Queuing Theory Equations

Definition

λ = Arrival Rate

μ = Service Rate

$\rho = \lambda / \mu$

C = Number of Service Channels

M = Random Arrival/Service rate (Poisson)

D = Deterministic Service Rate (Constant rate)

M/D/1 case (random Arrival, Deterministic service, and one service channel)

Expected average queue length $E(m) = (2\rho - \rho^2) / 2 \ (1 - \rho)$

Expected average total time $E(v) = 2 - \rho / 2 \mu \ (1 - \rho)$

Expected average waiting time $E(w) = \rho / 2 \mu \ (1 - \rho)$

M/M/1 case (Random Arrival, Random Service, and one service channel)

The probability of having zero vehicles in the systems $P_o = 1 - \rho$

The probability of having n vehicles in the systems $P_n = \rho^n \ P_o$

Expected average queue length $E(m) = \rho / (1 - \rho)$

Expected average total time $E(v) = \rho / \lambda \ (1 - \rho)$

Expected average waiting time $E(w) = E(v) - 1/\mu$

M/M/C case (Random Arrival, Random Service, and C service channel)

Note : $\frac{\rho}{c}$ must be < 1.0

The probability of having zero vehicles in the systems

$$P_o = \left[\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^C}{c!(1-\rho/c)} \right]^{-1}$$

The probability of having n vehicles in the systems

$$P_n = P_o \frac{\rho^n}{n!} \qquad \text{for } n < c$$

$$P_n = P_o \frac{\rho^n}{c^{n-c} c!} \qquad \text{for } n > c$$

Expected average queue length

$$E(m) = P_o \frac{\rho^{c+1}}{cc!} \frac{1}{(1-\rho/c)^2}$$

Expected average number in the systems

$$E(n) = E(m) + \rho$$

Expected average total time $E(v) = E(n) / \lambda$

Expected average waiting time $E(w) = E(v) - 1/\mu$

M/M/C/K case (Random Arrival, Random Service, and C service Channels and K maximum number of vehicles in the system)

The probability of having zero vehicles in the systems

For $\frac{\rho}{c} \neq 1$ $\quad P_o = \left[\sum_{n=0}^{c-1} \left(\frac{1}{n!} \rho^n \right) + \left(\frac{\rho^c}{c!} \right) \left(\frac{1 - \left(\frac{\rho}{c} \right)^{K-c+1}}{1 - \frac{\rho}{c}} \right) \right]^{-1}$

For $\frac{\rho}{c} = 1$ $\quad P_o = \left[\sum_{n=0}^{c-1} \left(\frac{1}{n!} \rho^n \right) + \left(\frac{\rho^c}{c!} \right) (K - c + 1) \right]^{-1}$

$$P_n = \frac{1}{n!} \rho^n P_o \qquad \text{for } 0 \leq n \leq c$$

$$P_n = \left(\frac{1}{c^{n-c} c!} \right) \rho^n P_o \qquad \text{for } c \leq n \leq k$$

$$E(m) = \frac{P_o \rho^c \left(\frac{\rho}{c} \right)}{c! \left(1 - \frac{\rho}{c} \right)^2} \left[1 - \left(\frac{\rho}{c} \right)^{k-c+1} - \left(1 - \frac{\rho}{c} \right) (k - c + 1) \left(\frac{\rho}{c} \right)^{k-c} \right]$$

$$E(n) = E(m) + c - P_o \sum_{n=0}^{c-1} \frac{(c-n)\rho^n}{n!}$$

$$E(v) = \frac{E(n)}{\lambda (1 - P_K)}$$

$$E(w) = E(v) - \frac{1}{\mu}$$