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REVERSE-ENGINEERING COUNTRY RISK RATINGS: A COMBINATORIAL NON-RECURSIVE MODEL

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Importance of Country Risk Ratings

- Globalization → Expansion and diversification of investment possibilities
- “Sovereign credit ratings are a condensed **assessment of a government’s ability and willingness to repay** its public debt both in principal and in interests on time” (Afonso et al. 2007); “pivot of all other country’s ratings” (Ferri et al. 1999), i.e., **ceiling or upper bound** on the other ratings
- Ratings influence the **interest rates** at which countries can obtain credit on the international financial markets
- Ratings also influence **credit ratings of national banks and companies**, and affect their attractiveness to foreign investors
- Institutional investors are sometimes contractually restricted on the degree of risk they can assume, i.e., they **cannot invest in debt rated below a prescribed level**

Two Approaches to Country Risk

- Country risk has both **financial/economic** and **political** components
- The **debt-service capacity** approach focuses on the deterioration of **solvency** of a country, which prevents it from fulfilling its commitments
- The **cost-benefit** approach views a default as a **deliberate choice** of the country, which may prefer this alternative over repayment

Moody's		S&P
Aaa		AAA
Aa1		AA+
Aa2		AA
Aa3		AA-
A1		A+
A2		A
A3		A-
Baa1		BBB+
Baa2	Investment	BBB
Baa3	Grade	BBB-
Ba1	High Yield	BB+
Ba2		BB
Ba3		BB-
B1		B+
B2		B
B3		B-
Caa1		CCC+
Caa		CCC
Caa3		CCC-
Ca		CC
		C
C		D

Critiques of Present Rating Systems - I

- **Comprehensibility (opaqueness)**: Rating agencies do not specify the factors that are used for determining their ratings, nor the way they are aggregated in a rating
- **Regional bias**: Haque et al. (1997) claim that (some) rating agencies favor certain regions (e.g., Asian and European countries)
- **Predictive power**: Some recent failures (no warning ahead of several financial crises) have challenged the trustworthiness of country risk ratings

Critiques of Present Rating Systems - II

- **Overreactions**: Rating agencies are considered to sometimes react in panic after realizing they fail to warn about a crisis, leading to the so-called procyclicality effect
- **Negative impact of rating changes**: The reluctance of raters to downgrade a country stems from the fact that a downgrade announcement can precipitate a country into crisis
- **Conflicts of interest**: Raters, charging fees to rated countries, can be suspected of reluctance to downgrade them, because of the possibility of jeopardizing their income sources

Recursive versus Non-recursive Models

- The set of independent variables used by many empirical studies includes directly or indirectly the **lagged sovereign ratings** of S&P, Moody's, or The Institutional Investor
- The 98% correlation level between The Institutional Investor ratings published in September 1997 and September 1998 confirms the **stability of sovereign ratings**
- The excellent correlation levels achieved by utilizing lagged ratings among the independent variables can be attributed to a large extent to ratings stability, and may not necessarily indicate the **predictive power** of the economic and political variables used as predictors
- A major drawback of recursive rating models is the impossibility of applying them to **not-yet-rated countries**

Objectives and Main Results

- The central objective of this paper is to develop a **transparent, accurate, non-recursive, and stable rating system**, closely approximating the learned (S&P's) country risk ratings
- This study:
  - reverse-engineers S&P's country risk ratings using Logical Analysis of Data (LAD) which derives a new rating system only from the qualitative information representing pairwise comparisons of country riskiness (**relative creditworthiness** approach)
  - develops an L_2-approximation of the LAD relative preferences to derive the **Logical Rating Scores** from the relative preferences in a straightforward way, by a single run of standard linear regression
  - generates a rating system that has the **granularity** (number of categories) desired by the user of ratings
  - allows to evaluate the importance of variables and to rate previously **unrated countries**

Data Sources and Variable Selection

- We use the S&P foreign currency country **ratings of 69 countries published at the end of December 1998** converted into a numerical scale (from 21 to 0)
- Values of **economic and financial variables** considered in this paper come from the International Monetary Fund (World Economic Outlook database, 2001), the World Bank (World Development Indicators database, 2000) and Moody's (2001)
- Values of **political variables** are provided by Kaufmann et al. (1999).
- **Variables used**: Gross domestic product per capita (GDPc), Inflation rate (IR), Trade balance (TB), Exports' growth rate (EGR), International reserves (RES), Fiscal balance (FB), Debt to GDP (DGDP), Exchange rate (ER), Financial depth and efficiency (FDE), Political stability (PS), Government effectiveness (GE), Corruption (COR) (9+3)

Pairwise Comparison of Countries: Pseudo-observations

- We associate to every country i in *I = {1,…,69}* the 13-dimensional vector C_i; the first component of C_i is the country risk rating given by S&P
- This study is based on the idea that a risk rating system can be constructed solely from the knowledge of (**pre)order of obligors with respect to their creditworthiness**
- The **pseudo-observations** are represented as 13-dimensional vectors; there are ***| I |*(| I | - 1)*** pseudo-observations
- The first component is an indicator which takes the value *"1" ("-1")* if the country i in the pseudo-observation P_{ij} has a higher (lower) rating; $P_{ij}[k] = C_i[k] - C_i[k], k = 2,\ldots,13$
- **Not independent**: $P_{ij} + P_{jk} = P_{ik}$

Logical Analysis of Data (LAD)

- Positive (negative) patterns are combinatorial rules which impose upper and lower bounds on the values of a subset of variables, such that a sufficient proportion of the positive (negative) observations in the dataset satisfy all the conditions of the pattern, and a sufficient proportion of the negative (positive) observations violate at least one of them.
- If *p* and *q* represent the number of positive and negative patterns in a model, and if *h* and *k* represent the numbers of positive, respectively negative patterns in the model covering a new observation θ, then the value of the discriminant is

$$\Delta(\theta) = h/p - k/q,$$

and the classification is determined by the sign of $\Delta(\theta)$

From Pseudo-observations to Relative Preferences

- After having constructed the LAD model, we compute the discriminant $\Delta(P_{ij})$ for each pseudo-observation P_{ij}
- The values $\boldsymbol{\Delta(P_{ij})}$ of the discriminant are called the ***relative preferences***, and the [69 x 69]-dimensional anti-symmetric matrix Δ having them as components will be called the *relative preference matrix*
- A large positive value of $\Delta(P_{ij})$ can be interpreted as country *i* being more creditworthy than country *j*, while the opposite conclusion can be drawn from a large negative value of $\Delta(P_{ij})$
- The interpretation of the sign of relative preferences as an indicator of rating superiority can result in the violation of the **transitivity requirement** of country ratings order relation

	Japan	**Canada**	**Belgium**
Japan		0.00625	-0.00625
Canada	-0.00625		0.03125
Belgium	0.00625	-0.03125	

From Relative Preferences to Logical Rating Scores

- If the sovereign ratings *β* are interpreted as cardinal values, it is natural to view the relative preferences *Δ* as differences of the corresponding ratings (allowing for inconsistencies):

$$\Delta(P_{ij}) = \beta_i - \beta_j + \varepsilon_{ij}, \text{for all } i, j \in I, i \neq j$$

- The determination of those values of the β_k's which provide the best L_2 approximation of the *Δ*'s can be found as a solution of the following multiple linear regression problem:

$$\Delta(\pi) = \sum_{k \in I} \beta_k * x_k(\pi) + \varepsilon(\pi)$$

$$\pi = \{(i, j) \mid i, j \in I, i \neq j\} \qquad x_k(i, j) = \begin{cases} 1, & \text{for } k = i \\ -1, & \text{for } k = j \\ 0, & \text{otherwise} \end{cases}$$

LRS-Based Rating System with Variable Granularity

- To define LRS-based ratings R_k of country k, one has to find cutpoints $\boldsymbol{x_0 \leq x_1 \leq \ldots < x_j \leq \ldots \leq x_{20} \leq x_{21}}$ to partition the range of LRS values into rating intervals (**arbitrary** number instead of 21!)
- Consistent partitioning may not exist → introduce **adjusted LRS** scores $\boldsymbol{\delta_k}$ to **approximate** the LRS scores β_k of country k
- The **number of countries** for which an adjustment of the LRS score is necessary has to be **minimized**; the **decision variables** $\boldsymbol{\alpha_k}$ take value *1* if an LRS adjustment is needed
- N – the set of countries, J ($|J|$) – the set (number) of rating categories, $j(k)$ is the S&P's rating category of country k
- M and ε respectively represent a large and an infinitesimal positive numbers
- The highest (smallest) LRS scores assigned to a country:

$$\overline{\beta} = \max_{k \in N} \beta_k \quad (\underline{\beta} = \min_{k \in N} \beta_k)$$

MIP for LRS-Based Rating System

$$\begin{aligned}
&\text{minimize} && \sum_{k \in I} \alpha_k && \\
&\text{subject to} && \delta_k \le x_{j(k)} && k \in I \\
& && x_{j(k)-1} + \varepsilon \le \delta_k && k \in I \\
& && \delta_k - \beta_k \le M\alpha_k && k \in I \\
& && \beta_k - \delta_k \le M\alpha_k && k \in I \\
& && \underline{\beta} = x_0 \le x_1 \le x_2, \ldots, \le x_j, \ldots, x_{20} \le x_{21} = \overline{\beta} && \\
& && \underline{\beta} \le \delta_k \le \overline{\beta} && k \in I \\
& && \alpha_k \in \{0,1\} && k \in I
\end{aligned}$$

$$R_k = j \text{ if } \begin{cases} \beta_k \le x_j^* \\ \beta_k > x_{(j-1)}^* \end{cases}, \quad k \in N, j = 1, \ldots, 21$$

Evaluation of Logical Rating Scores

- The LRS regression is highly significant, with the R^2 = 95.2%
- The correlations between LRS and the ratings of S&P's, Moody's and the Institutional Investor range from 94.11% to 95.54%
- To identify discrepancies between LRS and 1998 ratings:
  - LRS are first transformed to the scale of S&P ratings using a linear transformation $a^*\beta_i + c$ (i.e., solving simple linear regression to minimize the mean square difference between the transformed LRS and the S&P's ratings)
  - 90% confidence interval is constructed around the transformed LRS
- 5 countries' S&P's 1998 ratings are discrepancies; subsequent changes of their S&P ratings are in agreement with 1998 LRS

Evaluation of LRS-based Ratings

- In the most disaggregated LRS-based rating system (with the same number of rating categories as S&P - 22):
  - 18 countries have a 1-notch discrepancy
  - 3 countries have a 2-notch discrepancy
  - Person, Kendall, and Spearman correlations between the 1998 S&P's and LRS-based ratings are equal to 94.4%, 94.5 %, and 96.2%, resp.
- In the LRS-based rating system with commonly used 3 rating categories (investment-, speculative- and default grade):
  - 65 countries (94.2%) receive the same LRS-based ratings as the S&P's
  - 3 countries are border-line under-rated
  - 1 country is border-line over-rated
  - Subsequent S&P's rating changes for 2 of these 4 countries are in agreement with the LRS-based ratings

Temporal Stability of the LRS Model

- LRS-based model inferred from 1998 S&P's ratings is used to calculate ratings based on 1999 variable values, which are compared with 1999 S&P's ratings
- 94.12% correlation between LRS and 1999 S&P's ratings
- 2 countries have S&P ratings outside the 90% confidence interval of the transformed LRS
- Subsequent S&P's rating changes for these 2 countries are in agreement with the LRS
- In the 21-category LRS-based rating system a one-notch rating adjustment is needed for 19 countries and a two-notch adjustment is needed for 3 countries
- In the 3-category LRS-based rating system, 2 countries have different S&P's ratings than LRS-based ones
- Person, Kendall, and Spearman correlations between the 1999 S&P's and LRS-based ratings are equal to 94.1%, 94.4%, and 95.9%, resp.

LRS Ratings of Countries Previously Not Rated by S&P

- Since the LAD discriminantdoes not involve in any way the previous years' S&P's ratings, one can calculate the LRS and LRS-based rating of an unrated country by solving a single multiple linear regression model
- Thus **predicted** LRS values are compared (after a linear transformation) with the **subsequent** S&P's ratings when they **first** become **available**
- For 3 of the studied 4 countries, their initial S&P's ratings are within the 90%-confidence interval of the transformed LRS
- Ecuador's first S&P's rating (SD) given in July 2000 was too harsh, since only one month later S&P raised its rating to B-, thus justifying the LRS prediction

Cross-validation of Relative Preferences

- LAD can be susceptible to overfitting → cross-validate!
- Cross-validation by "**jackknife**" (JK): remove 1 country from the dataset, infer the LAD discriminant, and then re-calculate the relative preferences for all pseudo-observations involving the removed country, and all the LRS; repeat for other countries
- **Canonical** relative preferences based on LRS: $d_{ij} = \beta_i - \beta_j$
- In-sample and cross-validated correlations (**no overfitting**!):

	$d^{S\&P}$	Δ	d^{LRS}	Δ^{JK}	d_{JK}^{LRS}
$d^{S\&P}$	100%	93.21%	95.54%	92.98%	95.26%
Δ	93.21%	100%	97.57%	96.48%	96.36%
d^{LRS}	95.54%	97.57%	100%	96.35%	96.89%
Δ^{JK}	92.98%	96.48%	96.35%	100%	97.43%
d_{JK}^{LRS}	95.26%	96.36%	96.89%	97.43%	100%

Importance of Variables

- In LAD, the importance of variables is associated with their **participation in the patterns** of the discriminant
- In 1998 LAD model the three variables that appear most frequently in the patterns of the LAD discriminant are:
  - **financial depth and efficiency** (appearing in 47.5% of the patterns),
  - **political stability** (appearing in 39.4% of the patterns), and
  - **gross domestic product per capita** (appearing in 35.6% of the patterns)
- Most studies on country risk ratings acknowledge the key importance of gross domestic product per capita in evaluating the solvency of a country
- A political variable appearing among the three most significant ones in the selected set provides additional justification for the inclusion of **political variables** in country risk rating models

Concluding Remarks

- The proposed LRS-based rating model is highly **accurate**, having a 95.5% correlation level with the actual S&P's ratings
- The model **avoids overfitting**, as demonstrated by the 96% correlation between in- and out-of-sample rating predictions, and exhibits **temporal stability** → capable of predictions
- The model is **transparent** since it makes the role of the economic-financial and political variables explicit
- The model is **non-recursive** since it does not rely on any information derived from lagged ratings, and is capable of rating **previously unrated countries**
- The few discrepancies between the S&P's and the model's ratings were resolved by subsequent changes in S&P's ratings
- The methodology allows for the construction of a discrete rating system with the **number of rating categories specified by the user**