Competitive Evaluation of Threshold Functions and Game Trees in the Priced Information Model

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Abstract

In [Charikar *et al.* 2002] the authors proposed a new model for studying the function evaluation problem based on a variant of the classical decision tree for Boolean functions. In this variant each variable of the function to evaluate has an associated cost which has to be paid in order to read the value of the variable. Given a function f and an assignment σ to the variables of f, the performance of an algorithm for evaluating f is measured via the competitive ratio, i.e., the ratio of the total cost spent by the algorithm and the cost of the cheapest set of variables constituting a certificate for the value of the function on the given assignment.

In [Cicalese and Laber 2008] a new LP based approach (the $\mathcal{LPA}$) was introduced for designing competitive algorithms in the framework described by Charikar *et al.* The $\mathcal{LPA}$ is based on the solution of a linear program defined on the set of certificates of the function in question. Cicalese and Laber proved that for any monotone Boolean function the $\mathcal{LPA}$ provides algorithms with the best extremal competitive ratio (i.e., w.r.t. the worst case costs). Moreover, for some proper subclasses of the monotone Boolean functions, this competitiveness is achievable with efficient (polynomial time) implementation of the $\mathcal{LPA}$. Among such classes is the class of functions admitting a compact circuit representation with k-out-of-n gates (aka threshold trees). This class also includes the classic read once functions (aka AND-OR trees). The existence of an efficient implementation of the $\mathcal{LPA}$ for general monotone Boolean functions remains a major open problem.

We shall present some new developments in this line of research [5, 6].

- We extend the above result to the case when the cost of reading a variable depends on the value of the variable.
- We study the class of threshold functions, which generalize k-out-of-n functions and have applications in several contexts. We show an interesting connection between the separating structures of threshold functions and the solution of the LP used by the $\mathcal{LPA}$. A direct consequence of this result is the existence of a polynomial implementation of the $\mathcal{LPA}$ with the best competitiveness against the worst case costs for threshold functions given via a separating structure. We also show that a pseudo-polynomial implementation of the $\mathcal{LPA}$ exists for the class of functions that are representable by read once formulas whose connectives are threshold functions given by their separating structure. In the case the threshold functions are provided via their complete DNF our algorithm runs in polynomial time.
- We characterize the best possible extremal competitive ratio for the class of game tree functions, which are a generalization of AND/OR tree functions where AND and OR are replaced by MIN and MAX and variables are not restricted to be Boolean valued.

References

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