Secret correlation with pure automata

Olivier Gossner[1] and Penélope Hernández[2]

January 19, 2005

Let G be a 3-player game with actions sets X_1, X_2, X_3 and payoff function g for player 3. The min max in correlated strategies for player 3 is:

$$\underline{v} = \min_{d \in \Delta(X_1 \times X_2)} \max_{x_3 \in X_3} \mathbf{E}_d g(x_1, x_2, x_3) = \max_{s \in \Delta X_3} \min_{(x_1,x_2) \in X_1 \times X_2} \mathbf{E}_s g(x_1, x_2, x_3)$$

where the equality is a consequence of the min max theorem.

Let $\mathcal{A}_i(m_i)$ be the set of automata for player i of size m_i such that $\mathcal{A}_i(m_i)$ inputs at each stage an element of $\prod_{j \neq i} X_j$ and outputs an element of X_i. An *oblivious* automaton is an automaton which transitions are independent of other player's actions.

An triple of automata (A_1, A_2, A_3) induces an eventually periodic sequence of actions, and let $\gamma(A_1, A_2, A_3)$ be the average payoff of player 3 over a period of this sequence.

A consequence of [BP93] is that whenever m_3 is subexponential in m_1 and in m_2, there exist correlated automata of 1 and 2 against which player 3 cannot obtain significantly more than $\underline{v}$. Formally:

Proposition 1 *If* $\min(m_1(k), m_2(k)) \gg \ln m_3(k)$*, then:*

$$\min_{\sigma^{12} \in \Delta(\mathcal{A}_1(m_1(k)) \times \mathcal{A}_2(m_2(k)))} \max_{A_3 \in \mathcal{A}_3(m_3(k))} \mathbf{E}_\sigma \gamma(A_1, A_2, A_3) \to_{k \to \infty} \underline{v}$$

Furthermore, the correlated strategies in the proposition may have support the set of oblivious automata of players 1 and 2.

When players 1 and 2 are limited to rely on pure strategies, the following result obtains a consequence of [Ney97].

[1]PSE (Paris-Jourdan Sciences conomiques), Unit Mixte de Recherche CNRS - EHESS - ENPC - ENS, 48 boulevard Jourdan, 75014 Paris.

[2]Departamento de Fundamentos del Análisis Económico, Universidad de Alicante. Campus de Sant Vicente, 03071 Alicante.

Proposition 2 *If* $\min(m_1(k), m_2(k)) \gg m_3(k). \ln m_3(k)$, *then:*

$$\min_{(A_1,A_2)\in\mathcal{A}_1(m_1(k))\times\mathcal{A}_2(m_2(k))} \max_{A_3\in\mathcal{A}_3(m_3(k))} \gamma(A_1, A_2, A_3) \to_{k\to\infty} \underline{v}$$

Furthermore, the automata of players 1 and 2 can be chosen to be oblivious. We prove the following result, which strengthens the previous one:

Proposition 3 *If* X_1 *and* X_2 *are not singletons, and if* $\min(m_1(k), m_2(k)) \gg m_3(k)$, *then:*

$$\min_{(A_1,A_2)\in\mathcal{A}_1(m_1(k))\times\mathcal{A}_2(m_2(k))} \max_{A_3\in\mathcal{A}_3(m_3(k))} \gamma(A_1, A_2, A_3) \to_{k\to\infty} \underline{v}$$

The automata of players 1 and 2 we design in the proof of this result are not oblivious, but do not need to observe player 3's rely on techniques introduced in [GH03], and the proof that player 3 cannot obtain significantly more than $\underline{v}$ on large deviation techniques as in [Ney97].

Note finally that there is no hope of getting a result of this type if $m_3 > \min(m_1, m_2)$.

References

[BP93] E. Ben Porath. Repeated games with finite automata. *Journal of Economic Theory*, 59:17–32, 1993.

[GH03] O. Gossner and P. Hernández. On the complexity of coordination. *Mathematics of Operations Research*, 28:127–141, 2003.

[Ney97] A. Neyman. Cooperation, repetition, and automata. In S. Hart and A. Mas-Colell, editors, *Cooperation: Game-Theoretic Approaches*, volume 155 of *NATO ASI Series F*, pages 233–255. Springer-Verlag, 1997.