Homework for optimal control part of ASI, 2007, Lenhart

Problems 1-8 goes with first day's lectures and problems 9-11 goes with second day's lectures.

1. Solve for the optimal control and corresponding state and adjoint functions.
$$\min_u \int_1^2 tu(t)^2 + t^2 x(t)\, dt$$
$$x'(t) = -u(t),\ x(1) = 1.$$

2. Solve
$$\min_u \int_0^1 x^2(t) + x(t) + u^2(t) + u(t)\, dt$$
$$x'(t) = u(t),\ x(0) = 0.$$

3. Let $y(t) = t + 1$. Solve
$$\min_u \frac{1}{2}\int_0^1 (x(t) - y(t))^2 + u(t)^2\, dt$$
$$x'(t) = u(t),\ x(0) = 1.$$

4. Solve
$$\min_u \frac{1}{2}\int_0^1 u(t)^2\, dt + x(1)$$
$$x'(t) = -u(t),\ x(0) = 1.$$

5. Find all the necessary conditions but do not solve for explicit constants in the solutions of the differential equations.
$$\min_u \frac{1}{2}\int_0^1 x_1(t)^2 + x_2(t)^2 + u(t)^2\, dt + 5x_1(1)^2$$
$$x_1'(t) = x_2(t) + u(t),$$
$$x_2'(t) = x_1(t) - x_2(t),\ x_2(1) = 1.$$

6. Solve using cases on the size of a and b
$$\min_u \frac{1}{2}\int_0^1 u(t)^2\, dt + \frac{1}{2}x(1)^2$$
$$x'(t) = u(t),\ x(0) = 1,$$
$$a \le u(t) \le b.$$

7. Formulate an optimal control problem for a population with an Allee effect growth term, in which the control is the proportion of the population to be harvested. This means that differential equation has an Allee effect term. Choose an objective functional which maximizes revenue from the harvesting while minimizing the cost of harvesting. The revenue is the integral of amount harvested per time. Assume the cost of harvesting has a quadratic format. (Just formulate, not solve.)

8. Formulate an optimal control problem for a system of three ordinary differential equations. This system represents three interacting populations. Population 1 and population 2 compete. Population 3 cooperates with the other two populations. Population 1 has logistic growth. The growth function of population 2 has an Allee effect. Population 3 has exponential growth. The control is to harvest a proportion of population 3. The objective functional should maximize the population harvested over time and minimize the cost of the harvest. Assume the cost of harvesting is a quadratic function of the control. Set-up the optimal control problem and the corresponding necessary conditions, but do not solve the conditions.

9. Solve
$$\min_u \sum_{k=0}^{4} u_k^2$$
$$x_{k+1} = 2x_k + u_k, \quad k = 0, 1, \ldots, 4,$$
$$x_0 = 3, \ x_5 = 0.$$

10. Solve
$$\min_u \int_0^2 (2x - 3u)dt$$
subject to $x' = u + x$ and $x(0) = 5$ and $0 \le u(t) \le 2$.

11. Solve
$$\min_u \int_0^1 u(t)\, dt$$
$$x'(t) = x(t) - u(t),$$
$$x(0) = 1, \ x(1) = 0,$$
$$1 \le u(t) \le 2$$