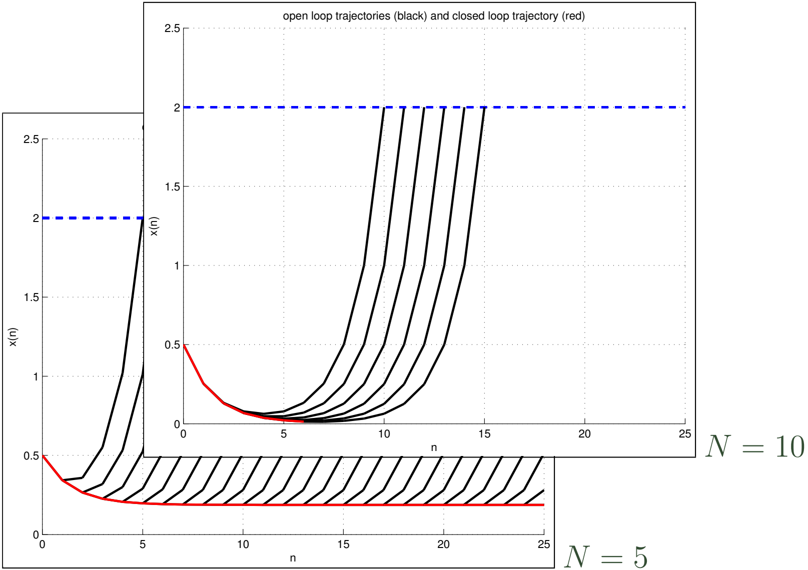
Optimal invariance example



$N = 10$

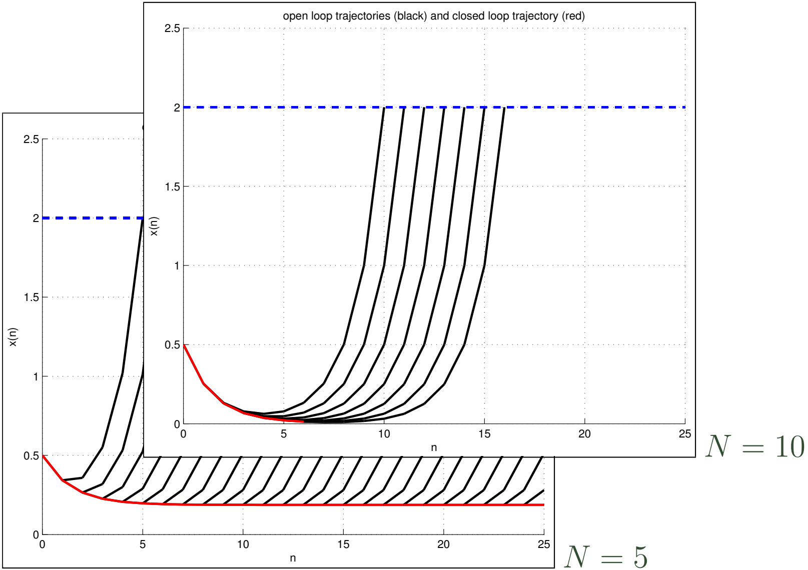
$N = 5$

Optimal invariance example

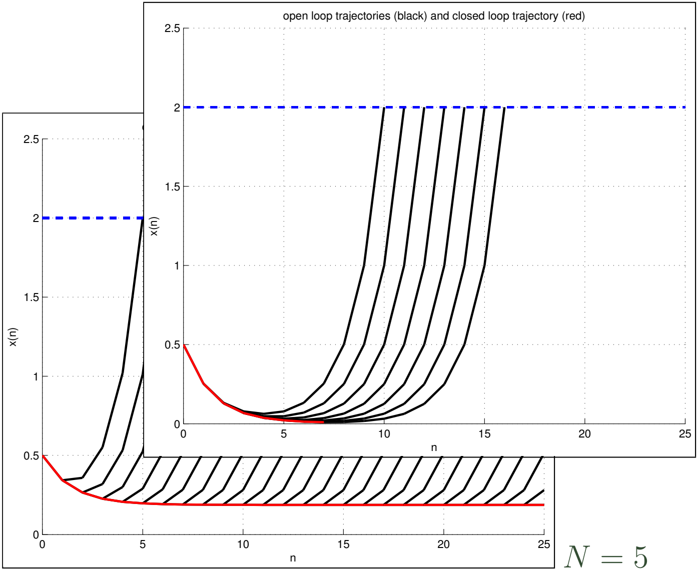


UNIVERSITÄT BAYREUTH

Optimal invariance example



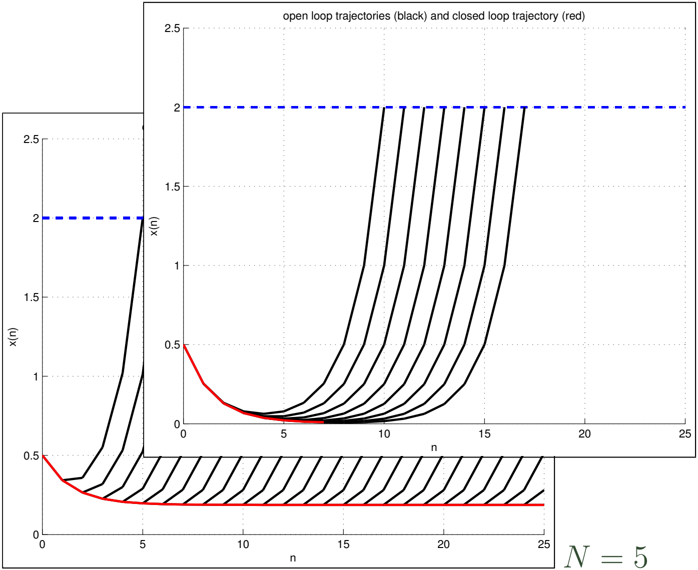
Optimal invariance example



$N = 10$

$N = 5$

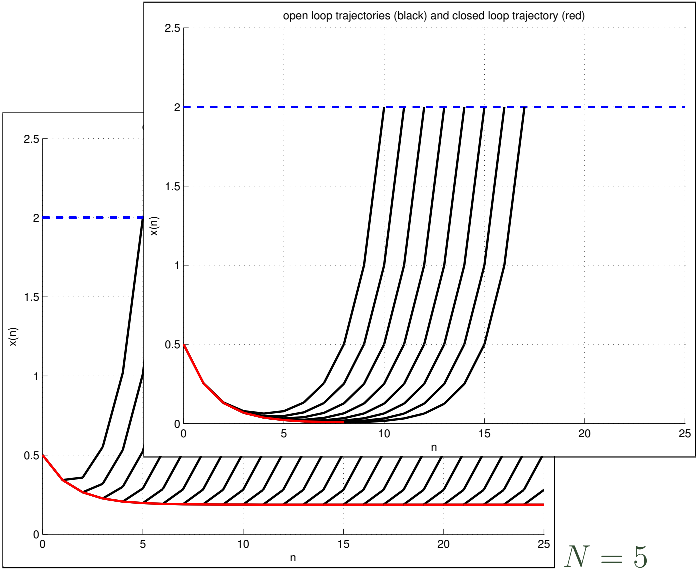
Optimal invariance example



$N = 10$

$N = 5$

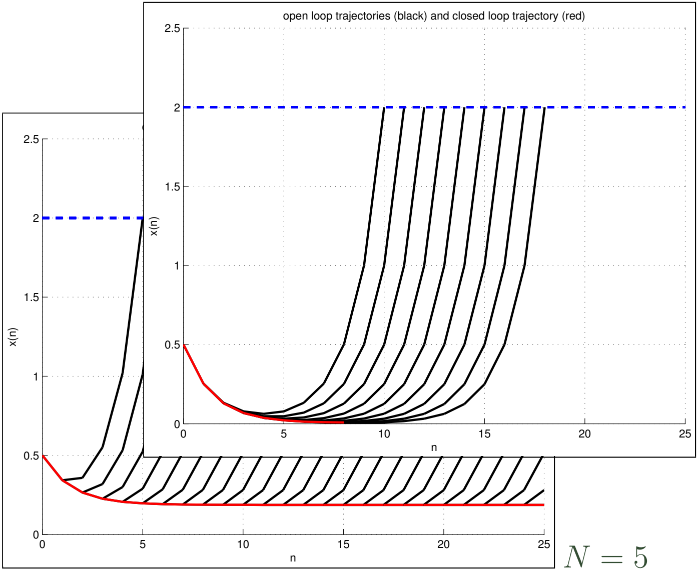
Optimal invariance example



$N = 10$

$N = 5$

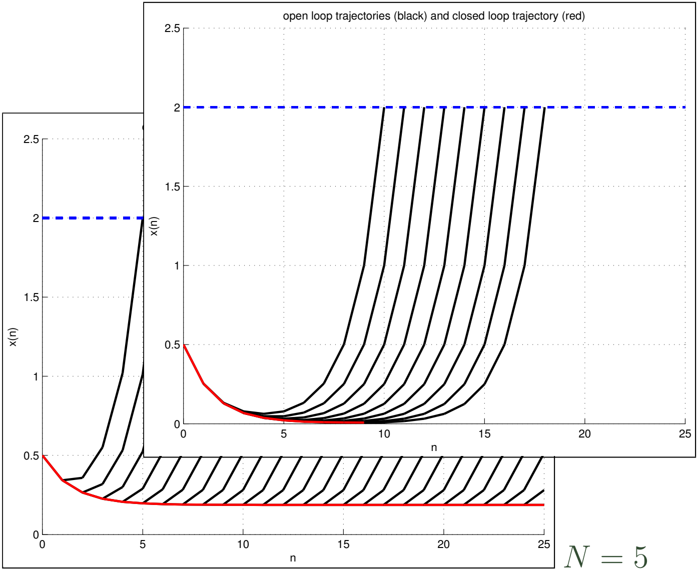
Optimal invariance example



$N = 10$

$N = 5$

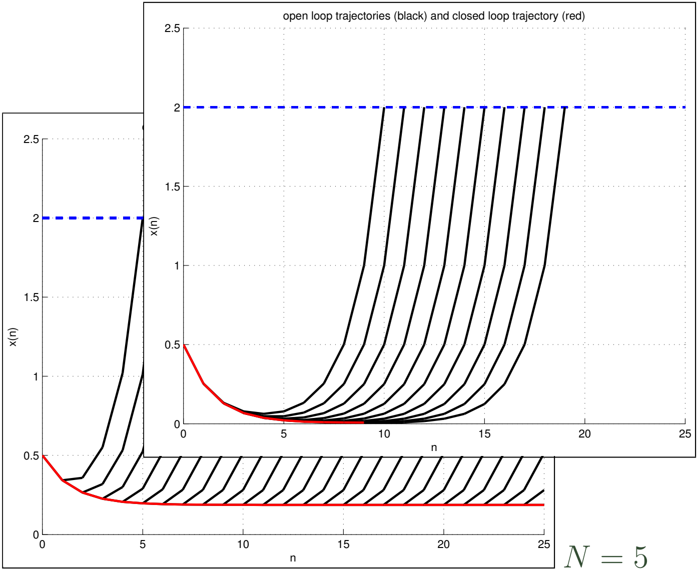
Optimal invariance example



$N = 10$

$N = 5$

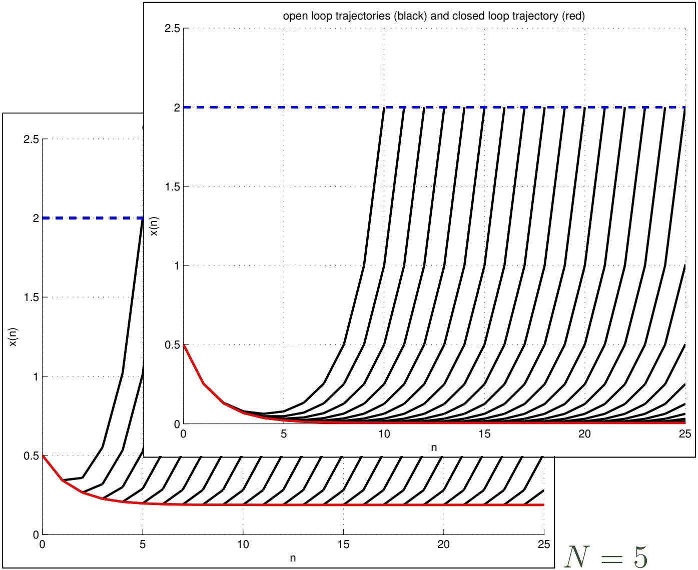
Optimal invariance example



$N = 10$

$N = 5$

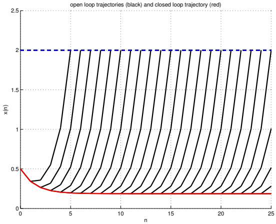
Optimal invariance example



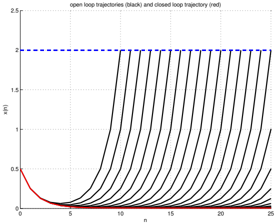
$N = 10$

$N = 5$

Optimal invariance: observations

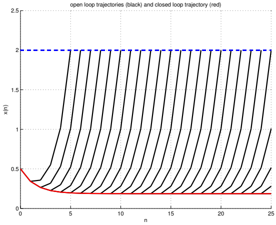


$N = 5$

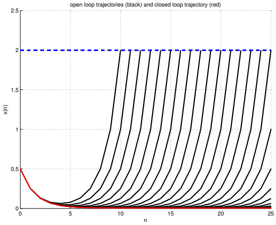


$N = 10$

Optimal invariance: observations



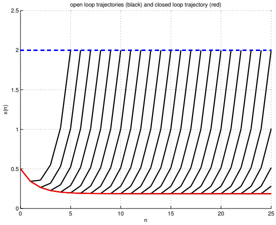
$N = 5$



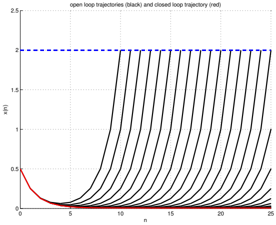
$N = 10$

- optimal open loop trajectories first approach the optimal equilibrium and then tend to the boundary of $\mathbb{X} = [-2, 2]$

Optimal invariance: observations



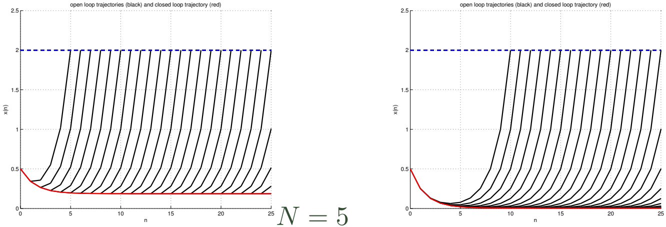
$N = 5$



$N = 10$

- optimal open loop trajectories first approach the optimal equilibrium and then tend to the boundary of $\mathbb{X} = [-2, 2]$
- closed loop trajectories follow the "good part" of the open loop trajectories

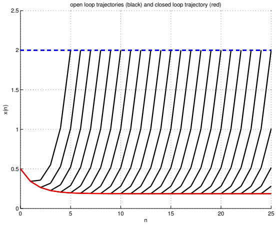
Optimal invariance: observations



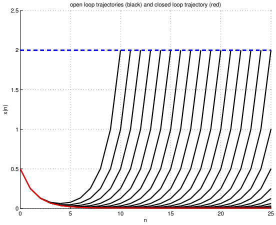
$N = 5$ $N = 10$

- optimal open loop trajectories first approach the optimal equilibrium and then tend to the boundary of $\mathbb{X} = [-2, 2]$
- closed loop trajectories follow the "good part" of the open loop trajectories
- the larger N, the "better" the closed loop trajectories.

Optimal invariance: observations



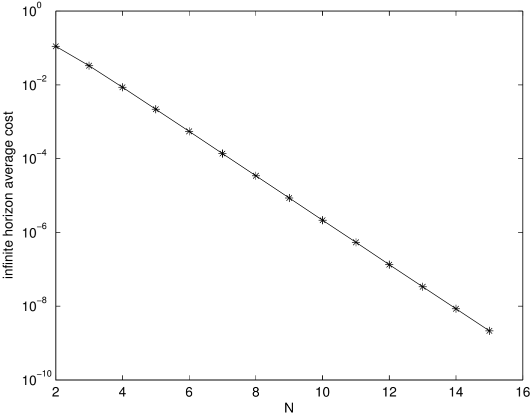
$N = 5$



$N = 10$

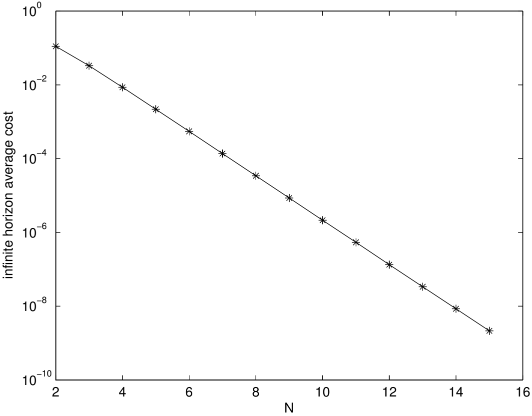
- optimal open loop trajectories first approach the optimal equilibrium and then tend to the boundary of $\mathbb{X} = [-2, 2]$
- closed loop trajectories follow the "good part" of the open loop trajectories
- the larger N, the "better" the closed loop trajectories. This is also reflected in the average closed loop costs

Optimal invariance: closed loop performance



$\overline{J}_\infty(0.5, F_N)$ depending on N, logarithmic scale

Optimal invariance: closed loop performance



$\overline{J}_\infty(0.5, F_N)$ depending on N, logarithmic scale

Can we prove this behavior?

Optimal invariance result

Theorem [Gr. 11] Assume that there are $N_0 \geq 0$, $\ell_0 \in \mathbb{R}$ and $\delta_1, \delta_2 \in \mathcal{L}$ such that for each $x \in \mathbb{X}$ and $N \geq N_0$ there exists a control sequence $u_{N,x} \in \mathbb{U}^{N+1}$ satisfying

- $x_{u_{N,x}}(k,x) \in \mathbb{X}$, $\quad k = 0, \ldots, N+1$ admissibility

UNIVERSITÄT BAYREUTH

Optimal invariance result

Theorem [Gr. 11] Assume that there are $N_0 \geq 0$, $\ell_0 \in \mathbb{R}$ and $\delta_1, \delta_2 \in \mathcal{L}$ such that for each $x \in \mathbb{X}$ and $N \geq N_0$ there exists a control sequence $u_{N,x} \in \mathbb{U}^{N+1}$ satisfying

- $x_{u_{N,x}}(k,x) \in \mathbb{X}, \quad k = 0, \ldots, N+1$ admissibility
- $\overline{J}_N(x, u_{N,x}) \leq \inf_{u \in \mathbb{U}^\infty} \overline{J}_N(x,u) + \delta_1(N)/N$ near optimality

UNIVERSITÄT BAYREUTH

Optimal invariance result

Theorem [Gr. 11] Assume that there are $N_0 \geq 0$, $\ell_0 \in \mathbb{R}$ and $\delta_1, \delta_2 \in \mathcal{L}$ such that for each $x \in \mathbb{X}$ and $N \geq N_0$ there exists a control sequence $u_{N,x} \in \mathbb{U}^{N+1}$ satisfying

- $x_{u_{N,x}}(k,x) \in \mathbb{X}, \quad k = 0, \ldots, N+1$ — admissibility
- $\overline{J}_N(x, u_{N,x}) \leq \inf_{u \in \mathbb{U}^\infty} \overline{J}_N(x,u) + \delta_1(N)/N$ — near optimality
- $\ell(x_{u_{N,x}}(N,x), u_{N,x}(N)) \leq \ell_0 + \delta_2(N)$ — small terminal value

Optimal invariance result

Theorem [Gr. 11] Assume that there are $N_0 \geq 0$, $\ell_0 \in \mathbb{R}$ and $\delta_1, \delta_2 \in \mathcal{L}$ such that for each $x \in \mathbb{X}$ and $N \geq N_0$ there exists a control sequence $u_{N,x} \in \mathbb{U}^{N+1}$ satisfying

- $x_{u_{N,x}}(k,x) \in \mathbb{X}, \quad k = 0, \ldots, N+1$ — admissibility
- $\overline{J}_N(x, u_{N,x}) \leq \inf_{u \in \mathbb{U}^\infty} \overline{J}_N(x,u) + \delta_1(N)/N$ — near optimality
- $\ell(x_{u_{N,x}}(N,x), u_{N,x}(N)) \leq \ell_0 + \delta_2(N)$ — small terminal value

Then $\overline{J}_\infty(x, F_N(x)) \leq \ell_0 + \delta_1(N-1) + \delta_2(N-1)$ follows for all $x \in \mathbb{X}$.

Optimal invariance result

Theorem [Gr. 11] Assume that there are $N_0 \geq 0$, $\ell_0 \in \mathbb{R}$ and $\delta_1, \delta_2 \in \mathcal{L}$ such that for each $x \in \mathbb{X}$ and $N \geq N_0$ there exists a control sequence $u_{N,x} \in \mathbb{U}^{N+1}$ satisfying

- $x_{u_{N,x}}(k,x) \in \mathbb{X}, \quad k = 0, \ldots, N+1$ admissibility
- $\overline{J}_N(x, u_{N,x}) \leq \inf_{u \in \mathbb{U}^\infty} \overline{J}_N(x,u) + \delta_1(N)/N$ near optimality
- $\ell(x_{u_{N,x}}(N,x), u_{N,x}(N)) \leq \ell_0 + \delta_2(N)$ small terminal value

Then $\overline{J}_\infty(x, F_N(x)) \leq \ell_0 + \delta_1(N-1) + \delta_2(N-1)$ follows for all $x \in \mathbb{X}$.

These assumptions can be ensured by suitable controllability conditions plus bounds on the performance of certain trajectories.

Optimal invariance result

Theorem [Gr. 11] Assume that there are $N_0 \geq 0$, $\ell_0 \in \mathbb{R}$ and $\delta_1, \delta_2 \in \mathcal{L}$ such that for each $x \in \mathbb{X}$ and $N \geq N_0$ there exists a control sequence $u_{N,x} \in \mathbb{U}^{N+1}$ satisfying

- $x_{u_{N,x}}(k,x) \in \mathbb{X}, \quad k = 0, \ldots, N+1$ — admissibility
- $\overline{J}_N(x, u_{N,x}) \leq \inf_{u \in \mathbb{U}^\infty} \overline{J}_N(x,u) + \delta_1(N)/N$ — near optimality
- $\ell(x_{u_{N,x}}(N,x), u_{N,x}(N)) \leq \ell_0 + \delta_2(N)$ — small terminal value

Then $\overline{J}_\infty(x, F_N(x)) \leq \ell_0 + \delta_1(N-1) + \delta_2(N-1)$ follows for all $x \in \mathbb{X}$.

These assumptions can be ensured by suitable controllability conditions plus bounds on the performance of certain trajectories. For our invariance example, this allows to rigorously prove $\overline{J}_\infty(x, F_N) \to 0$ as $N \to \infty$

Summary and outlook

- MPC without terminal constraints shows excellent results both for stabilizing and for economic problems

Summary and outlook

- MPC without terminal constraints shows excellent results both for stabilizing and for economic problems
- for stabilizing MPC, a controllability based analysis helps to identify and design stage costs ℓ for obtaining stability with small control horizons

Summary and outlook

- MPC without terminal constraints shows excellent results both for stabilizing and for economic problems
- for stabilizing MPC, a controllability based analysis helps to identify and design stage costs ℓ for obtaining stability with small control horizons
- for economic MPC, under suitable conditions an average performance close to that of an optimal equilibrium without a priori knowledge of this equilibrium can be achieved

Summary and outlook

- MPC without terminal constraints shows excellent results both for stabilizing and for economic problems
- for stabilizing MPC, a controllability based analysis helps to identify and design stage costs ℓ for obtaining stability with small control horizons
- for economic MPC, under suitable conditions an average performance close to that of an optimal equilibrium without a priori knowledge of this equilibrium can be achieved
- Future work:

Summary and outlook

- MPC without terminal constraints shows excellent results both for stabilizing and for economic problems
- for stabilizing MPC, a controllability based analysis helps to identify and design stage costs ℓ for obtaining stability with small control horizons
- for economic MPC, under suitable conditions an average performance close to that of an optimal equilibrium without a priori knowledge of this equilibrium can be achieved
- Future work:
  - extension of economic MPC results to more general problem classes and optimal periodic orbits

Summary and outlook

- MPC without terminal constraints shows excellent results both for stabilizing and for economic problems
- for stabilizing MPC, a controllability based analysis helps to identify and design stage costs ℓ for obtaining stability with small control horizons
- for economic MPC, under suitable conditions an average performance close to that of an optimal equilibrium without a priori knowledge of this equilibrium can be achieved
- Future work:
  - extension of economic MPC results to more general problem classes and optimal periodic orbits
  - (practical) asymptotic stability analysis of economic MPC without terminal constraints

Happy Birthday Eduardo!