A Bidder Aid Tool for Dynamic Package Creation in the FCC Spectrum Auctions[1]

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Abstract

We propose a bidder aid tool that will allow bidders to more effectively participate in combinatorial FCC spectrum auctions by enabling concise expression of preferences. In addition to logical relationships between items, bidders may express spectrum-specific preferences such as those related to minimum population coverage, bandwidth, and budget. The tool can be used to simultaneously generate and valuate the optimal set biddable packages, both at the start of the auction and dynamically before each round. Preliminary testing suggests that the use of this tool may significantly simplify bidders' efforts in generating packages of interest, and thus lead to more efficient auction outcomes.

1. Introduction

In combinatorial auctions, bidders face the daunting task of generating the optimal set of biddable packages, often requiring the enumeration of a vast number of alternatives. In this paper, we propose a bidder aid tool that will allow bidders to concisely express their preferences. This tool interprets these preferences to simultaneously generate and valuate a set of package bids at each auction round. Our design is intended for the Federal Communications Commission's (FCC) spectrum auctions, and is based on ideas from Cramton (2002, 2003) and Ausubel (2003), as well as interviews with several participants in past spectrum auctions (*e.g.* Wilkie (2002) and Tarnutzer (2002)).

In the past, the FCC conducted non-combinatorial Simultaneous Multiple Round (SMR) auctions, where bidders could only place bids on individual items. Recently, the FCC implemented a combinatorial auction design that allows the placing of bids on combinations or packages of items in order to mitigate some of the problems associated with single item bidding, for example the exposure problem where bidders run the risk

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of winning only a subset of the items they are truly interested in, and paying much more than their value for this subset. Also, combinatorial bidding allows bidders to express their valuations on items that are substitutes (where bidders will accept a greater number of items, but at a decreasing price) and complements (where bidders value the combination of items more than the sum of the individual item values). Even though combinatorial bidding alleviates these problems, it introduces the difficulty of generating packages of items that fully express the bidders' preferences. Several bidding languages have been suggested to help bidders in this task, for example OR, XOR, OR-of-XOR, XOR-of-OR and OR* languages. Nisan (2000) gives a thorough analysis of these bidding languages and determines that the OR* language is both expressive and compact. While these languages are very expressive, they are not the most natural way in which bidders in a spectrum auction would frame their business plans. Such plans are generally constrained by budget, and require a certain level of bandwidth and population coverage in a specific geographic region. Bidders need to translate these high-level preferences into logic that explicitly refers to the individual items being auctioned in order to use existing bidding languages.

Another approach to eliciting bidder preference has been proposed by Conen and Sandholm (2001). In their approach, an auctioneer agent asks bidders questions regarding package preference, package valuations, and package ranking, starting with all the bidders' packages, but only asking questions about those the agent deems potentially desirable according to the problem's inherent structure - the agent will ask bidders to consider only the packages that are possibly part of a Pareto efficient allocation. Their method reduces the number of packages that need to be valued, but does not ease the burden of estimating a value for a package or generating all packages of interest, and requires that the bidders release value information to the auctioneer agent.

The bidding language currently used by the FCC is an XOR language, implying that a bidder may win at most one of her bids. A bidder therefore has to enumerate and valuate all possible combinations of items she is interested in winning, a task often too complex for most bidders to take on. In addition, the auctioneer may limit the number of package bids allowed due to the computational complexity associated with combinatorial auctions. This forces the bidder to guess which of the packages she values have the greatest potential to win, and therefore increases the likelihood of inefficient allocations. Our goal is to simplify the task of generating and valuating packages by providing a bidder aid tool that bidders can use to concisely express their preferences and values. The proposed design will allow bidders to more effectively participate in ascending combinatorial FCC spectrum auctions, by enabling quicker bid generation and valuation both at the start of the auction, and within each round, when the bidder must decide which packages to bid based on the current price set by the auctioneer. Confidentiality of all the private information required by the bidder aid tool can be ensured by using local versions of the tool that reside on the bidders' computers. This bidder aid tool may also be used in a sealed-bid auction to generate constraints that will be applied directly to the winner determination problem, thereby circumventing package creation. The idea of applying constraints directly to the winner determination problem is similar to the concept proposed by Boutilier and Hoos (2001), and is the subject of future research.

2. Requirements Overview

To be useful to bidders participating in the FCC spectrum auctions, the bidder aid tool should enable bidders to express complex business plans in terms of logical relationships between items and other preferences such as minimum population coverage, bandwidth requirements, and budget. From information gathered during interviews conducted with previous auction participants, we concluded that bidders tend to group markets into sets of equivalent markets with associated unit values expressed in dollar per MHz-Pop (a product of bandwidth and population coverage). We refer to these sets as equivalence classes in keeping with the terminology used by Ausubel (2003) and Cramton (2002, 2003). Equivalence classes may be characterized as (a) primary, consisting of markets that are core to the bidder's business plan, (b) secondary, consisting of markets that may provide added value but are not essential, such as markets adjacent to the primary markets, or (c) tertiary, consisting of markets that are not of much interest, but that the bidder may still accept at a sufficiently low price. For each of these equivalence classes, bidders may wish to define the minimum population coverage, minimum and maximum bandwidth requirements, a maximum unit price, and a budget. Bidders may also wish to specify different unit prices based on the quantity of bandwidth acquired. This will enable them to express their marginal preferences. In addition, bidders may wish to express synergies for markets that are complementary due to reasons such as geographical adjacency. They may also wish to express logical relationships among markets, for example that markets from a secondary equivalence class are only of interest if all or a subset of the markets from the corresponding primary equivalence class is acquired.

The FCC currently auctions bandwidth in the form of licenses, with each license consisting of a fixed amount of bandwidth in a specific market area (geographic region). In this case the assumption is that bandwidth is not fungible, and is therefore structured as licenses of pre-defined frequency bands before the start of the auction. However, there is also the possibility of considering bandwidth to be fungible and auctioning quantities of bandwidth unrelated to specific frequencies as opposed to licenses with pre-determined frequency bands. The bidder aid tool should be able to function regardless of whether bandwidth is considered fungible or non-fungible. In the case where bandwidth is not considered fungible, the bidder may also need to express preferences regarding the frequency bands being auctioned. She may, for example, require bandwidth to be on the same frequency for all her markets, or may prefer one band to another if she already owns that band in an adjacent market. Finally, bidders need to be able to update their preferences as the auction progresses, keep track of existing packages, and keep private information hidden from the auctioneer.

To address these requirements, we present two aspects involved in the design of this tool, namely an interface that the bidder can use to input her preferences in a concise manner, and the optimization model that can be derived from the bidder's input and solved iteratively in order to simultaneously valuate and generate a set of suggested packages. We simulated our proposed design using BidBots – a simulation tool that was developed by Decisive Analytics Corporation for spectrum auction research at the FCC and that is capable of simulating a variety of auction mechanisms, bidder types and bidding strategies. The goal of the simulation is to test the feasibility of such a tool, and its impact on allocative efficiency. Our simulations to date focused on translating a generic bidder's preferences, as they would be input through the bidder aid tool's user interface, into package valuations by using a mathematical model, and determining the optimal set of package bids before each round according to a myopic best response strategy. Future simulation will include multiple bidder types using different bidding strategies. Note that bidders are not restricted to bid on the package generated by the bidder aid tool, and will have the opportunity to review the suggested packages and adjust their input to the tool before deciding on the final set of bids to submit in each round.

The proposed design assumes that information on bids by other bidders is not available. Therefore, the tool does not specifically enable bidders to compose packages that are intended to form coalitions with other bidder's bids. Such a capability may be desired by smaller bidders to overcome the "threshold problem" (the problem of determining how much each of a collection of smaller bidders must pay to overcome the total price of a larger package). A study on using this bidder aid tool to determine competitive coalitions, as well as the effect of having such a capability on strategic behavior and the incentive properties of the auction, is left for future research.

3. Bidder Aid Tool Interface

This section describes a high-level design of the bidder aid tool's user interface. The preference elicitation tool has two main parts. The first part collects information regarding the bidder's values and preferences related to markets and bandwidth assuming fungible bandwidth, while the second optional part collects information regarding the bidder's additional value for specific frequency bands, to be used in the case where bandwidth is not fungible and specific licenses are auctioned for each band. Each part consists of a number of steps the bidder should go through to input data in a concise and structured manner. This input is then used to derive an optimization model that will be solved iteratively to simultaneously generate and valuate a number of most profitable package bids before each round. A detailed description of each step is given below. An example is used throughout this section for illustration purposes, and all tables that follow will eventually be operated with drop-down lists so that bidders are restricted to the relevant choices with minimal effort required on their part. Please refer to the Appendix for the key to regional abbreviations.

Part 1: Markets and Bandwidth

Step 1.1: Input the bidder's overall budget, a limit on the number of packages to be generated, and a lower bound on the profit required.

The limit on the number of packages generated can be defined as either (a) a fixed number of packages, (b) the set of most profitable packages (with equal profitability), or (c) the set of packages with profit within x% of the profit of the most profitable package. The lower bound on the profit required may be the bidder's existing profit in the case where she is a provisional winner from the previous round, or any other number.

Step 1.2: Group markets of interest into equivalence classes and input the minimum population required, minimum and maximum bandwidth required, unit price (based on the minimum bandwidth), and budget associated with each equivalence class (see Example 1.2).

Each group may contain up to three equivalence classes, namely primary, secondary, and tertiary. The number of groups will depend on the type of bidder. For example, a bidder who wants nationwide coverage will likely have only one group, while a regional bidder may have several groups, each focusing on a number of related markets. Primary markets are those forming the core of the package. Secondary markets are contingent on choosing at least one of the primary markets in the same group, while tertiary markets are contingent on choosing at least one of the secondary markets in the same group. Each market may be present in at most one equivalence class. Bidders may also specify budgets and minimum population coverage for all primary markets, all secondary markets, and all tertiary markets. If a bidder wishes to obtain a market of secondary importance without also obtaining one of primary importance, he can simply put these markets in a separate primary group, as shown in group 3 in the example below. Note that the "Price" is the bidder's maximum unit bid price based on her value, in other words, the maximum price she is willing to pay for one unit of bandwidth and one unit of population, assuming she wins the minimum specified amount of bandwidth.

Example 1.2: Minimum population coverage, minimum and maximum bandwidth requirements, and budgets.

Group	**Equivalence Class**	**Markets**	**Min Pops (mil.)**	**Min Bandwidth (MHz)**	**Max Bandwidth (MHz)**	**Price ($/MHzPop)**	**Budget ($ mil.)**
1	P1	{NY,BP}	5	20	40	0.2	80
	S1	{BR, PH}	2	10	20	0.1	4
	T1	{CL}	0.5	10	10	0.08	2
2	P2	{LA,SF,PL}	5	20	40	0.15	40
	S2	{SL, PX, DV}	2	10	20	0.1	4
	T2	{SB, OM}	0.5	10	10	0.07	2
3	P3	{HW, PR}	2	10	10	0.1	3
Combined		All primary	5				100
		All secondary	0				8
		All tertiary	0				2
P=Primary; S=Secondary; T=Tertiary							

Step 1.3: Input any market-specific exceptions to the data input in Step 1.2 (see Example 1.3).

The markets in the first column of the table in Example 1.3 will be chosen from a drop-down list based on the input from Step 1.2. When choosing a market, the data in the other columns will automatically be filled in with the defaults specified in Step 1.2. The defaults that have been overridden are shown in bold. The "MinPops" and "Budget" fields in Example 1.2 cannot be overridden, seeing that the population per market is fixed and the budget is class specific and not market specific. The amount paid for a specific market is limited by the price for that market.

Example 1.3: Market specific exceptions for bandwidth requirements and price.

Market-Specific Exceptions	**Min Pops (mil.)**	**Min Bandwidth (MHz)**	**Max Bandwidth (MHz)**	**Price* ($/MHzPop)**	**Budget ($ mil.)**
SF		**30**	40	0.15	
SL		10	20	**0.11**	
DV		**20**	**30**	**0.12**	

Step 1.4: For each equivalence class, input price increments based on different quantities of bandwidth (see Example 1.4).

In Example 1.4 (and all examples thereafter), the grey fields are generated automatically whenever previous information cannot be edited. The bidder can specify any size of increment and number of increments, and the quantity of bandwidth is then defined within the bounds of the increments. The prices are in units of $/MHzPop, and not cumulative. For example, the bidder is willing to pay 0.2 $/MHzPop for 20MHz, and 0.18 $/MHzPop for any amount of bandwidth between 21 and 30 MHz for any market in equivalence class P1. Thus, if she wins 20 MHz, she pays (20*0.2) = 4 $/pop, while if she wins 23 MHz she pays (23*0.18) = 4.14 $/pop.

Example 1.4: Bandwidth-price increments.

Equivalence Class	Increment	Min Bandwidth (MHz)	Max Bandwidth (MHz)	Price ($/MHzPop)
P1	1	20	20	0.2
	2	21	30	0.18
	3	31	40	0.15
S1	1	10	10	0.1
	2	11	15	0.1
	3	16	20	0.09
T1	1	10	10	0.08
P2	1	20	30	0.2
	2	31	35	0.18
	3	36	40	0.15
S2	1	10	14	0.1
	2	16	20	0.11
T2	1	10	10	0.07
P3	1	10	10	0.1
	2	11	30	0.08

Step 1.5: For each exception in Step 1.3, input price increments based on the amount of bandwidth. Add any additional market specific exceptions to Step 1.4 (see Example 1.5).

Example 1.5: Exceptions to bandwidth-price increments.

Market	Increment	Min Bandwidth (MHz)	Max Bandwidth (MHz)	Price ($/MHzPop)
SF	1	30	35	0.2
	2	36	40	0.15
SL	1	10	15	0.11
	2	16	20	0.1
DV	1	20	24	0.12
	2	25	30	0.1

Step 1.6: Supply synergy information to reflect complements that result from regional adjacency.

The extent to which regional adjacency can be enforced will depend on the size of the auction. Here we assume that a bidder may choose to either (a) enforce that each market is adjacent to at least N other markets, (b) not enforce adjacency but give a preference to adjacent markets in terms of an additional price she's willing to pay due to the synergy, or (c) not enforce or encourage adjacency in any way. Option (b) is shown in Example 1.6 where the bidder is willing to pay an additional 0.01 $/MHzPop for any adjacent regions within P1, or between P1 and S1. There is no added synergy for adjacency within S1.

Example 1.6: Additional price to indicate a preference for adjacency.

Class 1 markets	**adjacent to**	**Class 2 markets**	**pay an additional**	**Price ($/MHzPop)**
P1		P1		0.01
P1		S1		0.01

Step 1.7: Supply information regarding any remaining logical relationships between equivalence classes or markets that have not been covered in previous steps.

The bidder may state whether all groups are mutually exclusive in the sense that each package may only contain markets from one group. Bidders may indicate additional logical relationships between groups and markets (see Example 1.7). Note that contingencies within groups are implicit as discussed in Step 1.2. The exact format of the table in Example 1.7 may need some adjustment depending on the bidder requirements, but the example gives the basic idea. This step is intended to cover any remaining logic that could not be captured in Steps 1.1 through 1.6 and is not intended to be an exhaustive listing of all possible relationships.

Example 1.7: Additional logic between groups and markets.

Equivalence Class 1	**Logical Operator**	**Equivalence Class 2**	**Logical Operator**	**Equivalence Class 3**	**etc.**
S1	XOR	S2			
P3	contingent on	[P1	OR	P2]	

Step 1.8: If bandwidth is considered fungible, trigger the model generation, solution, and report, else continue to Part 2.

Part 2: Band Selection (Optional)

Step 2.1: Supply "same band" requirements.

The bidder may indicate whether she requires adjacent markets in a package to be on the same frequency band, or all licenses to be on the same band.

Step 2.2: Supply band preferences.

Indicate any additional frequency band preferences for bands that are not required, but considered more valuable. In Example 2.2, the bidder prefers frequency band D for NY and all of the P2 markets and is therefore willing to pay an additional 0.025 \$/MHzPop if she is able to win this band. She absolutely does not want band E for the S1 markets, and therefore values it at 0.

Example 2.2: Band preferences

Equivalence Class	**Market**	**Band**	**Price (\$/MHzPop)**
P1	NY	D	0.025
S1	All	E	0
P2	All	D	0.025

Step 2.3: Trigger the model generation, solution, and report.

4. Model

The model presented here is derived from the bidder input in Section 3, and can be solved repeatedly to simultaneously generate and valuate a number of packages up to the limit stated in Step 1.1. A cut is added after each solution to ensure that the same package is not generated more than once in the same round. A detailed discussion of the constraints follow below, including an indication of the interface step that requires the respective constraint. The names with a horizontal line on top represent fixed parameters, while the names without the line represent variables. Please refer to the nomenclature list at the end of the paper.

The objective (1.1) is to maximize the profit, where profit is defined as the difference between the package value and the cost of the package at the current price (Constraint (1.2)):

$$\max \, profit \tag{1.1}$$

$$profit = value - cost \tag{1.2}$$

For a package to be accepted, its profit needs to be greater than the minimum profit from Step 1.1, as stated in Constraint (1.3).

$$profit \geq \overline{minProfit} \tag{1.3}$$

The total cost for the package equals the sum of the costs over all equivalence classes (Constraint (1.4)), where the cost of an equivalence class is defined as the sum over all markets in that class of the minimum acceptable bid for one MHz of bandwidth (from Steps 1.2 through 1.5), multiplied by the amount of bandwidth included in that package for that market (Constraint (1.5)).

$$cost = \sum_{c \in C} cost_c \tag{1.4}$$

$$cost_c = \sum_{r \in R(c)} \overline{MAB_r} bandWidth_r \qquad \forall c \in C \tag{1.5}$$

The value of a package equals the sum of the values of all the equivalence classes in the package (Constraint (1.6)), with the value of an equivalence class being the sum of the values of all markets in that class plus any additional value derived from synergies with adjacent markets (Constraint (1.7)). For each market, the value is calculated in Constraint (1.8) as the population of the market, multiplied by the product of the bandwidth and the price in the price increment the bandwidth falls into (from Steps 1.2 through 1.5). The synergy value for each market is calculated in Constraint (1.9) as the population of that market, multiplied by the product of the shared quantity of bandwidth with each adjacent market and the additional price the bidder is willing to pay for this synergy (from Steps 1.2 and 1.6).

$$value = \sum_{c \in C} value_c \tag{1.6}$$

$$value_c = \sum_{r \in R(c)} (value_r + synergyValue_r) \qquad \forall c \in C \tag{1.7}$$

$$value_r = (\sum_{i \in I(r)} \overline{price_{ir}} bandWidth_{ir}) \overline{pops_r} \qquad \forall r \in R \tag{1.8}$$

$$synergyValue_r = \sum_{r' \in ADJ(r)} synBandWidth_{rr'} \overline{synergyPrice_{rr'}} \, \overline{pops_r} \qquad \forall r \in R \tag{1.9}$$

The bandwidth for each market falls into at most one price increment, as shown in Constraint (1.10), where the Boolean variable x_r takes a value of 1 if market r is included in the package, and 0 otherwise, and the Boolean variable y_{ir} takes a value of 1 of the bandwidth for market r falls in increment i and 0 otherwise.

$$\sum_{i \in I(r)} y_{ir} = x_r \qquad \forall r \in R \tag{1.10}$$

Constraints (1.11) and (1.12) state that for the bandwidth to fall into a certain increment, it has to be less than the upper bound and greater than the lower bound of bandwidth associated with that increment as defined in Steps 1.2 through 1.5. The bandwidth associated with a market is then the sum of the bandwidths of all that market's increments, as shown in Constraint (1.13), where at most one of the increment bandwidths will have a positive value as enforced by Constraint (1.10). Constraint (1.14) states that for a market to be chosen, its associated bandwidth has to be at least the minimum unit of bandwidth to be auctioned as specified by the auctioneer.

$$bandWidth_{ir} \le \overline{maxBandWidth_{ir}} y_{ir} \qquad \forall r \in R, i \in I(r) \tag{1.11}$$

$$bandWidth_{ir} \ge \overline{minBandWidth_{ir}} y_{ir} \qquad \forall r \in R, i \in I(r) \tag{1.12}$$

$$bandWidth_r = \sum_{i \in I(r)} bandWidth_{ir} \qquad \forall r \in R \tag{1.13}$$

$$bandWidth_r \ge \overline{minUnitBandWidth}(x_r) \qquad \forall r \in R \tag{1.14}$$

The synergy bandwidth is the amount of bandwidth shared between two adjacent markets, and is determined by Constraints (1.15) through (1.18). These inequalities result in the choice of a synergy bandwidth equaling the lesser of the bandwidths of the two adjacent markets.

$$synBandWidth_{rr'} \le bandwidth_r \qquad \forall r \in R, r' \in ADJ(r) \tag{1.15}$$

$$synBandWidth_{rr'} \le bandwidth_{r'} \qquad \forall r \in R, r' \in ADJ(r) \tag{1.16}$$

$$synBandWidth_{rr'} \ge bandwidth_r - U(1 - y_{rr'}) \qquad \forall r \in R, r' \in ADJ(r) \tag{1.17}$$

$$synBandWidth_{rr'} \ge bandwidth_{r'} - Uy_{rr'} \qquad \forall r \in R, r' \in ADJ(r) \tag{1.18}$$

Constraints (1.19) through (1.23) represent the various budget constraints, namely the overall budget, budget for each equivalence class, and budgets over all primary classes, all secondary classes, and all tertiary classes, as defined in Steps 1.1 and 1.2.

$$cost \le \overline{overallBudget} \tag{1.19}$$

$$cost_c \le \overline{budget}_c \qquad \forall c \in C \tag{1.20}$$

$$\sum_{c \in P} cost_c \le \overline{primaryBudget} \tag{1.21}$$

$$\sum_{c \in S} cost_c \le \overline{\sec ondaryBudget} \tag{1.22}$$

$$\sum_{c \in T} cost_c \le \overline{tertiaryBudget} \tag{1.23}$$

Value limiting constraints (not shown here) similar to these budget constraints may be included in the case where the bidder does not want to create a package that she values higher than her budget. This may not always be the case, seeing that a bidder may want to submit packages with values higher than her budget in the hope of winning such a package at a low price.

For each equivalence class, the total population of all markets chosen in that class has to be greater than the minimum population required for that class (from Step 1.2), as shown in Constraint (1.24), where the Boolean variable x_c takes a value of 1 if equivalence class c is included in the package, and 0 otherwise. Constraint (1.25) states that the equivalence class has to be chosen for any of its markets to be chosen. Constraints (1.26) through (1.28) enforce the minimum population requirements for all primary markets, all secondary markets, and all tertiary markets.

$$\sum_{r \in R(c)} \overline{pops}_r x_r \ge \overline{minPops}_c x_c \qquad \forall c \in C \tag{1.24}$$

$$\sum_{r \in R(c)} x_r \le |R(c)| x_c \qquad \forall c \in C \tag{1.25}$$

$$\sum_{r \in P} \overline{pops}_r x_r \ge \overline{minPops}^P \tag{1.26}$$

$$\sum_{r \in S} \overline{pops}_r x_r \ge \overline{minPops}^S \tag{1.27}$$

$$\sum_{r \in T} \overline{pops}_r x_r \ge \overline{minPops}^T \tag{1.28}$$

Constraint (1.29) states that secondary markets can only be chosen if at least one primary market in the same group is chosen. Similarly, Constraint (1.30) states that tertiary markets can only be chosen if at least on secondary market in the same group is chosen. These constraints use the class information from Step 1.2.

$$\sum_{r \in R(S(g))} x_r \le |S(g)| \sum_{r \in R(P(g))} x_r \qquad \forall g \in G \tag{1.29}$$

$$\sum_{r \in R(T(g))} x_r \le |T(g)| \sum_{r \in R(S(g))} x_r \qquad \forall g \in G \tag{1.30}$$

Constraints (1.31) and (1.32) are optional constraints to be applied if the bidder requires all the groups to be mutually exclusive, as specified in Step 1.7.

$$\sum_{c \in C(g)} x_c \le |R(c)| x_g \qquad \forall g \in G \tag{1.31}$$

$$\sum_{g \in G} x_g \le 1 \tag{1.32}$$

Any remaining logical constraints from Step 1.7 representing conditional choices between equivalence classes or markets are represented by Constraint (1.33).

$$\Omega(x_c, x_r) = true \qquad (1.33)$$

A bidder may require all the markets in the package to be adjacent, or only that groups of adjacent markets exist in the package. We consider only partial adjacency from Step 1.6, which can be achieved by applying the optional Constraint (1.34). This constraint states that a market may only be chosen if at least N_{adj} of its adjacent markets are chosen, i.e. the package will consist of groups of N_{adj} or more adjacent markets. Care should be taken, however, that N_{adj} is not too high, seeing that a market may have a limited number of directly adjacent markets.

$$x_r \le \frac{1}{\overline{N}_{adj}} \sum_{r' \in ADJ(r)} x_{r'} \qquad \forall r \in R \qquad (1.34)$$

In the case where fungible quantities of bandwidth are auctioned, Constraint (1.35) will be used to prevent the same combination of bandwidth increments and markets to be chosen more than once in the same round.

$$\sum_{(i,r)\in IR^1(n')} y_{ir} - \sum_{(i,r)\in IR^0(n')} y_{ir} \le \left| IR^1(n') \right| - 1 \qquad \forall n' < n \qquad (1.35)$$

In the case where bandwidth is not considered fungible, and a set of licenses are auctioned instead of flexible quantities of bandwidth, the package creation tool will include optional Constraints (1.36) through (1.42) to facilitate the choice of specific licenses based on the frequency band preferences, as well as some variations on Constraints (1.5) and (1.35). The bandwidth for each market equals the sum of bandwidths of all licenses chosen in that market, as shown in Constraint (1.36). Constraint (1.5alt) shows an alternative cost calculation for the case where specific licenses are auctioned instead fungible bandwidth. In this case, the cost of an equivalence class equals the sum of the minimum acceptable bids of all chosen licenses in that class, where the Boolean variable x_l takes a value of 1 if license *l* is included in the package, and 0 otherwise.

$$bandwidth_r = \sum_{l \in L(r)} \overline{bandwidth}_l \, x_l \qquad \forall r \in R \qquad (1.36)$$

$$cost_c = \sum_{l \in L(c)} \overline{MAB}_l \, x_l \qquad \forall c \in C \qquad (1.5alt)$$

A bidder may specify in Step 2.1 that licenses in adjacent markets should be on the same band. This requirement can be enforced by applying the optional Constraints (1.37) through (1.40). Constraint (1.37) states that any shared bandwidth between two adjacent markets ($synBandWidth_{rr'}$) has to be on the same bands, while Constraints (1.38) through (1.40) force the Boolean variable $x_{ll'}$ to take a value of 1 if two adjacent licenses *l* and *l'* are chosen and *vice versa*.

$$synBandWidth_{rr'} = \sum_{b | l \in L(r), l' \in L(r'), l, l' \in L(b)} \overline{bandWidth}_b \, x_{ll'} \qquad \forall r \in R, r' \in ADJ(r) \qquad (1.37)$$

$$x_{ll'} \ge x_l + x_{l'} - 1 \qquad \forall l, l' \in ADJ(l, l') \qquad (1.38)$$

$$x_{ll'} \le x_l \qquad \forall l, l' \in ADJ(l, l') \qquad (1.39)$$

$$x_{ll'} \le x_{l'} \qquad \forall l, l' \in ADJ(l, l') \qquad (1.40)$$

A bidder may also specify in Step 2.1 that all licenses should be from at most one band, and this requirement can be enforced by Constraints (1.41) and (1.42).

$$\sum_{l \in L} x_l \le |L| x_b \qquad \forall b \in B \tag{1.41}$$

$$\sum_{b \in B} x_b \le 1 \tag{1.42}$$

Any additional band preferences indicated in Step 2.2 are dealt with by excluding any license with zero value from the set of licenses, and by adding the following term to the objective function (1.1):

$$+\sum_{l \in L} preferencePrice_b x_l$$

In the case where licenses are auctioned, Constraint (1.35alt) will be used to prevent the same combination of licenses to be chosen more than once in the same round.

$$\sum_{l \in L^1(n')} x_l - \sum_{l \in L^0(n')} x_l \le \left| L^1(n') \right| - 1 \qquad \forall n' < n \tag{1.35alt}$$

Finally, optional constraints on eligibility and activity requirements may be added to the model if such rules have been specified by the auctioneer. These are shown here for the non-fungible case (Constraints (1.43) and (1.44)), where the sum of the bidding units of the chosen licenses has to be less than the bidder's eligibility, and greater than the bidder's activity requirement.

$$\sum_{l \in L} \overline{biddingUnits}_l x_l \le \overline{eligibility} \tag{1.43}$$

$$\sum_{l \in L} \overline{biddingUnits}_l x_l \ge \overline{activityRequirement} \tag{1.44}$$

Flexibility is achieved by allowing only the relevant subset of constraints to be triggered depending on the bidder's input into the user interface. Additional constraints may be added to this interface, and the decision to include these in the model will be in the bidder's hands.

5. Conclusions and Future Directions

We proposed a bidder aid tool that will allow bidders to more effectively participate in combinatorial FCC spectrum auctions by enabling concise expression of preferences. This tool will enable bidders to express complex business plans involving logical relationships between items, as well as population coverage, bandwidth, and budget constraints. In addition, the bidder aid tool will allow bidders to update their preferences and reevaluate their bid composition dynamically before each round based on the latest price information. Use of this tool will simplify the process of package valuation and likely lead to more efficient allocations.

The bidder preferences input through the user interface are converted into a well-defined optimization model. This model is solved iteratively to simultaneously generate and valuate a set of packages with decreasing profitability based on the latest price information at the start of each round. Our preliminary simulation studies show that the use of such a tool in each round enables bidders to quickly generate and valuate the set of most profitable packages, and improves allocative efficiency.

Future work includes a thorough testing of this tool. Refinements to the bidder aid tool will be made as required after we have had the opportunity to evaluate the tool's usefulness by using it within mock auctions. We also intend to test the use of the tool by agents with varying goals in combination with different bidding strategies. In addition, we will investigate the use of this tool for generating constraints that can be applied directly to the winner determination problem, as opposed to generating packages.

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Nomenclature

Sets

ADJ(r) Markets adjacent to market *r*.

ADJ(l,l') Licenses adjacent to license *l*, with adjacency implying regional adjacency on the same band.

B Set of bands.

C Equivalence classes.

C(g) Equivalence classes associated with group *g*.

G Groups.

I(r) Price increments associated with market *r*.

$IR^1(n)$ The set of Boolean variables, y_{ir}, that took a value of 1 in round n.

L Licenses valued by the bidder.

L(b) Licenses associated with frequency band *b*.

L(c) Licenses associated with equivalence class *c*.

$L^1(n)$ The set of Boolean variables, x_l, that took a value of 1 in round n.

L(r) Licenses associated with market *r*.

N Rounds in the auction.

P Primary groups.

R Markets.

R(c) Markets associated with class *c*.

S Secondary groups.

S(g) Secondary class associated with group *g*.

T Tertiary groups.

T(g) Tertiary class associated with group *g*.

$L^1(n)$ Set of discrete license variables that took a value of 1 in round *n*.

$L^0(n)$ Set of discrete license variables that took a value of 0 in round *n*.

Indices

b Frequency band $b \in B$.

c Class $c \in C$.

g Group $g \in G$.

i Increment $i \in I$.

r Market $r \in R$.

l License $l \in L$.

n Round $n \in N$.

Parameters

activityRequirement - Required bidding activity (bidding units).

$bandwidth_l$ - Bandwidth for license *l* (MHz).

$biddingUnits_l$ - Bidding units required for license *l*.

$budget_c$ - Budget for class *c* ($).

minProfit - The current profit resulting from any provisionally winning bids.

eligibility - The number of bidding units the bidder is eligible to bid on.

MAB_l - Minimum acceptable bid for license *l* ($).
MAB_r - Minimum acceptable bid for one MHz of bandwidth in market *r* ($).
$maxBandwidth_r$ - Maximum bandwidth required for market *r* (MHz).
$minBandwidth_r$ - Minimum bandwidth required for market *r* (MHz).
$minPops_c$ - Minimum pops for required for class *c*.
$minPops^P$ - Minimum pops for all primary markets.
$minPops^S$ - Minimum pops for all secondary markets.
$minPops^T$ - Minimum pops for all tertiary markets.
minUnitBandWidth - The minimum amount of bandwidth that can be bid on (MHz).
N_{adj} - Number of adjacent markets required for a market to be chosen.
overallBudget - Budget for the package ($).
$pops_r$ - Population of market *r*.
$preferencePrice_b$ – Additional price ($/MHzPop) for bandwidth on band b.
primaryBudget - Budget for all primary markets ($).
secondaryBudget - Budget for all secondary markets ($).
$synergyPrice_{rr'}$ - Additional price ($/MHzPop) if adjacent markets *r* and *r'* are chosen.
tertiaryBudget - Budget for all tertiary markets ($).

Continuous Variables

$bandwidth_r$ - Bandwidth chosen for market *r* (MHz).
$bandwidth_{ir}$ - Bandwidth falling in increment *i* for market *r* (MHz).
cost - Cost associated with purchasing the collection of licenses ($).
$cost_c$ - Cost associated with the licenses chosen in equivalence class *c* ($).
profit - Profit for the collection of licenses based on the current prices ($).
$synBandWidth_{rr'}$ - Bandwidth shared between market *r* and adjacent market *r'* (MHz).
$synergyValue_r$ - Value contributed by market *r* being adjacent to other chosen markets ($).
value - Total value of the package ($).
$value_c$ - Value associated with class *c* ($).
$value_r$ - Value associated with market *r* ($).

Discrete Variables

x_b 1 if band *b* is chosen, 0 otherwise.
x_c 1 if class *c* is chosen, 0 otherwise.
x_g 1 if group *g* is chosen, 0 otherwise.
x_l 1 if license *l* is chosen, 0 otherwise.
$x_{ll'}$ 1 if adjacent licenses *l* and *l'* are chosen, 0 otherwise.
x_r 1 if market *r* is chosen, 0 otherwise.
y_{ir} 1 if the bandwidth in market *r* falls in increment *i*, 0 otherwise.
$y_{rr'}$ 1 if the bandwidth in market *r* is less or equal than the bandwidth in market r', 0 otherwise.

Appendix

Table A.1: Market Key

Abbreviation	Market
BP	Boston-Providence
BR	Buffalo-Rochester
CL	Cleveland
DV	Denver
HW	Hawaii
LA	Los Angeles - San Diego
NY	New York
OM	Omaha
PH	Philadelphia
PL	Portland
PR	Puerto Rico
PX	Phoenix
SB	Spokane-Billings
SF	San Francisco-Oakland-San Jose
SL	Salt Lake City