# Models of voting power in corporate networks

*European Journal of Operational Research* (2007)

Yves Crama

HEC Management School, University of Liège

Luc Leruth

International Monetary Fund

# PLAN:

1. *Introduction: shareholder networks and measurement of control*
2. *Simple games, Boolean functions and Banzhaf index*
3. *Application to the analysis of financial networks*
4. *Further questions*

# Corporate networks

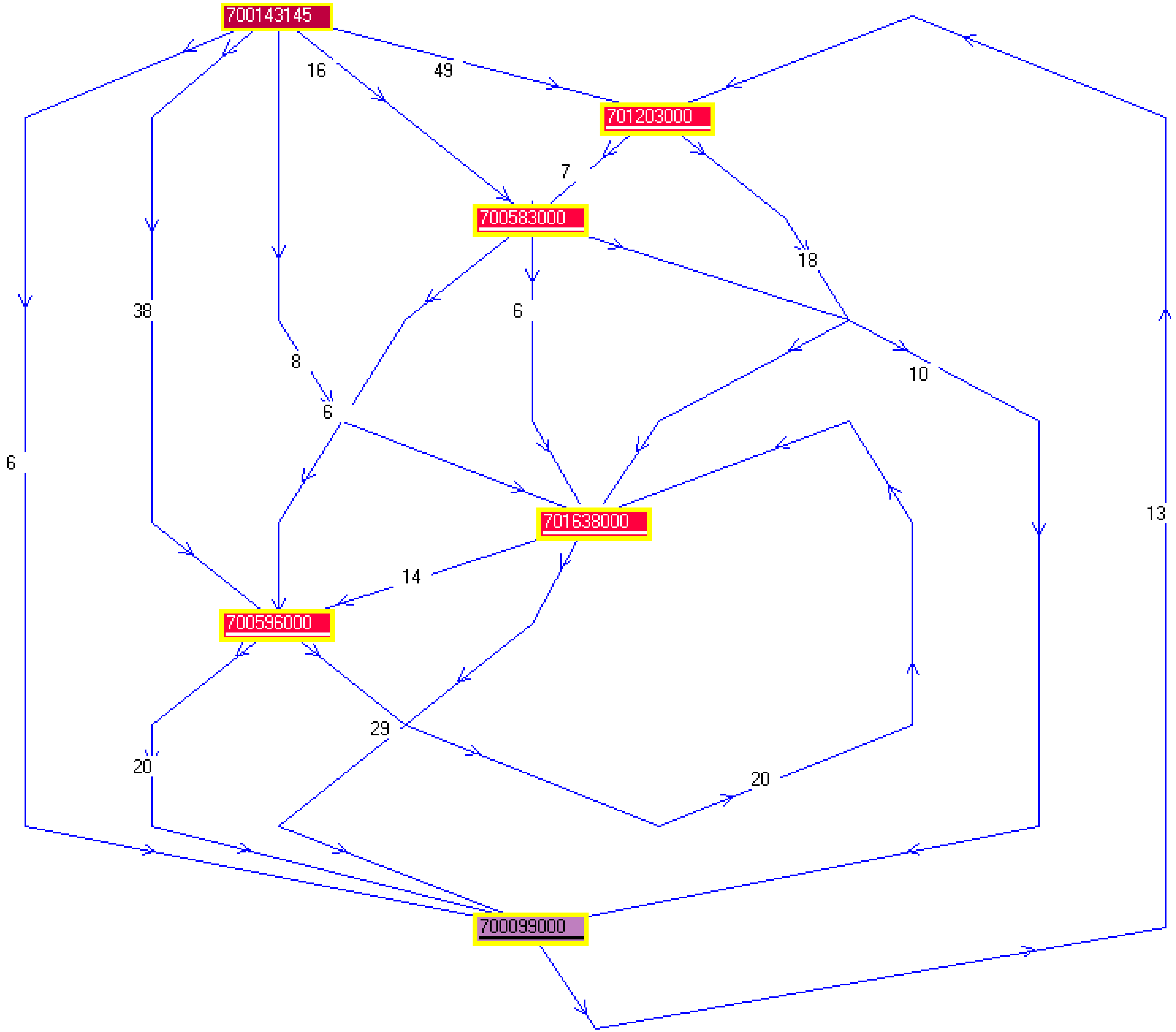
Objects of study:

- networks of entities (firms, banks, individual owners, pension funds,...) linked by shareholding relationships;
- their structure;
- notion and measurement of control in such networks.

# Corporate networks

Graph model:

- nodes correspond to firms
- arc $(i,j)$ indicates that firm $i$ is a shareholder of firm $j$
- the value $w(i,j)$ of arc $(i,j)$ indicates the fraction of shares of firm $j$ which are held by firm $i$



700143145
701203000
700583000
701638000
700596000
700099000
16
49
7
18
38
6
8
6
10
6
13
14
29
20
20

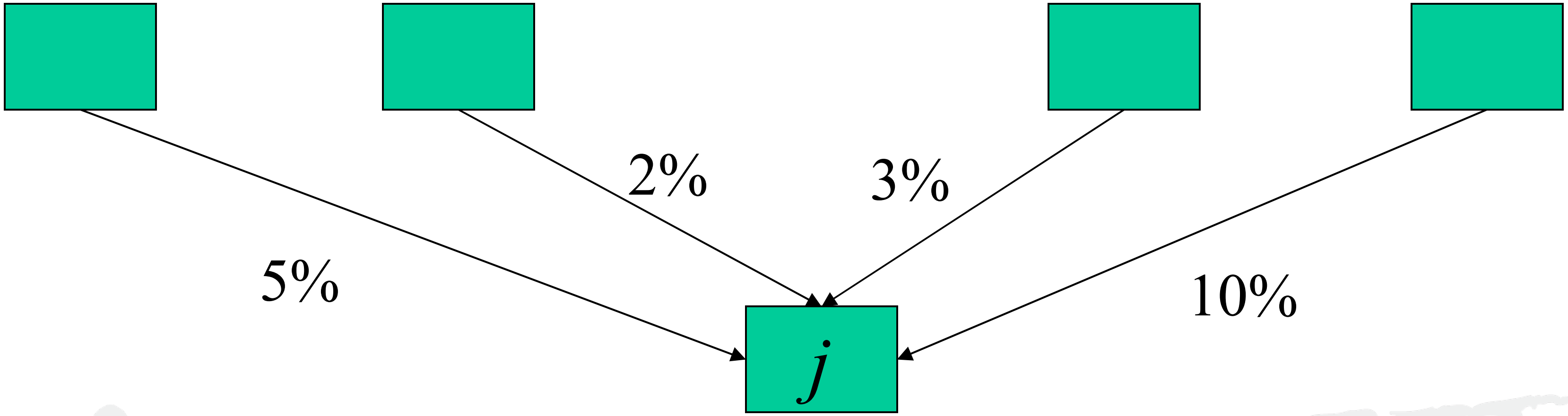
# Outsider vs insider system

Two types of systems are observed in practice:

# Outsider vs insider system

1. The outsider system:

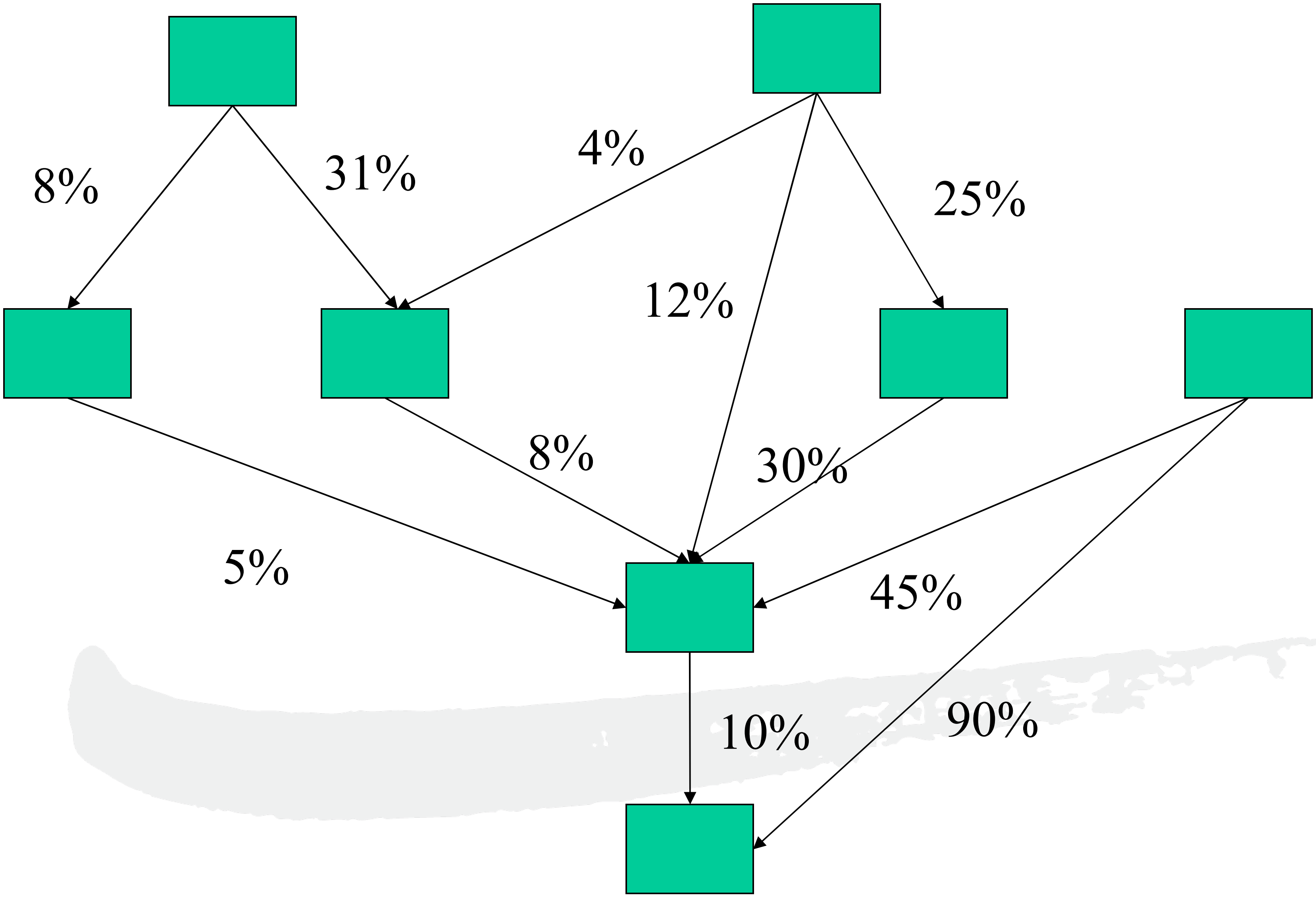
- single layer of shareholders;
- dispersed ownership, high liquidity;
- transparent, open to takeovers;
- weak monitoring of management;
- typical of US and British stock markets.

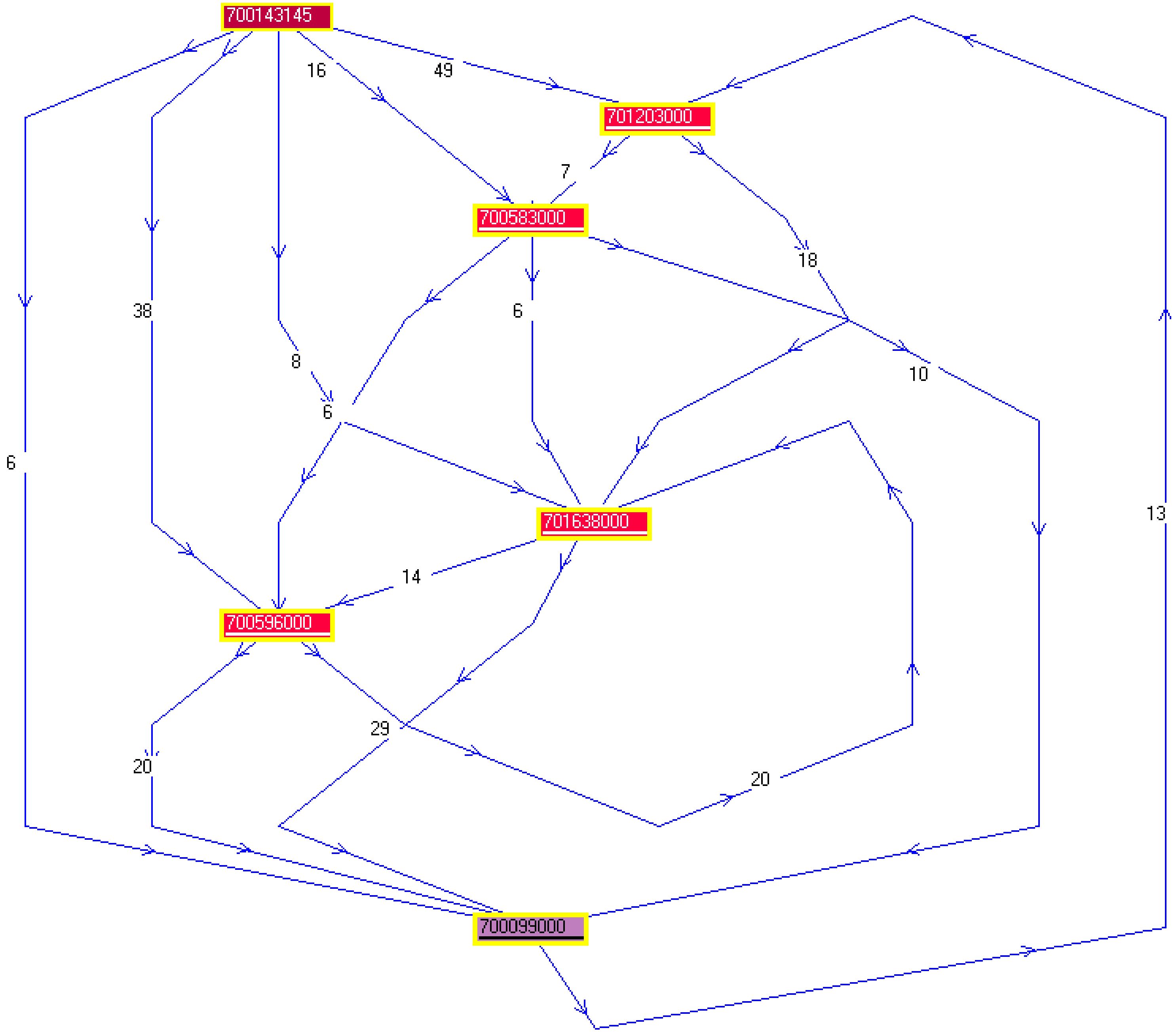

2%
3%
5%
10%
$j$

# Outsider vs insider system

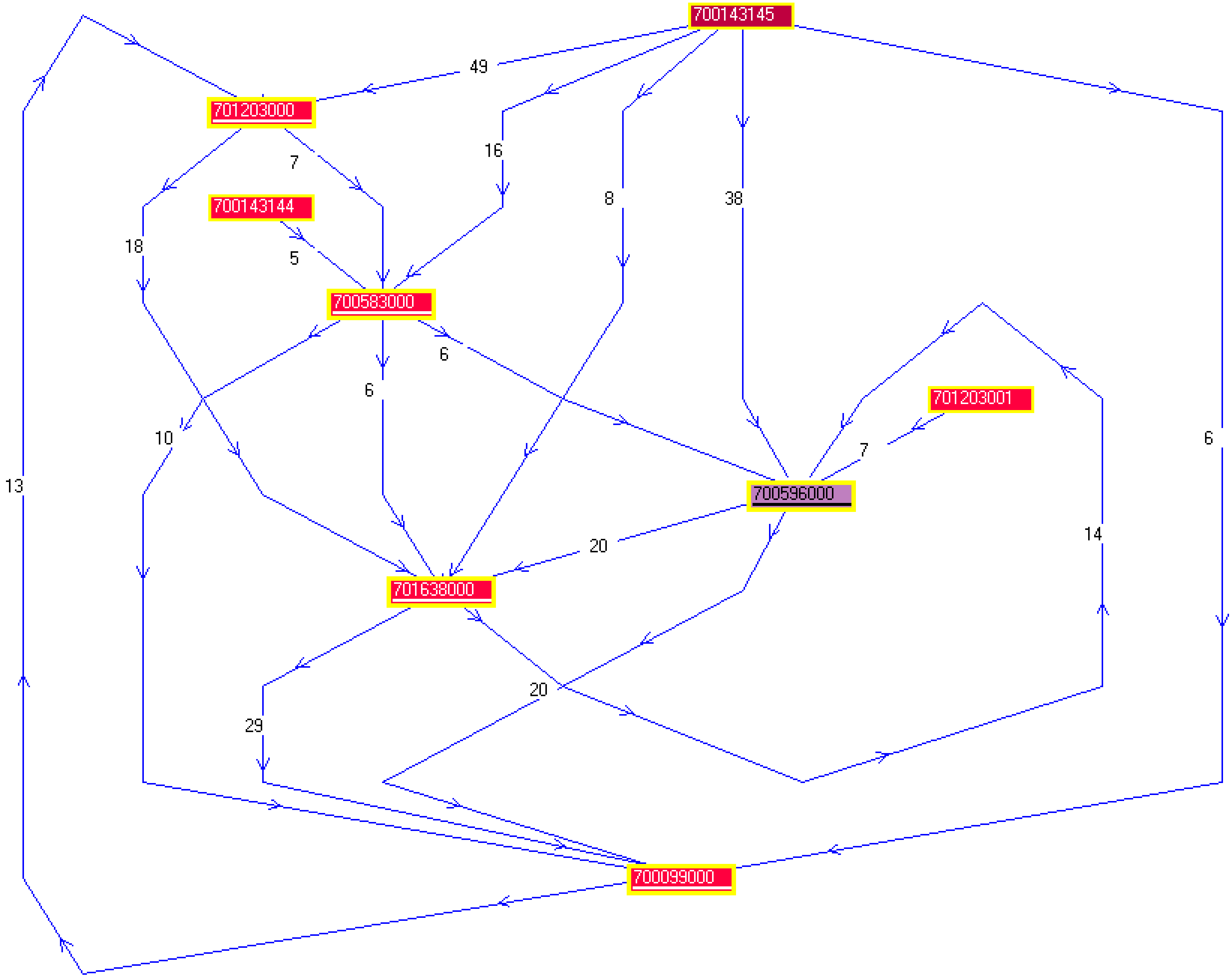
2. The insider system:

- multiple layers of shareholders, possibly involving cycles;
- concentrated ownership, low liquidity; controlling blocks;
- strong monitoring of management;
- typical of Continental Europe and Asia (Japan, South Korea, …).


8%
31%
4%
25%
12%
8%
30%
5%
45%
10%
90%



700143145
701203000
700583000
701638000
700596000
700099000
16
49
7
18
38
8
6
6
10
6
13
14
29
20
20



700143145
701203000
49
16
8
38
7
700143144
18
5
700583000
6
6
701203001
10
7
6
13
700596000
20
14
701638000
20
29
700099000

# Control in financial networks

Numerous authors have analyzed the issue of control in financial networks. Note: it is not necessary to own more than 50% of the shares in order to control a firm. It has been argued that 20% to 30% are often sufficient.

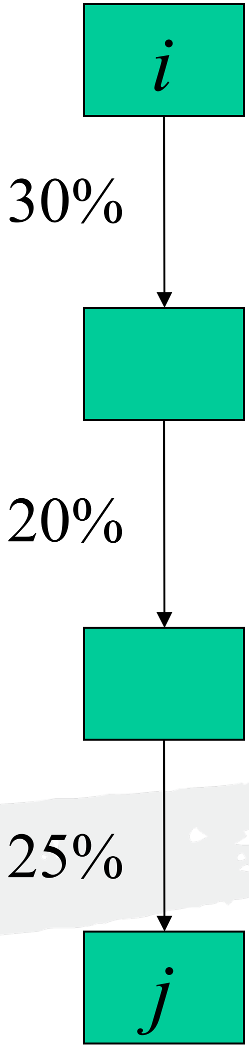
# Control in financial networks

Three main types of models.

1. Consider that firm $i$ controls firm $j$ if there is a « chain » of shareholdings, each with value at least $x$%, from firm $i$ to firm $j$.

Control:
$x = 20\%$

$i$ controls $j$

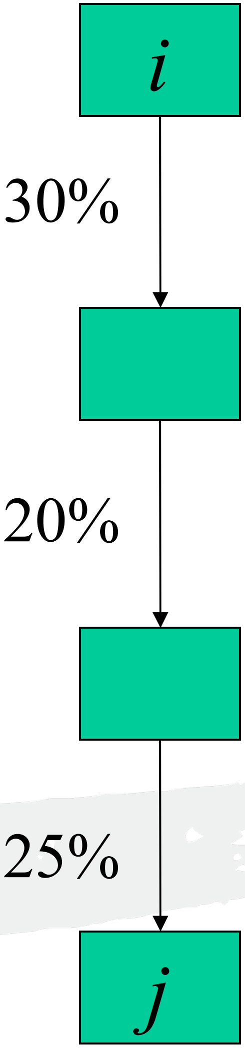


# Control in financial networks

This (or similar) models suffer from several weaknesses.
In particular, they cannot easily be extended to more complex networks because they do not account for the whole distribution of ownership.
Compare the following networks…

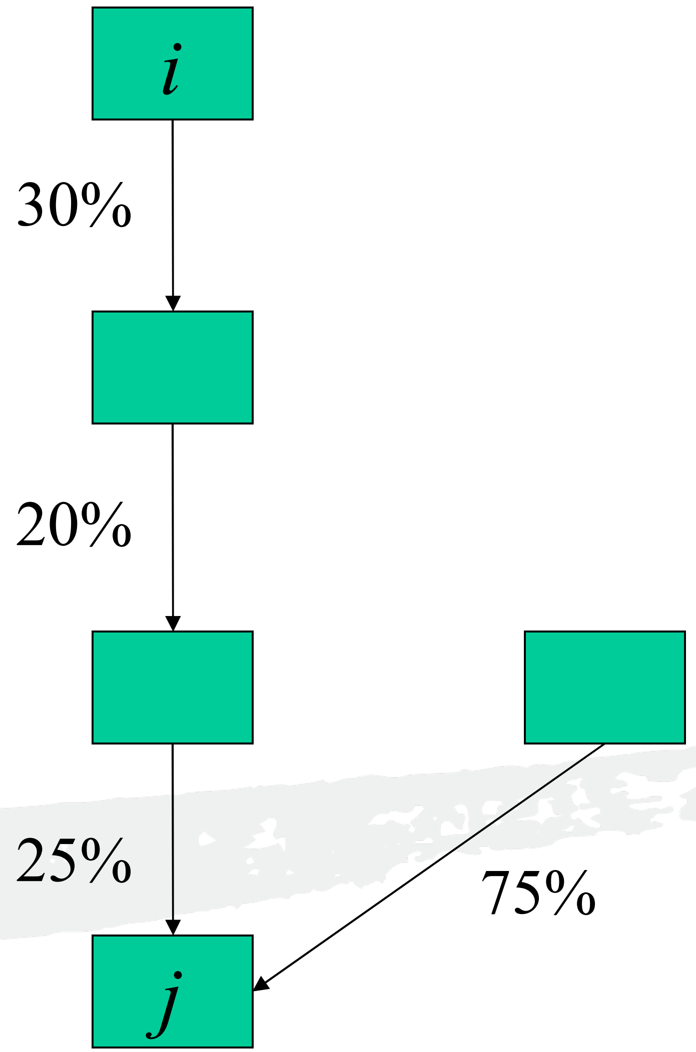
Control:
$x = 20\%$

$i$ controls $j$



Control:
$x = 20\%$

*i* controls *j* ??

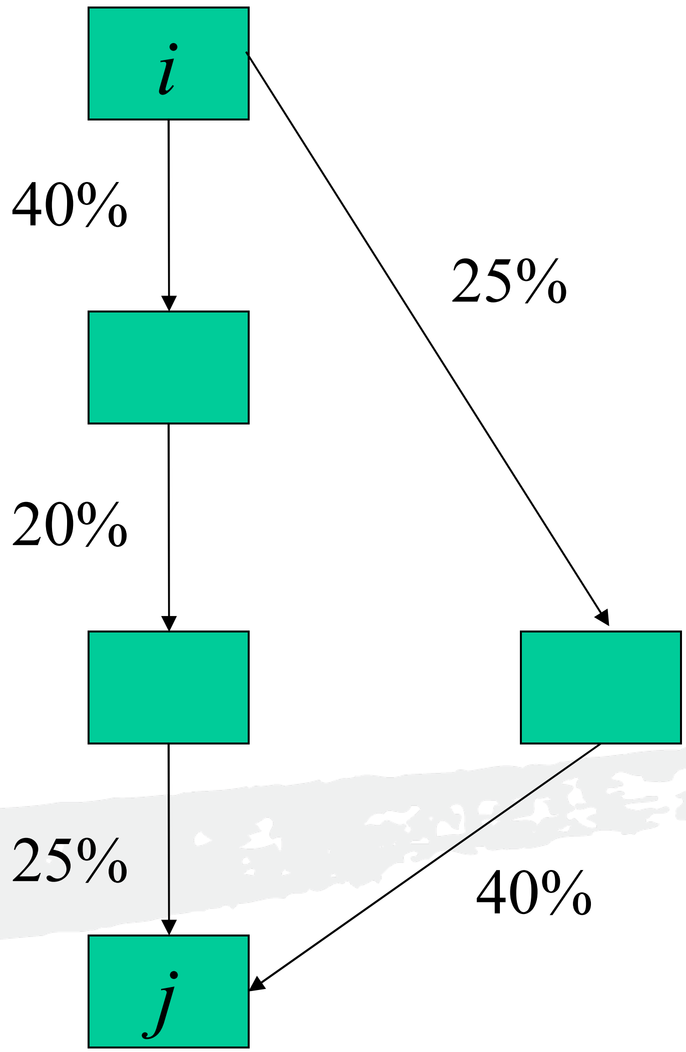


# Control in corporate networks

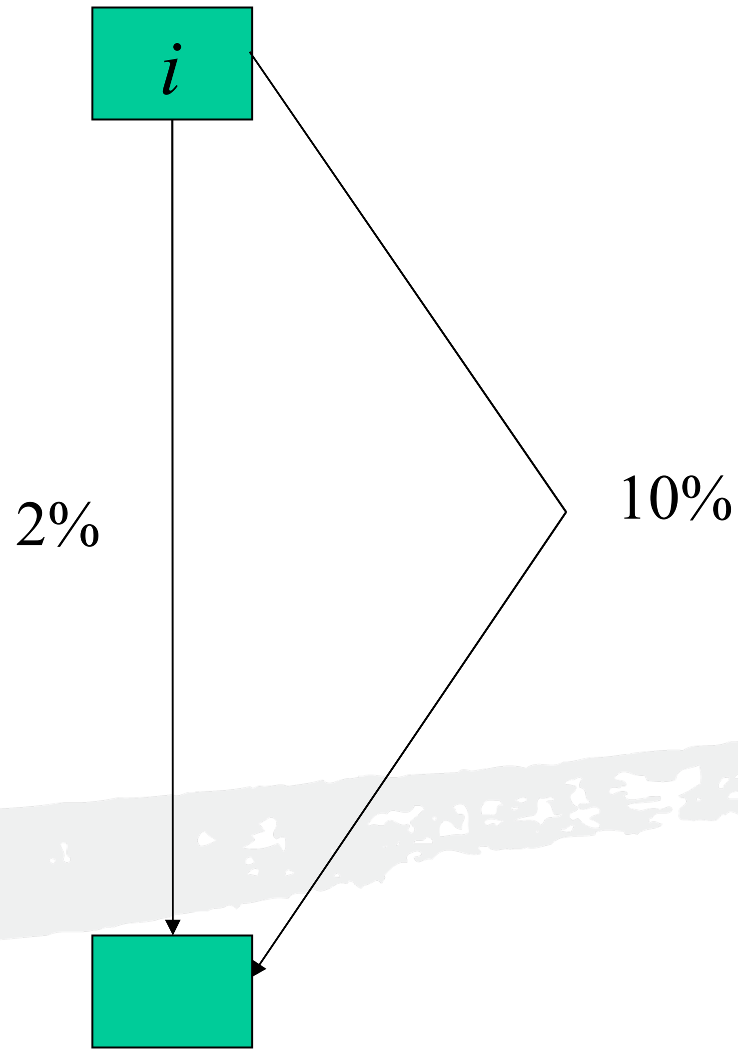
A second type of model:

2. Multiply the shareholdings along each path of indirect ownership; add up over all paths.

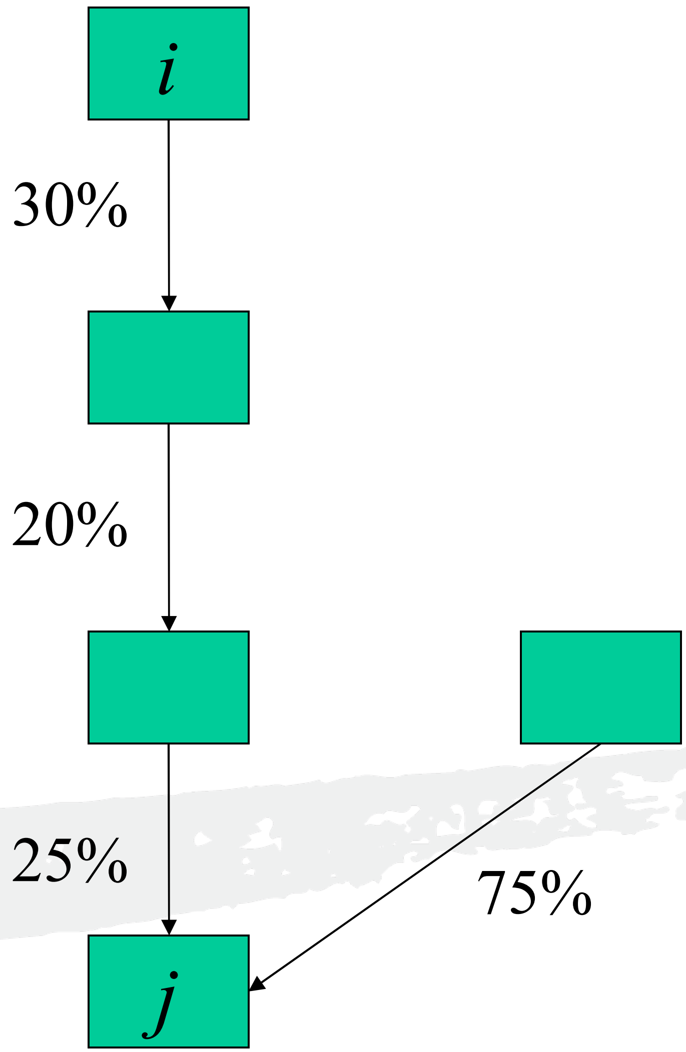
Direct ownership

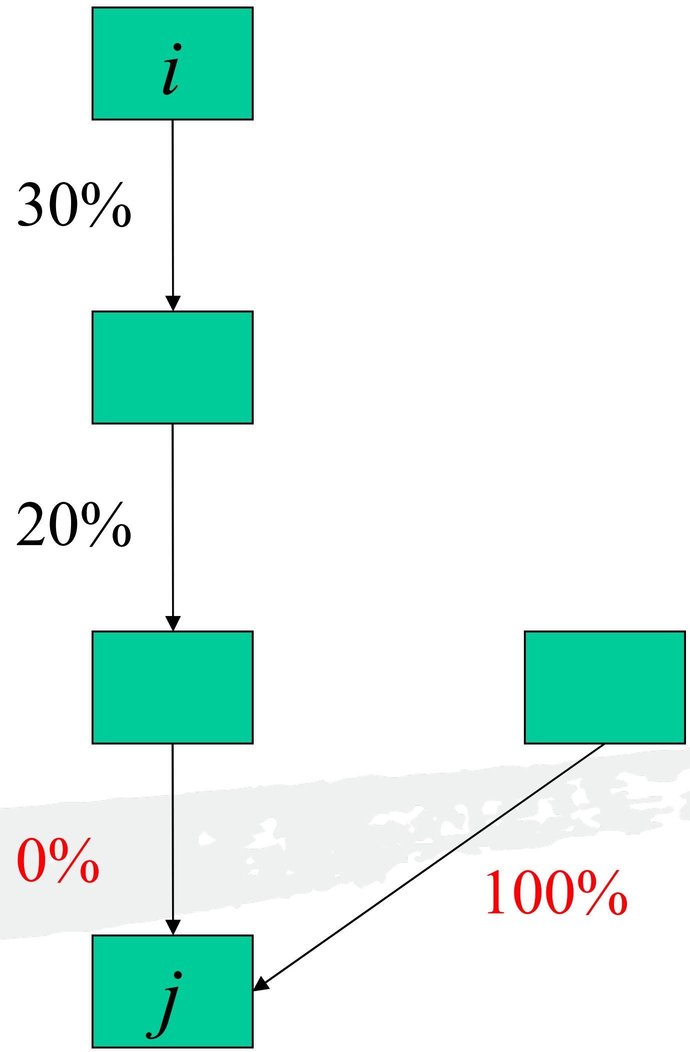


# Indirect ownership



From the point of view of control, however, several authors observe that the following situations are equivalent (e.g. Chapelle and Szafarz 2005) :


i
30%
20%
25%
75%
j



i
30%
20%
0%
100%
j

# Control in corporate networks

A third type of model:

3. Look at the shareholders of firm $j$ as playing a weighted majority game whenever a decision is to be made by firm $j$.

# Reminder: Simple games

A *simple game* on the player-set $N=\{1,2,\ldots,n\}$ is a monotonically increasing function

$$v : 2^N \rightarrow \{0,1\},$$

where $2^N$ is the power set of $N$.

# **Simple games** (2)

Interpretation: v describes the voting rule which is adopted by the set of actors $N$ in order to make a decision on any given issue.

If $S$ is a subset of players, then $v(S)$ is the outcome of the voting process when all players in $S$ vote Yes.

# Weighted majority games

A common example : weighted majority games

- player $i$ carries a *voting weight* $w_i$

- $q$ is the *quota* required to pass a resolution.

- $v(S) = 1$ iff $\sum_{i \in S} w_i > q$

# Weighted majority games (2)

Example:

- Shareholder meeting: $w_i$ is the number of (voting) shares held by shareholder $i$; $v(S) = 1$ iff $S$ holds at least one half of the shares.

# **Boolean functions** (1)

Connection with Boolean functions:
identify every set of players $S$ with its characteristic vector.

Example:

$$S = \{3,5,6\} \leftrightarrow X = (0,0,1,0,1,1),$$

$$v(S) = 1 \leftrightarrow v(0,0,1,0,1,1) = 1.$$

# **Boolean functions** (2)

A simple game is a monotonically increasing (or positive) Boolean function.
A weighted majority game is a threshold function.

# Simple games (3)

The *Banzhaf index Z* of player *k* is the probability that, in a random voting pattern (uniformly distributed), the outcome of the game changes (e.g. from 0 to 1) when player *k* changes her mind (e.g., from 0 to 1).
Or: probability that player *k* is a *swing player*.

# Simple games (4)

The *Banzhaf index* $Z_k$ of player $k$ is given by

$$Z_k = \Sigma_{k \in T \subseteq N} [v(T) - v(T \setminus k)] / 2^{n-1}$$

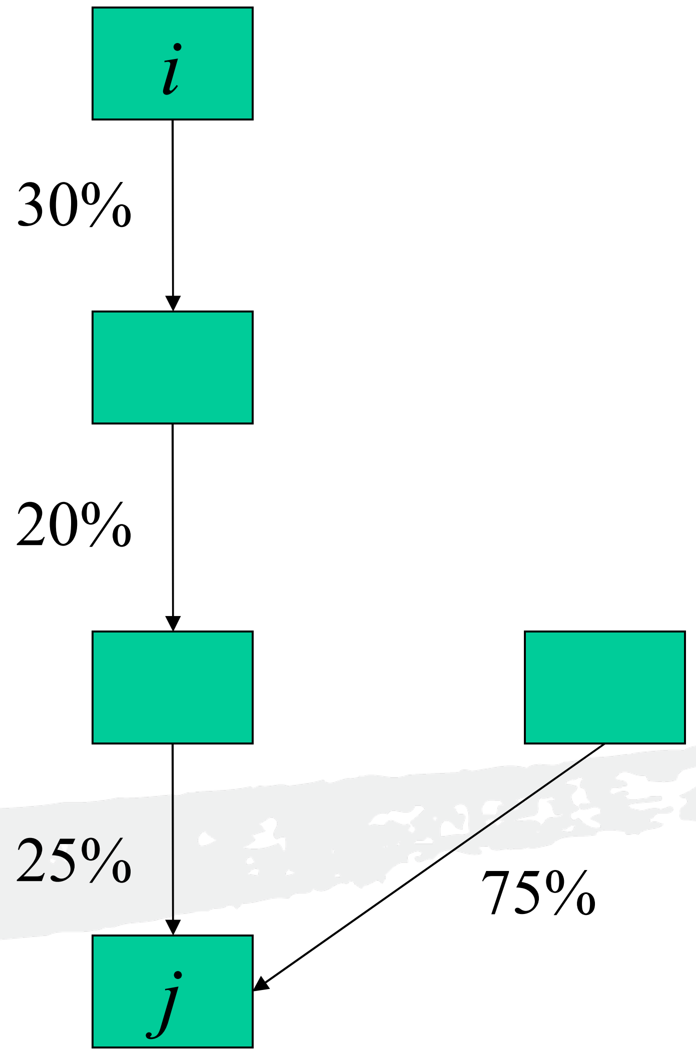
# Simple games (5)

The Banzhaf index provides a measure of the *influence* or *power* of player *k* in a voting game. (Banzhaf, *Rutgers Law Review* 1965) The index is related to, but different from the Shapley-Shubik index

# Simple games (6)

For a weighted majority game, the Banzhaf index $Z_k$ is usually different from (and is not proportional to) the voting weight $w_k$ of player $k$.

This is OK: remember the example.



i
30%
20%
25%
75%
j

# Boolean functions (3)

Chow has introduced $(n+1)$ parameters associated with a function $f(x_1,\dots,x_n)$

$$(\omega, \omega_1, \dots, \omega_n)$$

where

- $\omega$ is the number of « true points » of $f$
- $\omega_k$ is the number of « true points » of $f$

where $x_k = 1$.

# Boolean functions (4)

<u>Theorem</u> (Chow): Within the class of threshold functions, every function is uniquely characterized by its Chow parameters (i.e., no two functions have the same Chow parameters).

# Boolean functions (5)

$\omega_k$ is the number of « true points » of $f$ where $x_k = 1$.

Hence $\omega_k / 2^{n-1}$ is the probability that $f$ takes value 1 when $x_k$ takes value 1.

This can be interpreted as a measure of the *importance* or the *influence* of variable $k$ for $f$.

# Boolean functions (6)

Not surprisingly, the Banzhaf indices are simple transformations of the Chow parameters:

$$Z_k = (2\ \omega_k - \omega) / 2^{n-1}.$$

# Back to corporate networks…

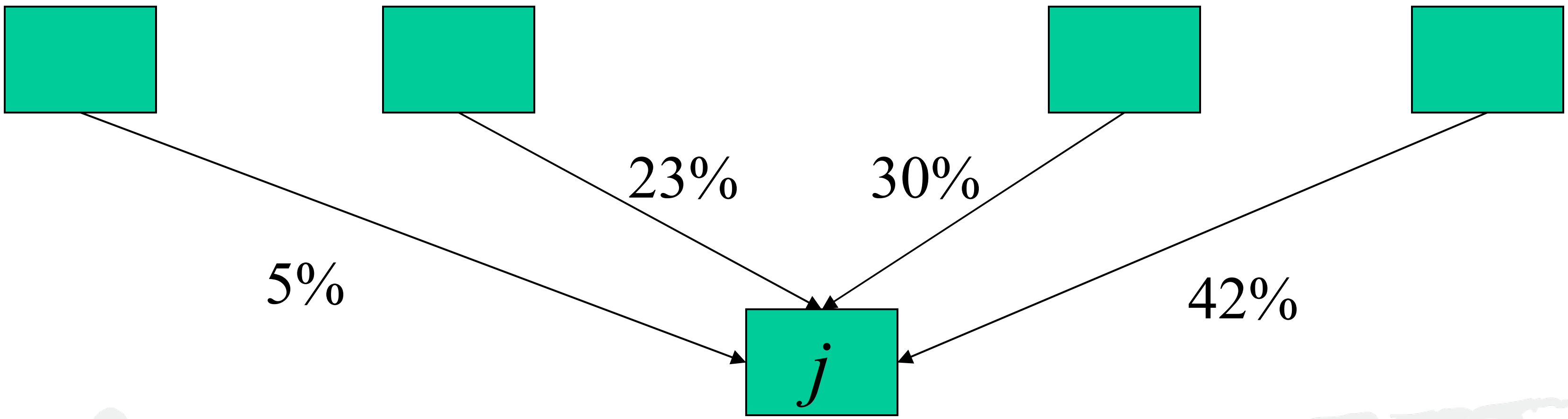
Look at the shareholders of firm $j$ as playing a weighted majority game (with quota 50%) whenever a decision is to be made by firm $j$.

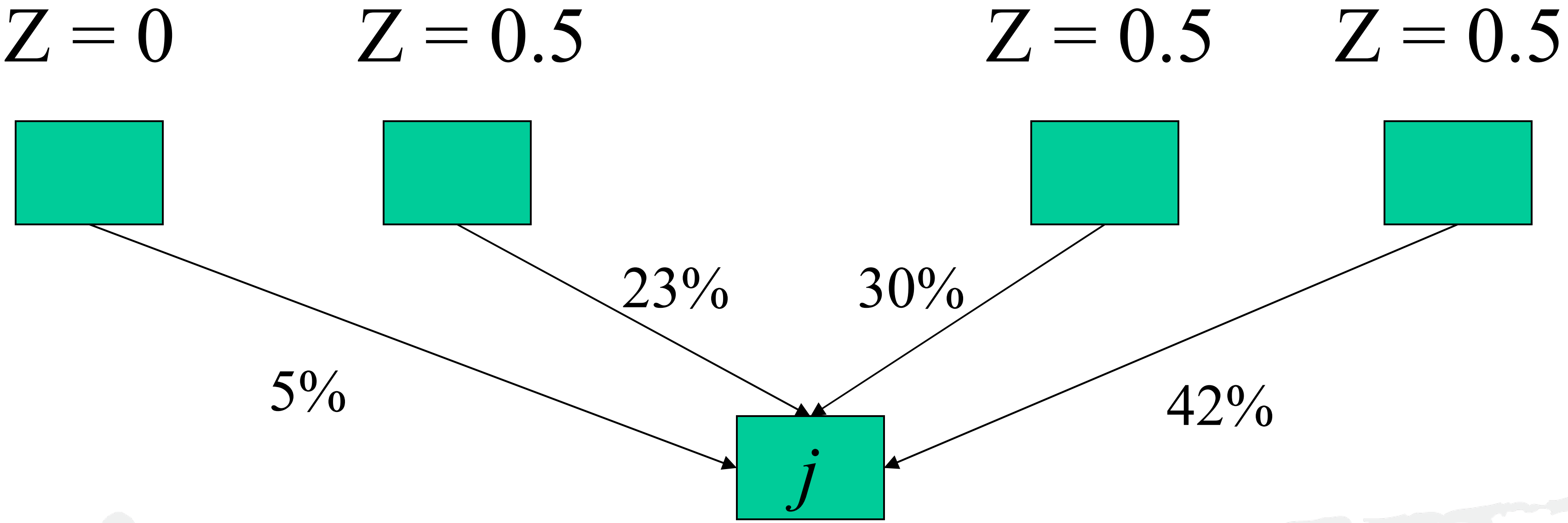
In this model, the level of control of firm $i$ over firm $j$ can be measured by the Banzhaf index $Z(i,j)$ of player $i$ in the game associated with $j$.

# Banzhaf index of control

The index Z($i,j$) is equal to 1 if firm $i$ owns more than 50% of the shares of $j$.

More generally, Z($i,j$) is not proportional to the shareholdings $w(i,j)$.


23%
30%
5%
42%
$j$


Z = 0
Z = 0.5
Z = 0.5
Z = 0.5
23%
30%
5%
42%
j

# Banzhaf index of control

Power indices have been proposed for the measurement of corporate control by several researchers (Shapley and Shubik, Cubbin and Leech, Gambarelli, Zwiebel,…)

# Banzhaf index of control

Most applications have been restricted to single layers of shareholders (weighted majority games, outsider system).

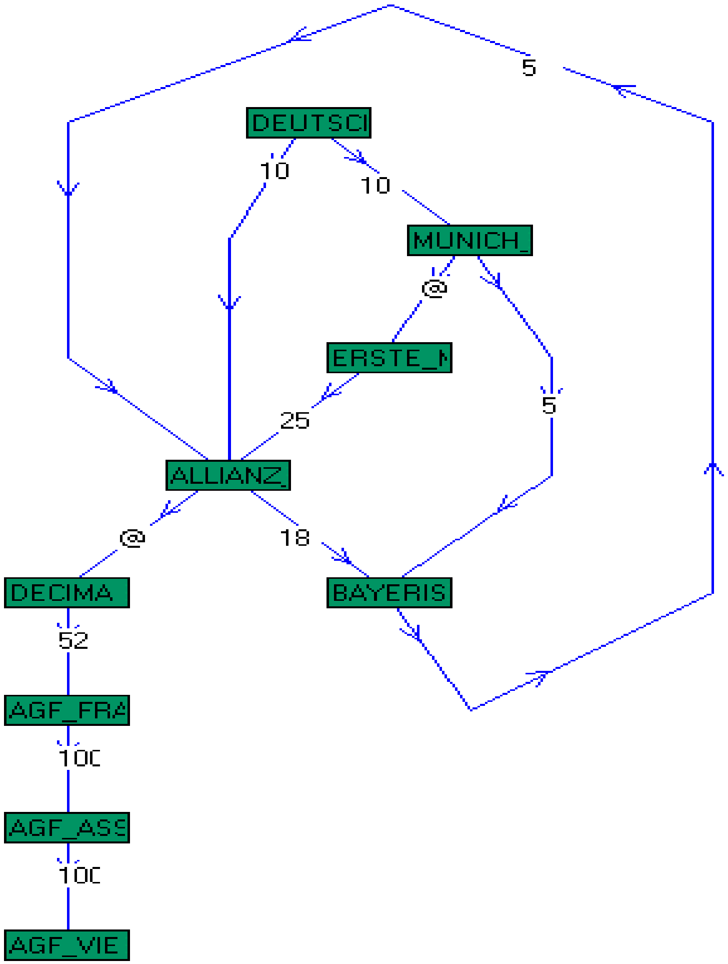
In this case, Banzhaf indices can be "efficiently" computed (dynamic programming pseudo-polynomial algo).

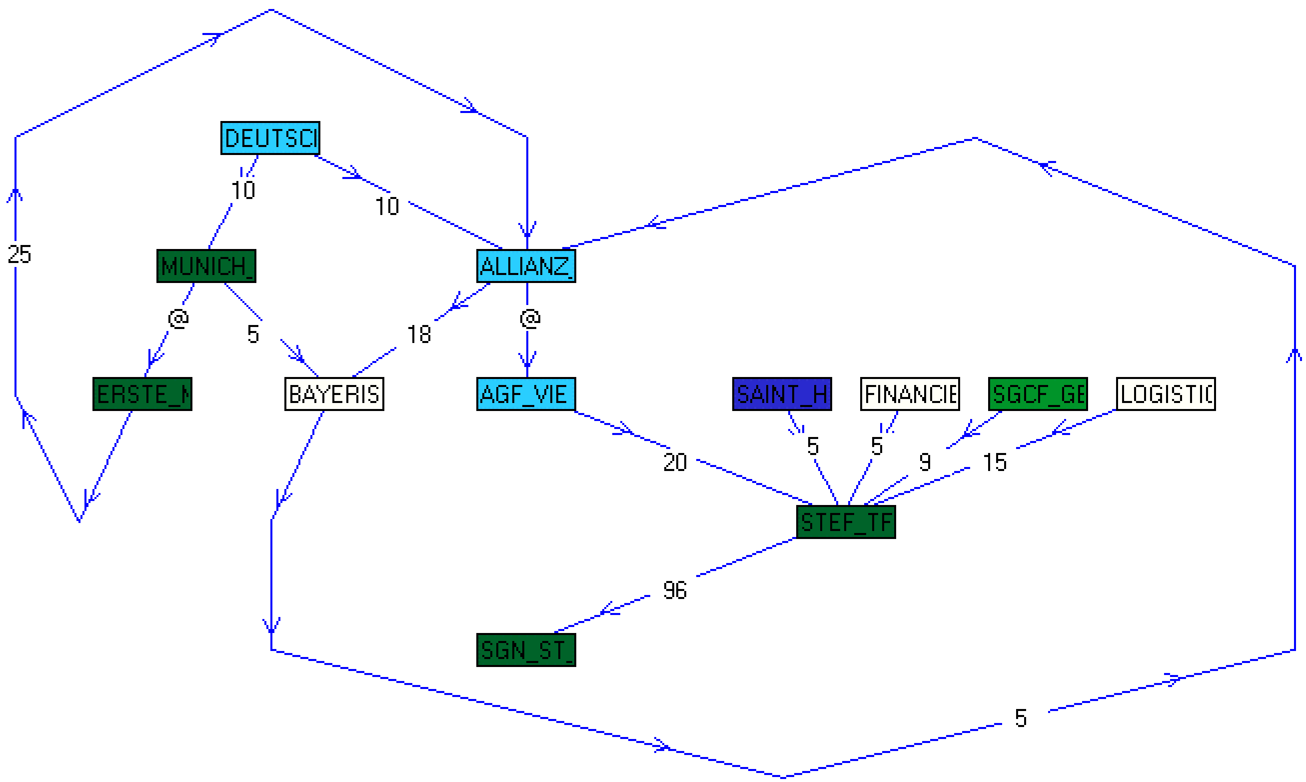
# Banzhaf index of control

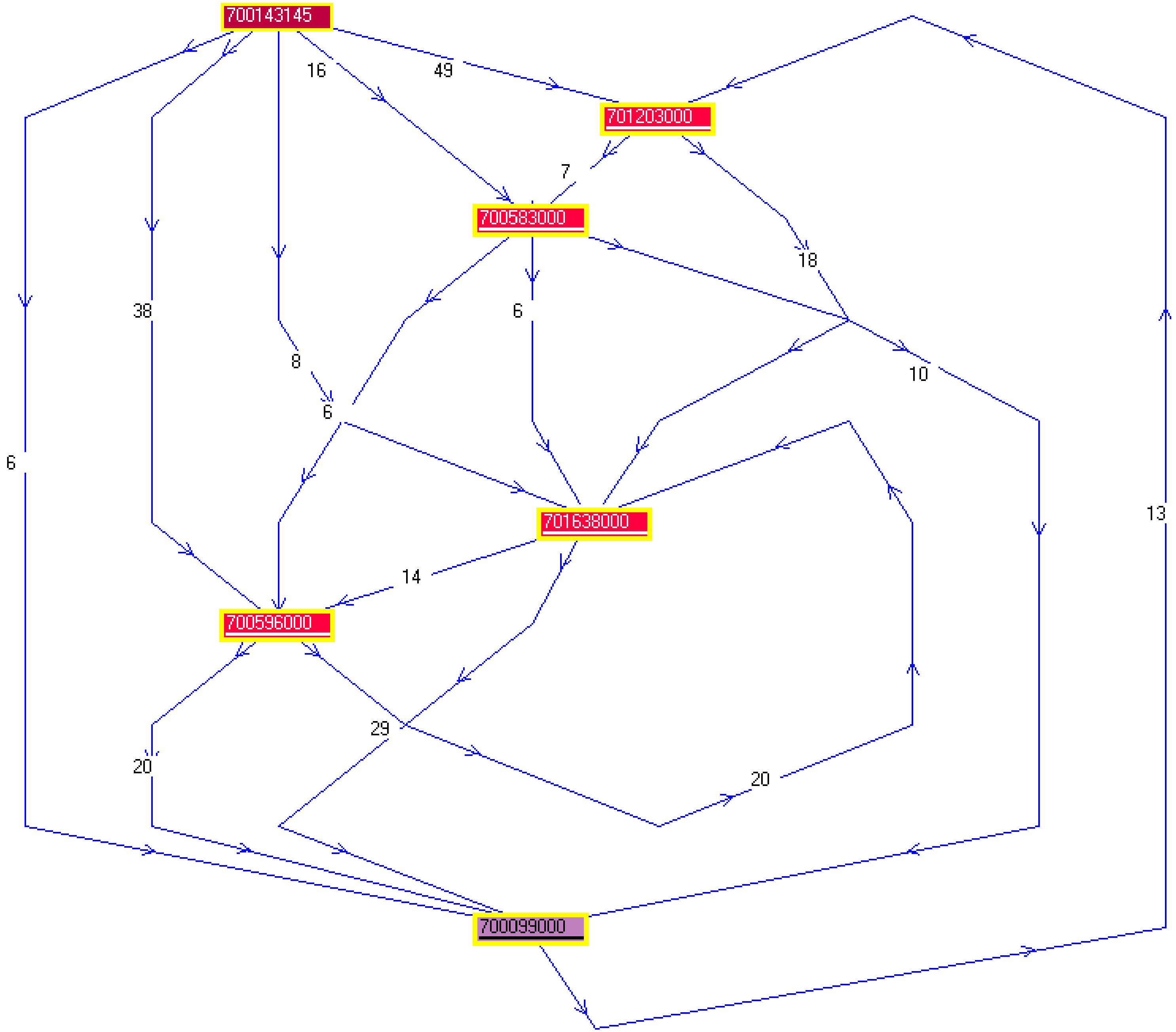
But real networks are more complex…

# Banzhaf index of control

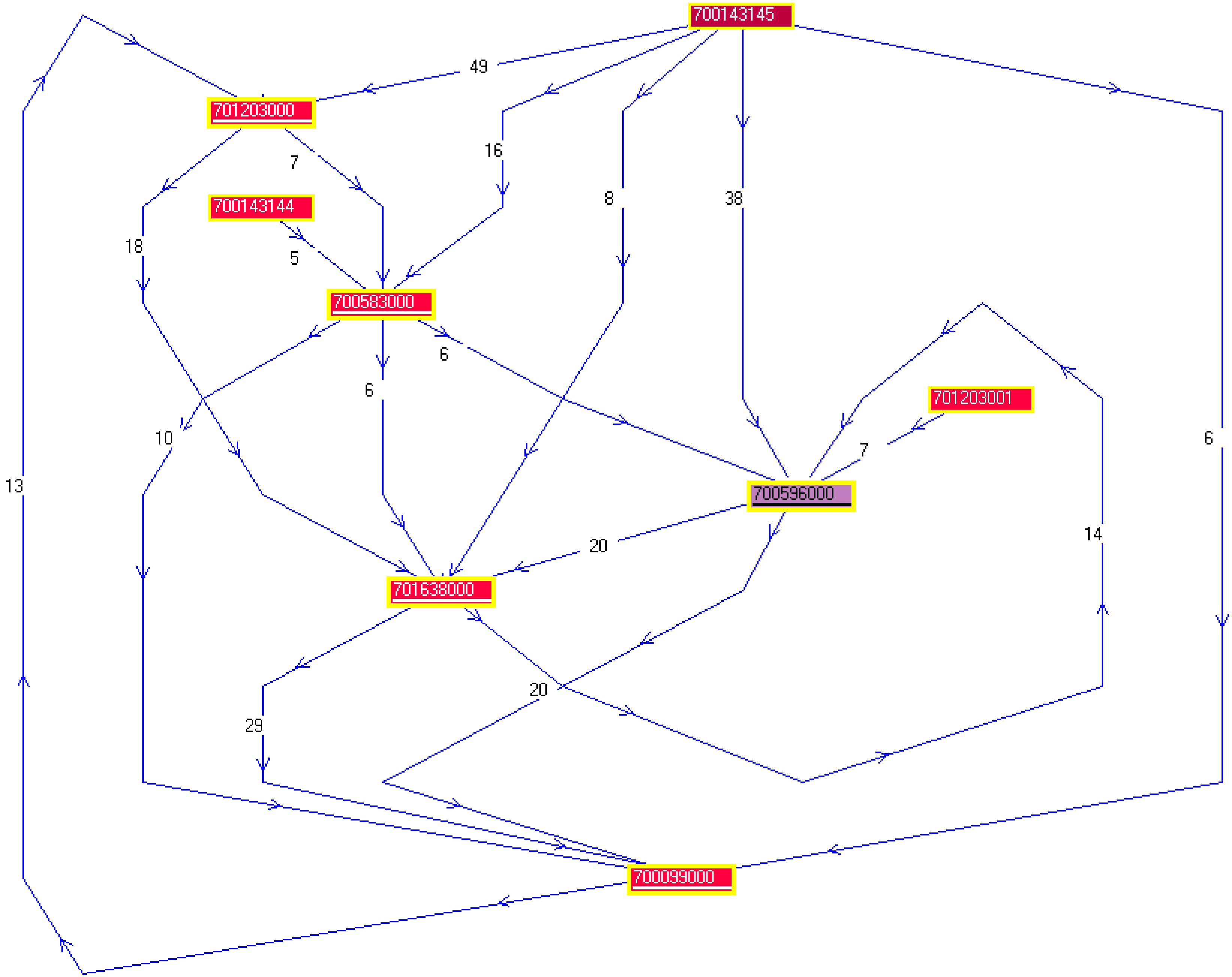
- Up to several thousand firms.
- Incomplete shareholding data (small holders are unidentified).
- Multilayered (pyramidal) structures.
- Cycles.
- Ultimate relevant shareholders are not univoquely defined.


5
DEUTSCI
10
10
MUNICH_
@
ERSTE_M
5
25
ALLIANZ_
@
18
DECIMA
BAYERIS
52
AGF_FRA
10C
AGF_ASS
10C
AGF_VIE


DEUTSCI
10
10
MUNICH_
ALLIANZ_
25
@
5
18
@
ERSTE_M
BAYERIS
AGF_VIE
SAINT_H
FINANCIE
SGCF_GE
LOGISTIC
5
5
9
15
20
STEF_TF
96
SGN_ST_
5

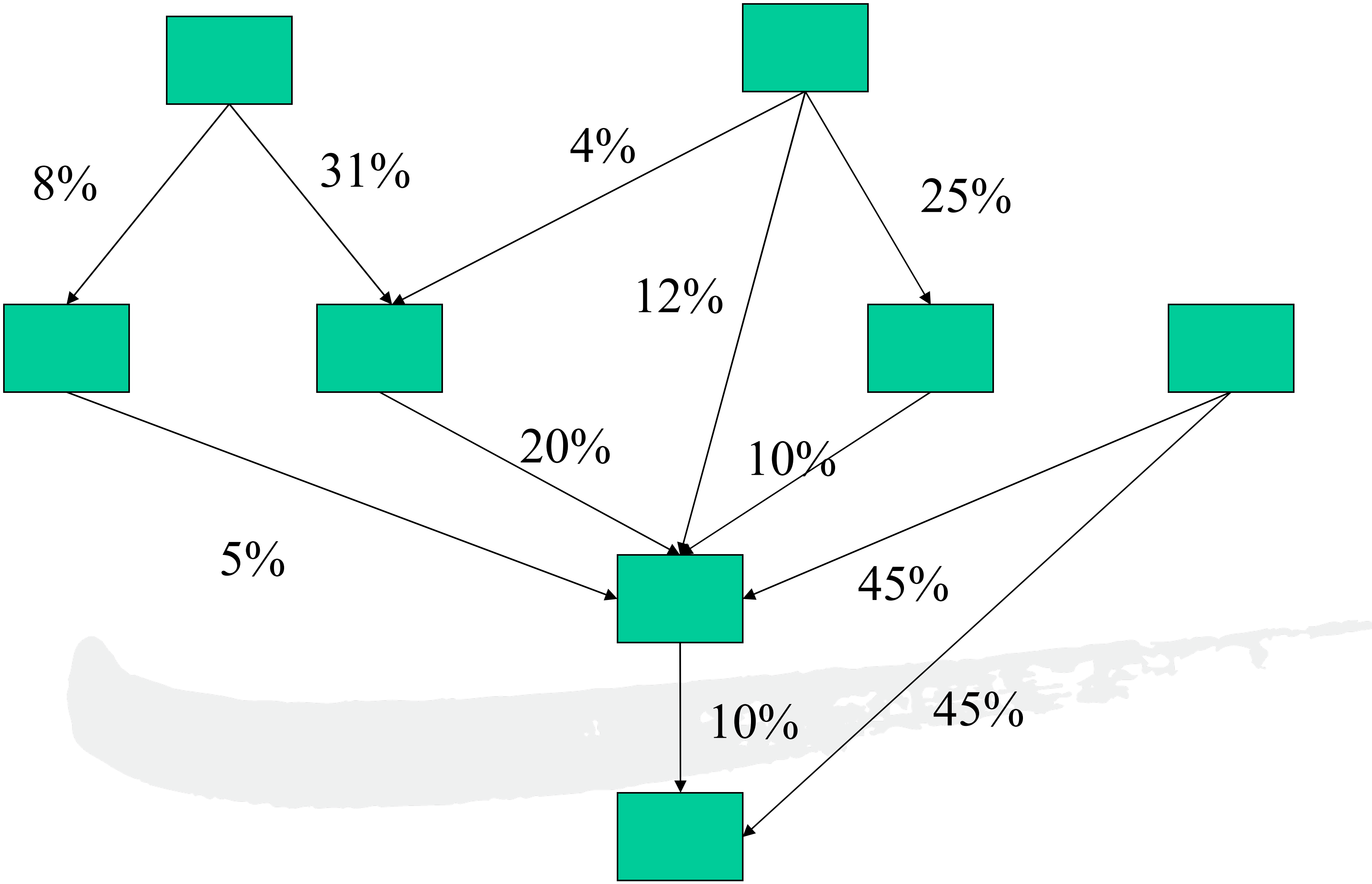


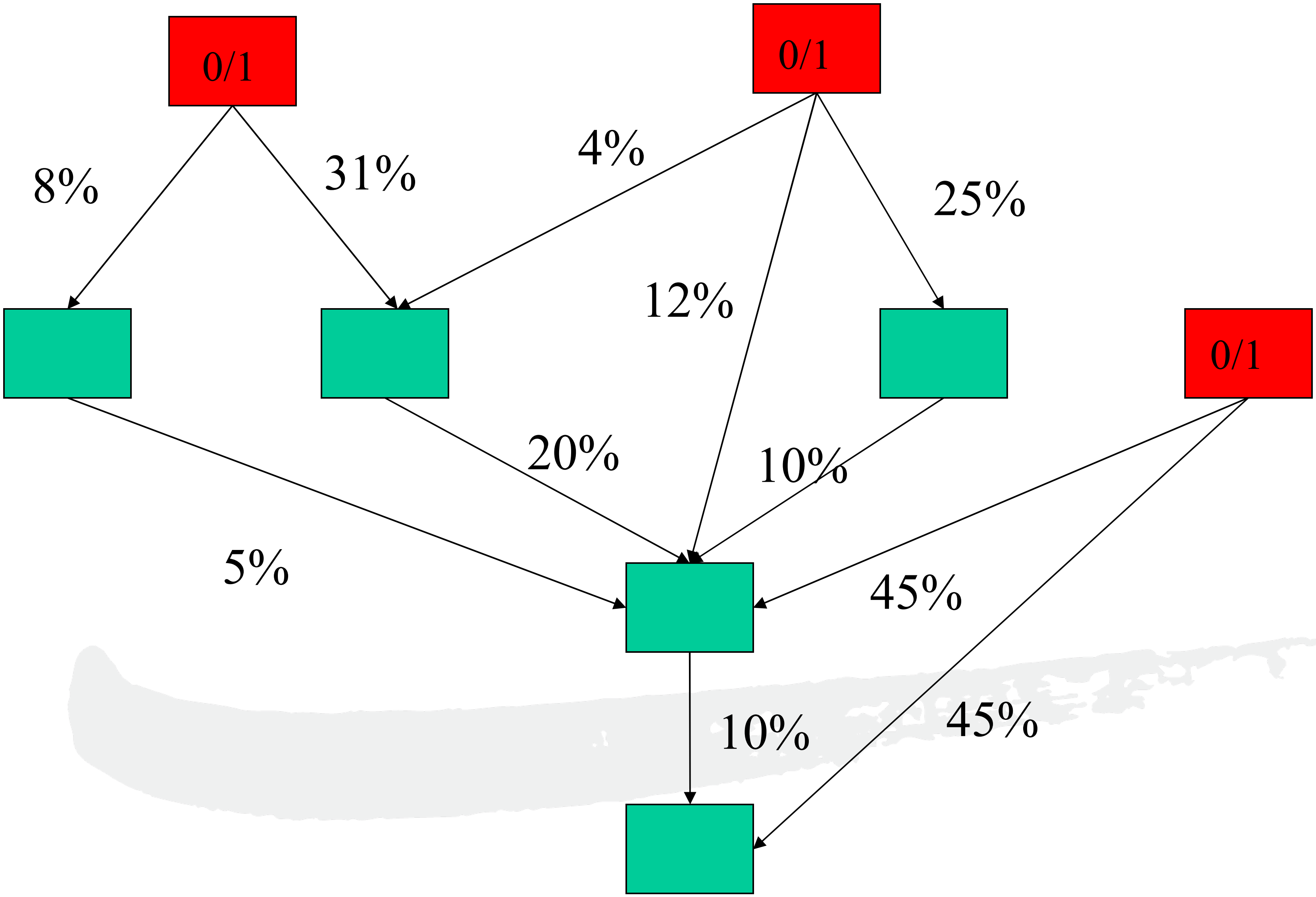
700143145
16
49
701203000
7
700583000
18
38
6
8
10
6
6
701638000
13
14
700596000
29
20
20
700099000

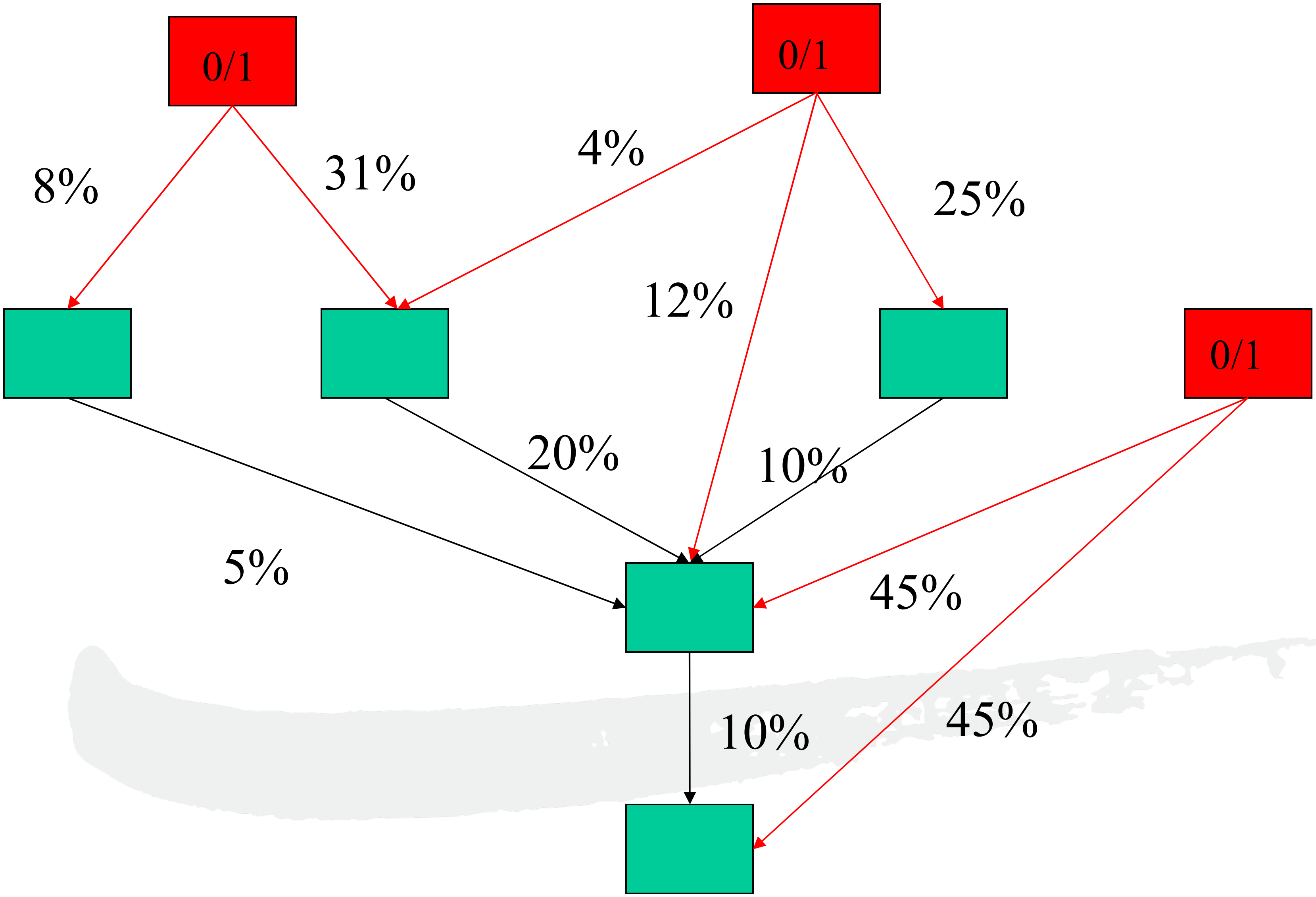

700143145
49
701203000
16
8
38
7
700143144
18
5
700583000
6
6
701203001
10
7
6
13
700596000
14
20
701638000
20
29
700099000

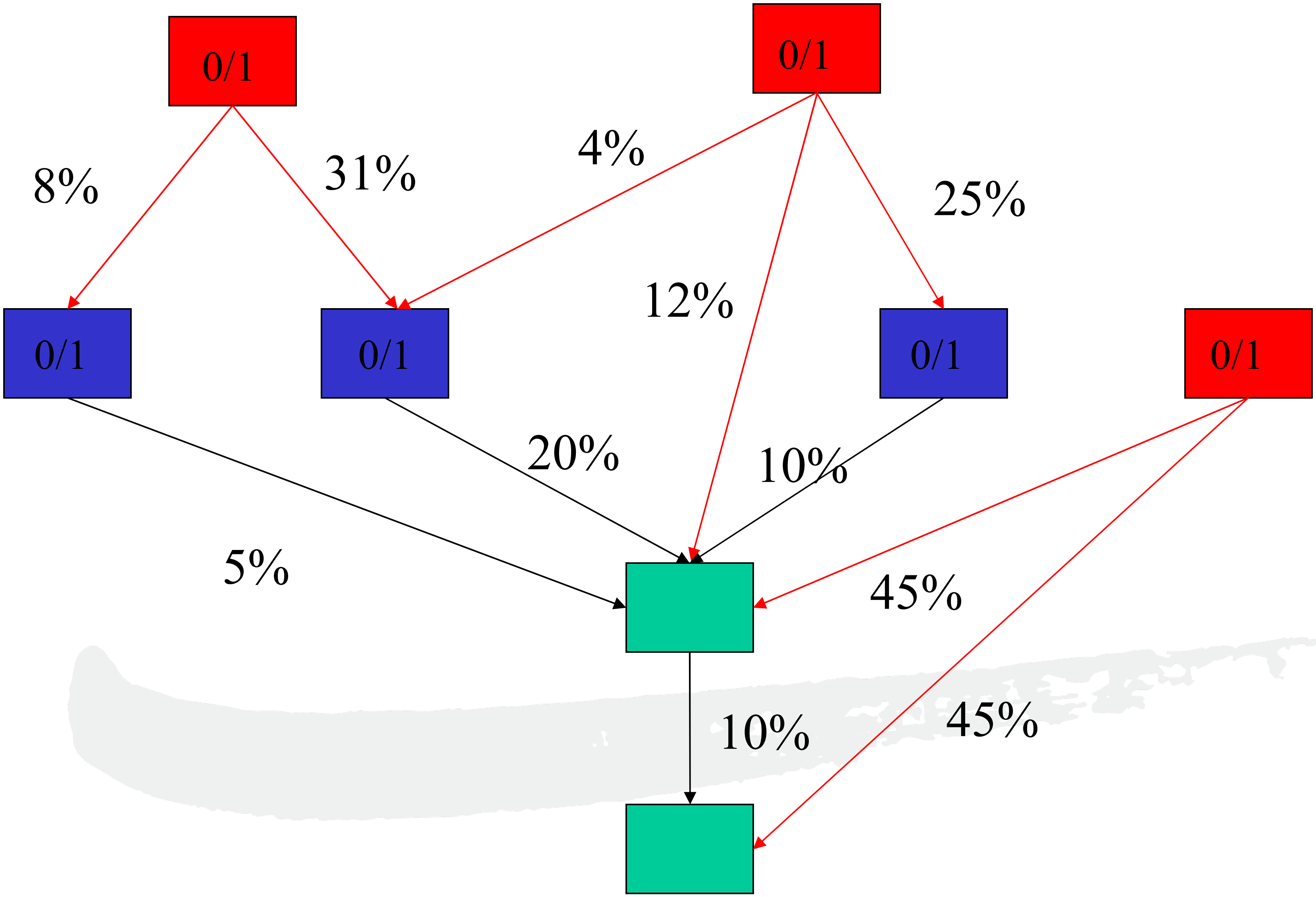
# Analysis of complex networks

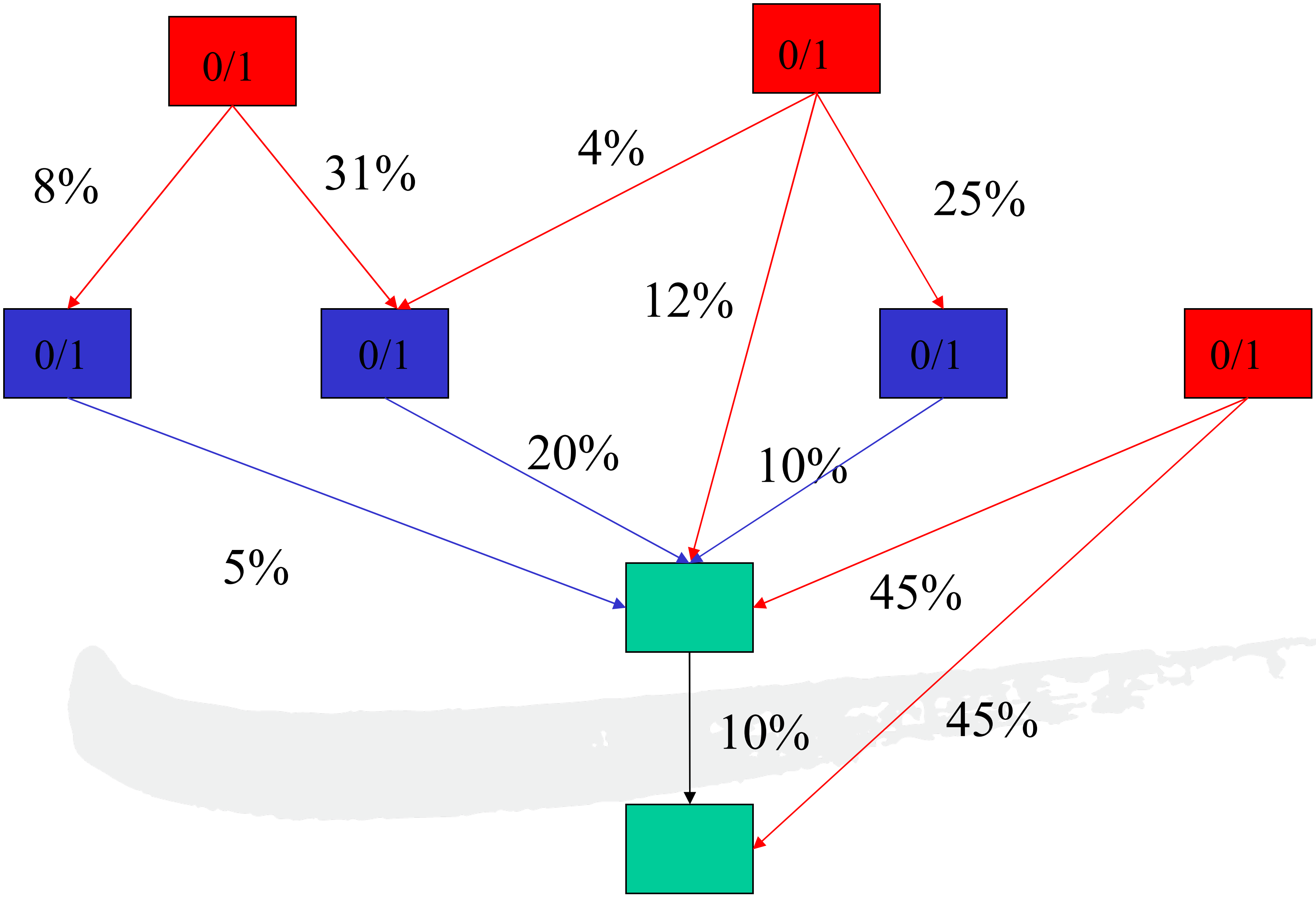
We extend previous studies:
- look at multilayered networks as defining compound games, i.e. compositions of weighted majority games;

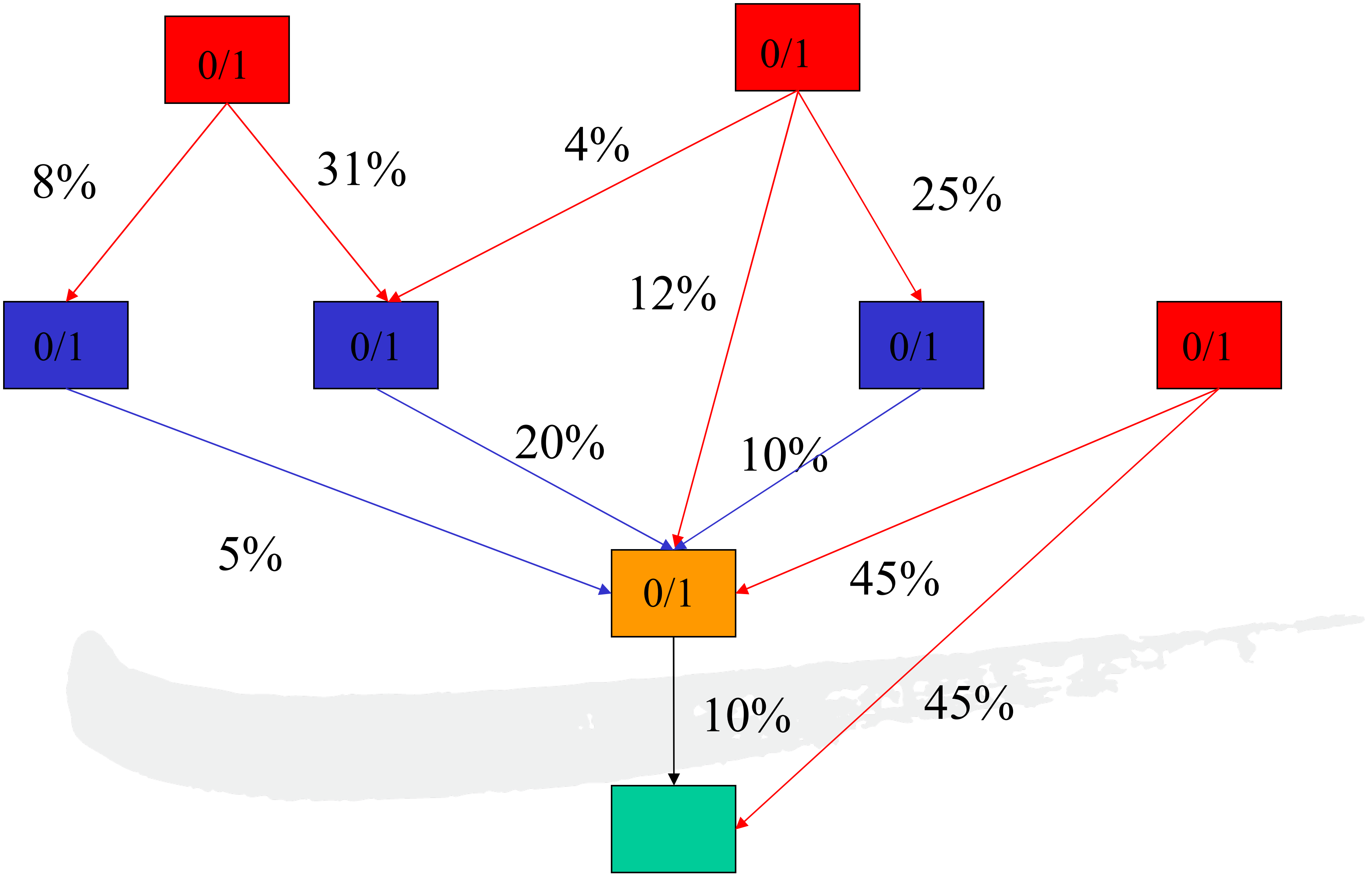

8%
31%
4%
25%
12%
20%
10%
5%
45%
10%
45%

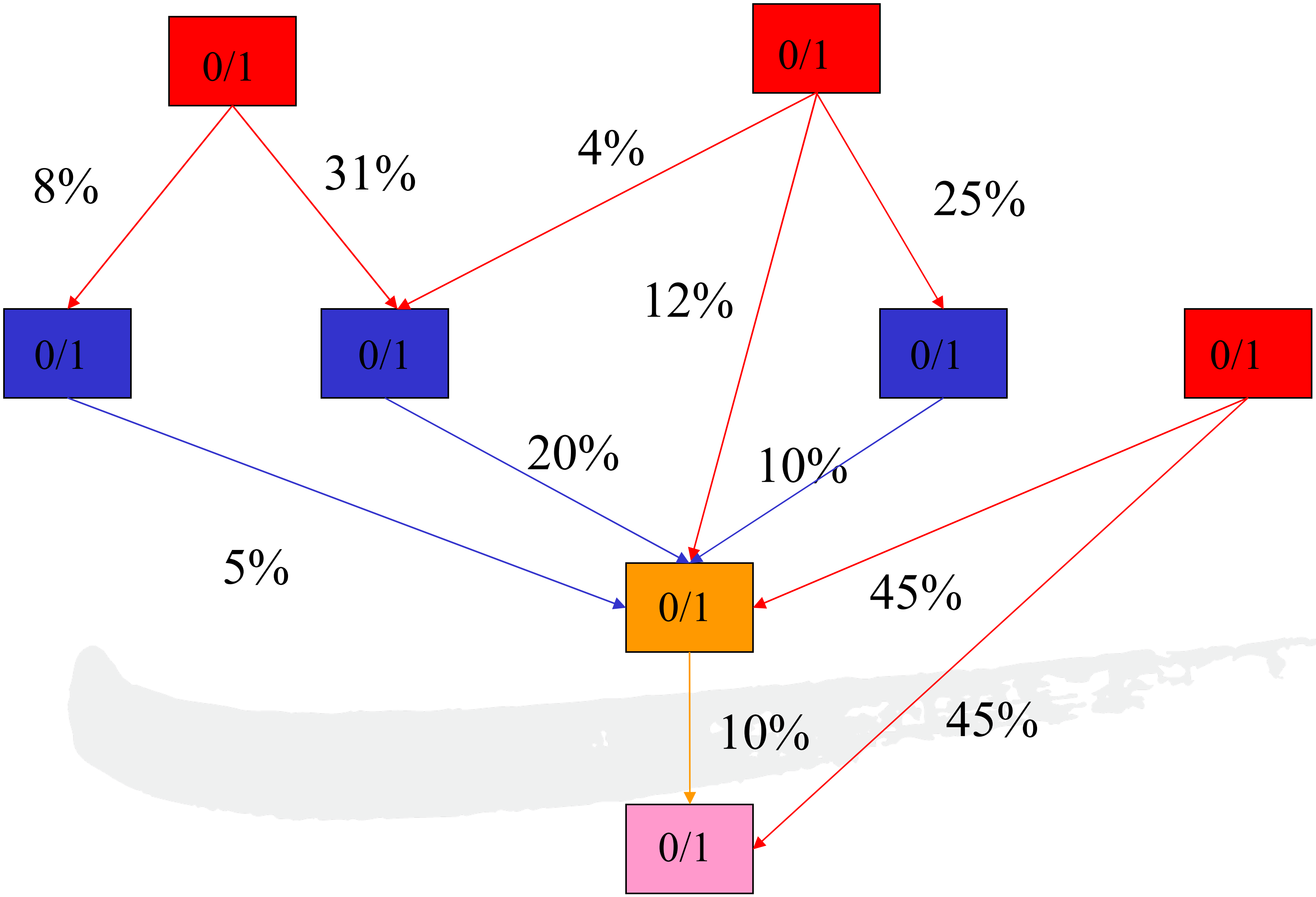

0/1
0/1
8%
31%
4%
25%
12%
0/1
20%
10%
5%
45%
10%
45%

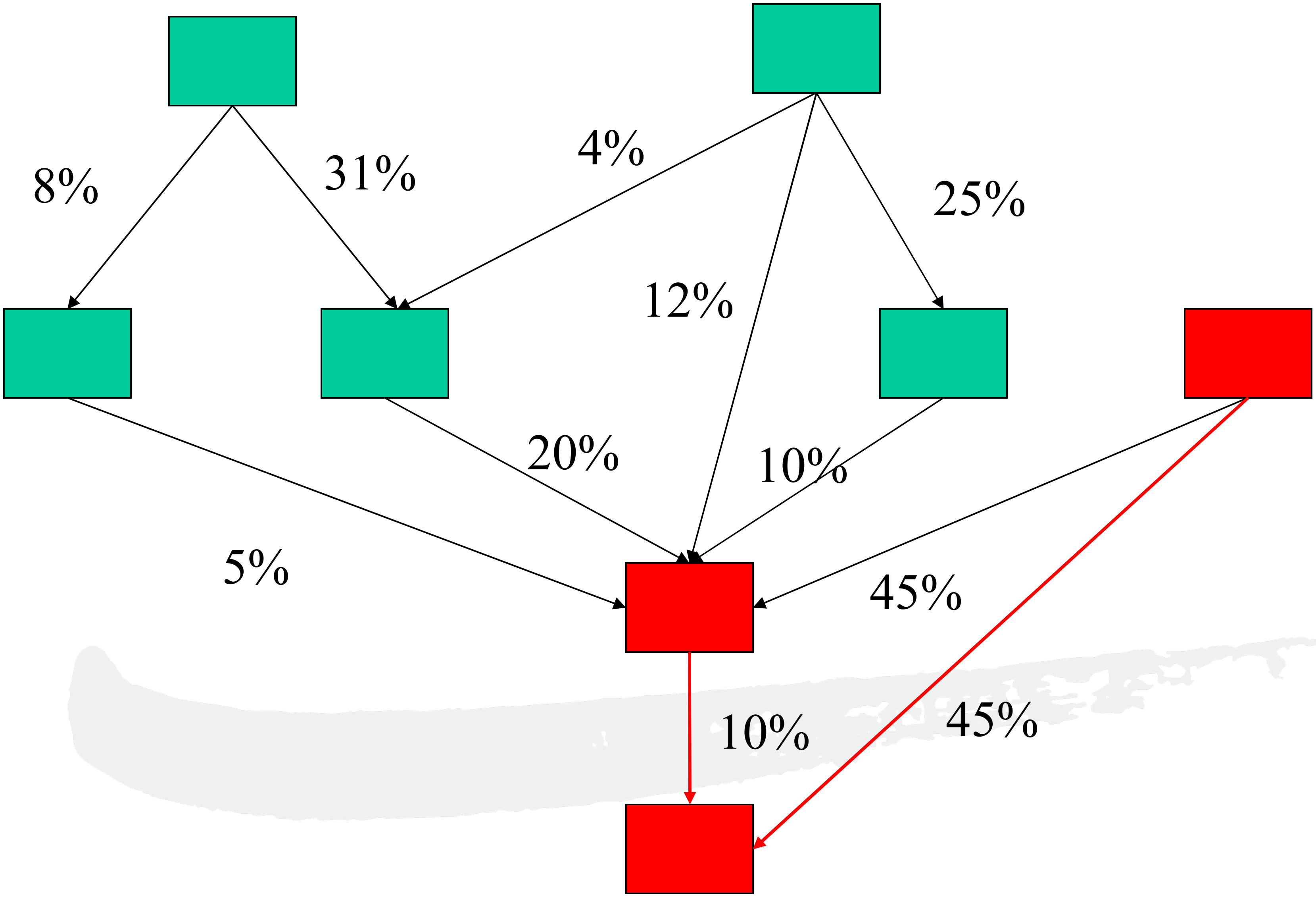

0/1
0/1
8%
31%
4%
25%
12%
0/1
20%
10%
5%
45%
10%
45%

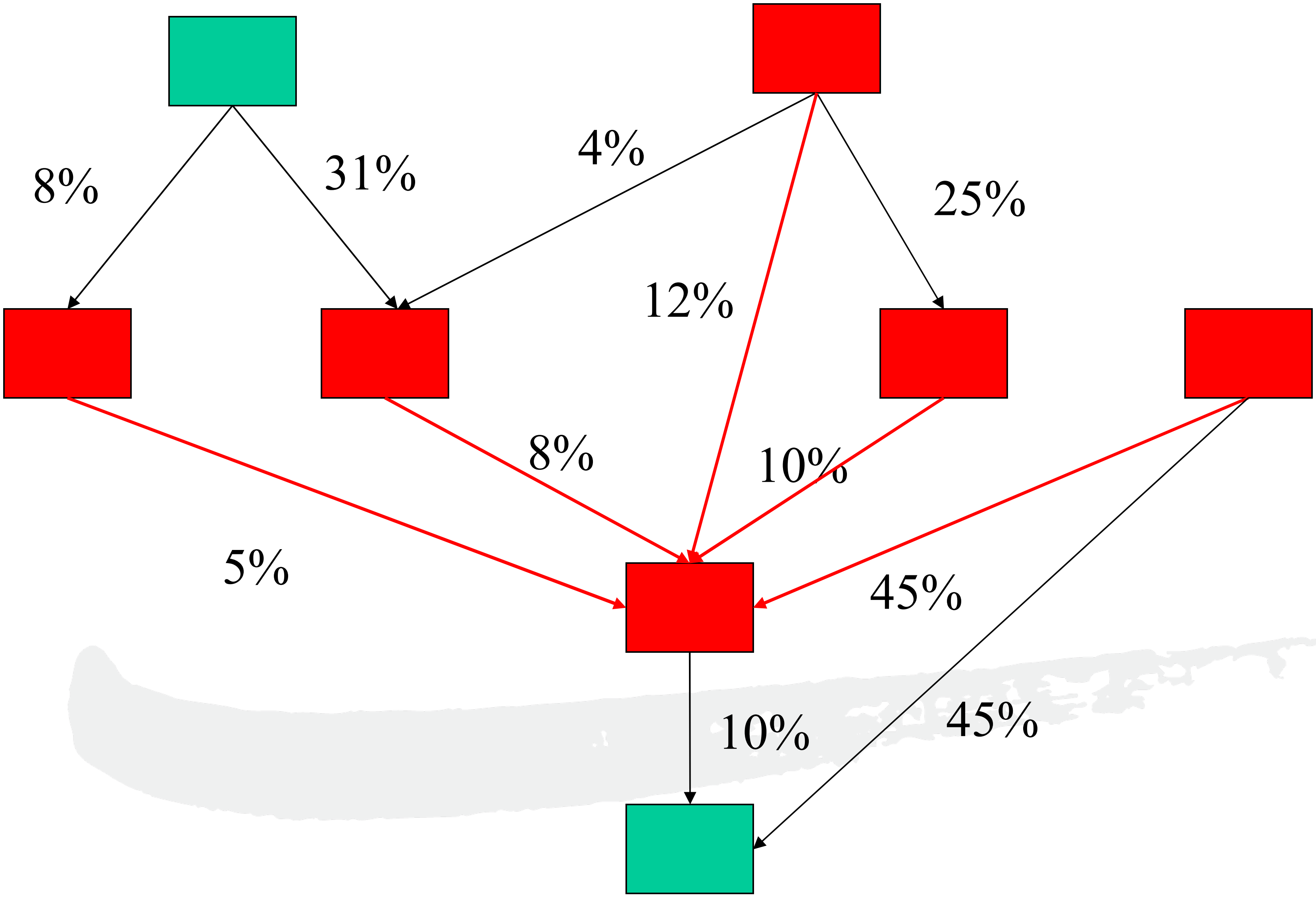

0/1
0/1
8%
31%
4%
25%
12%
0/1
0/1
0/1
0/1
20%
10%
5%
45%
10%
45%

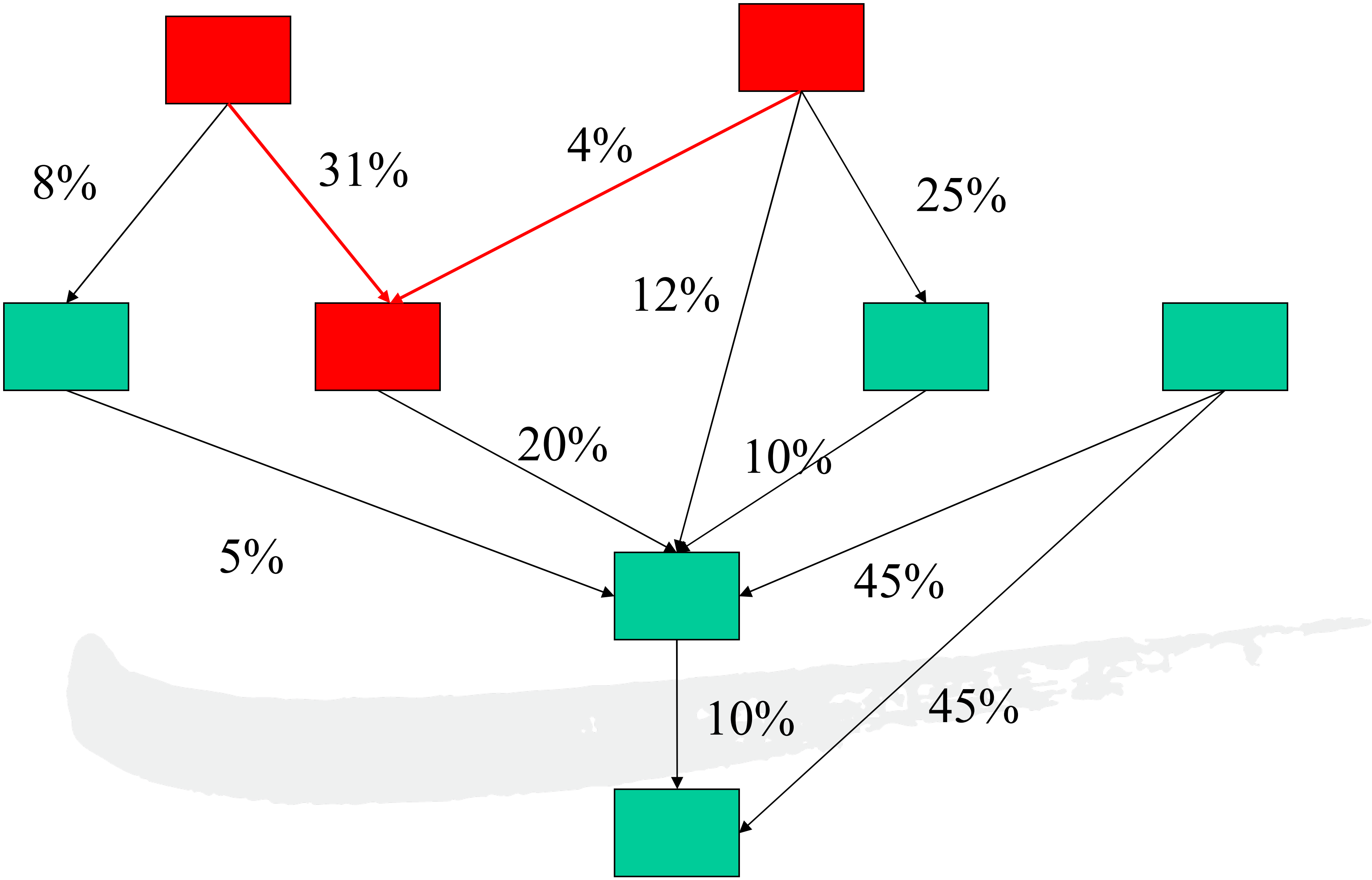

0/1
0/1
8%
31%
4%
25%
12%
0/1
0/1
0/1
0/1
20%
10%
5%
45%
10%
45%


0/1
0/1
8%
31%
4%
25%
12%
0/1
0/1
0/1
0/1
20%
10%
5%
0/1
45%
10%
45%


0/1
0/1
8%
31%
4%
12%
25%
0/1
0/1
0/1
0/1
20%
10%
5%
0/1
45%
10%
45%
0/1


8%
31%
4%
12%
25%
20%
10%
5%
45%
10%
45%


8%
31%
4%
25%
12%
8%
10%
5%
45%
10%
45%


8%
31%
4%
25%
12%
20%
10%
5%
45%
10%
45%

# Computation of Banzhaf indices for complex networks

Main ingredients:

# Computation of Banzhaf indices

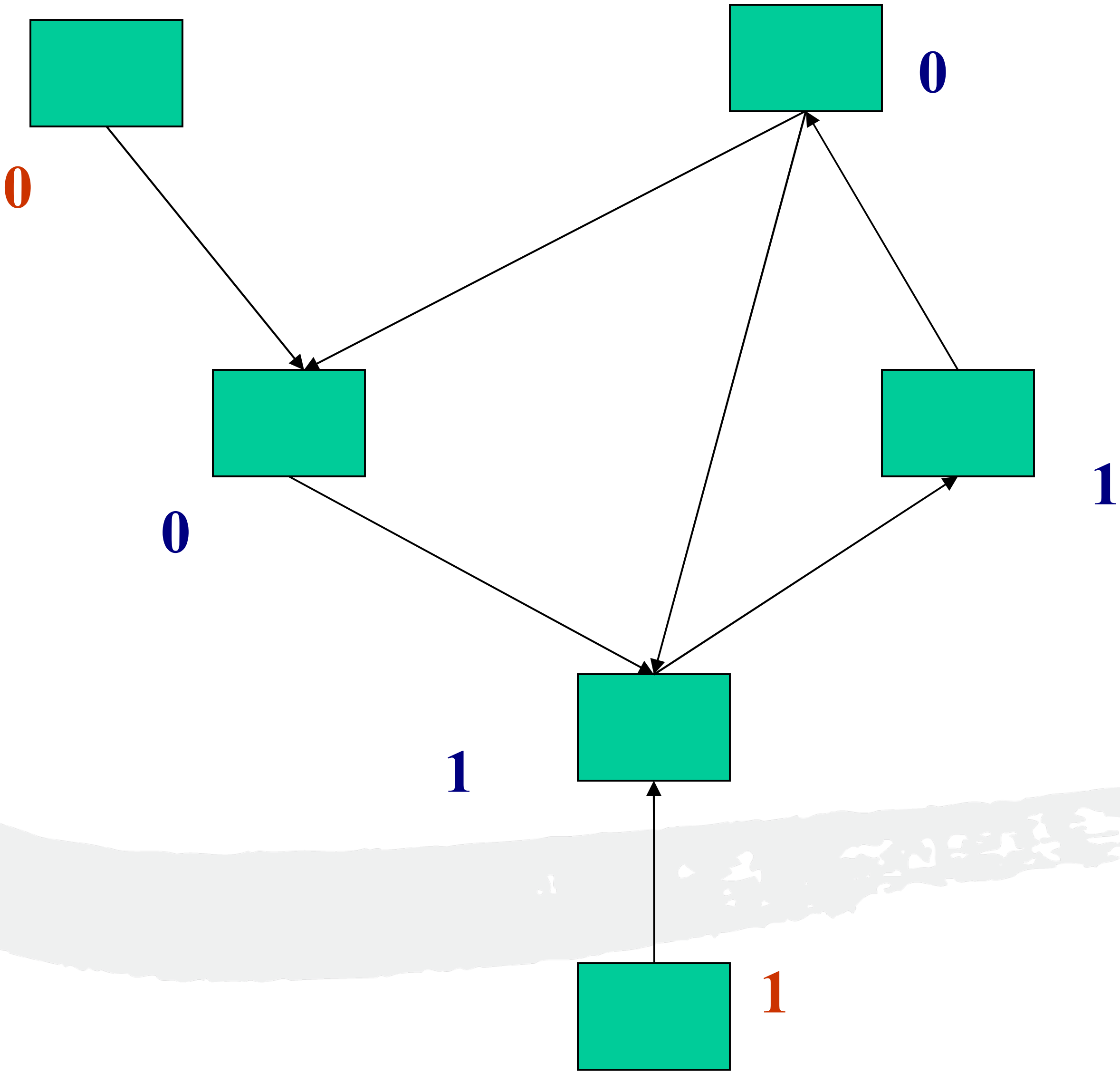
- handle large networks by Monte Carlo methods (simulation of votes) to estimate

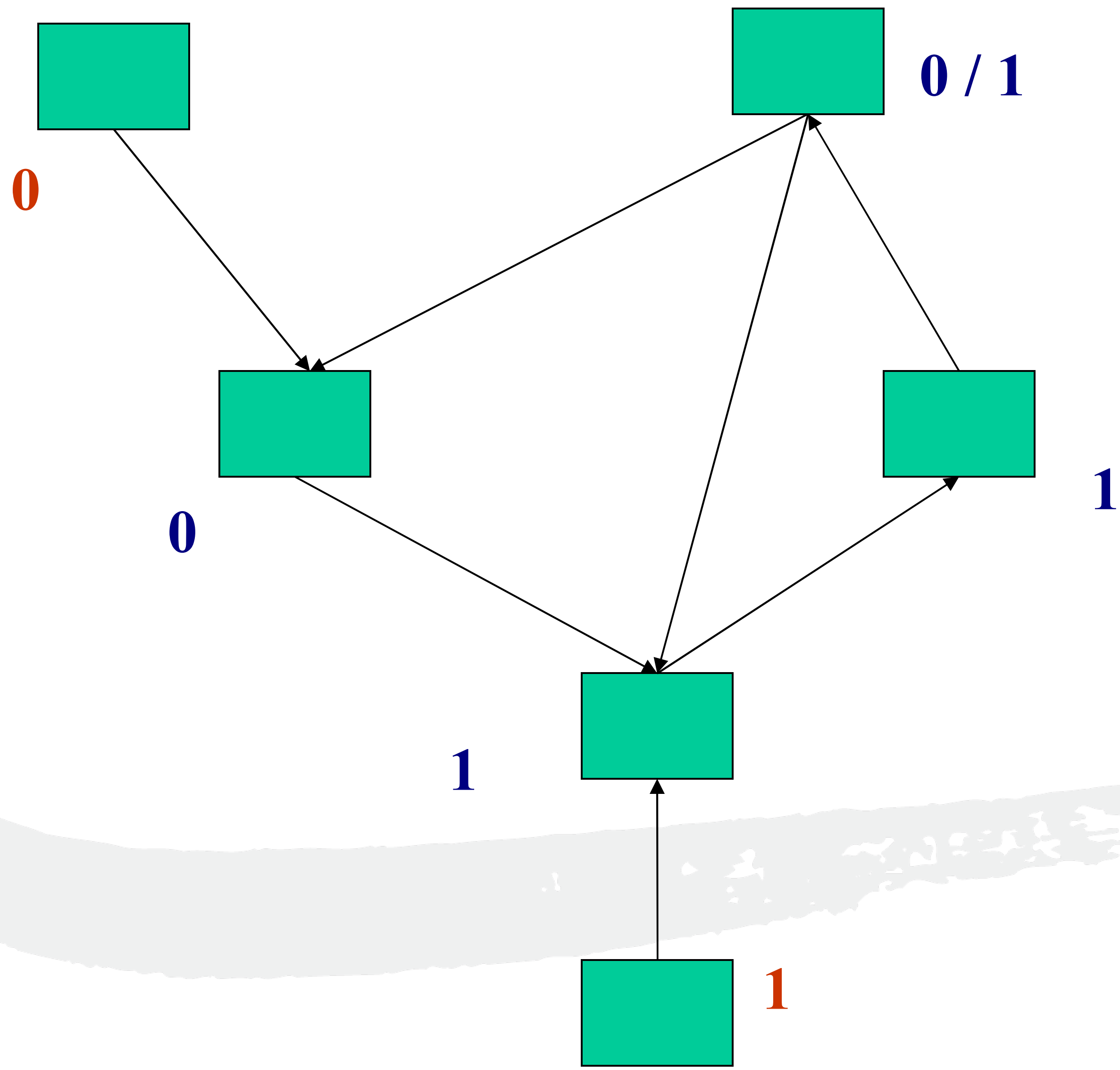
$$Z_k = \sum_{k \in T \subseteq N} [v(T) - v(T \setminus k)] / 2^{n-1}$$

- approximate small unknown shareholders (float) by normally distributed random votes

# Analysis of complex networks (4)

- handle cycles by generating iterated sequences of votes (looking for « fixed point » patterns, or sampling from the resulting distribution)


0
0
0
1
1
1


0 / 1
0
0
1
1
1

# Computation of Banzhaf indices

- Integrated computer code:

- takes as input a database of shareholdings;

- returns the Banzhaf indices of ultimate shareholders for every firm.

- First approach allowing to compute Banzhaf indices for large corporate networks in a systematic fashion.

# Applications

- Automatic identification of corporate groups (groups of firms controlled by a same firm).
- Use of control indices in econometric models of financiel performance.
- Computation of market liquidity indices

# Future research

- Improve the computation of the Banzhaf indices in this framework.

Special feature:
- the game is defined as a composition of weighted majority games;

# Future research

- in MC simulation, we want to « learn » the value of $f$ in many points; how efficiently can this be done?
- draw on results from learning theory or from reliability analysis?

# Future research

• Develop a formal model to account for cycles.

• Additional applications (check of transparency compliance by SE's: availability and reliability of data is an issue).