Optimization Models for Container Inspection

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Joint work with L. Fedzhora and P.B. Kantor (Rutgers),

and K. Saeger and P. Stroud (LANL)

Container Inspection

Problem

- Finding ways to intercept illicit nuclear materials and weapons destined for the U.S. via the maritime transportation system is an exceedingly difficult task. Today, only a small percentage of containers arriving to U.S. ports are inspected.
- Inspection involves checking paperwork, using various imaging sensors, and manual inspection.
- Objectives involve maximizing **detection rate**, minimizing **unit cost of inspection**, **rate of false positives**, **time delays**, etc.

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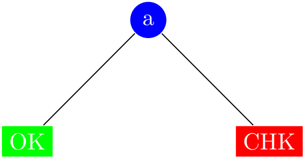
A small example involving two sensors



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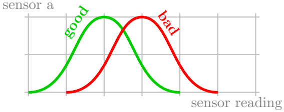

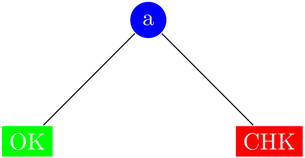

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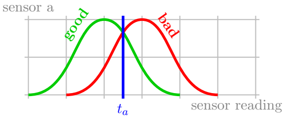

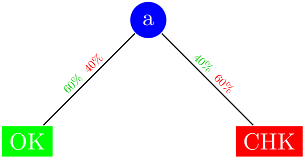

Inspection cost	$\mathbf{0.4C_{CHK}}$ $\mathbf{+C_a}$
Detection rate	**60%**

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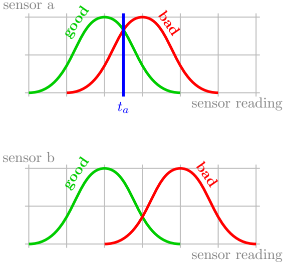

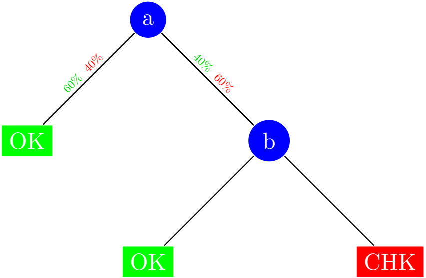

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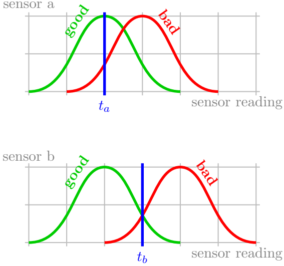

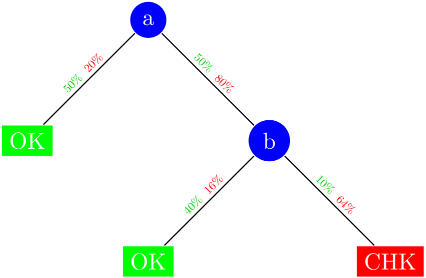

Inspection cost	$0.4C_{CHK} + C_a$	$\mathbf{0.1C_{CHK} + C_a + 0.5C_b}$
Detection rate	60%	**64%**

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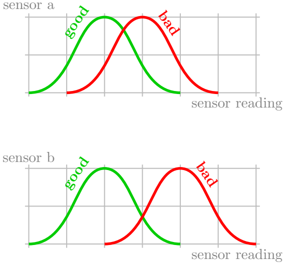

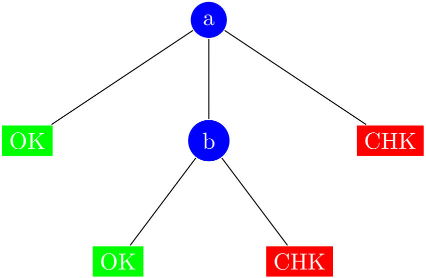

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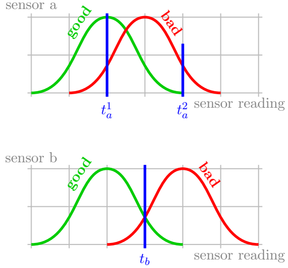

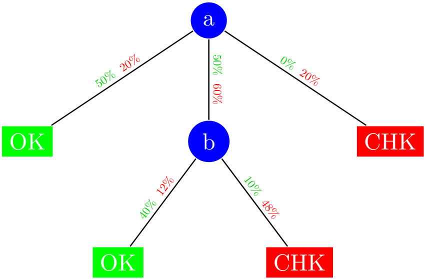

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Detection rate	60%	64%	**68%**

Mathematical Model

Maximize detection rate $\mathbf{\Delta(D, t)}$

- over all **decision trees D** and **threshold selections t**
- subject to **budget**, **capacity**, and **delay** constraints

A possible solution (Stroud and Saeger, 2003)

- Enumerate all possible (binary) **decision trees** and compute best possible **threshold selections** for each.

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  - **Number of decision trees is doubly exponential!**
  - **Enumeration is possible only for $s \leq 4$!**
  - **Too expensive to analyze tradeoffs!**
  - **Why only 1-1 thresholds?**
  - **Why a single decision tree?**

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Large Scale LP Formulation

- Developed a polyhedral description of all possible decision trees.
- Formulated a large scale LP model for optimal inspection policy; maximization of **detection rate**, while limiting **unit cost of inspection**, **rate of false positives**, and **time delays**, etc.

- Off the shelf LP packages can find optimal inspection strategies up to 6-8 sensors.
- Detection rate – unit inspection cost ROC curve can be tabulated.
- Effects of capacity and time delay limitations can be analyzed.
- Benefits of new sensor technologies can be evaluated.

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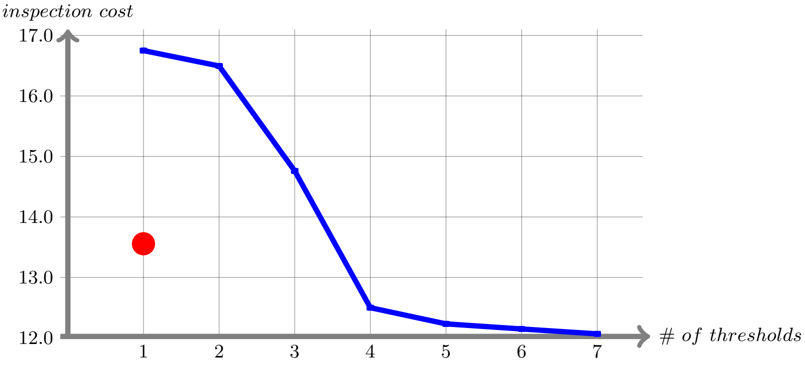
Experiments with 4 sensors (Stroud and Saeger, 2003)



- Detection rate $\geq$ **81.5**%
- *Threshold-optimized* pure strategy found by Stroud and Saeger (2003)
- *Non-optimized* threshold grid; savings of $\approx$ **10**%

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