Multiple Clothing Part Placement: Direct Representation of Curves vs. Polygonal Approximation

Victor J. Milenkovic
University of Miami

Techniques exist for numerically robust cascaded set operations on planar polygonal regions: Boolean operations (union, intersection, difference) and Minkowski sum[1]. Numerically robust means that the symmetric difference with the ideal exact output is small. Cascaded means that the numerical representation has bounded complexity so that the output can become the input again. So called ECG (Exact Computation Geometry) is obviously numerically robust but cannot be cascaded with bounded bit-complexity, and thus some form of geometric rounding is required. Existing rounding techniques for polygons include nearest pair rounding [2], shortest path rounding [4], and snap rounding [1].

Recently, the speaker and Elisha Sacks of Purdue University have developed a numerically robust inconsistency-sensitive arrangement algorithm for implicit polynomial curves. This algorithm is efficient and accurate both in theory and practice (Figure 1). This result implies numerically robust cascaded Boolean set operations on planar regions bounded by implicit polynomial curves. The Minkowski sum presents a problem in general because it raises the algebraic degree of the curves and hence the complexity of the representation. However, the set of circular polygons (planar regions bounded by straight line segments and circular arcs) is closed under Boolean operations and the Minkowski sum. Thus the new arrangement algorithm implies numerically robust cascaded set operations on circular polygons.

This talk addressed the following practical question: are the new results good for anything? The apparel industry, for example, almost exclusively uses polygonal representations, even for parts with curved boundaries. With robust cascaded set operations on polygons, one can create practical algorithms for minimal rectangle enclosure of multiple[2] clothing parts[3] [3] (Figure 2).

However, the representation of a clothing part to the required accuracy[4] requires up to 100 vertices per part. According to experts in the industry, at most a half dozen line segments and a half dozen circular arcs would suffice to represent any clothing part.

The new results permit us for the first time to implement minimal enclosure of multiple circular polygons. This talk will compare the running time of minimal enclosure of multiple parts represented using polygonal approximations vs. using circular polygons. The results of these experiments will help determine if direct representation of curves is a worthwhile endeavor or if polygonal approximations are sufficient for practical purposes.

References

[1] Leonidas Guibas and David Marimont. Rounding arrangements dynamically. In *Proc. 11th Annu. ACM Sympos. Comput. Geom.*, pages 190–199, 1995.

[2] V. J. Milenkovic. Rotational polygon containment and minimum enclosure using only robust 2d constructions. *Computational Geometry: Theory and Applications*, 13:3–19, 1999.

[3] V. J. Milenkovic and K. M. Daniels. Translational Polygon Containment and Minimal Enclosure using Mathematical Programming. *International Transactions in Operational Research*, 6:525–554, 1999.

[4] Victor J. Milenkovic. Shortest path geometric rounding. *Algorithmica*, 27(1):57–86, 2000.

[1] $A \oplus B = \{a + b \mid a \in A \text{ and } b \in B\}$.
[2] But not too many, since minimal enclosure is NP-hard.
[3] Under translation: because fabric has a grain, arbitrary rotation is not permitted.
[4] $1/10^4$ for the apparel industry

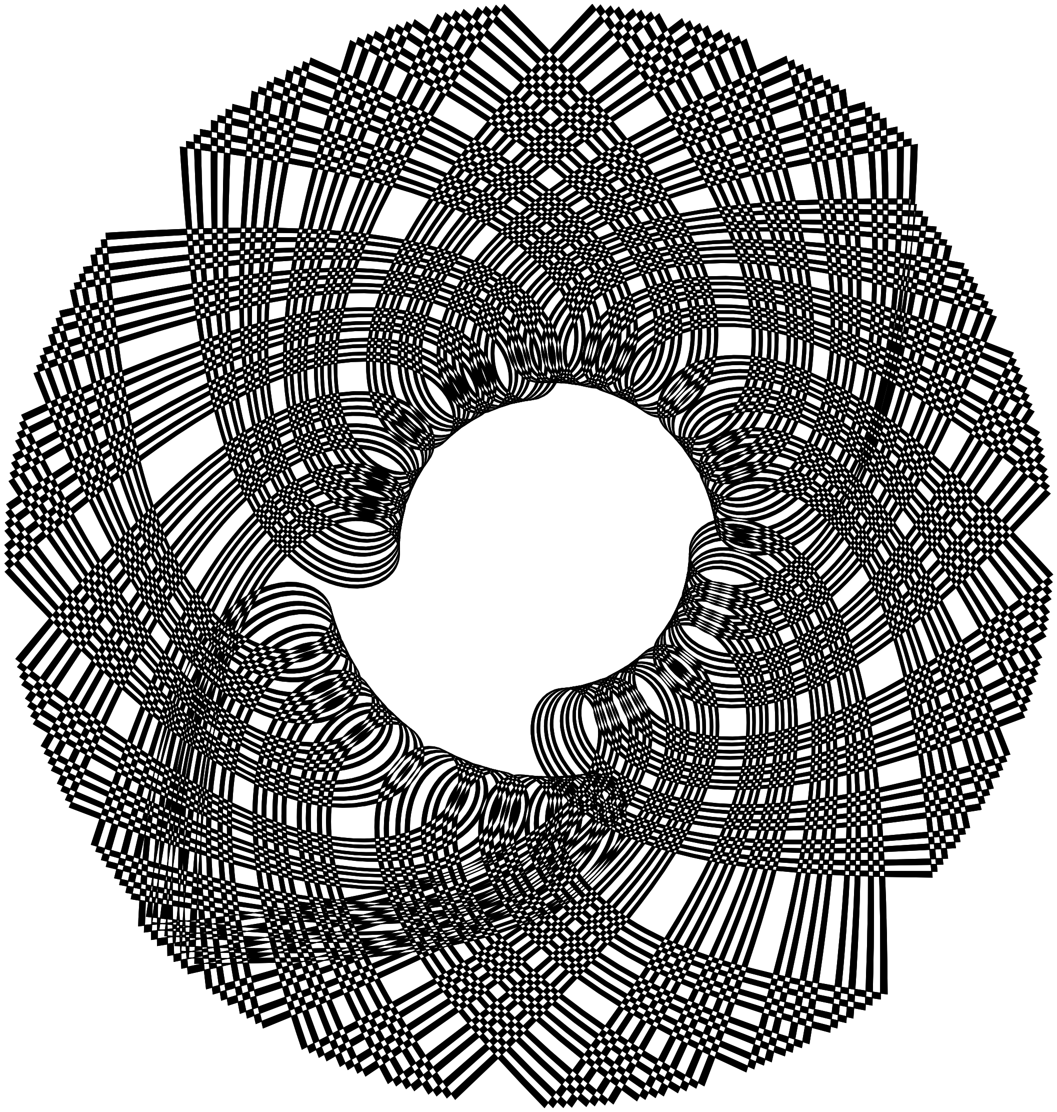

Figure 1: Region bounded by implicit cubic curve, rotated and XORed with itself 8 times, each rotation half the one before. Number of vertices $= 23971$, edges $= 47185$, and cells $= 23216$.

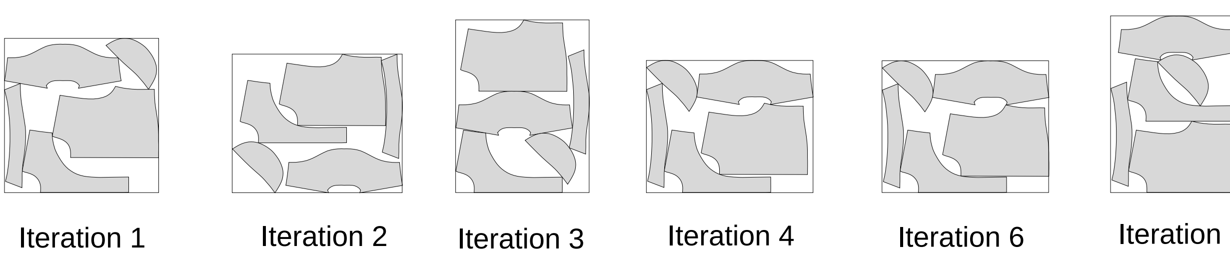

Figure 2: Minimal enclosing rectangle of five polygons with 55, 61, 66, 65 and 72 vertices. Iteration 1 is a square container with compaction applied. For iterations 2-5, the algorithm set the target area to be 1% less than the previous layout after compaction. Iteration 5 was infeasible. The algorithm set the target for iteration 6 to be 0.01% smaller than the area of iteration 4, and similarly, iteration 7 and 8 have targets 0.01% than the previous layouts after compaction. Iteration 8 was infeasible, and therefore iteration 7 is within 0.01% of optimum.