On the chromatic number of a random 5-regular graph

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Abstract

The chromatic number is a well-studied parameter of graphs, and in particular random graphs. For the uniform probability space $\mathcal{G}_{n,d}$ of d-regular graphs on n labelled vertices, recent work has focussed on the chromatic number for fixed d. Achlioptas and Moore have announced that if k is the smallest integer such that $d < 2k \log k$ then a.a.s. $\mathcal{G}_{n,d}$ has chromatic number k, $k+1$, or $k+2$. (We say an event holds *asymptotically almost surely* (a.a.s.) if it holds with probability tending to 1 as $n \to \infty$. In these asymptotics for $\mathcal{G}_{n,d}$ we assume nd is always even for feasibility.) Shi and the second author narrowed this range for several small values of d. In particular, for $d = 5$ they prove the chromatic number is a.a.s. 3 or 4, and for $d = 6$ it is a.a.s. 4. After those results, the biggest open problem in this area is to settle the question of the chromatic number of random 5-regular graphs.

In this paper we show that if a certain 4-variable function has a unique maximum at a given point in a bounded domain, then the chromatic number of a random 5-regular graph is a.a.s. 3. The main tool is the small subgraph conditioning method applied to a special type of colouring. Computational experiments by Díaz, Kirousis, Kaporis, and Perez strongly suggest that the function does have the required property.

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